Comparison of Two Types of Structural Optimization Procedures for Flutter Requirements

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A comparison is made of results obtained from mathematical programing and optimality criteria procedures for the minimum-mass design of typical aircraft wing structures to satisfy prescribed flutter requirements. The mathematical programing method is based on an interior penalty function approach. A Langrangian optimality criterion and an intuitive optimality criterion based on uniform strain energy density are considered. An intuitive resizing procedure is used for both optimality criterion solutions. All results are calculated using the same computer program, changing only the optimization procedure. Both high- and low-aspect-ratio wings are examined. Finite elements are used for structural modeling, and the generalized coordinates for the flutter solution are based on the natural vibration modes of the structure. Second-order piston theory aerodynamics is used for supersonic conditions and kernel function aerodynamics for subsonic conditions. Convergence of the optimality criteria procedures with respect to the number of natural modes is considered.

Nomenclature

$[A]$ = aerodynamic force matrix
$[B]$ = $[K]-\omega^2[M]-[A]$, Eq. (2)
$[B_{,i}]$ = derivative of $[B]$ with respect to the ith variable mass
$[B_{,\omega}]$ = derivative of $[B]$ with respect to the flutter frequency
c_i = modal amplitudes of the flutter eigenvector
c_i^* = modal amplitudes of the adjoint flutter eigenvector
E_{AV} = average of E_v
E_{AVAV} = average of E_v above E_{AV}
E_{AVG} = average of E_v of active design variables for final iteration of design process (Fig. 5b)
E_v = measure of Lagrangian optimality criterion, see Eq. (12)
e_{AV} = average of e_v
e_{AVAV} = average of e_v above e_{AV}
e_{AVG} = average of e_v of active design variables for final iteration of design process (Fig. 5a)
e_v = strain energy density from Eq. (5)
$[K]$ = stiffness matrix
$[K_i]$ = change in stiffness matrix per unit mass for the ith variable mass
$[K_0]$ = stiffness matrix for the nonvariable structure
$\{L\}$ = adjoint flutter eigenvector in terms of nodal displacements
M = Mach number
$[M]$ = mass matrix
$[M_i]$ = change in mass matrix per unit mass for the ith variable mass
$[M_0]$ = mass matrix for the nonvariable structure
m = total mass of the structure
m_i = mass of the ith variable element, the ith design variable
m_0 = mass of the nonvariable structure
n = total number of variable elements
t_{new} = new value of thickness increment given by Eq. (6)
t_{old} = value of thickness increment from previous resizing iteration
V_f = computed flutter speed for supersonic condition
V_R = required flutter speed for supersonic condition
$\{X\}$ = flutter eigenvector in terms of nodal displacements
$\{\bar{X}\}$ = complex conjugate of $\{X\}$
Δm = total mass increment
Δm_{MP} = total mass increment for the optimum design from mathematical programing
Δt = thickness increment above minimum gage
ρ_f = air density at computed flutter altitude
ρ_R = air density at required flutter altitude
ω = flutter frequency
ω_i = ith natural frequency

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I. Introduction

EFFICIENT and dependable structural design of new aircraft demands improved optimization procedures for a variety of constraint conditions. Growing interest in optimum design with flutter constraints is evidenced by the number of recent publications[1-12] on the subject. The number of different approaches to the problem of structural optimization with flutter constraints is almost equal to the number of

publications. This multitude of approaches may be roughly divided into two categories: gradient methods (of which mathematical programing is perhaps the most general) and optimality criteria methods.

Although both mathematical programing and optimality criteria approaches to flutter optimization appear to be useful, there is currently little information available to permit a direct evaluation and comparison of these two procedures. The objective of this paper is to compare a mathematical programing procedure with two optimality criteria procedures for the minimum-mass design of typical aircraft wing structures to satisfy prescribed flutter requirements. No attempt is made to conduct a comprehensive comparison of all mathematical programing and optimality criteria optimization procedures. Rather, the paper briefly describes the specific mathematical programing technique and optimality criteria techniques considered, and compares results obtained by these techniques. The results presented are for wings that initially satisfy strength requirements and that are then resized (with the strength-defined thicknesses used as minimum gage) to satisfy flutter requirements for subsonic or supersonic conditions.

II. Optimization Problem

The optimization problem that is solved by all methods considered in this study is to minimize the mass m of a wing structure subject to a constraint on the flutter speed or flutter altitude. The design variables are the thicknesses of n structural elements which, taken together, constitute the variable part of the structure. A miminum gage constraint, which may be selected to satisfy strength requirements, is imposed on all variables.

The total mass m can be represented by

$$m = m_0 + \sum_{i=1}^{n} m_i \tag{1}$$

where m_0 is the mass of the nonvariable part or the structure and m_i is the variable mass proportional to the ith design variable.

