

# Technical Notes

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## Shock Wave Focusing in a Log-Spiral Duct

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### Introduction

A LOGARITHMIC spiral (log-spiral) duct was contrived by Milton<sup>1</sup> as an effective tool to produce an extremely high pressure and temperature by focusing an initially plane shock wave. According to the ray shock theory,<sup>2</sup> no triple-point (or reflected shock wave) is expected to form in a log-spiral duct until the implosion is completed. In fact, no reflected shock wave was seen in shadowgraphs taken by Milton.<sup>1</sup> On the other hand, after implosion Milton observed an intense vortex in the flow behind the reflected shock wave. Then a question arises: What is the mechanism of the vortex formation? Milton suggests that a triple point may appear in a flowfield very close to the focus. The appearance of the triple point might be related to the formation of the vortex. At present, however, it seems pretty difficult to analyze experimentally the detailed flowfield close to the focus. The purpose of this study is to simulate numerically the shock wave focusing process in a log-spiral duct and to clarify the mechanism of the formation of a vortex behind the reflected shock wave after implosion.

### Numerical Method

A two-dimensional, log-spiral duct adopted in this study is presented in Fig. 1. The duct is assumed to be symmetric with respect to the plane BO. The duct is designed such that an initially plane shock wave focuses at a point O. The configuration of the duct is determined by the characteristic length  $L$  and the characteristic angle  $\chi$  and is described as follows<sup>1</sup>:

$$r = R \exp\left(\frac{\chi - \phi}{\tan \chi}\right) \quad (1)$$

where  $R = L/\cos \chi$ . Upstream of point A, the wall is assumed to be straight. In the following, lengths are nondimensionalized by  $L$ . The two-dimensional Euler equations were solved:

$$\frac{\partial Q}{\partial t} + \frac{\partial E}{\partial x} + \frac{\partial F}{\partial y} = 0 \quad (2)$$

where  $x$  and  $y$  are the coordinates, and  $t$  is the time. The symbols  $Q$ ,  $E$ , and  $F$  are flux vectors and are expressed as follows:

$$Q = \begin{bmatrix} \rho \\ \rho u \\ \rho v \\ e \end{bmatrix}, \quad E = \begin{bmatrix} \rho u \\ \rho u^2 + p \\ \rho uv \\ (e + p)u \end{bmatrix}, \quad F = \begin{bmatrix} \rho v \\ \rho vu \\ \rho v^2 + p \\ (e + p)v \end{bmatrix} \quad (3)$$

where  $u$  and  $v$  are the velocity components in  $x$  and  $y$  directions, respectively,  $\rho$  the density, and  $e$  the total energy per unit volume. The pressure  $p$  is given as  $p = (\gamma - 1)[e - \rho(u^2 + v^2)/2]$ , where  $\gamma$  is the specific heat ratio and is fixed to be 1.4 in this study. A MUSCL total variation diminishing (TVD) finite volume method with flux-vector splitting<sup>3</sup> was used. For time integration, the Runge-Kutta scheme was adopted. Computational accuracy is of the second order both in time and

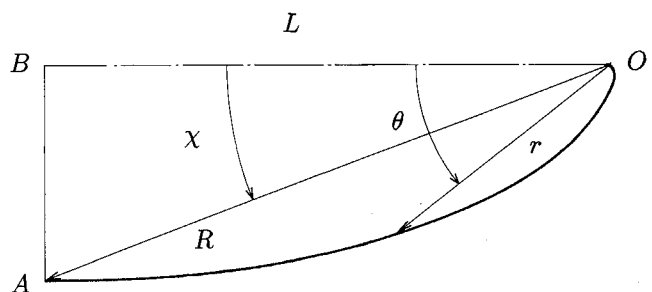


Fig. 1 Log-spiral duct.

