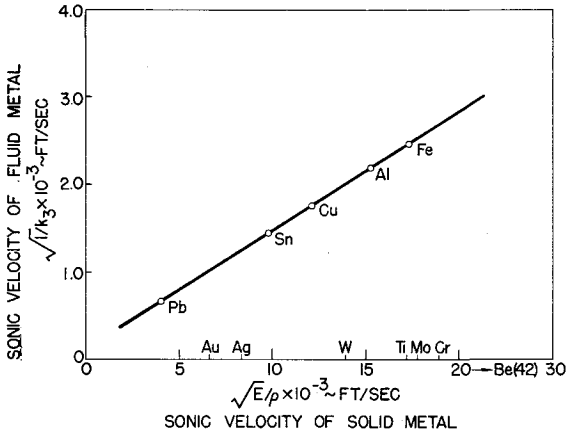

Fig. 1 Sonic velocity of various materials under hypervelocity impact condition and standard condition.

(2–5) of Ref. 1]. The body force is neglected in the foregoing equations [namely, in Eqs. (4) and (5)], but Zaker implies that there exists a body force in the equation of state. The numerical solution of the previous equations would give the velocity distribution in a shock layer (fluid state). Since the material properties at hypervelocity impact are unknown, it is not possible to obtain such a solution at the present time. Equation (8) in Ref. 1 is a terminating solution in which the penetration and impact velocity are related. The authors wish to point out that Zaker's argument regarding the condition on u at $x \rightarrow +\infty$ does not apply here because a solid state exists there. Even for a completely gaseous flow, the condition that the flow be uniform at positive infinity is only a necessary, but not sufficient, condition. Hence, the existence of other solutions is physically possible. For air at standard conditions, the thickness of the shock layer at high supersonic speeds, as calculated from Eq. (8), of Ref. 1, is in the order of magnitude of 10^{-5} in. As the temperature of the air increases, the thickness of the shock layer increases considerably.

Zaker remarks that the author's interpretation of the constant $A_2^2 = k_3$ in Eq. (9) of Ref. 1 has no rational physical basis. On the contrary, the significance of the constant k_3 is indicated in Fig. 1. The remarkable relation between the sonic velocities of five different materials at hypervelocity impact conditions and at standard conditions, as shown in Fig. 1, cannot be merely an accident.

The fictitious projectile diameter d, as referred to by Zaker, is introduced in Eq. (9) of Ref. 1 only because the experimental data are given in p/d vs V form. The experimental data used in Ref. 1 are limited to spherical projectiles. If other shapes of projectiles are used, the value of k_1 in Eq. (9) of Ref. 1 would be different. This is similar to the contraction coefficient used in hydraulics, and it can be determined only experimentally. The results given in Ref. 1 are valid for one-dimensional problems of hypervelocity impact where the projectile and target materials are the same. The authors did not claim that these results are applicable to an axially symmetric, three-dimensional impact problem.

As mentioned in Ref. 1, the six constants $A_1, A_2, \ldots, A_6$ in Eq. (8) are very complicated in nature, and they cannot be determined directly until the physical constants of materials at the hypervelocity impact conditions are known. A logical approach to this problem is to evaluate these constants from available experimental data. After neglecting all of the small terms, Eq. (8) in Ref. 1 reduces to Eq. (9). Actually, experimental data were used in reducing Eq. (8) to Eq. (9) by the method of steepest descent with the aid of the CDC 1604 digital computer. This type of iteration technique is well known; hence, Eq. (9) is not a truncated form of Eq. (8) as suggested by Zaker.

In conclusion, the authors wish to restate that the theory of hypervelocity impact is still in the infant stage of development, and more systematic experimental data are needed to aid the advancement of the theory. The analysis given in Ref. 1 is only exploratory in nature, but the results of the analysis may be applied intelligently to a limited number of practical problems. It is well known that the Prandtl mixing-length theory for a turbulent flow lacks rational physical basis, but it is the only practical tool, for the time being, which can be used to solve certain turbulent flow problems. Furthermore, the constants in the universal velocity distribution from Prandtl's theory are determined entirely from experimental data. Finally, with all of the interest, which Zaker has shown in his comment, it would be interesting to know any new theory of hypervelocity impact that he may offer in the near future.

References

[1] Yuan, S. W. and Bloom, A. M., "An analytical approach to hypervelocity impact," AIAA J. 2, 1667–1669 (1964).

[2] Davids, N. and Huang, Y. K., "Shock waves in solid craters," J. Aerospace Sci. 29, 550–557 (1962).

