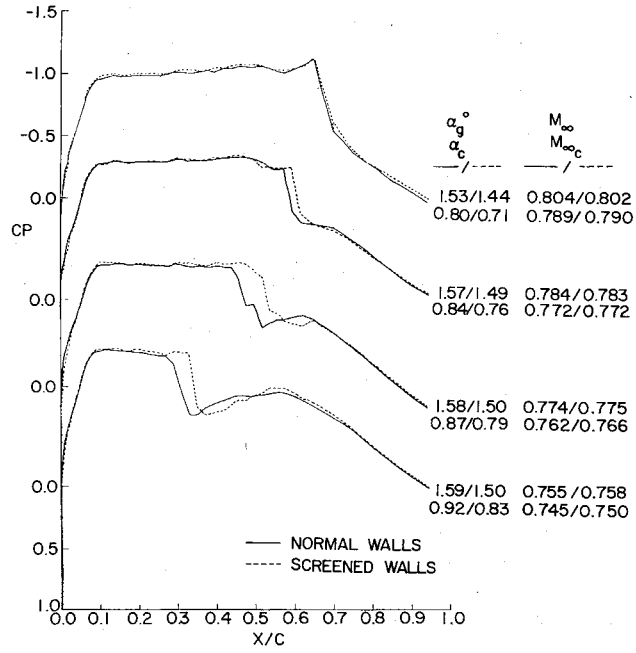

Fig. 4 Upper surface pressure distributions for BGK No. 1 airfoil.

shock is located over the "flat" portion of the airfoil, say 20-65% chord. Sample results of the upper surface pressure distribution for a fixed nominal incidence of 1.5 deg at various Mach numbers are given in Fig. 4. The wind-tunnel freestream Mach number M_∞, the geometric incidence α_g, and their values M_{∞_c} and α_c corrected for wall interference following Mokry,[4] are listed in the figures. The differences in M_{∞_c} for the two lower cases are in the direction that would cause the shock wave to move further downstream for the screened wall case. In the one case where the two M_{∞_c} are identical, there is still a noticeable shift in shock location. At the highest Mach number the shocks coalesce. In all cases the differences in the angle of attack, whether corrected or not, are in the direction that would suggest a reversal in the observed shock positions.

Computational results, using the SCW II method,[5] have been obtained to determine the sensitivity of shock position to small changes in Mach number and angle of attack. These may or may not be all that meaningful because of an inexact treatment of the shock/boundary-layer interaction. In any case, they show in general that the observed differences in shock location cannot be explained by the small differences in Mach number and angle of attack. The computational results are presented and discussed in more detail in Ref. 2. Thus we are led to conclude that at some supercritical conditions the shock position can be affected by the presence of edge-tone noise. The mechanism of such an effect is far from clear. One may surmise that the edge-tone noise interacts in some way with the disturbed boundary layer in the shock/boundary-layer interaction zone, so as to cause the observed effect.

As a final comment, it is worth relating the present findings to those of Dougherty and Steinle.[6] Using the so-called AEDC 10 deg cone to determine transition Reynolds number in various wind tunnels, they found that by covering the perforated walls of the AEDC 16T wind tunnel with tape, thus eliminating the edge-tone noise, the transition Reynolds number increased compared to that for the open perforated walls. This appears to be in conflict with the present finding.

However, one can reason that they are not necessarily in conflict. In their case the unit Reynolds number was 6.6×10^6/m, which is an order to magnitude smaller than for the present NAE test ($33\text{-}69\times10^6$/m). This lower unit Reynolds number implies a correspondingly lower "dangerous" frequency for the Tollmien-Schlichting (T-S) waves. But, the edge-tone center frequency in the 16T wind tunnel is also much lower than in the NAE wind tunnel (600 and 7400 Hz, respectively, at $M=0.75$). In neither case are these frequencies close to the estimated T-S dangerous frequencies, which are an order of magnitude higher. It is still possible that higher harmonics of the edge tones may excite the T-S waves. The increase in transition Reynolds number attributed to edge-tone suppression in the 16T wind tunnel is only 10-15% in $M=0.7\text{-}0.8$ range. Such an increase would hardly be detectable in the NAE two-dimensional drag data. Transition location at the high Reynolds number NAE investigation is known to be close to the leading edge. A shift in transition from 10 to 11% chord, say, would result in less than 0.0001 decrease in the drag coefficient, which is within normal data scatter. Thus, the results from the two investigations may not be conflicting.

