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Robust Design for Unconstrained Optimization Problems Using the Taguchi Method

Kwon-Hee Lee,* In-Sup Eom,* Gyung-Jin Park,† and Wan-Ik Lee‡
Hanyang University, Seoul 133-791, Republic of Korea

Engineering optimization has been developed for the economic design of engineering systems. The conventional optimum is determined without considering noise factors. Thus, applications to practical problems may be limited. Within current design practice, noises tend to be allowed for by specification of closer tolerances, or the use of safety factors. However, these approaches may be economically infeasible. Thus, the inclusion of design-variable noises is required for practical design in optimization. A method is developed to find robust solutions for unconstrained optimization problems. The method is applied to problems with discrete variables. The orthogonal array based on the Taguchi concept is utilized to arrange the discrete variables. Through several examples, it is verified that the solutions from the suggested method are more insensitive to noise than the conventional optimum within the range of variations for design variables.

I. Introduction

THE automatic design and the reduction of manufacturing cost for the design of an engineering system have been accomplished with the development of engineering optimization techniques.[1] The methods find design variables that minimize the cost function and satisfy given constraints. However, a system designed by conventional optimization techniques can have nonrobust results, which are attributed to uncertain noise factors.[2,3] Noise factors are parameters that vary with environment and usage. Therefore, they are not controlled by the designer. On the other hand, control factors can be set and determined by the designer.[3,4] In modern design, noise factors are essential for system performance. Thus, designers should consider the noises within the design process. In current practice, the noise factors are considered after the design stage, i.e., with tolerances. However, that approach is economically infeasible and does not guarantee the enhancement of system performance. Therefore, attention is being given to robust design technology, which includes noise and cost during the design stage.

Robust design is an engineering method for finding a solution. The variation of responses is minimized, and the mean represents a target value.[4] The existing robust design technology, implementing the Taguchi method, uses the analysis of variance (ANOVA) and analysis of mean (ANOM) on the signal-to-noise (SN) ratio, to determine the optimum levels of the design variables. The Taguchi-method control factors are equivalent to the optimization design variables. The concept of the robust design is formulated into a pseudo-objective function. This is acquired from the original objective function and its standard deviation. The standard deviation or variance determines the robust capacity. Weighting factors are utilized to represent the tradeoff between the degree of robustness and minimization. The pseudo-objective function is an index to find the design insensitivity of noises and the original objective function's minimum. Minimizing the pseudo-objective function is equivalent to simultaneously minimizing the objective function and the variance. This differs from the existing Taguchi method in that the procedures of ANOVA and ANOM using the SN ratio are not required in the present robust design, since the pseudo-objective function includes both characteristics.

The suggested robust design is applied to unconstrained optimization problems, which have discrete design variables. The parameter design scheme of the Taguchi method is used to obtain a solution. There are two methods for the robust design within a discrete design space. One is postprocessing of conventional optimization, which is useful for structural problems. The robust design strategy is applied after an optimum is obtained by the conventional method. The second method is iterative robust design, which is expected to be a topic of active research.[4] The robust design strategy is applied iteratively within the range of concern for design variables. In this study, methods for postprocessing are examined.

Various examples have been solved to illustrate the results of the research. Robust design is applied to a mathematical problem. For engineering applications, standard structural problems are selected: the three- and ten-bar truss problems.[1,5] These structural problems are treated using robust design within a discrete space, and the results are presented.

II. Robust Design Using the Taguchi Method

A. Taguchi Method

By the end of the 1940s the Taguchi method had been developed by G. Taguchi for quality improvement. Taguchi suggested that the steps for a product or a process design be composed of three levels: system, parameter, and tolerance designs.[4]

The system design is a step where new ideas are generated to provide products to customers. Within the parameter design step, the designer determines the optimum setting for control factors using orthogonal arrays and SN ratios. The manufacturing cost will not be affected by the parameter design, since the tolerances are fixed. The final goal of the parameter design is that the designer makes products insensitive to noise factors without eliminating them. The tolerance design is implemented to improve quality at a minimum cost. It should be used when the sensitivity of responses resulting from the parameter design is not satisfactory.

In particular, the parameter design scheme of the Taguchi method is adopted for robust design. The details of the Taguchi method can be found in the references.[4,6]

B. Pseudo-Objective Function for Robust Design

Unconstrained optimization finds design variables, while minimizing objective functions without any constraints.[1] Conventional optimization is based on the assumption that design variables are completely under the control of the designer. However, a design variable can deviate from its nominal value in a practical design. Therefore, the response obtained from the unconstrained optimization may be sensitive to the variation of design variables, given that the solution is sensitive. The deviations from the nominal values of design variables can be considered as noises. A pseudo-

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*Graduate Student, Mechanical Design and Production Engineering, College of Engineering, 17 Haengdang-Dong, Seongdong-Ku.

†Associate Professor, Mechanical Design and Production Engineering, College of Engineering, 17 Haengdang-Dong, Seongdong-Ku.

‡Professor, Mechanical Design and Production Engineering, College of Engineering, 17 Haengdang-Dong, Seongdong-Ku.

objective function, including the effects of noises, is defined as follows:

$$\Phi(x) = \alpha f(x) + (1-\alpha)\sigma_f \tag{1}$$

where

x = nominal-value vector of design variables
$f(x)$ = original objective function
σ_f = standard deviation of functions generated from noises
α = weighting factor ($0 \le \alpha \le 1$)

For example, the robust design in a discrete design space comprises the standard deviation and mean for Eq. (1). It is expressed as follows:

$$\sigma_f = \sqrt{\sum_{i=1}^{N}(f_i - \mu_f)^2/N} \tag{2}$$

$$\mu_f = \sum_{i=1}^{N} f_i/N \tag{3}$$

where

N = number of function calculations considering the deviation from the nominal value at a current design point
f_i = function value at the ith deviation

In Eq. (1), the first and second terms of the right-hand side are defined, respectively, for minimization and variance. The weighting factor is used for the tradeoffs between the two terms.