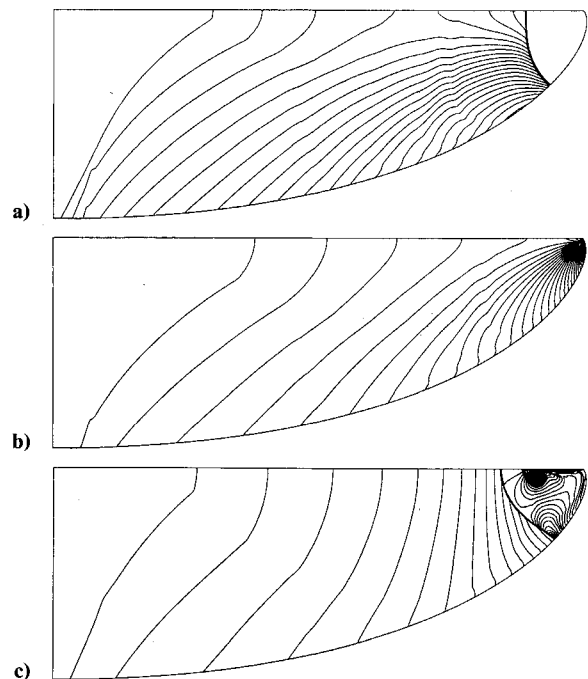


Fig. 2 Focusing process in a log-spiral duct; isopycnics,  $M_0 = 2.3$ ,  $\chi = 22.5^\circ$ : a)  $t = 0.3854$  (before focusing), b)  $t = 0.4303$  (just before focusing), and c)  $t = 0.5165$  (after focusing).

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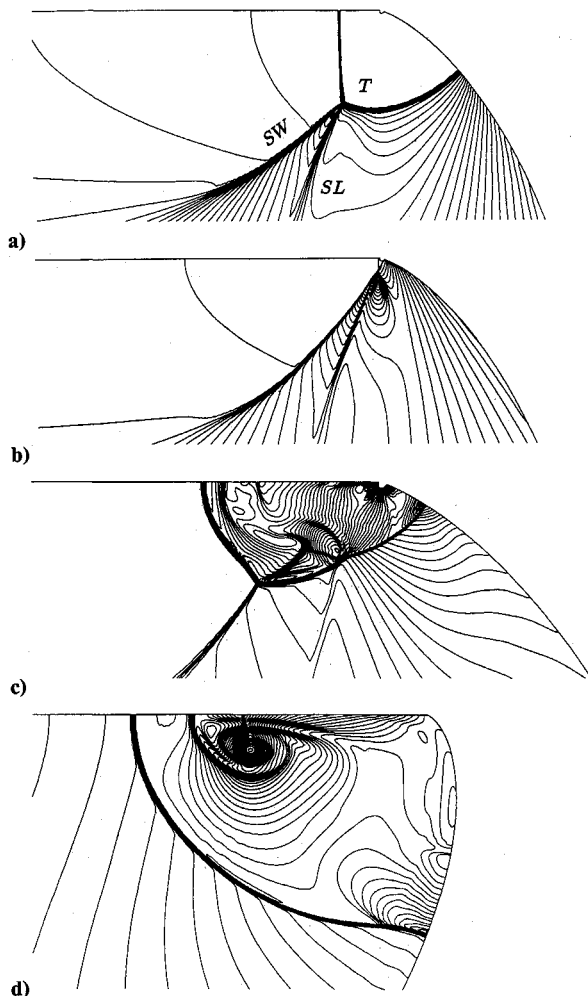


Fig. 3 Detailed mechanism of vortex formation; isopycnics,  $M_0 = 2.3$ ,  $\chi = 22.5^\circ$ : a)  $t = 0.4303$  (before focusing,  $m = 50$ ), b)  $t = 0.4308$  (just before focusing  $m = 100$ ), c)  $t = 0.4313$  (after focusing,  $m = 100$ ), and d)  $t = 0.4654$  (after focusing,  $m = 6.25$ ).

space. The number of grids are 32,000 (1600 along the log-spiral wall by 200 perpendicular to the wall).

### Results and Discussion

The shock wave focusing process was simulated for several different configurations of a log-spiral duct by changing  $\chi$  or equivalently the Mach number  $M_0$  of the incident shock wave. However, qualitative features of the focusing process were not dependent on the configurations. A typical example of focusing is shown in Fig. 2. As the flow is symmetric with respect to the plane BO in Fig. 1, only the lower half is presented in Fig. 2 (and also in Fig. 3). No reflected shock wave seems to form until the implosion is completed (Figs. 2a and 2b). After implosion, on the other hand, a single reflected shock moves outward and a vortex can be seen behind the reflected shock (Fig. 2c). The flow feature shown in Fig. 2 is close to that observed by Milton.<sup>1</sup> The detailed structures in the flowfield near the focus are presented in Fig. 3 for the same case as in Fig. 2. Each view of the flowfield shown in Fig. 3 is magnified, respectively, and the magnification rate is given by the symbol  $m$ . An initially plane shock wave moves at a constant speed along the symmetric plane (BO), whereas it accelerates along the log-spiral wall. As a result, the shock wave bends (Fig. 2a), and with increasing distance the pressure gradient along the shock wave becomes larger. With a sufficiently large pressure gradient, a second shock wave (SW in Fig. 3a) is formed, and a slip line (SL in Fig. 3a) is emanated from the triple point (T). The second shock wave and the slip line survive through the

