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Saturation Effects on Stagnation Radiative Heating for the Jupiter Probe

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The equations for nonequilibrium radiation transport in the stagnation ablation layer for conditions typical of entry of spacecraft into the atmosphere of a major planet are developed. The equations are simplified and shown to contain two parameters which are used to classify the radiation transfer as equilibrium or nonequilibrium. The parameter ω represents the ratio of the radiative de-excitation rate to the sum of the radiative de-excitation rate plus the collisional excitation rate. The second parameter is ω' and it represents the ratio of the radiative excitation rate, due to external radiation, to the sum of the radiative de-excitation rate plus the collisional excitation rate. Both parameters vary between zero and unity. It is shown that the population ratio of the two electronic states of a molecular band approaches its equilibrium value as ω approaches zero. When ω and ω' are greater than zero the population ratio becomes greater than its equilibrium value. An order of magnitude analysis is undertaken for conditions expected in the shock layer near the maximum heating point in the entry trajectory of the Jupiter Probe. It shows that saturation effects are likely to occur in the ablation layer.

Nomenclature

$A(u,\ell)$	= Einstein spontaneous emission coefficient
$B(\ell,u)$	= Einstein absorption coefficient
$B(u,\ell)$	= Einstein induced emission coefficient
c	= speed of light
C_2	= molecular carbon molecule
C_{ex}	= reference collisional excitation rate
C_{dex}	= reference collisional de-excitation rate
$g(i)$	= degeneracy of state i
h	= Planck's constant
I_ν	= radiative intensity at frequency ν
J_ν	= average radiative intensity at frequency ν
k	= Boltzmann's constant
$K(\ell,u)$	= collisional excitation rate
$K(u,\ell)$	= collisional de-excitation rate
K_0	= reference collisional rate
ℓ	= lower state of transition
L	= thickness of ablation layer
m_{C_2}	= mass of C_2 molecule
M	= any atom or molecule
N	= total number density
$N(i)$	= number density of state i
P	= pressure
$\dot{r}(\ell)$	= excitation rate
S_ν	= source function
T	= temperature
u	= upper state of transition
x	= distance
β	= collision cross section
γ	= parameter in Eq. (27)
δ	= parameter in Eq. (20)
$\epsilon_{\Delta\nu}$	= emissivity of hydrogen shock layer in spectral region $\Delta\nu$
θ	= angle between intensity direction and x axis
$\kappa_\nu(\ell,u)$	= absorption coefficient
μ	= $\cos^{-1}(\theta)$
ν	= frequency
$\nu_{\ell u}$	= frequency of band center
$\xi(u,\ell)$	= emission coefficient
$\sigma_\nu(\ell,u)$	= absorption cross section
τ_ν	= optical depth
$\phi_\nu(\ell,u)$	= spectral shape of absorption band
ω	= parameter in Eq. (21)
$d\Omega$	= differential solid angle
ω'	= parameter in Eq. (31)

Subscripts

0	= reference quantity, evaluated at $x=0$
b	= center frequency of molecular band
ν	= frequency variation

Superscripts

$-$	= nondimensional quantity
$'$	= dummy variable

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Introduction

THE National Aeronautics and Space Administration is scheduled to launch a probe to the planet Jupiter in early 1982 (Project Galileo). The probe will enter Jupiter's atmosphere in 1985; its scientific objectives are to define the nature and composition of the atmosphere. To accomplish its objectives the probe must survive a severe aerothermal environment caused by strong radiative heating during the atmospheric entry. The radiative heating is so intense that the heat shield comprises nearly half of the probe's design weight. Current experimental facilities are not able to simulate the thermal environment completely; consequently, it has been necessary to design the heat shield based on analytical predictions of the heating environment.

Considerable effort has been expended to predict the radiative flux reaching the heat shield of the Jupiter Probe. Solutions were first obtained for the stagnation point,[1] then for the off-stagnation inviscid flow,[2,3] and finally for off-stagnation viscous flowfields.[4-6] The solutions show that the ablation layer is effective in protecting the vehicle from its severe thermal environment. The ablation layer absorbs about half of the radiation energy incident on it. The design of the Jupiter Probe heat shield depends critically upon this radiation blockage factor.

Equilibrium conditions have been assumed to exist in the ablation layer for most of the calculations to date; that is: 1)

equilibrium chemistry is used to calculate the composition throughout the ablation layer, 2) a single equilibrium temperature is defined at each position in the ablation layer flowfield, and 3) the various states of the molecules, atoms, and ions in the ablation layer are assumed to be populated with a Boltzmann distribution at the local temperature.

Lincoln et al.[7] investigated the assumption that two of the ablating species, C_2 and C_3, exist together in equilibrium. They found that the mole fraction of C_3 is very large near the heat shield and that the C_3 might not have time to dissociate to form C_2 before it is swept away. This C_2 deficiency, as compared to equilibrium solutions, leads to a decrease in the optical thickness of the ablation layer of about one order of magnitude. This decrease in optical thickness decreases the ability of the ablation layer to block radiation and leads to higher heating rates at the heat shield.

There is concern about the existence of nonequilibrium population distributions in the ablation layer. These population distributions can affect the ability of the ablation layer to block radiation and protect the spacecraft. Horton[8] presented results for the transfer of radiation through a layer of an atomic gas with a non-Boltzmann population distribution of its electronic states. He obtained analytic solutions for high-speed planetary entry by using a very simple flow model. These solutions show that non-Boltzmann population distributions are likely to reduce the ablation layer's ability to block radiation in the 1-10 eV spectral region, which is where most of the radiation energy is concentrated during typical entry trajectories in the atmosphere of the major planets.