The flutter equations of motion for steady-state oscillation may be expressed in terms of the nodal displacements $\{X\}$ as

$$[B]\{X\} \equiv ([K] - \omega^2[M] - [A])\{X\} = 0 \tag{2}$$

where $[K]$ is the stiffness matrix of the structural model, $[M]$ is the mass matrix, $[A]$ is the aerodynamic force matrix, and ω is the frequency at flutter. Because of the finite-element structural modeling that was used in this study, the matrices $[K]$ and $[M]$ are linear functions of the design variables and can be expressed as

$$[K] = [K_0] + \sum_{i=1}^{n} m_i [K_i] \tag{3}$$

and

$$[M] = [M_0] + \sum_{i=1}^{n} m_i [M_i] \tag{4}$$

In Eqs. (3) and (4) $[K_0]$ and $[M_0]$ are the stiffness and mass matrices for the nonvariable part of the structure, and $[K_i]$ and $[M_i]$ represent the changes in stiffness and mass per unit mass of the ith design variable. Second-order piston theory aerodynamics is used for supersonic conditions and kernel function aerodynamics for subsonic conditions.

III. Optimization Techniques Compared

The structural optimization procedures compared in this study are an intuitive optimality criterion based on uniform strain energy density, a Langrangian optimality criterion, and a mathematical programing method based on an interior penalty function. An intuitive resizing algorithm is used in conjunction with both of the optimality criterion solutions.

A. Strain Energy Density Optimality Criterion and Intuitive Resizing Algorithm

From an optimality criterion point of view, the minimum-mass structure, subjected to a static loading condition and sized to satisfy strength requirements, exhibits a stress state in which the strain energy density in every structural element is a constant.[13] Siegel[10] has used a similar optimality criterion for flutter optimization. His criterion is based on the intuitive concept that the most efficient distribution of structural material for flutter requirments is the one that provides constant strain energy per unit volume throughout the structure when it is deformed in the critical flutter mode. The strain energy density optimality criterion (referred to hereafter as the energy method) expressed in terms of the strain energy per unit mass is

$$(e_v)_i = \{\bar{X}\}^T [K_i]\{X\} = \text{Constant} \quad i = 1,2,\ldots,n \tag{5}$$

In this study, only one material is considered for each structural model, so the material density is constant throughout the wing. Therefore, Eq. (5) is equivalent to the criterion proposed by Siegel[10] which is based on strain energy per unit volume.

In Siegel's method,[10] the required flutter speed V_R is achieved by means of an iterative procedure in which, at each step, a complete flutter analysis is performed and the structural changes needed to raise the flutter speed are calculated. To insure that skin gages of the structure are not reduced below those necessary to satisfy the strength requirements, they are only allowed to increase or remain unchanged during each iteration. Incremental structural changes are calculated by first determining e_{AV}, the average e_v over the structure. Next, a second average e_{AVAV} is obtained from those elements whose e_v exceed e_{AV}. The new values of the design variables, t_{new}, whose e_v exceed e_{AVAV} are then determined by means of a resizing algorithm presented in Ref. 10 that includes the square of the ratio of the required flutter speed to the calculated flutter speed V_f and can be expressed as

$$(t_{\mathrm{new}})_i = (t_{\mathrm{old}})_i (V_R/V_f)^2 [(e_v)_i / e_{\mathrm{AVAV}}]^{1/2} \tag{6}$$

As used in Ref. 10, the subscript i in Eq. (6) refers *only* to those elements whose e_v exeeed e_{AVAV}, and t_{old} is the thickness of the element from the previous resizing iteration. The thickness of each element whose e_v is less than e_{AVAV} remains unchanged.

One shortcoming of the previous procedure is that, during the iterative resizing process, the thickness of an element can never be decreased, even if it is substantially above the thickness dictated by the strength requirements. Thus, if early in the iterative process an element becomes too thick, its thickness cannot be decreased later on. This shortcoming is overcome in the present calculations by considering *all* design variables in the resizing process but, after each resizing iteration, imposing the constraint

$$(t_{\mathrm{new}})_i \geq (\text{minimum gage})_i \tag{7}$$

This constraint prevents any thickness from being decreased below minimum gage, which can be the thickness distribution necessary to satisfy the strength requirements. For subsonic application, the velocity ratio in Eq. (6) is replaced by an air density ratio that corresponds to the ratio of the density at the required altitude to the density at the computed altitude for a given iteration.