Furthermore, they both show, in the author's opinion, that the edge-tone noise, in spite of its very significant contribution for the overall noise levels in the two wind tunnels, has a surprisingly small effect on aerodynamic measurements, and demonstrably so only at high subsonic speeds.

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Correlation of Hypersonic Stagnation Point Heat Transfer at Low Reynolds Numbers

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Nomenclature

a	= sound velocity
C^*	$=\mu^* T_\infty/\mu_\infty T^*$
CH	= Stanton number, $\dot{q}/\rho_\infty u_\infty (H-H_w)$
H	= altitude or total enthalpy
Kn	= Knudsen number
Kr^2	= Cheng's parameter of Eq. (7)
$\dot{q}$	= heat-transfer rate
Re	= Reynolds number

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Rn = nose radius
T = temperature
T^* = $(T_2+T_w)/2$
Δ = shock detachment distance
δ = boundary-layer thickness
ϵ = ρ_1/ρ_2
η = accommodation coefficient
λ = mean free path

Subscripts

CON = value at continuum limit
FM = value at free molecule flow limit
BL = value of laminar boundary layer
0 = value at stagnation point
2 = local value after shock wave
∞ = value at freestream condition

Superscript

* = value evaluated at T^*

Introduction

It is still difficult to estimate the stagnation point heat-transfer rate at low Reynolds numbers with reasonable accuracy for engineering purposes.

A semiempirical correlation for the aerodynamic coefficient X at low Reynolds numbers was proposed first by Martino[1] as

$$X=\frac{X_{\mathrm{CON}}+KnX_{\mathrm{FM}}}{1+Kn} \tag{1}$$

For a coefficient in the slip or transition regime, this equation is a bridge between the limits of both continuum and free molecule flow using the Knudsen number as a governing parameter.

Martino showed that the recovery factor on a circular cylinder at $Kn\sim 0.1$ was correlated well by Eq. (1). For heat-transfer coefficients averaged over the surfaces of a cylinder and a sphere, Reeves et al.[2] introduced a correlation equation from Eq. (1) in terms of $Nu/\sqrt{Re_2}$ and the local Knudsen number $Kn_2=M_\infty/\sqrt{Re_2}$. Blick[3] and Guy et al.[4] showed that aerodynamic force coefficients were correlated by Eq. (1) reasonably well at low Reynolds numbers.

The purpose of this Note is to introduce a simple correlation equation in order to estimate heat-transfer rates at the stagnation point of a blunted vehicle flying at high velocities and high altitudes. From this standpoint, a correlation equation has been investigated such that $\dot{q}_0$ can be formulated only in terms of freestream conditions.

Correlation

Regarding a governing parameter for the stagnation point heat-transfer rate at low Reynolds numbers, several parameters have been proposed. Depending on the flow regime, the Knudsen number can be defined generally as

$$Kn\equiv\lambda_\infty/Rn\sim M_\infty/Re_\infty \quad \text{for} \quad Re_\infty\sim 1 \tag{2}$$

$$Kn\equiv\lambda_\infty/\delta\sim M_\infty/\sqrt{Re_\infty} \quad \text{for} \quad Re_\infty\gg 1 \tag{3}$$

For the heat-transfer coefficient to blunt bodies, Reeves et al.[2] used the local value of

$$Kn_2\sim M_\infty/\sqrt{Re_2} \tag{4}$$

Bloxsom-Rhode[5] and Blick[3] evaluated the heat-transfer correlation after the shock wave as

$$Kn_s=\lambda_2/\Delta\sim\lambda_2/\epsilon Rn \tag{5}$$

and showed that the drag coefficients of several geometries were well correlated. Guy et al.,[4] however, used Eq. (3) successfully for the drag coefficient of a regular hexagonal prism in the transition regime. On the other hand, Potter[6] suggested the use of the local Reynolds number

$$Re_2\equiv Re_\infty\mu_\infty/\mu_2, \qquad Re_w\equiv Re_\infty\mu_\infty/\mu_w \tag{6}$$

in order to include the influences of the bow shock formation and the T_w/T_∞ effect. In thin shock-layer theory, Cheng[7] introduced a rarefaction parameter defined as