Two recent papers by Tiwari et al.[9,10] consider nonequilibrium effects on the radiative heating of a Jovian entry body. The first paper[9] considers the influence of precursor heating on chemical and radiative nonequilibrium viscous flow in the entry body shock layer. It shows that nonequilibrium effects increase both the electron and heavy particle temperature. This, in turn, increases the radiative source function and leads to higher radiative heating at the heat shield. The second paper[10] investigates the effects of non-Boltzmann electronic state populations on the radiative transfer to the entry body heat shield. The non-Boltzmann population distribution was assumed to occur in the inviscid part of the shock layer just behind the shock wave. The study shows that non-Boltzmann population distributions tend to reduce the magnitude of the radiative source function and, in turn, the radiative heating of the entry vehicle. It should be noted that Tiwari et al.[9,10] did not consider nonequilibrium effects in the ablation layer, which are emphasized herein.

A considerable amount of work has appeared in the astrophysics literature on the influence of non-Boltzmann population distributions on the transfer of radiation.[11-15] The effect of the competition between the collisional and radiative rates in excitation and ionization has been investigated. The studies have considered atomic gases that are modeled as having two or three electronic states and the dynamic conditions representative of stellar atmospheres.

A recent investigation of the uncertainties in the calculation of the Jupiter Probe's heat shield recession rate indicates that the heat shield recession is very sensitive to the population distribution and absorption coefficient of C and C_2 (Ref. 16). These ablation layer species present a significant problem in heat shield design because: 1) there is considerable doubt as to whether these species are in local thermochemical equilibrium, 2) their absorption coefficients are not well known, especially those for C_2, and 3) the populations of the electronic levels may be non-Boltzmann distributed. Each of these factors influences the ablation layer radiation blockage predictions and, in turn, the spacecraft heat shield design.

Recent studies show that C_3 is also a very important species in the ablation layer.[17,18] New absorption coefficient data show that the C_3 absorption coefficient is considerably smaller, but the spectral band is wider than was previously thought. These new absorption coefficient data increase the sensitivity of the heat shield recession calculations to the C_3 population.

Saturation effects in the Swan band of C_2 have been investigated experimentally by observing fluorescence.[19] Fluorescence occurs when a gas is excited with monochromatic radiation and then reemits radiation. The absorption of the incident radiation causes molecules to be excited to higher energy levels, which then decay radiatively. Pulsed laser induced fluorescence has been developed as a diagnostic tool for investigating excited state population of molecules in combustion.

Saturation spectroscopy has been used to measure the number density of the C_2 Swan band ground state in oxyacetylene flames.[20] This was accomplished by inducing electronic fluorescence in the 0-0 band of the C_2 Swan band system ($^3\pi_g - {}^3\pi_u$) at about 5165°A, using a tunable pulsed laser. The laser was strong enough to saturate the transition. When saturation occurred, the fluorescence of the excited molecules became independent of the laser intensity.

The objective of the present study is to investigate the possibility of saturation of the Swan band transition of C_2 at the maximum heating point in the trajectory of the Jupiter Probe as it enters the Jovian atmosphere. The analysis will be done in the ablation layer at the stagnation point.

Mathematical Development

A. Radiative Transfer Equation

The determination of the effects of nonequilibrium population distributions on radiative heating for a stagnation shock layer is very complicated. Many simplifications are needed to formulate a workable model. The shock layer will be divided into two parts: an isothermal, inviscid, atomic hydrogen layer and a nonisothermal, viscous, ablation layer, as shown schematically in Fig. 1. The equilibrium stagnation point solutions of Moss et al.[4,5] will be used to give realistic flowfield and thermodynamic conditions for the entire shock layer. The radiation emitted by the inviscid portion of the shock layer will serve as the boundary condition for the equations of radiative transfer in the ablation layer. This radiation is the driving potential for the problem. Only the C_2 molecular electronic states involving the Swan band transitions will be allowed to be out of equilibrium. In addition, because there are very few electrons in the ablation layer, the collisional rate will be dominated by heavy particle interactions.

The equation of radiative transfer for a single electronic band transition can be written

$$\mu\frac{\mathrm{d}I_\nu}{\mathrm{d}x} = \frac{h\nu}{4\pi}N(\ell)B(\ell,u)\phi_\nu(\ell,u)\left\{-\left[1-\frac{N(u)B(u,\ell)}{N(\ell)B(\ell,u)}\right]I_\nu + \frac{N(u)A(u,\ell)}{N(\ell)B(\ell,u)}\right\} \tag{1}$$

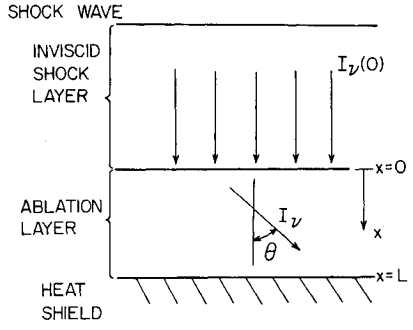

Fig. 1 Schematic of planetary entry body stagnation point shock layer. In the analysis the inviscid shock layer is assumed to be hydrogen and the ablation layer is assumed to be diatomic carbon.

where the emission and absorption band shapes are assumed to be identical. If one defines the absorption cross section

$$\sigma_\nu(\ell,u) = (h\nu/4\pi)B(\ell,u)\phi_\nu(\ell,u) \tag{2}$$

the absorption coefficient

$$\kappa_\nu(\ell,u) = \sigma_\nu(\ell,u)N(\ell)\left\{1-\frac{N(u)g(\ell)}{N(\ell)g(u)}\right\} \tag{3}$$

and the emission coefficient

$$\xi_\nu(u,\ell) = \frac{2h\nu^3}{c^2}\sigma_\nu(\ell,u)N(u)\frac{g(\ell)}{g(u)} \tag{4}$$

one can write the radiative transfer equation in its fundamental form as

$$\mu\frac{\mathrm{d}I_\nu}{\mathrm{d}x} = -\kappa_\nu(\ell,u)I_\nu + \xi_\nu(u,\ell) \tag{5}$$

