Two Types of Instream Stagnation

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Unlike separation from a solid body, the instream recirculation regions in laminar incompressible flows with a single vorticity component are associated with two different types of stagnation region. They can be distinguished by the null vorticity contour and the angle of streamline saddlepoint intersection. Examples are drawn from the two-dimensional and axisymmetric literature over a wide range of Reynolds numbers.

Introduction

THIS paper continues the study of recirculation via associated stagnation regions.[1] There is only one type of two-dimensional separation from a solid stationary boundary, the familiar Prandtl type.† In connection with flow reversals and recirculations that occur away from a solid boundary, there are saddlepoints of streamlines through instream stagnation points. There are two types.[1] One is an orthogonal intersection of streamlines where the null vorticity contour locally coincides with a streamline. (Vorticity $\Omega = |\nabla \times q|$ where q is the two-dimensional velocity vector.) The other type is an oblique intersection of streamlines at the saddlepoint. One purpose of this article is to present additional documentation of the oblique saddlepoint streamline intersection (from old and new publications) and also to point out the analogous situations that occur for axisymmetric flows. The relations are valid for all Reynolds numbers, for steady and unsteady flows, because near the stagnation point Stokes' approximation reduces the momentum equation to a linear form.[1]

The orthogonal streamline intersection is related to a longstanding Moore-Rott-Sears (MRS)[2-4] conjecture on velocity shear at the stagnation point. At the stagnation point $U = V = 0$, according to MRS, $\partial U/\partial y = 0$ where the x coordinate is aligned with a nearby solid boundary. When the null vorticity contour coincides with a dividing streamline at the stagnation point $\Omega = (\partial V/\partial x) - (\partial U/\partial y) = 0$ where the local streamline x,y coordinates need not be aligned with the solid boundary. If they are aligned and V is identically zero along the dividing streamline the two criteria coincide, of course, but MRS is a special case, not a general criterion. The wake closure of the vortex pair behind a circular cylinder in aligned moderate Reynolds number uniform flow has (polar velocity component) $U_\theta = 0$ on the symmetry plane ($\Omega = 0$); the normal velocity component $V_R(R) = 0$ determines the stagnation location. The streamline intersection is orthogonal in steady flow[5-8] or the transient response to impulsive flow.[9-12]

The oblique saddlepoint at stagnation is virtually ignored in the theoretical literature of separation. In Telionis' recent monograph on unsteady viscous flow,[13] the oblique saddlepoint type is only represented by a schematic sketch (see Ref. 13, Fig. 7.2b). Unfortunately, the stagnation point is mislabeled MRS; it does not satisfy MRS requirements stated above.

Both theoretical and experimental data were quoted in Ref. 1 to prove the existence of the oblique streamline saddlepoint. In addition, Fig. 1a illustrates steady uniform flow U past a rotating circular cylinder. The streamlines are drawn from a low Reynolds number ($Re = Ua/\nu = 5$, and normalized cylinder surface speed $U_\theta = 1$) solution by Townsend.[14] The approximate value of normalized vorticity at the stagnation point is $\Omega(s) = 1$. The vorticity contours are given in the report, so it is readily apparent that none coincides with the dividing streamline at the stagnation point. The oblique type of instream intersection is preserved as the Reynolds number increases[14]; see also Taneda.[15] Prandtl showed a similar tracer photograph to the British Royal Aeronautical Society in 1927.[16] Another example is shown in Fig. 1b, where the wall and a rotating cylinder move in opposite directions. This is a Stokes flow with an exact analytic solution.[17] At higher Reynolds numbers (based on shear rate and gap distance) the vorticity contours would display asymmetries with respect to the centerplane but would not be zero in the gap region.

The oblique saddlepoint is seen in various high Reynolds number numerical theoretical studies where individual vortices are shed into the wake region (see Ref. 1), but it is not confined to dividing streamlines enclosing a single closed streamline (vortex) region. A side-by-side pair of like-signed vortices is induced in a shallow wide cavity by steady planar shear flow over it,[18] Fig. 2. The topography of calculated streamline contours[19] includes the oblique saddlepoint region. Opposing shear flows that are allowed to interact over a short distance also create oblique intersections[20]; Fig. 3 shows a flow visualization. These oblique saddlepoints have also been predicted by finite difference flow simulations (see Ref. 20, Fig. 28).

Telionis has published a flow visualization of a steady oblique saddlepoint stagnation region.[21] He noted in that paper that the stagnation point was not MRS on the "upstream-moving side" of a circular cylinder rotating in uniform flow. As previously discussed, vorticity was not measured. There are also unsteady oblique saddlepoints revealed in the article.

In unsteady flows the location of the saddlepoint usually varies with time, as do the vorticity contours. The location and angle of intersection can be captured by flow visualization, but the time-dependent vorticity is seldom measured. This quantity can be predicted reliably by a solution of the full momentum equation at moderate Reynolds numbers.