B. Lagrangian Optimality Criterion

Turner[11] and Pines and Newman[12] present sets of optimality equations that provide minimum-mass designs for

structures that must satisfy flutter requirements. These equations are derived by minimizing a functional which is the sum of the total mass of the structure and an expression that introduces the flutter equations of motion by means of Lagrange multipliers. Such a derivation is given in Appendix A of Ref. 14. The following four sets of nonlinear equations are obtained:

$$[B]\{X\}=0 \tag{8}$$

$$\{L\}^T[B]=0 \tag{9}$$

$$\mathrm{Re}(\{L\}^T[B_{,i}]\{X\})=\delta \tag{10}$$

$$\mathrm{Re}(\{L\}^T[B_{,\omega}]\{X\})=0 \tag{11}$$

The matrix $[B_{,\omega}]$ represents the derivative of $[B]$ with respect to ω and $[B_{,i}]$ is the derivative of $[B]$ with respect to the ith variable mass m_i. Any solution that satisfies Eqs. (8-11) is a rigorous solution to the problem and represents a minimum-mass design. Equation (8) is simply the equation of motion [same as Eq. (2)]; Eq. (9) identifies the vector of Lagrange multipliers $\{L\}$ as the left-hand, or adjoint, eigenvector of the system; Eq. (10) is the statement of the rigorous Lagrangian optimality criterion; and Eq. (11) is a normalization relation between $\{L\}$ and $\{X\}$ which guarantees that ω satisfies the flutter equation. Equations (8-11) are a set of nonlinear equations which have to be solved under the side constraint of minimum gages for the design variables. The solution of this set of nonlinear equations should be recognized as a standard mathematical programing problem (it is equivalent to the minimization of the residual of the equations). This mathematical programing problem is of much higher degree of complexity than the penalty function mathematical programing problem discussed in the Sec. IIIC because the flutter eigenvector and the adjoint flutter eigenvector are additional unknown variables. Thus, a standard solution technique for Eqs. (8-11) cannot be expected to be as efficient (in terms of computing effort) as the solution of the minimization problem by means of a penalty function formulation.

Previously, solutions were obtained to Eqs. (8-11) by making some simplifying assumptions. Turner[11] did not impose a minimum gage constraint and solved the equations by successive linearizations (Newton's method). He pointed out that this method works only if the initial design is close enough to the optimum. Pines and Newman[12] represent the aerodynamics by a quasi-steady theory, so that the matrix $[A]$ is real and is independent of the frequency; thus the derivative of $[A]$ with respect to ω in Eq. (11) is zero. The resulting system of equations is therefore real. Pines and Newman also used Newton's method for the solution of the system of equations. They included the minimum gage constraint in the formulation, but it is not clear how the constraint is accounted for by the algorithm.

In the present work two approaches are tried for the solution of Eqs. (8-11). The first approach is the successive linearization approach.[11,12] The first iteration of the solution is started with the values of m_i, ω, $\{X\}$, and $\{L\}$ corresponding to the initial design and each iteration thereafter is started from the values obtained from the previous iteration. The equations are linearized about the starting point and solved for increments to m_i, ω, $\{X\}$, and $\{L\}$. If a negative increment is obtained for a design variable at minimum gage, that design variable is eliminated from the equations and the corresponding equation of Eq. (10) is also eliminated for that iteration. The new set of linearized equations is solved and the elimination process is repeated until no negative increments to the minimum gage design variables are obtained. The variables m_i, ω, $\{X\}$, and $\{L\}$ are incremented and a new iteration is started using the entire set of design variables and equations.

This solution process was implemented only for piston theory aerodynamics. It was found to be unreliable when the initial design is not close to the optimum, and the computational effort involved is large. It was decided, therefore, to apply Siegel's resizing algorithm [Eq. (6)] to the Lagrangian optimality criterion of Eq. (10). Using Eqs. (3) and (4), and the fact that the matrix $[A]$ is independent of m_i when the governing equations are expressed in nodal coordinates, the Lagrangian optimality criterion of Eq. (10) is

$$(E_v)_i=|\{L\}^T([K_i]-\omega^2[M_i])\{X\}|=\text{Constant}$$

$$i=1,2,\ldots,n \tag{12}$$

The absolute value is taken instead of the real part to insure that $(E_v)_i$ is positive so that Siegel's resizing algorithm may be used with $(E_v)_i$ replacing $(e_v)_i$ in Eq. (6). This approach is referred to hereafter as the Lagrangian method.

C. Mathematical Programing

The mathematical programing procedure used in this study is the sequential unconstrained minimization technique (SUMT)[15] in which design constraints are introduced by means of an interior penalty function. For this procedure, it is necessary to start the optimization process with an initial design that has a flutter speed V_f greater than the required flutter speed V_R. (For the optimality criteria solutions the starting design is not required to satisfy the flutter constraint.) Newton's method with approximate second derivatives[5] is the optimization algorithm used for each unconstrained minimization.