Further, defining the optical depth

$$\tau_\nu = \int_0^x \kappa_\nu(\ell,u)\mathrm{d}x \tag{6}$$

allows one to write the radiative transfer equation as

$$\mu\frac{\mathrm{d}I_\nu}{\mathrm{d}\tau_\nu} = -I_\nu + \frac{\xi_\nu}{\kappa_\nu} = -I_\nu + S_\nu \tag{7}$$

where S_ν is the radiative source function. The solution of this equation in the ablation layer as shown schematically in Fig. 1 is:

$$I_\nu(\tau_\nu,\mu) = I_\nu(0,\mu)\exp[-\tau_\nu/\mu] + \int_0^{\tau_\nu} S_\nu(\tau_\nu')$$

$$\times\exp[-(\tau_\nu-\tau_\nu')/\mu]\frac{\mathrm{d}\tau_\nu'}{\mu} \quad \mu\geq 0 \tag{8a}$$

$$I_\nu(\tau_\nu,\mu) = I_\nu(L,\mu)\exp[-(\tau_\nu(L)-\tau_\nu)/\mu] - \int_{\tau_\nu}^{\tau_\nu(L)} S_\nu(\tau_\nu')$$

$$\times\exp[-(\tau_\nu'-\tau_\nu)/\mu]\frac{\mathrm{d}\tau_\nu'}{\mu} \quad \mu<0 \tag{8b}$$

The average intensity is defined as

$$J_\nu(\tau_\nu) = \frac{1}{4\pi}\int_{4\pi} I_\nu(\tau_\nu)\mathrm{d}\Omega = \frac{1}{2}\Big\{I_\nu(0)E_2(\tau_\nu)$$

$$+I_\nu(L)E_2(\tau_\nu(L)-\tau_\nu) + \int_0^{\tau_\nu(L)} S_\nu(\tau_\nu')E_1[|(\tau_\nu-\tau_\nu')|]\mathrm{d}\tau_\nu'\Big\} \tag{9}$$

where the radiation incident on the boundary at $x=0$ and $x=L$ is assumed to be independent of direction.

B. Rate Equation

Collisional rates of the type

$$C_2(\ell)+M \underset{K(u,\ell)}{\overset{K(\ell,u)}{\rightleftharpoons}} C_2(u)+M \tag{10a}$$

and radiative rates

$$C_2(\ell)+h\nu_{\ell u} \underset{A(u,\ell)+B(u,\ell)I_\nu}{\overset{B(\ell,u)I_\nu}{\rightleftharpoons}} C_2(u) \tag{10b}$$

are considered in the analysis. The general rate equation for transition between the lower and upper state of an electronic transition in a C_2 molecule can be written

$$\dot{r}(\ell) = -\int_\nu\int_{4\pi} N(\ell)\sigma_\nu(\ell,u)\Big\{\Big[1-\frac{N(u)g(\ell)}{N(\ell)g(u)}\Big]I_\nu$$

$$-\frac{2h\nu^3}{c^2}\frac{N(u)g(\ell)}{N(\ell)g(u)}\Big\}\frac{\mathrm{d}\Omega\mathrm{d}\nu}{h\nu} + K(u,\ell)N(u)N$$

$$-K(\ell,u)N(\ell)N \quad 1/\mathrm{cm}^3\mathrm{s} \tag{11}$$

The integral term represents the change in the lower state population by radiative processes and the two algebraic terms represent the gain and loss of lower state population by collisional processes, respectively. The total number density N can be related to the pressure and temperature by

$$P = NkT \tag{12}$$

If one assumes steady state, $\dot{r}(\ell)$ becomes zero and one can rewrite Eq. (11) as

$$4\pi N(\ell)\int_\nu \sigma_\nu(\ell,u)\Big\{\Big[1-\frac{N(u)g(\ell)}{N(\ell)g(u)}\Big]J_\nu - \frac{2h\nu^3}{c^2}\frac{N(u)g(\ell)}{N(\ell)g(u)}\Big\}\frac{\mathrm{d}\nu}{h\nu}$$

$$= \frac{P}{kT}[K(u,\ell)N(u) - K(\ell,u)N(\ell)] \tag{13}$$

where Eqs. (9) and (12) were used. Equation (13) is the steady-state rate equation. It can be solved for the ratio of $N(u)/N(\ell)$ throughout the ablation layer if $I_\nu(0)$, $I_\nu(L)$, are known and if J_ν, $K(u,\ell)$, $K(\ell,u)$, $\sigma_\nu(\ell,u)$, P, and T are known throughout the ablation layer.

C. Source Function

The source function is defined as the ratio of the emission coefficient to the absorption coefficient in Eq. (7). It involves the population ratio. In order to solve for the source function it is assumed that

$$N(u)g(\ell)/N(\ell)g(u) \ll 1 \tag{14}$$

so that the optical depth as defined by Eq. (6) becomes

$$\tau_\nu(x) = \int_0^x \sigma_\nu(\ell,u)N(\ell)\mathrm{d}x' \tag{15}$$

This assumption removes the population ratio from the optical thickness which appears in the expression for J_ν. It will limit the application of the theory as developed; however, the simplification that it yields justifies its use in the present study. For the C_2 Swan band the equilibrium population ratio is

$$\frac{N(u)g(\ell)}{N(\ell)g(u)} = \exp\left(\frac{-29014}{T}\right) \tag{16}$$

Consequently, it is less than 0.01 for temperatures less than 6300 K. In other words, the upper state could be populated to 10 times its equilibrium value and the assumption indicated by Eq. (14) would still hold in the ablation layer near the heat shield. Therefore, even though the assumption is restrictive, it is not overly so.

