

The peak values of u' fluctuations u'_m follow a near-Gaussian distribution for all of the flows studied when \bar{u}'_m is nondimensionalized with \bar{u}'_m . The distributions are again identical for all of the flows (Fig. 5) and the values of σ and μ are 0.6 ± 0.05 and 0.09 ± 0.1 , respectively. Certain deviations could be observed at low values of u'_m , but this falls within experimental uncertainty. One should remember that in present procedure the mean of u'_m is obtained as an arithmetic average of the modulus of u'_m without taking into consideration the sign of the fluctuations. Thus, the negative and positive parts of the Gaussian distribution in this case are artificially created and do not represent the sign of the signal in the real sense. Only in this way could a single relation be obtained for all of the flows investigated.

The results presented in this Note are somewhat preliminary in nature. Further investigations are required to understand the significance of these results in relation to the physics of turbulence. The major conclusion of the present work is that the velocity fluctuations in a variety of turbulent flows exhibit some similarity when the duration of the zero crossings and the maximum amplitude of the velocity fluctuations between the crossings are considered as the basic time and velocity scales.

Acknowledgment

The author would like to thank C. S. Subramanian for the help rendered in conducting some of the experiments.

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Peak Strouhal Frequency of Subsonic Jet Noise as a Function of Reynolds Number

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Introduction

IN principle, the sound power of jet noise W is functionally dependent on both Reynolds number and Mach number, viz.,

$$W / (\frac{1}{2} \rho U^3 D^2) = f(M, Re)$$

At high Reynolds number, the acoustic power satisfies the approximate relationship

$$W / (\frac{1}{2} \rho U^3 D^2) = KM^5$$

where K is a constant. This relationship is confirmed by experiments conducted at moderate to high Reynolds num-

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Index categories: Aeroacoustics; Jets, Wakes, and Viscid-Inviscid Flow Interactions.

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bers. However, there is some evidence that even at Reynolds numbers of the order of 10^5 , the spectral characteristics of jet noise are dependent on Reynolds number, although such evidence has been largely ignored. Mollo-Christensen et al.¹ studied the Reynolds number effects and found that the rms value of the far-field sound pressure is proportional to $(Re)^m$ where m has values of $1 \leq m \leq 3$. Furthermore, they observed that the nondimensional narrow band spectra of the noise also varied with Reynolds number. Mollo-Christensen² conjectured that jet noise will be insensitive to Reynolds number when the boundary layer at the wall is turbulent, which corresponds to Reynolds number of about 2×10^5 based on jet diameter. Above this value of Reynolds number, noise spectra will be similar. Ahuja and Bushall³ noted that at low Reynolds numbers there is less variation of peak frequency with observation angle. This may be related to the observation by McLaughlin et al.⁴ in which the low Reynolds number spectrum was dominated by a few discrete modes.

This Note describes an extension of the previous work. Emphasis is placed on the narrow band spectral characteristics of jet noise. The data reported herein were obtained during an extensive study of the acoustic field generated by subsonic jets at low Reynolds numbers. The results are presented in terms of a dimensionless power spectral density defined by

$$S\left(\frac{fD}{U_j}\right) = \frac{p_j^2 / (\rho_0 U_j^2)}{D \cdot \Delta f / U_j} \cdot \left(\frac{R}{D}\right)^2$$

where p_j^2 represents the mean square of the pressure fluctuations at frequency f observed in the far field at a distance R .