$$Kr^2\equiv p_\infty Rn/\mu_\infty u_\infty C^* \tag{7}$$

As shown by Cheng, this parameter is related to the local Reynolds number and local Knudsen number as

$$Kr^2=\epsilon_\infty(\rho_\infty u_\infty Rn/\mu^*)\cdot T^*/T_0 \tag{8}$$

$$=\epsilon_\infty Rn/\lambda^* \tag{9}$$

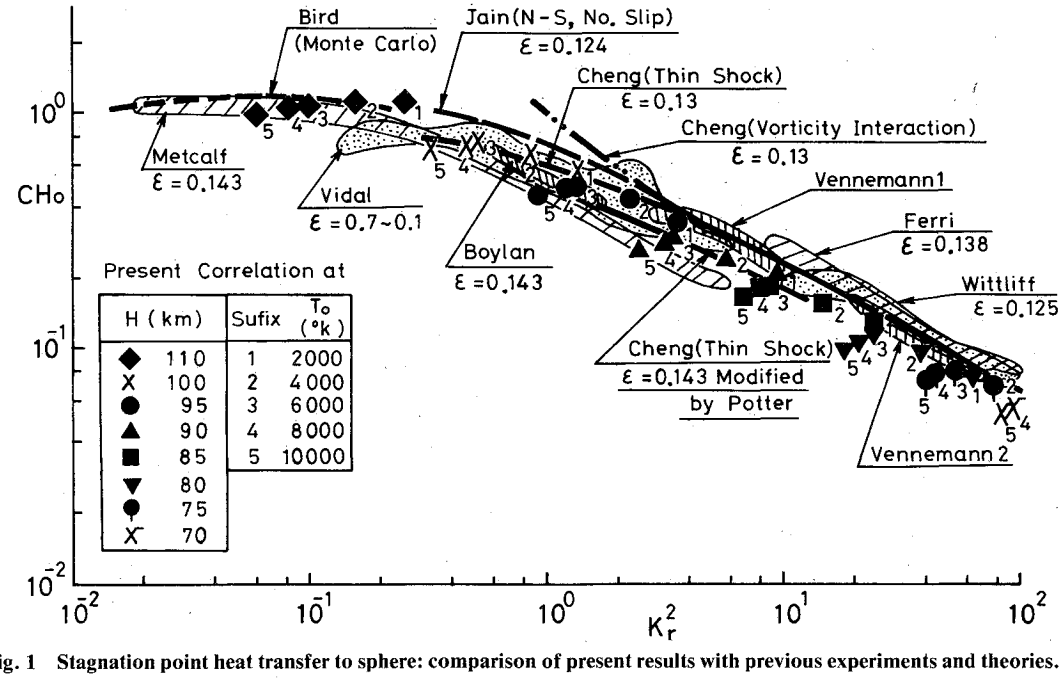

Fig. 1 Stagnation point heat transfer to sphere: comparison of present results with previous experiments and theories.

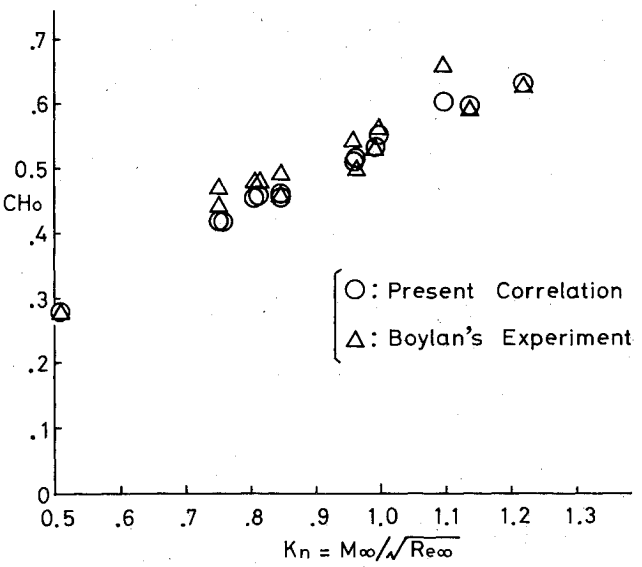

Fig. 2 Comparison of present results with Boylan's experiments.