The assumption indicated by Eq. (14) allows one to solve Eq. (13) for the population ratio

$$\frac{N(u)g(\ell)}{N(\ell)g(u)} = \frac{4\pi\int_\nu \sigma_\nu(\ell,u)J_\nu \frac{d\nu}{h\nu} + \frac{P}{kT}K(\ell,u)}{4\pi\int_\nu \frac{2h\nu^3}{c^2}\sigma_\nu(\ell,u)\frac{d\nu}{h\nu} + \frac{P}{kT}K(\ell,u)\exp[h\nu_{\ell u}/(kT)]} \tag{17}$$

where the collisional rates are related by using detailed balancing

$$K(u,\ell) = K(\ell,u)\frac{g(\ell)}{g(u)}\exp\left(\frac{h\nu_{\ell u}}{kT}\right) \tag{18}$$

Note that when collision processes dominate the radiation process ($\sigma_\nu \approx 0$), the population ratio approaches the Boltzmann population distribution $\exp[-h\nu_{\ell u}/(kT)]$. The assumption indicated by Eq. (14) allows the source function to be written as

$$S_\nu = \frac{2h\nu^3}{c^2}\frac{N(u)g(\ell)}{N(\ell)g(u)} \tag{19}$$

with the population ratio defined by Eq. (17). For equilibrium conditions the source function represents the Wien approximation to the Planck function.

It is convenient to define a parameter

$$\delta = \frac{P}{kT}K(\ell,u)\exp\left(\frac{h\nu_{\ell u}}{kT}\right)\Big/4\pi\int_\nu \frac{2h\nu^3}{c^2}\sigma_\nu(\ell,u)\frac{d\nu}{h\nu} \tag{20}$$

where δ represents the ratio of the collisional de-excitation rate to the radiative de-excitation rate.[11-13] Note that δ increases as pressure increases. At high pressures, δ becomes large and the collision rate becomes much larger than the radiation rate. At low pressures δ becomes small and the radiation rate becomes much larger than the collisional rate.

It is common in the astrophysics literature to define a parameter[11-13]

$$\omega = 1/(1+\delta) \tag{21}$$

so that ω varies between zero and unity as δ varies from zero to infinity. When radiation is important $\omega \to 1$ and when collisions are important $\omega \to 0$.

The source function can be written in terms of ω as

$$S_\nu = \frac{2h\nu^3}{c^2}\frac{\omega}{2} \times \frac{\int_\nu \frac{\sigma_\nu(\ell,u)}{h\nu}I_\nu(0)E_2(\tau_\nu)d\nu + \int_\nu \frac{\sigma_\nu(\ell,u)}{h\nu}I_\nu(L)E_2(\tau_\nu(L)-\tau_\nu)d\nu}{\int_\nu \frac{2h\nu^3}{c^2}\sigma_\nu(\ell,u)\frac{d\nu}{h\nu}}$$

$$+\frac{2h\nu^3}{c^2}\frac{\omega}{2}\frac{\int_\nu \frac{\sigma_\nu(\ell,u)}{h\nu}\left\{\int_0^{\tau_\nu(L)} S_\nu(t)E_1[\,|\tau_\nu - t|\,]dt\right\}d\nu}{\int_\nu \frac{2h\nu^3}{c^2}\sigma_\nu(\ell,u)\frac{d\nu}{h\nu}}$$

$$+\frac{2h\nu^3}{c^2}(1-\omega)\exp[-h\nu_{\ell u}/(kT)] \tag{22}$$

where Eq. (9) was used for J_ν. The first term in Eq. (22) represents the influence of the radiation at the boundaries of the ablation layer. The second term represents the effect of emission elsewhere in the ablation layer on the value of the source function at the specific point of interest in the ablation layer. The last term represents the effect of local emission. It is $(1-\omega)$ times the Planck function for the Wien approximation.

Equation (22) shows that when the ablation layer is dominated by collisions ($\omega \to 0$) the source function becomes the Planck function. This is the equilibrium situation. As ω approaches unity the radiation source function becomes controlled by the radiation from the ablation layer boundaries and by the radiation emitted elsewhere in the plasma. This is the nonequilibrium situation. Consequently, the value of the parameter ω is very important in determining whether or not the ablation layer will be in equilibrium.

D. Method of Solution

The solution to the set of equations as formulated is very complicated. Equation (17) for the population ratio is coupled to the radiation field by the average intensity $J_\nu(\tau_\nu)$, which in turn is coupled to the radiation emitted at each point in the flowfield by the source function, $S_\nu(\tau'_\nu)$, and the incident radiation on both sides of the ablation layer $I_\nu(0)$ and $I_\nu(L)$. This coupling makes the solution for the population ratio in the ablation layer and the radiative flux at the wall very complicated. The population distribution of excited states at a point in the ablation layer cannot be determined until the source function is known throughout the ablation layer. Yet the source function distribution cannot be determined until the excited state population distribution is known.

There are two possible ways out of this dilemma. The first is to solve the problem by the method of iteration-perturbation.[21] The second possible way is to formulate the problem as an integral equation for the source function.[22,23] Either method is very complicated. The approach in the current study is to do an order of magnitude analysis by investigating the magnitudes of the parameters that arise in the formulation of the problem. This allows one to simply determine the range of thermodynamic conditions for which non-Boltzmann population ratios will appear. Detailed numerical solutions can be obtained once it is determined that non-Boltzmann populations do exist. The objective of the present analysis is to determine if non-Boltzmann population ratios do indeed exist in the Jupiter Probe ablation layer.