IV. Implementation of Algorithms

The WIDOWAC (WIng Design Optimization With Aeroelastic Constraints) computer program[5,16] was used to obtain the results for this study, WIDOWAC was developed to study techniques for sizing minimum-mass symmetric-airfoil structures to satisfy flutter, strength, and minimum gage requirements. WIDOWAC contains the SUMT mathematical programing algorithm to perform the optimization. The aerodynamic representation, flutter equations, finite elements, and design variables as applied in WIDOWAC are discussed in detail in Ref. 16.

The mathematical programing results for this study are obtained from the basic WIDOWAC program. The optimality criteria results are obtained by replacing the mathematical programing subroutines of the basic program by appropriate optimality criteria subroutines. Thus, it is possible to compare results for which the only difference in the design process is the optimization algorithms. The number of Eqs (8) and (9) is reduced by using a finite number of natural vibration modes as generalized coordinates, and modalized approximations for the flutter and adjoint flutter eigenvectors are obtained by solving these reduced flutter equations. Approximations to $\{X\}$ and $\{L\}$ are than obtained by transforming the modalized flutter and adjoint flutter eigenvectors back into nodal coordinates for use with the energy and Lagrangian methods.

V. Description of Wing Models

In this study a high-aspect-ratio wing and a low-aspect-ratio wing are investigated. Since WIDOWAC was developed for symmetric airfoils, only the upper half of the wing needs to be modeled.

A. High-Aspect-Ratio Wing

The high-aspect ratio wing, shown in Fig. 1a, is a diamond-airfoil aluminum full-depth sandwich wing which is clamped at the root. A finite-element model, shown in Fig. 1b, represents the wing with 36 degrees of freedom. The core is

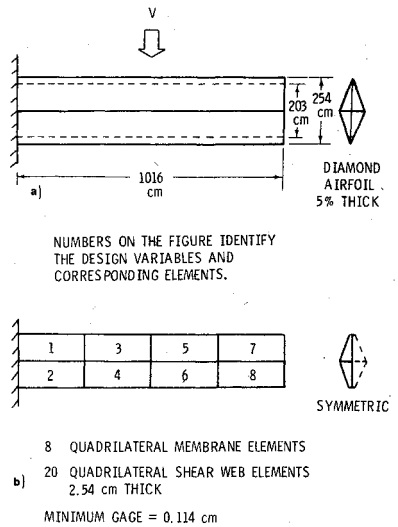

Fig. 1 High-aspect-ratio wing model. a) Aerodynamic planform, airfoil, and structural box. b) Finite-element model.

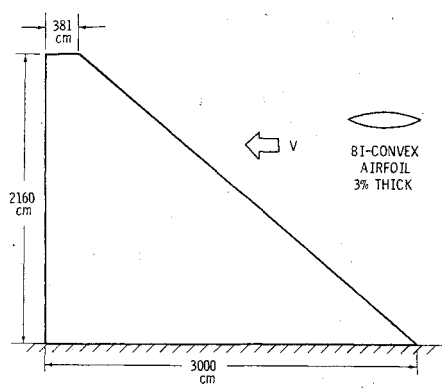

Fig. 2 Aerodynamic planform and airfoil section for low-aspect-ratio wing model.

simulated by very stiff, constant-thickness shear webs. The thicknesses of the eight cover-panel elements are used as the design variables. Additional information about the wing structure is given in Ref. 14. The wing is required to be flutter free in standard atmosphere for $M=2.5$ at an altitude of 12192 m, which corresponds to a speed of 738 mps.

B. Low-Aspect-Ratio Wing

A titanium full-depth sandwich clipped-delta wing which is clamped at the root is studied as an example of a low-aspect-ratio wing and is shown in Fig. 2. The core is simulated by very stiff, constant thickness shear webs. This wing has also been studied by Stroud, Dexter, and Stein,[1] Haftka,[5] and Haftka and Yates.[17] The wing is required to satisfy a maximum allowable stress requirement of 862 NM/m^2 for a uniform pressure loading of 3.45 kN/m^2. The core of the wing is assumed to be filled with fuel having a mass density of 0.47 g/cm^3. Both subsonic and supersonic studies are made of this wing. For the subsonic study, the wing is required to be flutter free in standard atmosphere for $M=0.6$ at an altitude of 1524 m, which corresponds to a speed of 201 mps; and for the supersonic study, the wing is required to be flutter free in standard atmosphere for $M=2.48$ at an tltitude of 7620 m, which corresponds to a speed of 770 mps.