Experimental Observations

Figures 1 and 2 contain dimensionless spectra plotted as a function of the Strouhal number $Sh = fD/U_j$ for various Reynolds numbers and two Mach numbers. The variation of the sound spectrum with Reynolds number is evident. Three important features should be mentioned. First, the magnitude of the spectrum is an increasing function of the Reynolds number. This is consistent with the observation by Mollo-Christensen et al.¹ Second, the maximum level of the spectrum is not a linear function of Reynolds number. One could conjecture that, at a sufficient low Reynolds number, the flow is completely laminar and no sound is radiated. However, it is questionable whether a jet flow can be completely laminar over all space at any Reynolds number greater than zero. For our purposes this conjecture is probably irrelevant. Third, the spectra are insensitive to Reynolds number above a critical value of about 10^5 . The results are inconsistent with the view adopted by Meecham.⁵ He argues that the large-scale characteristics of jet turbulence are dependent on the way the jet is generated. Since most of the low-frequency sound is related to the large eddies, the low-frequency end of the sound spectrum will depend upon the driving mechanism of the jet turbulence. On the other hand, he suspects that the high-frequency end of the spectrum may be independent of the driving forces. This view is not supported by the results shown in Figs. 1 and 2. The high-frequency end of the spectra is obviously affected. The sound intensity appears to drop off more rapidly with frequency with the smaller nozzles. Unfortunately, the observed Reynolds number dependence does not follow a simple relationship. It should also be noted that at low Reynolds number, the flow is likely to be very sensitive to the fluctuations of upstream conditions, however small they may be. Thus, the observed acoustic characteristics may vary from one jet to another, even at constant Reynolds number.

One of the most important findings in our jet noise study was that the narrow band peak frequency of the radiated sound is an increasing function of the Reynolds number. It is

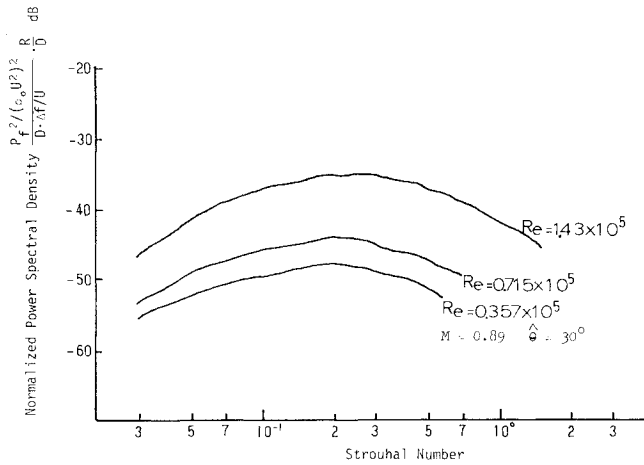


Fig. 1 Dimensionless power spectral density of jet noise.

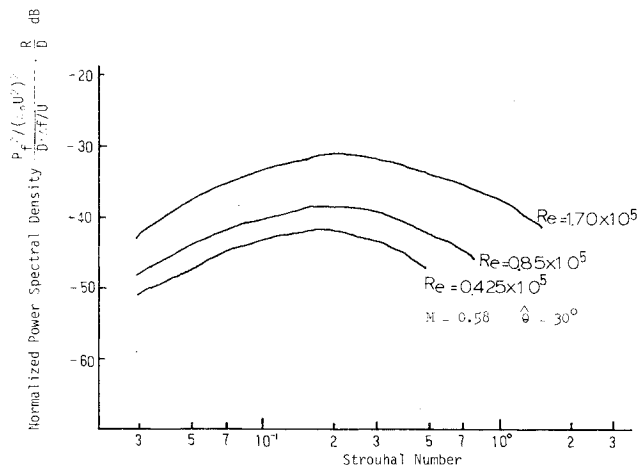


Fig. 2 Dimensionless power spectral density of jet noise.