[3] Yuan, S. W. and Courter, R. W., "An investigation of crater formation by hypervelocity impact," Proceedings of the Seventh Symposium on Hypervelocity Impact, Tampa, Fla. (November 1964).

Comments on "Hypersonic Viscous Flow Near the Stagnation Streamline of A Blunt Body: I. A Test of Local Similarity"

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In a recent paper,[1] Kao has remarked upon the rather unstable nature of certain approximations to the Navier-Stokes equations. The character of this instability is discussed briefly here. The coupled continuity, state, and y-momentum equations [Kao's Eqs. (10) and (12)] may be written, as $y \equiv r - 1 \rightarrow 0$ (i.e., approaching stagnation point)

$$V_1'' = B(V_1' - 2U_1) + 0(1) \qquad B \equiv \tfrac{3}{4}(P_1 Re/\mu_1 V_1) > 0 \tag{1}$$

An identical expression governs stability in the earlier work of Ho and Probstein.[2] With the assumptions $V_1 = \frac{1}{2}V_{1b}''y^2 + 0(y^3)$ and $U_1 = U_{1b}'y + 0(y^2)$ the preceding equation may be perturbed to study the growth of small numerical errors near $y = 0$. The dominant solution is e^{Ay} with $A \equiv -\frac{3}{2}(P_1 Re/\mu_1 V_1'')_b < 0$, which grows enormously for increasing y. For larger y, the perturbed growth over short intervals is e^{By}, again growing rapidly for increasing y since $B \gtrsim 10\ Re$. For decreasing y, the integration step-size h must be controlled to maintain stability according to the criterion $|\partial V''/\partial V'|h = Bh \leq K$, where K is a constant for a given integration scheme. Emanuel[3] lists the values $K = 2.8$ for Kutta's Simpson's rule, and $K = 1.28$ for the fourth-order Adams predictor-corrector.

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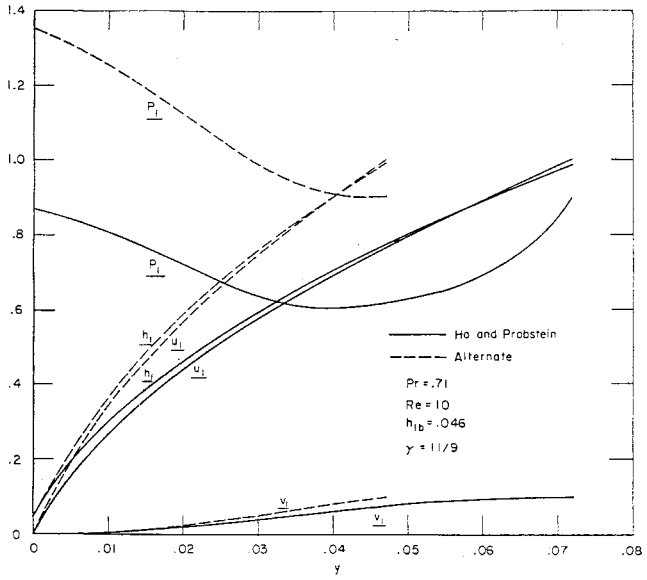

Fig. 1 Example of nonuniqueness.

One is tempted to speculate that, since the numerical instabilities only grow for increasing y, it is permissible to integrate the system [Kao's Eqs. (10–14)] inward from the shock to the body. On closer inspection, however, it becomes clear that one is merely trading existence difficulties at one boundary for uniqueness trouble at the other. That is, if arbitrarily small errors in conditions near the body grow to very large errors at the shock, then there are certain large changes that may be made at the shock which will have no effect at the body. Indeed, reference to the preceding stability equation (1) shows that, as $V_1 \rightarrow 0$, V_1' decays more and more rapidly toward the limit $2U_1$. Since $V_{1b} = U_{1b} = 0$ are prescribed as boundary conditions in both Refs. 1 and 2, one concludes that $V_{1b}' = 0$ as the solution of the equations. Nevertheless, both papers treated $V_{1b}' = 0$ as an independent boundary condition. In the absence of further conditions, the resulting solutions are not unique.

The lack of uniqueness is illustrated in accompanying Fig. 1, which shows a solution from Ref. 2 at $Re = 10$, together with an alternate solution of the same equations and boundary conditions. A quite similar solution to Ref. 2 is given in Ref. 1 (e.g., the standoff distance is identical). The steep pressure drop immediately behind the shock is a particularly suspicious behavior, since it indicates greater viscous stresses there than elsewhere. Such a pressure drop is not present in the case of a related treatment of the viscous blunt body problem in Ref. 4, where the solutions are made unique by the introduction of an additional boundary condition consisting of mass balance at the shock.