A finite-element model represents the wing structure with 93 degrees of freedom and is shown in Fig. 3a. The model is studied using both six and 51 design variables. For the six-design-variable study, the cover-panel elements are grouped into six segments, as shown in Fig. 3b. The vertices of the segments are represented by circles at the appropriate structural grid points of Fig. 3a. One design variable represents the

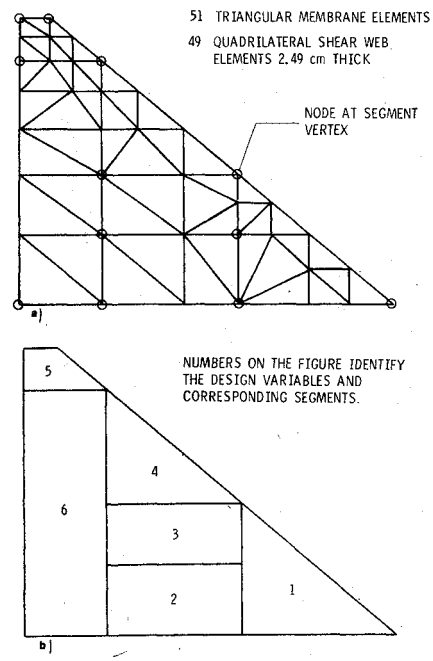

Fig. 3 Low-aspect-ratio wing model. a) Finite-element model. b) Segments.

change in thickness of each segment, and the design variables are identified by the numbers shown in Fig. 3b. The minimum gages of the six segments are based on an optimized strength design that satisfies the strength requirements only, and are given in Table 1a of Ref. 14. For the 51-design-variable study, one design variable represents the change in thickness of each of the 51 elements, and the minimum gages are the same as for the six-design-variable model.

VI. Results

The results obtained from the optimality criteria analyses are discussed and compared with the mathematical programing results for the high-aspect-ratio wing model at supersonic speeds and for the low-aspect-ratio wing model at subsonic and supersonic speeds. Six natural vibration modes are used as the generalized coordinates of the reduced flutter equations except when studying modal convergence. The computer core storage requirements are nearly the same for the mathematical programing and optimality criteria procedures; therefore, computing times are discussed as a measure of the computing effort required for the procedures.

A. Wing Design Results

1. High-aspect-ratio wing model

Variations of the ratios V_f/V_R and $\Delta m/\Delta m_{\mathrm{MP}}$ with the number of iterations are shown in Fig. 4 as obtained from the energy method for the high-aspect-ratio wing. The quantity Δm is the total mass increment above the minimum-gage design given by the optimality criteria and Δm_{MP} is the total mass increment above the minimum-gage design for the optimum design given by the mathematical programing solution. For the first few iterations, the increases in the ratios V_f/V_R and $\Delta m/\Delta m_{\mathrm{MP}}$ are primarily due to the velocity ratio term $(V_R/V_f)^2$ appearing in the resizing algorithm given by Eq. (6). In some instances, when the increases predicted by the resizing algorithm were large enough to cause difficulties in the iterative flutter solution technique, a number less than the actual velocity ratio term was used in the resizing algorithm. Examples of a solution with the velocity ratio term limited to a value of 1.1 and a solution without a limit on the velocity ratio term are shown in Fig. 4. As the flutter velocity ratio approaches unity, the incremental thickness changes become much smaller, and are essentially a function only of the energy density ratio term in the resizing algorithm.

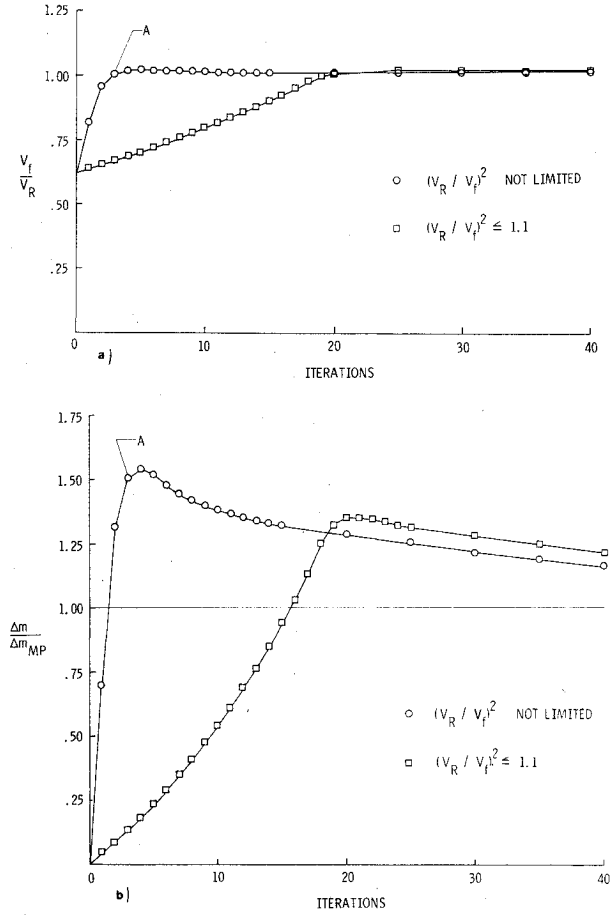

Fig. 4 Variation of flutter speed and variable mass with number of iterations for the high-aspect-ratio wing. Results obtained from the strain energy density optimality criterion. a) Flutter speed. b) Mass increments.