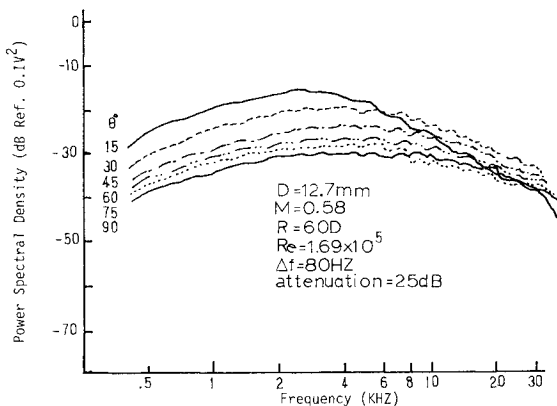


Fig. 3 Narrowband power spectral density of jet noise.

well known that the peak frequency increases with increased observation angles measured from the downstream jet axis. This was confirmed in the present study⁶ at Reynolds numbers greater than 10^5 . However, at lower Reynolds numbers the spectral peak lacks distinct directivity. This can be seen in Figs. 3 and 4. Ahuja and Bushell³ also report a similar trend, as shown in Fig. 5. It is quite obvious that variation of peak frequency with emission angle decreases with decreasing Reynolds number and that there is a tendency for the peak frequency to stabilize at a constant value for a given Mach number.

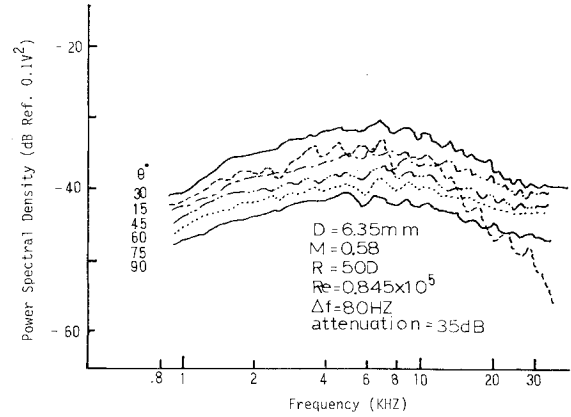


Fig. 4 Narrowband power spectral density of jet noise.

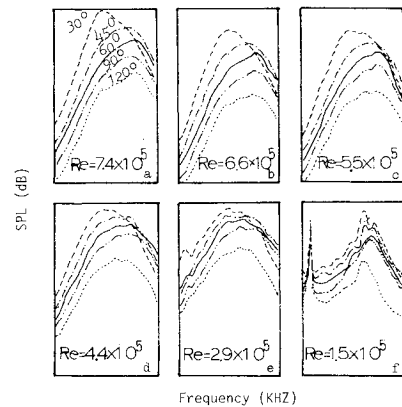


Fig. 5 1/3 octave spectra of jet noise.³

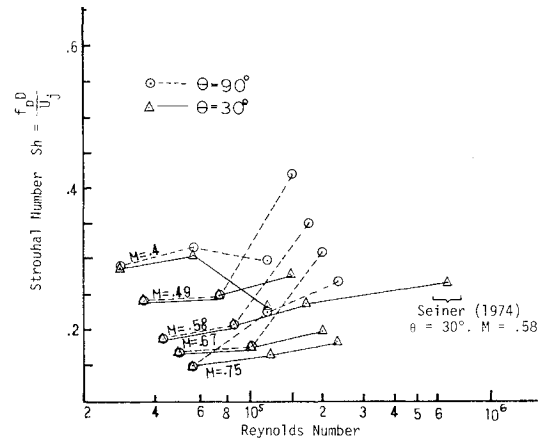


Fig. 6 Narrowband peak frequency of jet noise.

Figure 6 contains the narrow band peak frequency data as a function of Reynolds number at various Mach numbers. Seiner's⁷ results are also presented in the figure. Up to $Re = U_j D / \nu \approx 10^5$, the peak Strouhal frequency observed at $\theta = 30$ deg is identical with that at $\theta = 90$ deg for any exit Mach number. At higher Reynolds numbers the peak Strouhal frequency increases with increasing emission angle. Although peak Strouhal frequency is a weak function of Reynolds number, the effect is definitely measurable. Lush⁸ found that the peak frequency does not scale very well with U_j/D at angles between 15 and 45 deg, and that the peak moves progressively to higher frequencies as the emission angle is

increased. At the sideline, the peak scales as a Strouhal number. In this case (see Fig. 20 of Lush⁸), the peak Strouhal numbers lay between 0.75 and 0.93 at $\theta = 90$ deg. Below a Reynolds number of about 10^5 , the peak Strouhal number is independent of emission angle and falls in the range of 0.15 to 0.3.