The role of instream singularities that may or may not appear in high Reynolds number boundary-layer calculations is not clear.[13,22] If we accept a definition of separation often used by aerodynamicists as "the abrupt growth in δ, the displacement boundary layer thickness," the sketch of observed steady streamlines in the vicinity of an upstream-moving wall, Fig. 4, after Figs. 9 and 13 of Ref. 21, indicates such a point upstream of an oblique stagnation saddlepoint. (See, also, their instantaneous Figs. 25b and 25c.) A recent

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†The beginning of the reversed flow region (separation or detachment) is where the wall shearing stress and vorticity go to zero.[1] At wall reattachment (if it exists), the quantities are zero again.

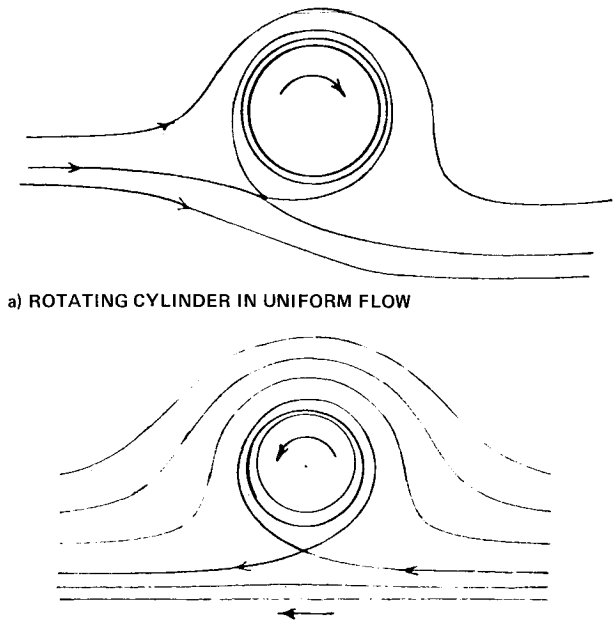

Fig. 1 Oblique saddlepoints in flows with steadily rotating cylinders

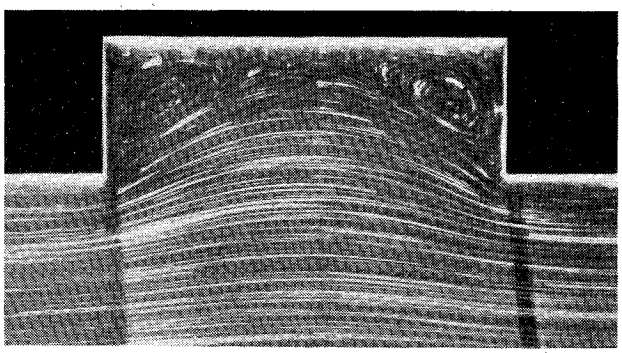

Fig. 2 Oblique saddlepoint in a shallow cavity under a steady shear flow.[7]

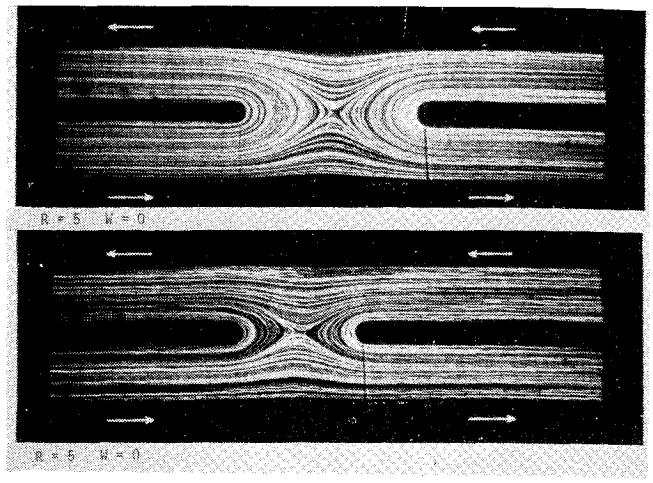

Fig. 3 Oblique saddlepoints where two opposing Newtonian shear flows interact.[20] $Re = 5$.

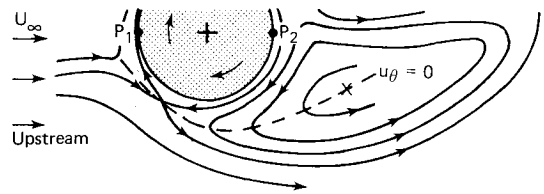

Fig. 4 Streamlines for a rotating cylinder in a uniform stream U_∞ ($Re = 50$). The wall velocity magnitude $|U_\theta|_w = 0.80\ U_\infty$ and between P_1 and P_2 the wall moves upstream (after Ref. 21). The null u_θ curve passes through the oblique streamline intersection and the vortex center.