Nondimensional Equations

In order to estimate the ratio of the population of the upper state to the lower state of the electronic band (Swan band of C_2), it is convenient to first nondimensionalize the equations. This yields nondimensional variables whose magnitudes are of order unity. The nondimensional variables are defined in the Appendix. These definitions allow one to write the average radiation intensity as

$$\bar{J}(\bar{x},\bar{\nu}) = \tfrac{1}{2}\{\bar{I}(0,\bar{\nu})E_2[\tau_{\nu_b}\bar{\tau}(\bar{x},\bar{\nu})]$$

$$+\int_0^1 \bar{S}(\bar{x}',\bar{\nu})E_1[\tau_{\nu_b}|\bar{\tau}(\bar{x},\bar{\nu})-\bar{\tau}(\bar{x}',\bar{\nu})|]\tau_{\nu_b}d\bar{\tau}(\bar{x}',\bar{\nu})\} \tag{23}$$

where it is assumed that $I_\nu(L)$ is negligible compared to $I_\nu(0)$. The Swan band center frequency is denoted by ν_b. The characteristic optical thickness becomes

$$\tau_{\nu_b} = \sigma_{\nu_b}(0)N_0L \tag{24}$$

as defined in Eq. (A14).

The radiative source function becomes

$$\bar{S}(\bar{x},\bar{\nu}) = \omega\bar{\nu}^3 \int_0^\infty \bar{\sigma}(\bar{x},\bar{\nu})\,\bar{J}(\bar{x},\bar{\nu})\,\mathrm{d}(\ln\bar{\nu}) \Big/ \int_0^\infty \bar{\nu}^3\bar{\sigma}(\bar{x},\bar{\nu})\,\mathrm{d}(\ln\bar{\nu})$$

$$+ (1-\omega)\,\bar{\nu}^3\bar{I}_B(\bar{x},\nu_b) \tag{25}$$

where $\nu_b = \nu_{\ell u}$. The nondimensional population ratio, Eq. (17), becomes

$$\frac{\bar{N}_u(\bar{x})g(\ell)}{\bar{N}_\ell(\bar{x})g(u)} = \omega' \int_0^\infty \bar{\sigma}(\bar{x},\bar{\nu})\,\bar{J}(\bar{x},\bar{\nu})\,\mathrm{d}(\ln\bar{\nu}) \Big/ \int_0^\infty \bar{\nu}^3\bar{\sigma}(\bar{x},\bar{\nu})\,\mathrm{d}(\ln\bar{\nu})$$

$$+ (1-\omega)\exp\left[-\frac{h\nu_b}{kT_0\bar{T}}\right] \tag{26}$$

where $\omega' = \omega\gamma$ and where

$$\gamma = \left[\frac{4\pi\sigma_{\nu_b}(0)N_0 I_{\nu_b}(0)}{h}\right]\left[\frac{c^2}{8\pi\nu_b^3\sigma_{\nu_b}(0)N_0}\right] = [R_{\mathrm{ex}}]\left[\frac{1}{R_{\mathrm{dex}}}\right] \tag{27}$$

The nondimensional parameter γ represents the ratio of the radiative excitation rate (R_{ex}), due to absorption of external radiation to the radiative de-excitation rate (R_{dex}) due to local emission.

If one defines a reference collisional excitation rate

$$C_{\mathrm{ex}} = N_0^2 K_0 \tag{28a}$$

and a reference collisional de-excitation rate

$$C_{\mathrm{dex}} = N_0^2 K_0 \exp(h\nu_b/kT_0) \tag{28b}$$

one can write δ as

$$\delta = \left[\frac{C_{\mathrm{dex}}}{R_{\mathrm{dex}}}\right] \frac{\dfrac{\bar{P}(\bar{x})}{\bar{T}(\bar{x})}\bar{K}(\bar{x})\exp\left[\dfrac{h\nu_b}{kT_0}\left(\dfrac{1}{\bar{T}}-1\right)\right]}{\displaystyle\int_0^\infty \bar{\nu}^3\bar{\sigma}(\bar{x},\bar{\nu})\,\mathrm{d}(\ln\bar{\nu})} \tag{29}$$

One can write ω as

$$\omega = \frac{\dfrac{R_{\mathrm{dex}}}{C_{\mathrm{dex}}}\displaystyle\int_0^\infty \bar{\nu}^3\bar{\sigma}(\bar{x},\bar{\nu})\,\mathrm{d}(\ln\bar{\nu})}{\dfrac{R_{\mathrm{dex}}}{C_{\mathrm{dex}}}\displaystyle\int_0^\infty \bar{\nu}^3\bar{\sigma}(\bar{x},\bar{\nu})\,\mathrm{d}(\ln\bar{\nu}) + \dfrac{\bar{P}(\bar{x})}{\bar{T}(\bar{x})}\bar{K}(\bar{x})\exp\left[\dfrac{h\nu_b}{kT_0}\left(\dfrac{1}{\bar{T}}-1\right)\right]} \tag{30}$$

and one can write ω' as

$$\omega' = \frac{\dfrac{R_{\mathrm{ex}}}{C_{\mathrm{dex}}}\displaystyle\int_0^\infty \bar{\nu}^3\bar{\sigma}(\bar{x},\bar{\nu})\,\mathrm{d}(\ln\bar{\nu})}{\dfrac{R_{\mathrm{dex}}}{C_{\mathrm{dex}}}\displaystyle\int_0^\infty \bar{\nu}^3\bar{\sigma}(\bar{x},\bar{\nu})\,\mathrm{d}(\ln\bar{\nu})\,\dfrac{\bar{P}(\bar{x})}{\bar{T}(\bar{x})}\bar{K}(\bar{x})\exp\left[\dfrac{h\nu_b}{kT_0}\left(\dfrac{1}{\bar{T}}-1\right)\right]} \tag{31}$$