According to McCartney,⁹ the peak frequencies of self and shear noise spectra can be expressed in terms of the derivatives of a Fourier transform of the second-order moving frame correlation tensor of turbulent velocities at two points at different times. He further proposed that if the space and time dependence of the correlation tensor can be separated, the peak frequency can be calculated for any given time decay of the correlation tensor. It follows that if the observed shift in peak frequency with Reynolds number is an intrinsic property of jet noise, the decaying profile of the second-order moving frame correlation tensor of turbulent velocities would also be dependent on Reynolds number. No attempt was made to confirm this hypothesis experimentally during this study.

It is difficult to quantify the critical value of 10^5 for Reynolds number. Apparently the criterion is one of whether or not transition to turbulence occurs in the nozzle boundary layer or in the initial free shear layer. Presumably, the jet noise characteristics stabilize when the nozzle boundary layer is turbulent. This is supported by the fact that crude calculations indicate that transition to turbulence occurs in the nozzle boundary layer at Reynolds numbers higher than about 10^5 .

McLaughlin et al.⁴ found that a few discrete modes at a Strouhal number of about 0.2 in a low Reynolds number, supersonic jet are powerful noise generators. This compares favorably with the present study in which the peak Strouhal number was also found to be of order 0.2. The detailed information reported herein would not be uncovered in octave and third octave analysis of the noise signal. Apparently, narrow band spectral analysis is necessary to unveil certain details of jet noise which have received scant attention in the literature.

Acknowledgment

This work was supported by the NASA Lewis Research Center under Grant NGR 39-009-270 and the Air Force Office of Scientific Research.

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Numerical Solution for Perpendicular Sonic Hydrogen Injection into a Ducted Supersonic Airstream

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Nomenclature

c_v	= specific heat at constant volume
e	= total internal energy per unit volume
f	= hydrogen mass fraction
p	= static pressure
T	= static temperature
u	= streamwise velocity
v	= normal velocity
x	= streamwise coordinate
y	= normal coordinate
Le	= Lewis number
Pr	= Prandtl number
Sc	= Schmidt number
μ	= total viscosity

Introduction

SCRAMJET engine concepts being investigated at the NASA Langley Research Center¹ incorporate both parallel and perpendicular hydrogen fuel injection. This approach tailors the injection to an optimum heat release schedule in the engine over a flight Mach number range. Efficient engine design requires a detailed understanding of the flowfield near each type fuel injector. In order to limit the broad area of injector design that must be considered experimentally, an analytical tool is needed to survey a number of attractive options. This Note discusses a computer program being developed to study the flowfield near opposing perpendicular fuel injectors and presents some preliminary results used during the initial evaluation of the code.

Analysis

The flowfield near a perpendicular fuel injector (Fig. 1) is complex, and presents a very challenging problem for numerical analysis. Regions of separation and recirculation exist both in front of and behind the jet, and the flow undergoes severe turning through a bow shock upstream of and an expansion behind the jet. The analysis of such a flow requires the solution of the full (elliptic) compressible Navier-Stokes, energy and species equations. In conservative form, these equations in two dimensions are^{2,3}

$$\frac{\partial U}{\partial t} + \frac{\partial F}{\partial x} + \frac{\partial G}{\partial y} = 0 \quad (1a)$$

$$U = \begin{Bmatrix} \rho \\ \rho u \\ \rho v \\ e \\ \rho f \end{Bmatrix} \quad (1b)$$

Received Jan. 16, 1979. This paper is declared a work of the U.S. Government and therefore is in the public domain.

Index category: Aerodynamics.

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