Stagnation Point Heat Transfer for Jet Impingement to a Plane Surface

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Nomenclature

C = const, Eq. (2)
C_H = Stanton number, $C_H = h/\rho_\infty V_\infty C_p$
C_p = specific heat
D_i = local jet diameter
D_j = jet diameter
D_N = nozzle diameter
h = heat-transfer coefficient
H = nozzle-to-plate spacing
k = thermal conductivity
L = characteristic length, $L = V_\infty/\beta$
Pr = Prandtl number, $Pr = C_p\mu/k$
$\dot{q}_s$ = stagnation point heat-transfer flux, Eq. (1)
r = radial distance along surface
Re_{D_j} = Reynolds number, $Re_{D_j} = \rho_\infty V_\infty D_j/\mu_\infty$
Re_L = Reynolds number, $Re_L = \rho_\infty V_\infty L/\mu_\infty$
Re_{W_j} = Reynolds number, $Re_{W_j} = \rho_\infty V_\infty W_j/\mu_\infty$
T_t = total temperature
T_w = wall temperature
U = potential flow velocity along surface
V = potential flow velocity normal to surface
V_∞ = freestream velocity normal to surface
W_j = width of infinite two-dimensional jet
β = stagnation point velocity gradient, $\beta = (\mathrm{d}U/\mathrm{d}r)_{r=0}$
μ_w = fluid viscosity at wall temperature
μ_∞ = fluid viscosity at freestream temperature
ρ_w = fluid density at wall temperature
ρ_∞ = fluid density at freestream temperature

Introduction

A PROBLEM of extreme interest in the solid rocket motor industry is the design of effective ignition systems which at the present time relies primarily on proprietary company experience rather than rigorous analytical procedures. In general, an igniter system consists of a pyrotechnic device which directs high-temperature, high-velocity gases, in the form of single or multiple jets, to impinge on the propellant surface. Although it is a highly complex phenomenon, any mathematical simulation of the ignition process must include a detailed knowledge of the local heat-transfer rates in the neighborhood of the jet stagnation point.

Stagnation point flows and the associated convective heat transfer afforded some of the earlier exact solutions of the boundary-layer equations.[1] However, these solutions were developed in the classical nondimensional form and include a velocity gradient parameter which is not defined except for special cases. During the formative years of supersonic and hypersonic flight, convective heating at the leading edge of aerodynamic surfaces[2] and blunt bodies of revolution[3] employed the velocity gradients derived from the potential flow about cylinders and spheres, respectively. For flat-faced bodies similar data are available from the complex potential for streaming motion past a two-dimensional rectangular section.[4]

The solutions described above are applicable to bodies immersed in an infinite stream and in the limit predict zero heat transfer to an infinite plane surface. For the problem of interest described herein the approach is to consider a finite jet impinging against an infinite plane, about which in comparison to the conventional convective flow and heat-transfer fields relatively little is known. A theoretical heat-transfer correlation has been derived that is an extension of the analyses presented by Squire[2] and Sibulkin[3] incorporating a modification of the potential flow for a two-dimensional jet normal to a flat plate as given by Milne-Thomson.[5]

Theoretical Discussion

In the neighborhood of a stagnation point the flow can be considered incompressible and laminar regardless of the mainstream flowfield. This leads to a marked simplification of both the momentum and energy equations that becomes amenable to closed-form solutions. Although the flow is considered incompressible, the analysis should be applicable to supersonic conditions, provided the properties just downstream of the normal shock are selected as freestream conditions.

Boundary-Layer Solutions

The general boundary-layer equations for incompressible plane flow can be transformed into ordinary differential equations by the Blasius similarity transformation. As reported in Ref. 1 the momentum equation was solved by Hiemenz for two-dimensional flow and Homann for the axisymmetric case. Squire and Sibulkin then integrated the energy equation for the two-dimensional and axisymmetric cases to yield,

$$\dot{q}_s = C(\rho_w\mu_w)^{1/2} Pr^{-3/5} \beta^{1/2} C_p (T_t - T_w) \tag{1}$$

or alternately for the Stanton number,

$$C_H = C\left(\frac{\rho_w\mu_w}{\rho_\infty\mu_\infty}\right)^{1/2} Pr^{-3/5} Re_L^{-1/2} \tag{2}$$

where C is a constant equal to 0.570 for two-dimensional flow and 0.763 for axisymmetric flow.