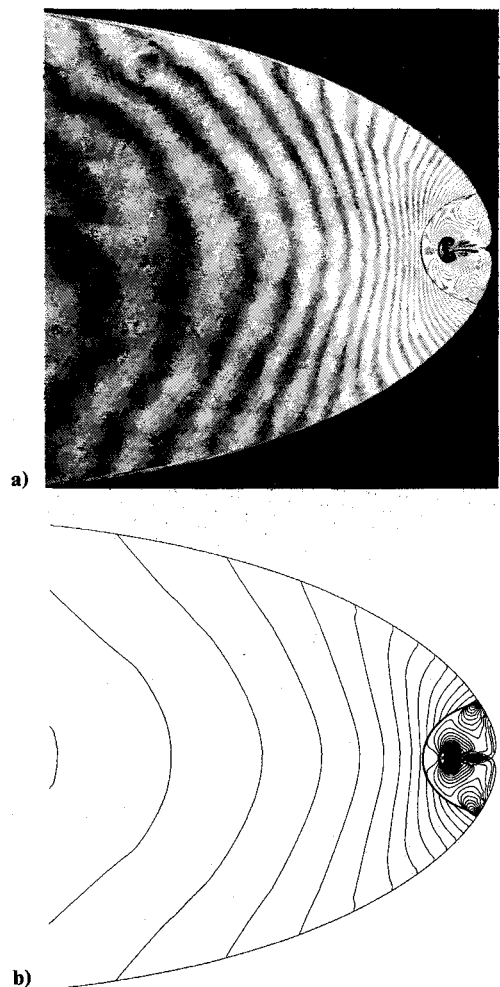


Fig. 4 Comparison with experiment; isopycnics: a) experiment,  $M_0 = 2.3$ ; b) computation,  $M_0 = 2.3$ ,  $t = 0.5165$ .

implosion (Figs. 3b and 3c). After the implosion, a reflected shock wave moves outward and interacts with the second shock wave and the slip line, respectively. These two interactions produce, respectively, new slip lines behind the reflected shock wave (Fig. 3c). Vorticity produced by the two slip lines merges into a single vortex (Fig. 3d). It may be interesting to see that a new shock wave is formed in the vortex. A comparison of a calculated flowfield with experimental observation by holographic interferometric photography, sufficiently after the implosion is completed, is presented in Fig. 4. As readily seen from Fig. 4, the two flowfields are very close to each other. Therefore, we may say that the present results simulate the focusing process very well.

The configuration of a log-spiral duct is determined such that the characteristics of the ray shock theory are centered at a focusing point (O in Fig. 1) and do not intersect with each other at any other point. Therefore, a triple point is not expected to appear before the implosion. The computational results in Fig. 3, as well as the results for other configurations examined in this study, however, show that the triple point does not appear before the implosion. Careful examination of the computational results indicates that the appearance of the triple point may not be related to the intersection of the characteristics. A possible reason for the failure of the ray shock theory to predict the appearance of the triple point may be due to the assumption of quasi-one-dimensionality adopted in characteristic rules. That is, immediately before the implosion, variation of flow quantities along the shock wave such as pressure and density gradients is very large (Fig. 3a), and thus the assumption of quasi-one-dimensionality does not hold. Here, it should be mentioned that the appearance of the triple

point before the implosion is confined within the range of a few percent of the characteristic length  $L$  from a focusing point. Therefore, we may say that the ray shock theory<sup>2</sup> gives a reasonable approximation to the flowfield so far as the flowfield far from the focus is concerned.

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### Introduction

THE flow separation aft of a blunt-based axisymmetric body at zero angle of attack creates an axisymmetric wake that is the most complex part of the flowfield. An axisymmetric wake consists of a near-wake region and a far-wake region. The near-wake region exists just after the body base, creating a recirculation region covered by the free-shear layer.