In the collision dominated limit when C_{dex} becomes very large, Eq. (30) and (31) show that ω and ω' go to zero and the population ratio as given by Eq. (26) approaches its equilibrium value of

$$\bar{N}_u(\bar{x}) = \bar{N}_\ell(\bar{x})\frac{g(u)}{g(\ell)}\exp\left[-\frac{h\nu_b}{kT_0\bar{T}(\bar{x})}\right] \tag{32}$$

In the opposite limit, when the collisional rate becomes very small, Eq. (30) and (31) show that ω and ω' go to unity. In this limit, the population ratio is determined by the radiative processes contained in the first term on the right-hand side of Eq. (26). For this case saturation of the upper state may occur. In other words, $\bar{N}_u(\bar{x})$ may become much greater than its equilibrium value as given by Eq. (32).

Application to Jupiter Entry

The nondimensional parameters ω and ω' will be evaluated to determine if saturation may occur in the ablation layer of the Jupiter Probe near the maximum heating point in its trajectory. Several assumptions are necessary to carry out this evaluation. They are: 1) consider only Swan band of molecular C_2, 2) ablation layer consists of C_2, 3) inviscid shock layer consists of hydrogen plasma at 15,000 K and 1 cm thick, 4) calculations at stagnation point, 5) no radiative emission from the probe heat shield, 6) nondimensional variables, such as $\bar{T}(\bar{x})$, $\bar{P}(\bar{x})$, etc., will be assumed to be unity, 7) the collisions in the ablation layer are predominantly molecule-molecule collisions.

The heavy particle collision excitation cross section is not well known for C_2 molecules; consequently, the argon atom-atom excitation cross section will be used. It is expected to be of the same order of magnitude as that for C_2. As is shown in the Appendix, one can write the reference collisional excitation rate as

$$K_0 = \beta_0\left(\frac{kT_0}{m_{c_2}}\right)^{1/2}(h\nu_b)\exp[-h\nu_0/(kT_0)],\ \mathrm{cm^3/s} \tag{33}$$

where the cross section β_0 is 7.50×10^{-8} cm^2/erg (Ref. 24).

The reference number density in the ablation layer, N_0, is related to the ablation layer pressure and temperature by

$$N_0 = \frac{P_0}{kT_0} = 7.34\times10^{21}\frac{(P_0,\ \mathrm{atm})}{(T_0,\ {}^\circ\mathrm{K})},\ \frac{1}{\mathrm{cm}^3} \tag{34}$$

The reference radiative absorption cross section for the Swan band is assumed to be $\sigma_{\nu_b}(0) = 5.00\times10^{-18}$ cm^2 at the frequency corresponding to 2.5 eV (Ref. 25).

The reference radiation intensity $I_{\nu_b}(0)$ is defined in the Appendix as an average emissivity times the Planck function evaluated at ν_b, where ν_b is the frequency that corresponds to 2.5 eV. The average emissivity was determined by summing the radiative intensity in the spectral interval from 1.8 to 3.25 eV for a 15,000 K, 1-cm-thick hydrogen plasma at several pressures[26,27] and dividing these values by the integral of the Planck function over the same spectral interval. Figure 2 shows the average emissivity as a function of pressure.

The value of γ can be calculated using Eq. (A-7) for $I_{\nu_b}(0)$ and Eq. (27). It becomes

$$\gamma = \epsilon_{\Delta\nu}\Big/\left[\exp\left(\frac{h\nu_b}{kT_H}\right)-1\right] = 0.169\epsilon_{\Delta\nu} \tag{35}$$

for $h\nu_b = 2.5$ eV and $T_H = 15{,}000$ K.

If one assumes an isothermal ablation layer at P_0, T_0 one can evaluate ω and ω'. The integral in Eq. (30) and (31) can be evaluated since the Swan band absorption cross section σ is given by $\sigma = 10^{\sigma'}/10^{18}$, where

$$\sigma' = 51.144 + 0.0059285T - 0.4332\times10^{-6}T^2 + h\nu[40.864$$

$$-0.004723T + 0.3538\times10^{-6}T^2] + (h\nu)^2[-8.0892$$

$$+0.009537T - 0.7368\times10^{-7}T^2] \tag{36a}$$

and where $h\nu$ is in eV units and T is K units.[28] The nondimensional absorption cross section becomes

$$\bar{\sigma} = 10^{\sigma'}/[(10^{18}\sigma_{\nu_b}(0)] \tag{36b}$$

This cross section is based on the population of C_2 molecules, rather than the population of the ground state of

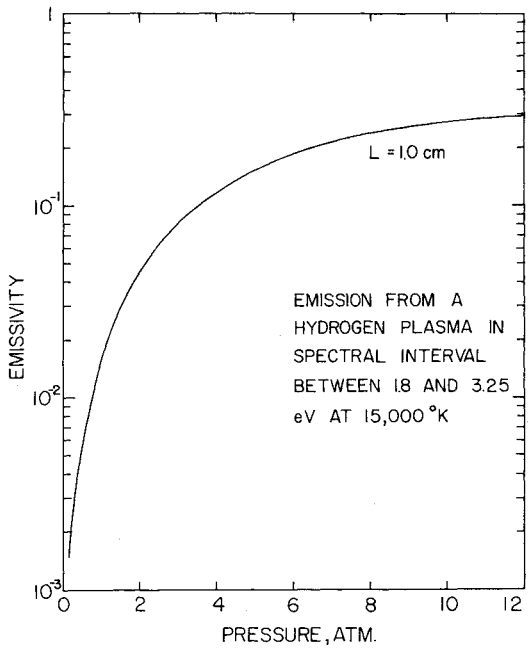

Fig. 2 Emission from a 1-cm-thick isothermal hydrogen plasma at 15,000 K in the spectral interval 1.8-3.25 eV as a function of the plasma pressure.