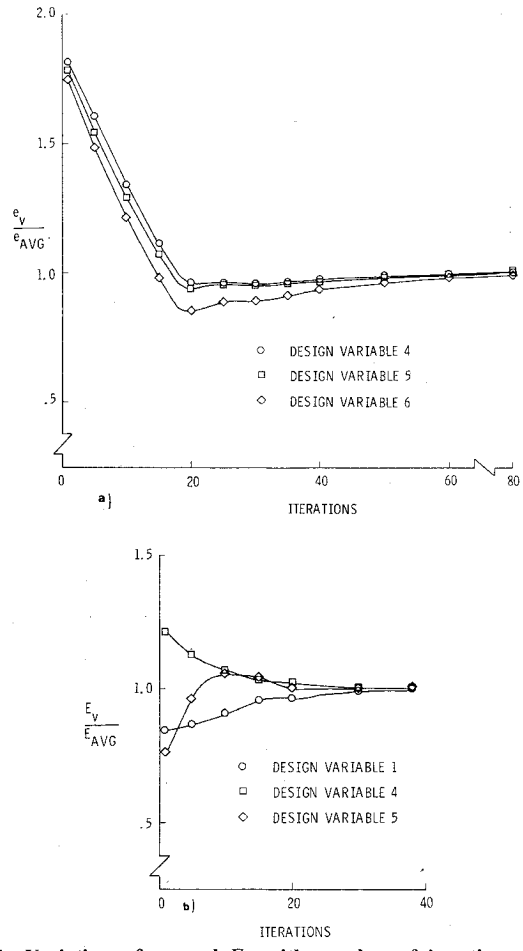

Fig. 5 Variation of e_v and E_v with number of iterations. High-aspect-ratio wing. a) Strain energy densities. b) Lagrangian optimality criterion.

It was stated in Sec. IIIA that the basic resizing approach presented in Ref. 10 was modified to obtain the results given herein. The modification allows thicknesses to be decreased, as well as increased. The effect of this modification is illustrated by the curve in Fig. 4 representing results for the case without any limit on the velocity ratio term in Eq. (6). The minimum-mass flutter-free design predicted by the original approach of Ref. 10 is obtained after only three iterations (point A in Fig. 4). For this design $\Delta m/\Delta m_{MP} = 1.50$. After 20 iterations with the modified resizing algorithm $\Delta m/\Delta m_{MP} = 1.28$, and after 40 iterations $\Delta m/\Delta m_{MP} = 1.16$, which indicates that obtaining the minimum-mass design requires more than just a few iterations.

Table 1 Optimum designs for high-aspect-ratio wing model, six-mode solution

	Mathematical programing		Energy method	Lagrangian method	
Δm, kg	38.1		42.5	37.8	
V_f/V_R	1.0010		1.0005	1.0002	
Segment	Δt, cm	Δt, cm	e_v/e_{AVG}	Δt, cm	E_v/E_{AVG}
1	0.0483	0	....	0.0435	0.9979
2	0.0010[a]	0	....	0	....
3	0.0012[a]	0	...	0	....
4	0.3144	0.3169	1.0007	0.3344	1.0014
5	0.1643	0.2358	1.0036	0.1512	1.0008
6	0.0009[a]	0.0423	0.9957	0	....
7	0.0017[a]	0	....	0	....
8	0.0009[a]	0	....	0	....

[a]The small numbers for the mathematical programing results are associated with the penalty function and result in a slightly larger value of Δm when compared with the Lagrangian method.

The variation of e_v and E_v for the active design variables with number of iterations is shown in Fig. 5 for the high-aspect -ratio wing. These results correspond to the curve in Fig. 4 with the velocity ratio term limited by 1.1. The quantities e_v and E_v are normalized by their average values for the final iteration of the design process; E_{AVG} for the Lagrangian, and e_{AVG} for the energy method. The E_v obtained from the Lagrangian method approaches a uniform state in about half the number of iterations required for the energy method for this case. However, for most of the cases in this study, the number of iterations required for a uniform state is similar for both criteria.