Delery and Siriex<sup>1</sup> and Delery and Lacau<sup>2</sup> outlined the axisymmetric wake physics based on earlier investigations. Merz et al.<sup>3</sup> investigated the effect of Mach number on the turbulent near-wake of a cylindrical blunt-based body over the entire subsonic Mach number range. Merz et al.<sup>4</sup> also experimentally investigated the near wake of an axisymmetric semi-elliptical afterbody. Atli<sup>5</sup> performed some hot-wire measurements to investigate the wakes of four complex bodies of revolution at zero angle of attack. Despite these and other works, new investigations are necessary for more information on the problem because of its fundamental and practical importance. New data may also be useful for comparison with computational predictions of the flowfield.

In the present work the wakes of three axisymmetric missile-type bodies with different afterbody geometry at zero angle of attack and at a low subsonic velocity were studied by using a constant temperature hot-wire anemometer. Mean velocity and turbulence measurements, both along and across the wake, were performed. Turbulence was considered as the time-averaged rms fluctuating velocities. Vortex shedding and instability in the free-shear layer were not treated.

## Wakes of Three Axisymmetric Bodies at Zero Angle of Attack

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### Nomenclature

$C_D$	= drag coefficient
$d$	= maximum diameter of the model
$l$	= total length of the model
$Re_d$	= Reynolds number based on maximum body diameter, $U_\infty d / \nu$
$r$	= radial distance from the wake centerline
$r_b$	= $r$ position where $\bar{U}_d = \frac{1}{2}(\bar{U}_d)_{\max}$
$U_\infty$	= freestream velocity
$\bar{U}$	= mean velocity in the $x$ direction
$\bar{U}_d$	= velocity defect, $\bar{U}_{\max} - \bar{U}$
$\sqrt{(u')^2}$	= rms velocity fluctuations in the $x$ direction
$u_0$	= $\sqrt{(u')^2} / U_\infty$
$u_1$	= $\sqrt{(u'_{CL})^2} / U_\infty$
$u_2$	= $\sqrt{(u'_{CL})^2} /  \bar{U}_{CL} $
$x$	= distance from the base of the model in the freestream direction
$x_{sp}$	= length of the recirculation region

### Subscript

CL = centerline of the wake

### Models and Experimental Technique

The models shown in Fig. 1 have the same fineness ratio,  $l/d = 16.4$ , with the same tangent-ogive nose and cylindrical main body. The first model has a flared afterbody with a neck, the second one a blunt base, and the third one a truncated conical afterbody (boat-tail). The surfaces of the models were smooth.

All of the tests were conducted at zero angle of attack in an open-circuit, low-speed wind tunnel with a closed test section of  $50 \times 50 \times 200$  cm. The freestream velocity  $U_\infty$  was 10 m/s ( $M_\infty \approx 0.03$ ) with the freestream turbulence intensity  $\sqrt{(u')^2} / U_\infty = 0.3\%$ . The Reynolds number  $Re_d$  based on maximum body diameter was  $2 \times 10^4$ . A hot-wire anemometer system of DISA (55M) was used with a personal computer and an Analog/Digital converter of Data Acquisition 380 Validyne. The models were supported from one side by two thin circular stings 5 mm in diameter. The support of the hot-wire probe was placed perpendicular to the model symmetry axis and to the model supports to minimize the effects of the supports. Blockage area ratio was less than 0.3% so that no correction for blockage effects has been made. The positional accuracy of the probe traverse was  $\pm 0.1$  mm, and the accuracy of the hot-wire anemometer outputs for mean velocity and fluctuations is about  $\pm 0.1$  and  $\pm 0.002$  m/s, respectively.

The outputs of a hot-wire anemometer do not indicate the flow direction; in other words, the mean velocity output of the

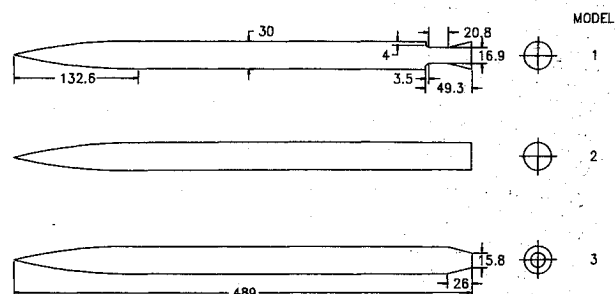


Fig. 1 Model geometries (dimensions in mm).

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