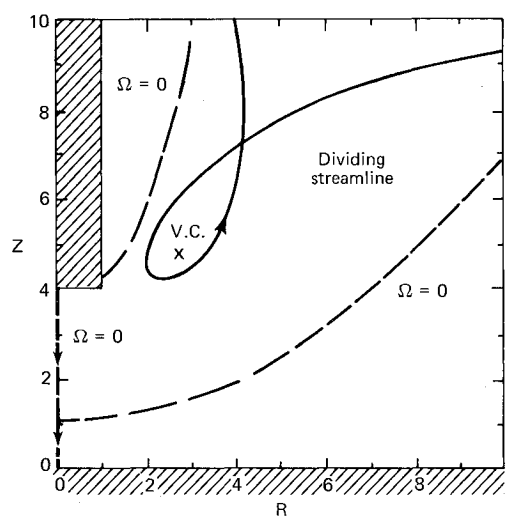

Fig. 5 Relationships of dividing streamline and null vorticity contours (*jet* $Re = 1$, $H = 4$), after Figs. 4a and 5a in Ref. 30. × marks the vortex center, "V.C."

steady theoretical calculation is in agreement that the stagnation point with a flat upstream-moving wall is not MRS because $\partial U/\partial y = 0$ and $U = 0$ curves do not intersect.[23] Regrettably, the dividing streamlines that enclose the recirculation regions are not shown in his figures.

By analogy simple axisymmetric flows should be expected to show the same types of median plane streamline intersection as the two-dimensional flows, because they also have only one vorticity component. They do. There are two kinds of instream stagnation regions. The orthogonal saddlepoint is predicted and seen on the axis ($\Omega = 0$) in the closed finite wake behind spheres[24] and ellipsoids[25-26] in uniform flow. A circulating fluid sphere in slow uniform translation has two such points, nose and tail.[27-29] At higher Reynolds number with a trailing vortex ring, there is a stagnation ring on the fluid sphere surface[30]; the dividing streamline in the median plane is orthogonal (saddlepoint vorticity is zero).

On the other hand, recent velocity field calculations of a steady jet from a pipe impinging normally on a plane showed a closed toroidal ring whose location depends on pipe Reynolds number.[31] Professor Deshpande (personal communication) was kind enough to plot the dividing streamline for pipe Reynolds number $Re = 25$, at a normalized distance $H = 4$ from the flat plane (corresponding to their Fig. 4b). It clearly showed the oblique intersection of 71 deg. The location of the saddlepoint is a function of Re and H (see their Fig. 12), but it is always located between the $\Omega = 0$ contours of the field, Fig. 5. Also, the oblique saddlepoints in shallow wide axisymmetric cavities around steady pipe Poiseuille shear flows[19] resemble those in Fig. 2 for the two-dimensional cavity.

Both instream saddlepoint types also occur in axisymmetric unsteady flows. Instream orthogonal saddlepoints occur and move on the axis ($\Omega = 0$) behind an impulsively started sphere.[32] The oblique saddlepoint occurs transiently during a pulsatile flow cycle through a constricted tube, Fig. 6, where

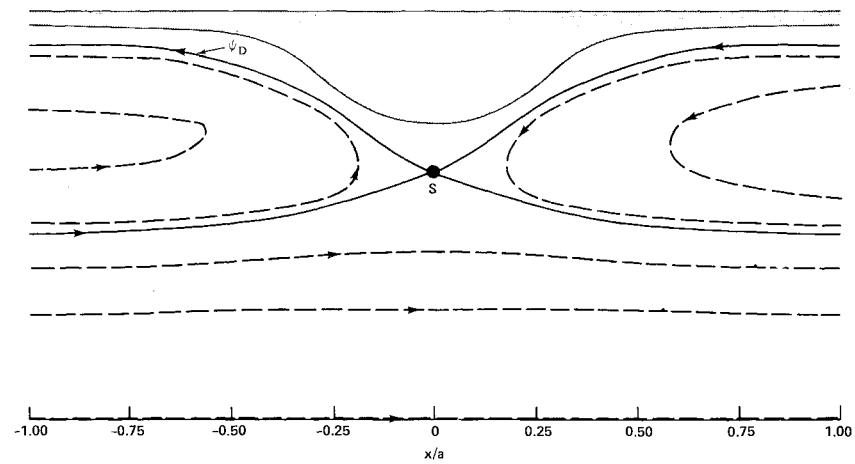

Fig. 6 Transient oblique saddlepoint within the simple pulsatile flow through a constricted tube (instantaneous streamlines near minimum flux when upstream shear is negative at the wall). Mean inlet $Re = 100$.

our numerical simulation solution[33] gives the instantaneous vorticity contours as well as streamlines.

The recognition and acceptance of two types of laminar instream stagnation regions may help to clarify computations at higher Reynolds numbers where quantitative comparison with experimental data is difficult. It does not seem likely that stagnation regions which occur in turbulent attached boundary layers or instream can be qualitatively different when the high-frequency random fluctuations are averaged over a long time or many periodic cycles.

Acknowledgments

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