References

[1] Kao, H. C., "Hypersonic viscous flow near the stagnation streamline of a blunt body: I. A test of local similarity," AIAA J. 2, 1892–1897 (1964).

[2] Ho, H-T. and Probstein, R. F., "The compressible viscous layer in rarified hypersonic flow," *Rarefied Gas Dynamics* (Academic Press, New York, 1961), pp. 525–552.

[3] Emanuel, G., "Problems underlying the numerical integration of the chemical and vibrational rate equations in a near-equilibrium flow," Arnold Engineering Developement Center AEDC-TDR-63-82 (1963).

[4] Shih, W. C. L. and Krupp, R., "Viscous nonequilibrium blunt body flows," Massachusetts Institute of Technology Aerophysics Lab. TR 100, Defense Documentation Center AD613-026 (1964).

Comments on "Nonequilibrium Sodium Ionization in Laminar Boundary Layers"

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TWO comments on Kane's[1] treatment of sodium ionization in boundary layers are warranted; his chemical system needs revision, and his method of calculation can be greatly simplified by an additional assumption.

In Lenard's work,[2, 3] Bortner's[4] updated chemical system was used; the principal reactions turned out to be two-body reactions:

$$\mathrm{Na} + \mathrm{O}_2 \rightarrow \mathrm{Na}^+ + \mathrm{O}_2^- \tag{1}$$

$$\mathrm{Na} + \mathrm{O} \rightarrow \mathrm{Na}^+ + \mathrm{O}^- \tag{2}$$

$$\mathrm{Na} + \mathrm{M} \rightarrow \mathrm{Na}^+ + e^- + \mathrm{M} \tag{3}$$

(The electrons detach almost instantly from the preceding negative ions; the "retarded" diffusion of these ions[3] had no significant effect.) All but one of the preceding reactions differ from the three that were considered by Kane,[1] which were, in addition to Eq. (3),

$$\mathrm{Na} + e^- \rightarrow \mathrm{Na}^+ + 2e^- \tag{4}$$

$$\mathrm{Na} + 2\mathrm{O} \rightarrow \mathrm{Na}^+ + e^- + \mathrm{O}_2 \tag{5}$$

The dominant reaction in Ref. 1 was Eq. (5), which is a three-body reaction. In Ref. 3, where a 10-reaction system was used, and reactions (3–5) had rates substantially different from those used by Kane,[1] reaction (2) was found dominant for conditions when there was sufficient atomic oxygen, otherwise reaction (1) dominated, with reaction (3) assuming a nonnegligible role.

In Lenard's calculations,[2, 3] the numerical integrations of Ref. 1 are not necessary in the treatment of the injection [Eq. (2) of Ref. 2 replacing Eq. (1) of Ref. 1] and the trace contaminant ionization [Eqs. (7) and (8) of Ref. 2 replacing Eqs. (4) and (5) of Ref. 1]. This simpler method is based on the additional assumptions of "similar profiles" for all reaction products and on the fact that the peaks of these profiles coincide with the peak temperature in the boundary layer. Using this method, only the peak concentrations of the various ions and other trace reaction products need to be calculated; their profiles throughout the boundary layer can then be reconstructed by the "similarity" assumption. The validity of these assumptions has been confirmed by examining a large number of results of exact nonsimilar calculations for pure air[5] and one representative case for the sodium-air system.[3] The factors that strengthen the validity of these simplifying assumptions are a catalytic wall, a considerable absence of equilibrium, sharply peaked temperature profiles, and temperature-sensitive rates for the principal productive reactions (true for all five sodium ionizing reactions considered in Ref. 3). Using this simpler method, the treatment of nonequilibrium ionization of sodium in laminar boundary layers is described by the expression

$$\frac{C_{\mathrm{Na}^+\max}}{C^{\mathrm{Na}}(\eta_{\max})} = \left[1.0 + \frac{10lu_e}{Sc\rho \times \left(\dfrac{k_1C_{\mathrm{O}_2}}{32} + \dfrac{k_2C_{\mathrm{O}}}{16} + \dfrac{k_3}{m}\right)}\right]^{-1.0} \tag{6}$$

where C_{Na^+} and C^{Na} refer to concentrations of sodium ions and total sodium (including ions and atoms), respectively. The numbering of the forward rate constants k_1, k_2, and k_3

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