Results for the final designs of the high-aspect-ratio wing are given in Table 1. Data tabulated include the total mass increments Δm (or flutter penalty), the ratio V_f/V_R, the thickness increments Δt above the minimum-gage strength design, and the e_v and E_v of the active design variables nor-

Table 2 Optimum designs of low-aspect-ratio wing model for $M = 0.6$, six-mode solution

	Mathematical programing		Energy method	Lagrangian method	
Δm, kg	930.5		989.7	1026.0	
ρ_f/ρ_R	1.0008		1.0007	1.0001	
Segment	Δt,cm	Δt,cm	e_v/e_{AVG}	Δt,cm	E_v/E_{AVG}
1	0.0005	0	....	0	....
2	0.0004	0	....	0.0383	0.9995
3	0.0830	0.1206	1.0063	0.1102	1.0010
4	0.1930	0.0793	0.9880	0.1239	0.9995
5	0.0007	0.0110	1.0003	0	....
6	0.0734	0.1140	1.0055	0.0903	1.0000

Table 3 Optimum designs of low-aspect-ratio wing model for $M = 2.48$, six-mode solution

	Mathematical programing		Energy method		Lagrangian method
Δm, kg	259		1299		292
V_f/V_R	1.000		1.000		1.000
Segment	Δt,cm	Δt,cm	e_v/e_{AVG}	Δt,cm	E_v/E_{AVG}
1	7.44×10^{-5}	0	....	0	....
2	5.05×10^{-5}	1.161	0.9998	0	....
3	0.128	0.228	0.9998	0.090	0.9995
4	9.39×10^{-5}	0	....	0.056	1.0005
5	12.47×10^{-5}	0	....	0	....
6	2.63×10^{-5}	0.086	1.0004	0	....

malized by their average values. The values of the Δm and the Δt given by the Lagrangian method and mathematical programing are essentially the same. The value of the Δm obtained from the energy method is about 11% higher than, and the values of the Δt of segments 1, 5, and 6 are substantially different from those obtained by the other approaches.

2. Subsonic low-aspect-ratio wing model

Results for the final designs of the low-aspect-ratio wing model for $M = 0.6$ are given in Table 2. The data tabulated include the values of the Δm, the ratio of the density ρ_f of the computed flutter altitude to the density ρ_R of the required flutter altitude, the values of the Δt for each segment, and the e_v and E_v of the active design variables normalized by their average values. The values of the Δm obtained from the energy and Lagrangian methods are about 6% and 10% greater, respectively, than the value given by mathematical programing. Differences in the individual segment thickness increments of the three designs are substantial, as can be seen in Table 2.

3. Supersonic low-aspect-ratio wing model

Results for the final designs of the low-aspect ratio wing model for $M = 2.48$ are given in Table 3. The values of the Δm obtained from the energy and Lagrangian methods are about 400% and 13% greater, respectively, than the value given by mathematical programing. The mathematical programing solution tends to lump all incremental mass into segment 3, whereas the Lagrangian method lumps all incremental mass into segment 3 and the adjoining segment 4. The energy method, however, nearly doubles the incremental thickness of segment 3 compared to the mathematical programing results and, in addition, indicates considerble thickness increases for segments 2 and 6.

The number of design variables used for the finite-element model was increased from six (the six segments shown in Fig. 3b) to one design variable for each of the 51 cover-panel finite elements shown in Fig. 3a. The total mass increment Δm predicted for the six- and 51-element model designs differ by less than 1%, but there are significant differences in segment thicknesses. However, both model designs tend to lump the increased thicknesses near the center of the wing.

4. Related results and comments

The optimality criteria results presented inTables 1-3 are obtained by starting from the strength designs, treating their segment thicknesses as minimum gage. Additional optimality criteria calculations were made with different starting points and, in all instances, essentially the same final designs were obtained. In particular, calculations were carried out for the high-aspect -ratio wing by starting with the final design obtained from mathematical programing. Neither optimality criterion was quite satisfied at this design, and the resulting algorithm in each case drove the designs to the optimality criterion results with segment thicknesses virtually identical to those obtained previously by starting from the strength design. This change from one minimum-mass design to another may be related to the differences in the design variable spaces in which each method searches.

The results in Table 1-3 indicate that the values of the Δm predicted by the Lagrangian method agree reasonably well with the values given by mathematical programing, although there may be differences in the thicknesses of individual elements. Comparison of the values of the Δm predicted by the energy method with the results obtained using mathematical programing indicate agreement ranging from good to very poor, which suggests that this method is not consistently reliable.

B. Modal Convergence

Modal convergence studies were made to provide an indication of the convergence of the flutter speed or altitude, incremental mass, and e_v and E_v with respect to the number of natural vibration modes used to represent the reduced flutter equations. The results of these studies indicate reasonable convergence for the high-aspect-ratio wing and the low-aspect ratio wing at $M = 0.6$. However, for the low-aspect ratio wing at $M = 2.48$, the results indicated only fair convergence for the flutter speeds and poor convergence for e_v, E_v, and mass increments.

The low-aspect-ratio wing considered herein is also studied in Ref. 17. In that study, the derivatives used by the mathematical programing technique are found to have poor convergence with respect to the number of natural modes. An expression related to the natural frequencies ω_i of the wing and the modal amplitudes c_i and c_i^* of the flutter and adjoint flutter eigenvectors, respectively, is derived and used to indicate the nature of the convergence difficulties of the flutter derivatives. This expression is obtained by considering the dependence of the derivatives on the quantities ω_i, c_i and c_i^* when there is only one design variable that scales the stiffness of the entire structure.