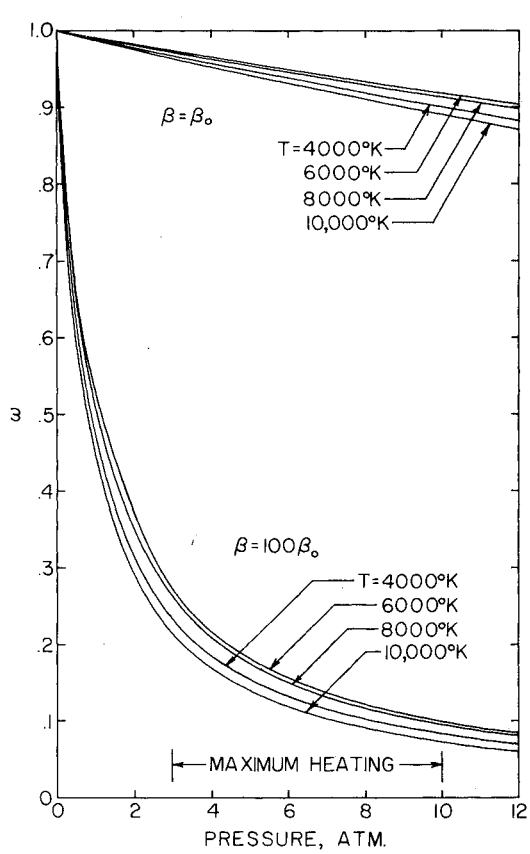

Fig. 3 Parameter ω as a function of pressure for two values of the collisional excitation cross section β. Values are shown for four ablation layer temperatures for each value of β. The maximum heating for the Jupiter Probe occurs between 3 and 10 atm.

the Swan band transition. However, it is suitable for estimation purposes, since the conversion factor is accounted for in the value of $\sigma_{\nu_b}(0)$.

Figure 3 shows the value of ω as a function of pressure for several ablation layer temperatures. Data are shown for the reference collisional cross section β_0 and for 100 β_0. Recall

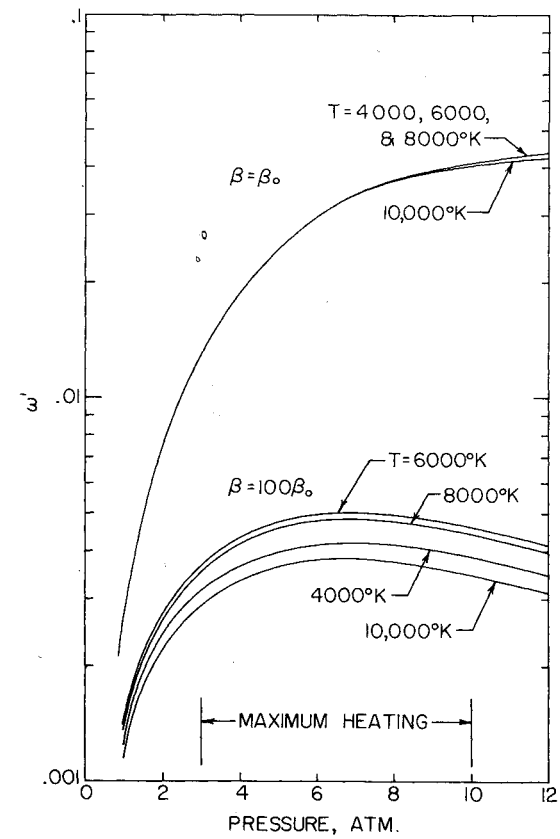

Fig. 4 Parameter ω′ as a function of pressure for two values of the collisional excitation cross section β. Values are shown for four ablation layer temperatures for each value of β. The maximum heating for the Jupiter Probe occurs between 3 and 10 atm.

that the population ratio approaches its equilibrium value as ω approaches zero. Thus, Fig. 3 shows that the population ratio can be expected to be out of equilibrium over most of the pressure-temperature range shown. The stagnation shock layer pressure is in the range of 3-10 atm at the maximum heating point of the Jupiter probe entry trajectory. If the collisional cross section β is increased beyond 100 β_0, the population ratio will approach its equilibrium value at pressures greater than 10 atm. If β is decreased from β_0, ω will approach unity and the population ratio will become increasingly different than its equilibrium value.

The population ratio also depends on ω', which is shown in Fig. 4 as a function of pressure. For $\beta = \beta_0$ this parameter is less than 0.05 for pressures less than 12 atm and ablation layer temperatures between 4000 and 10,000 K. Increasing β by a factor of 100 reduces the magnitude of ω', while also reducing the magnitude of ω, thus predicting population ratios closer to equilibrium ones.

The presence of heavy particle collision partners other than C_2 in the ablation layer is accounted for in the analysis by increasing the collision cross section β_0 by a factor of 100. The results shown in Figs. 3 and 4 indicate that nonequilibrium population distributions will still occur for these much larger collision cross sections. Of course, if the collisional cross section becomes less than β_0, nonequilibrium will become more important.

Figures 3 and 4 show that it is highly probable that saturation effects may exist in the Jupiter Probe ablation layer near the maximum heating point in its trajectory. The population ratio as given by Eq. (26) is very small for equilibrium (when ω approaches zero). Any situation for which the first term is not negligible will produce an above equilibrium upper state population, which could lead to saturation. Thus, this order of magnitude analysis indicates

that saturation is likely to occur during the Jupiter Probe atmospheric entry into Jupiter.