As for hypersonic heat-transfer rates at the stagnation point of a sphere, Boylan[8] showed that his experimental results in the form of CH_0 were well correlated by Kr^2 and agreed well with Cheng's theory, if the differences of ϵ were corrected by the Potter method.[6] In the same way, Metcalf et al.[9] compared their experimental data with previous experimental and theoretical results in the form of CH_0 vs Kr^2. They showed that all results were correlated fairly well for the regime from slip to free molecule flow beyond the limitation of Cheng's analysis [$Kr^2 \gg 0(1)$]. The use of the Stanton number in these studies can include the T_w/T_∞ effect as $\dot{q}_0/\rho_\infty u_\infty H_0 (1-H_w/H_0)$, whose importance was pointed out by Potter. Also, it is convenient to use CH_0 for the present purpose, since $\dot{q}_0$ is nondimensionalized only by freestream conditions.

Although Cheng's parameter seems to correlate the Stanton number well, its definition is not simple to use for engineering purposes. The squared Knudsen number $Kn^2 = (M_\infty/\sqrt{Re_\infty})^2$ can be related to Kr^2 as

$$Kn^2 = \frac{u_\infty \mu_\infty}{a_\infty^2 \rho_\infty Rn} \sim \frac{1}{Kr^2} \tag{10}$$

assuming the invariant constant to be in a linear relationship of μ to T. Then, with the same assumption, it can also be said that Kn^2 is related directly to the local Knudsen number of Eq. (5) and inversely to the local Reynolds number of Eq. (6). Therefore, the simple parameter of Kn^2 may be used in place of Kr^2 to correlate the Stanton number.

Using CH_0 and Kn^2 as the aerodynamic coefficient and a governing parameter in Eq. (1), respectively, we obtain a correlation equation for the stagnation point heat-transfer rate as

$$CH_0 = \frac{CH_{\mathrm{BL}} + (Kn/c)^2 CH_{\mathrm{FM}}}{1+(Kn/c)^2} \tag{11}$$

where a matching constant c is determined to be 3 through comparison with previous data, as shown later. Applying Eq. (11), the stagnation point heat-transfer rates of spheres with $Rn = 0.01$-1.0 m were calculated for the flight conditions $u_\infty = 2$-15 km/s and $H = 20$-110 km. CH_{BL} in Eq. (11) was calculated by Fay-Riddell's theory[10] and CH_{FM} by $CH_{\mathrm{FM}} = \eta/(1-H_w/H_0)$ for $\eta = 1.0$ and $T_w = 300$ K. For the commonly used surface in engineering practice[11] $\eta = 0.8$-0.98. The stagnation conditions after the normal shock wave were obtained by the method of Horton and Menard,[12] which numerically calculated the chemical equilibrium properties for the high-temperature air after the shock wave.

Results for a sphere with $Rn = 1$ m compared with previous work (taken from Ref. 9) are shown in Fig. 1, where the matching constant c in Eq. (11) was chosen as 3. Each calculated point in Fig. 1 corresponds to the flight conditions (H, T_0) shown. The agreement between Eq. (11) and previous results is good over the region from continuum to free molecule flow. Results for $Kr^2 > 10^2$ converge to a laminar boundary-layer solution. Results for spheres with other Rn showed almost the same trend. In order to show a more precise comparison, Boylan's[8] experimental data for a 51 mm (2 in.) diam hemisphere/cylinder are compared with the present results in Fig. 2. An excellent agreement is shown.

In applying Eq. (11) to a blunted vehicle flying along a trajectory, CH_{BL} can be evaluated by using a simple correlation equation formulated only by flight conditions as given by Detra et al.[13] Therefore, Eq. (11) satisfies the present purpose of obtaining a simple correlation equation for $\dot{q}_0$ only in terms of freestream conditions.

Considering that Kn^2 is directly related to the local Knudsen number defined by Eq. (5), the present result of Eq. (11) is similar to Blick's[3] where the drag coefficient C_D was given by

$$C_D = \frac{C_{D_{\mathrm{CON}}} + \beta Kn_s C_{D_{\mathrm{FM}}}}{1+\beta Kn_s} \tag{12}$$

with a matching constant β. Therefore, Kn_s in Eq. (12) may be replaced by Kn^2 using an appropriate constant, although the quantitative comparison cannot be made without detailed experimental conditions.

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