Table 4 Comparison of computing times, six-mode solutions

		Mathematical programing	Lagrangian method
High-aspect-ratio wing	CPU, s	304[a]	124[a]
	Iterations	....	38
Low-aspect ratio wing for $M = 2.48$	CPU, s	495[b]	225[b]
	Iterations	....	33
Low-aspect ratio wing for $M = 0.6$	CPU, s	452[b]	238[a]
	Iterations	....	28

[a] CDC 6600 central processor unit time.
[b] These cases were run on a CDC 6400. Times shown were converted to equivalent CDC 6600 central processor unit time by dividing CDC 6400 time by 2.5.

Table 5 Variation of computing time with number of design variables for low-aspect-ratio wing model for $M = 2.48$, six-mode solutions

	Design variables	6	51
Lagrangian method	CPU, s	225[a]	442[a]
	Iterations	33	40
Mathematical programing	CPU, s	495[a]	>5000 (est.)

[a] These cases were run on a CDC 6400. Times shown were converted to equivalent CDC 6600 central processor unit time by dividing CDC 6400 time by 2.5.

Using the same procedures as Ref. 17, expressions for the examination of the nature of the convergence of e_v and E_v with respect to increasing numbers of natural modes are derived in Appendix B of Ref. 14. For the energy method, the expressions is

$$e_v = \sum_{i=1}^{n} c_i c_i \omega_i^2 \tag{13}$$

and, for the Lagrangian method, the expression is

$$E_v = \sum_{i=1}^{n} c_i c_i^* \omega_i^2 \tag{14}$$

These expressions suggest that the nature of the convergence of e_v and E_v depends on whether or not the products $c_i c_i$ or $c_i c_i^*$ decrease faster than ω_i^2 increases. The criteria given by Eqs. (13) and (14) were used in Ref. 14 and indicated that the low-aspect-ratio at $M=2.48$ should, indeed, have poor convergence. Equation (14) has exactly the same form as the aforementioned expression used in Ref. 17; therefore, the Lagrangian method would be expected to experience the same type of convergence problems associated with using natural vibration modes as mathematical programing methods employing derivatives. It appears that any resizing technique used in conjunction with modalized coordinates could have similar convergence problems; that is, if there are convergence problems, then more modes may be required for an accurate representation of quantities necessary for the resizing techinque than are required for the calculation of the flutter speed.

C. Computing Times

The computing times and number of iterations required to obtain optimum six-mode designs for both the high- and low-aspect-ratio wings are shown in Table 4 for the Lagrangian method. These results indicate computing times on the order of 2-8 min, with roughly a factor of 2 increase in computing time required by mathematical programing compared to the Lagrangian method. For a given number of iterations, the computing time for the energy method is slightly less than the time required for the Lagrangian method for the examples of this study since calculation of the adjoint flutter eigenvector is required by the latter method.

The effect of the number of design variables on computing time is shown in Table 5 for the low-aspect-ratio wing model for $M=2.48$ using six natural modes. For the Lagrangian method, increasing the number of design variables from 6 to 51 caused an increase in computing time of about a factor of 2. A mathematical programing run with 51 design variables was not made. However, experience indicates the effect would have been an order of magnitude or more increase in run time.

While the comparisons just presented are indicative of the computing effort required by the two procedures, they should not be considered a strict comparison since other factors exist that have not been considered. The computing effort can be reduced for both methods. For example, the optimality criteria were programed only for the purpose of studying the procedures and, perhaps, could have been made more efficient. The mathematical programing calculations in this study used finite-difference techniques for derivative calculations. Implementation of the analytical expressions presented in Ref. 17 should reduce considerably the computing times for this method.

VII. Conclusions

A comparison is made of results obtained from a penalty function mathematical programing and optimality criteria procedures for the minimum-mass design of typical aircraft wing structures to satisfy prescribed flutter requirements. The results obtained using an intuitive resizing algorithm applied to an intuitive optimality criterion based on uniform strain energy density indicate that the intuitive criterion is not consistently reliable. Rigorously derived optimality equations can be obtained by the use of Lagrange multipliers. The solution of these equations subject to minimum gage constraints is a mathematical programing problem of higher complexity, requiring more computational effort, than solving the penalty function mathematical programing problem. It was found, however, that applying the intuitive resizing algorithm to a Lagrangian optimality criterion which is part of the optimality equations provides results that compare favorably with the results obtained from the penalty function mathematical programing procedure. A comparison of computing times indicates that the optimality criteria are more efficient than the penalty function mathematical programing technique considered. Thus, the Lagrangian optimality criterion as used in this paper is a useful tool for minimum-mass structural designs that must satisfy flutter requirements.

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