The nonequilibrium population distribution in the ablation layer will, in general, increase the heat transfer to the Probe's heat shield because there is: 1) less absorption of high temperature hydrogen radiation from the inviscid region of the shock layer in the ablation layer and 2) increased emission in the ablation layer because of the overpopulated upper states. The reduced optical thickness of the ablation layer, which allows much more of the inviscid layer radiation to be transmitted to the heat shield, is the more important of the two effects. The increased emission occurs in the relatively low temperature ablation layer, so its effect on the heat transfer is small. Other effects, such as 1) additional molecular bands of C_2, 2) molecular bands of the other molecules present, 3) overlapping of bands, 4) atomic lines, and 5) bound-free molecular and atomic processes, will further complicate the radiation heat transfer and need to be considered in future studies.

Summary and Conclusions

The basic equations for nonequilibrium radiative transport in the stagnation ablation layer for conditions typical of spacecraft entry into the atmosphere of Jupiter have been developed. The equations are simplified and two parameters are obtained which can be used to classify the radiation transfer as equilibrium or nonequilibrium. The parameter ω represents the ratio of the radiative de-excitation rate to the sum of the radiative de-excitation rate and the collisional excitation rate. The second parameter ω' represents the ratio of the radiative excitation rate due to absorption of external radiation to the sum of the radiative de-excitation rate and the collisional excitation rate. Both parameters vary between zero and unity. The population ratio of the upper and lower states of the molecular band approaches its equilibrium value as ω approaches zero. When ω and ω' are greater than zero the population ratio becomes greater than its equilibrium value.

An order of magnitude analysis of the resulting equations was undertaken to investigate the possibility of saturation of the upper state of the Swan band transition in molecular carbon. The parameters were evaluated for conditions expected for the Jupiter Probe near the maximum heating point in its trajectory and it appears likely that saturation will occur. More refined numerical solutions are needed to completely define the extent to which saturation occurs in the Jupiter Probe ablation layer during entry into the atmosphere of Jupiter.

Appendix: Definition of Nondimensional Variables

Nondimensionalization of the equations is as follows:

$$x = L\bar{x}; \quad dx = L d\bar{x} \tag{A1}$$

where L is the ablation layer thickness in centimeters.

$$T = T_0 \bar{T}(\bar{x}) \tag{A2}$$

and

$$P = P_0 \bar{P}(\bar{x}) \tag{A3}$$

where T_0 and P_0 are the temperature and pressure at the inviscid layer-ablation layer interface where $x = 0$.

The reference ablation layer number density is

$$N_0 = P_0 / (kT_0) \tag{A4}$$

where k is Boltzmann's constant. The upper and lower state populations of the transition of interest are

$$N(u) = N_0 \bar{N}_u(\bar{x}) \tag{A5}$$

$$N(\ell) = N_0 \bar{N}_\ell(\bar{x}) \tag{A6}$$

The reference frequency is

$$\nu = \nu_b \bar{\nu}$$

where ν_b is molecular band center frequency ($\nu_{\ell u}$) for the Swan band it corresponds to 2.5 eV.

The quantity

$$I_{\nu_b} = \epsilon_{\Delta\nu} \frac{2h\nu_b^3}{c^2} \frac{1}{\exp[h\nu_b/(kT_H)] - 1} \tag{A7}$$

is the reference radiation intensity. The quantity T_H is the temperature of the inviscid hydrogen layer just behind the shock wave. It is taken as 15,000 K. The emissivity $\epsilon_{\Delta\nu}$ is the average emission of a 15,000 K hydrogen plasma in the spectral interval $\Delta\nu$. It depends on the plasma thickness and pressure, as discussed in the text. Quantities which are nondimensionalized by I_{ν_b} are

$$I_\nu = I_{\nu_b} \bar{I}(\bar{x}, \bar{\nu}) \tag{A8}$$

$$S_\nu = I_{\nu_b} \bar{S}(\bar{x}, \bar{\nu}) \tag{A9}$$

$$J_\nu = I_{\nu_b} \bar{J}(\bar{x}, \bar{\nu}) \tag{A10}$$

$$I_B = I_{\nu_b} \bar{I}_B(\bar{x}, \bar{\nu}) \tag{A11}$$

The radiative cross section is defined as

$$\sigma_\nu(\ell, u) = \sigma_{\nu_b}(0) \bar{\sigma}(\bar{x}, \bar{\nu}) \tag{A12}$$

The optical depth is defined as

$$\tau_\nu(x) = \int_0^x \sigma_\nu(\ell, u) N(\ell) dx = \tau_{\nu_b} \int_0^x \bar{\sigma}(\bar{x}', \bar{\nu}) \bar{N}_\ell(\bar{x}') d\bar{x}' \tag{A13}$$

where

$$\tau_{\nu_b} = \sigma_{\nu_b}(0) N_0 L \tag{A14}$$

The collisional excitation rate is written as

$$K(\ell, u) = \beta_0 \left(\frac{kT_0}{m_{c_2}}\right)^{1/2} h\nu_b \times \exp\left(\frac{-h\nu_b}{kT_0}\right) (\bar{T})^{1/2} \exp\left[\frac{-h\nu_b}{kT_0}\left(\frac{1}{\bar{T}} - 1\right)\right] \tag{A15}$$

so that

$$K_0 = \beta_0 \left(\frac{kT_0}{m_{c_2}}\right)^{1/2} h\nu_b \exp\left(\frac{-h\nu_b}{kT_0}\right) \tag{A16}$$

and

$$\bar{K}(\bar{x}) = (\bar{T}(\bar{x}))^{1/2} \exp\left[-\frac{h\nu_b}{kT_0}\left(\frac{1}{\bar{T}(\bar{x})} - 1\right)\right] \tag{A17}$$

This allows one to write

$$K(\ell, u) = K_0 \bar{K}(\bar{x}) \tag{A18}$$

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