Velocity Gradient Parameter

On examining Eqs. (1) and (2) it is immediately apparent that the heat transfer is dependent on the velocity gradient β at the stagnation point. This parameter can be determined analytically using the incompressible relation for a two-dimensional jet (infinite slit) impinging against a plate as given by Milne-Thomson. Referring to Fig. 1 the potential flow analysis yields,

$$Z = \frac{W_j}{\pi}\left\{\log\frac{V_\infty - (U - iV)}{V_\infty + (U + iV)} + i\log\frac{V_\infty + i(U - iV)}{V_\infty - i(U - iV)}\right\} \tag{3}$$

and in the neighborhood of the stagnation point

$$U \ll V_\infty \quad \text{and} \quad V \ll V_\infty$$

Eq. (3) after rearrangement yields,

$$U = (\pi/4)(V_\infty/W_j)x \quad \text{and} \quad \beta = (\pi/4)(V_\infty/W_j) \tag{4}$$

Substituting for the velocity gradient into Eq. (2) yields for the two-dimensional case,

$$C_H = 0.570\left(\frac{\pi}{4}\right)^{1/2}\left(\frac{\rho_w\mu_w}{\rho_\infty\mu_\infty}\right)^{1/2} Pr^{-3/5} Re_{W_j}^{-1/2} \tag{5}$$

The ratio of the velocity gradient parameter for axisymmetric to two-dimensional (sphere to cylinder) flow is given by,

$$\beta_s/\beta_c = 3/4 \tag{6}$$

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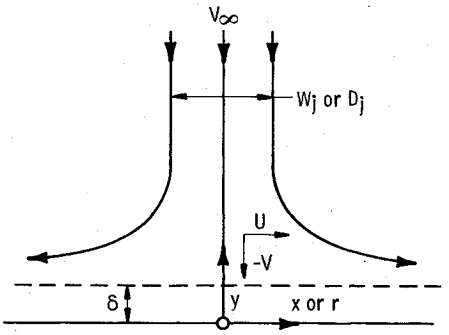

Fig. 1 Potential flow for jet impingement.

assuming the same relationship for planar flow and a circular jet yields,

$$\beta = (3\pi/16)(V_\infty/D_j) \tag{7}$$

and the heat-transfer coefficient,

$$C_H = 0.763\left(\frac{3\pi}{16}\right)^{1/2}\left(\frac{\rho_w\mu_w}{\rho_\infty\mu_\infty}\right)^{1/2} Pr^{-3/5} Re_{D_j}^{-1/2} \tag{8}$$

Equations (5) and (8) indicate that the heat-transfer coefficient is inversely proportional to the square root of the jet width and jet diameter. As in the flow about immersed bodies, the heat transfer reduces to zero in the limit for an infinite jet, implying that the flow approaches an overall stagnation condition with a corresponding degeneration of the convective process.

General Discussion

The heat-transfer coefficients given by Eqs. (5) and (8) are independent of the nozzle-to-plate spacing and would appear to be contrary to the physical evidence.[6] Experimental data by Tani and Komatsu[7] provide the radial velocity distribution along the surface as a function of the nozzle-to-plate spacing (H/D). The limited data show essentially a constant velocity gradient for the close spacing $(H/D=4,8)$ and then decreases for the larger spacing $(H/D=12)$. The Tani and Komatsu data lead to the following correlation for a circular jet,

$$\beta(D_N/V_\infty) = 0.77 \tag{9}$$

as compared to the derived values of

$$\beta\frac{D_N}{V_\infty} = \frac{\pi}{4} = 0.79(\text{2D}), \quad \beta\frac{D_N}{V_\infty} = \frac{3\pi}{16} = 0.59(\text{axi}) \tag{10}$$

Poreh and Cermak[8] derived the velocity gradient from experimental data for spacings greater than the potential core length,

$$\beta(D_N/V_\infty) = 98(\pi/r)^{1/2}(H/D_N)^{-2} \tag{11}$$

which includes the effect of nozzle-to-plate spacing. The potential flow given by Eq. (3) does not account for jet decay due to viscous effects and discussions with Hess[9] suggested that the free-jet diffusion parameters be used as the reference conditions. This leads to

$$\beta(D_N/V_\infty) = K(3\pi/16)(H/D_N)^{-2} \tag{12}$$

where the jet decay laws for laminar and turbulent circular jets are given by,

$$(V/V_\infty)\alpha(H/D_N)^{-1} \text{ and } (D/D_N)\alpha(H/D_N) \tag{13}$$

and the constant K is determined from either experimental data or jet decay theory.

Conclusions

A theoretical correlation has been developed to predict the stagnation point heat transfer for jet impingement on a plane surface. The correlation provides a closed-form solution for nozzle-to-plate spacing less than the potential core which agrees reasonably well with existing data. For spacings greater than the potential core, a correlation was developed that is similar to existing relationships; however, it includes an undefined constant that must be evaluated from either experimental data or free-jet analysis.

The sustained ignition process of a solid rocket motor is a highly complex phenomenon that is not clearly understood. It is readily apparent, however, that any mathematical modeling must include the transient heating due to jet impingement and the proposed correlation described herein can serve as a program input for predicting the time lag to propellant surface autoignition. Finally, the design of ignition systems suggests that the present study be extended to include the effects of jet impingement angle.

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