Taguchi recommends the implementation of the SN ratio for reflecting the amount of variance. The variability of the system is minimized by maximizing the SN ratio. For nominal-best-type characteristics that have the target values, the SN ratio is defined as[4]

$$\mathrm{SN} = 10\log_{10}\left(\mu_f^2/\sigma_f^2\right) \tag{4}$$

The μ_f and σ_f of Eq. (4) are the same as those defined in Eqs. (2) and (3). In Eq. (4), if any design variables increase the mean μ_f, instead of decreasing σ_f for maximizing the SN ratio, an incoherent answer can be obtained, because Eq. (4) is a function of mean and variance. Considerable risk of conflicting variability is involved.[7] Thus, it is important to use the standard deviation in Eq. (1).

C. Robustness and Robust Design

For robust design in a discrete design space, the variance is shown in the following example for a pseudo-objective function, which measures robustness. If a is the nominal value of a design variable, and Δx is the deviation from the nominal value, then the design variable can have three discrete values: low tolerance limit $(a - \Delta x)$, nominal value (a), and high tolerance limit $(a + \Delta x)$. It is assumed that the three levels have the same probability. Therefore, the variance and mean of responses is expressed as follows according to Eqs. (2) and (3):

$$\sigma_f^2 = \sum_{i=1}^{3}(f_i - \mu_f)^2/3 \tag{5}$$

$$\mu_f = \frac{f(a-\Delta x) + f(a) + f(a+\Delta x)}{3} \tag{6}$$

By substituting Eq. (6) into Eq. (5),

$$\sigma_f^2 = (\Delta x)^2 \frac{[f'_-(a)]^2 + [f'_+(a)]^2 + [f'_-(a) + f'_+(a)]^2}{3} \tag{7}$$

where

$$f'_-(a) = \frac{f(a) - f(a-\Delta x)}{\Delta x} \tag{8}$$

$$f'_+(a) = \frac{f(a+\Delta x) - f(a)}{\Delta x} \tag{9}$$

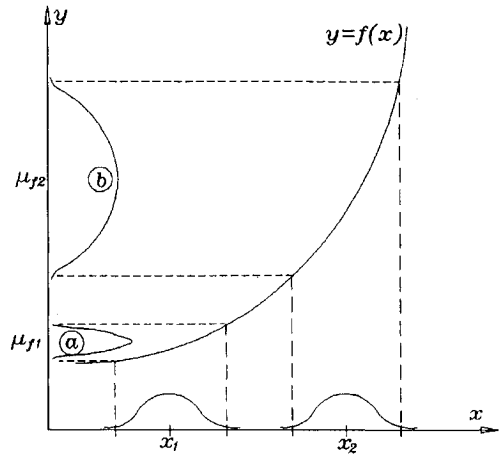

Fig. 1 Design variable vs its response.

The backward difference quotient of Eq. (8), the forward difference quotient of Eq. (9), and the central difference quotient of the function at the current design point are included in Eq. (7). When Δx is small, Eq. (7) approximates the partial derivative at a. For such small Δx, the robustness of the variance is ineffective. The solution from the robust design and the nonrobust optimization can be identical. On the other hand, if Δx is greater, Eq. (7) can express the robustness of distributions. In Eqs. (5–9), it is assumed that the standard deviation for Eq. (1) is an index of robustness.

The graphical representation of a robust design is shown in Fig. 1. The distributions by two levels of a design variable, x_1 and x_2, correspond to curves a and b. Based on this responses, curve b is more sensitive to noises than curve a, since the standard deviation of the level x_2 is larger than that of the level x_1. Furthermore, $f(x_1) < f(x_2)$, and $\Phi(x_1) < \Phi(x_2)$. Therefore, x_1 is the most feasible solution in regard to robust design. The goal of this study is to get a robust solution, such as x_1 in Fig. 1.

III. Robust Design in Discrete Design Space

Robust design strategies are developed in a discrete design space. The evaluation of the conventional optimum is in a continuous design space. However, a real value exists discretely in a practical design, such as structural design problems. Thus, a discrete design scheme is strongly required. Usually, one-step-larger values than the optimum evaluated in a continuous design space are taken for the final design, although this process is inefficient.[8,9] Therefore, the robust design technology can enhance the performance of the final design.

First, an optimum is evaluated by conventional optimization, which has a continuous design space. Near the optimum, the number of levels for each design variable (control factor in the Taguchi method) are determined by the designer's experience. However, it is usually set at three to five, because too many levels may lead to numerous function calculations. After the levels are determined, a series of design variable combinations, called an inner array, are implemented to obtain system responses. Considering all cases, the best condition can be selected from full factorial experiments. However, that is inefficient and time-consuming. The number of experiments increase with E^k, where E is the number of levels of design variables, and k the number of design variables. Furthermore, the outer array is needed to analyze the effects of noise factors. These two arrays are depicted in Fig. 2. Each row of the inner array generates a value from the pseudo-objective function, which is calculated from the outer array. To select an optimum level, $N_{\mathrm{in}} \times N_{\mathrm{out}}$ function calculations (where N_{in} is the number of experiments in the inner array, and N_{out} the number of experiments in the outer array) need to be carried out. This could be infeasible. However, orthogonal arrays may limit the many combinations of factor levels to tractable numbers. Therefore, the use of orthogonal arrays is strongly recommended.

The best results within the full combinations are not always obtained by orthogonal arrays. This is because the orthogonal arrays cannot replace the full combinations perfectly and neglect

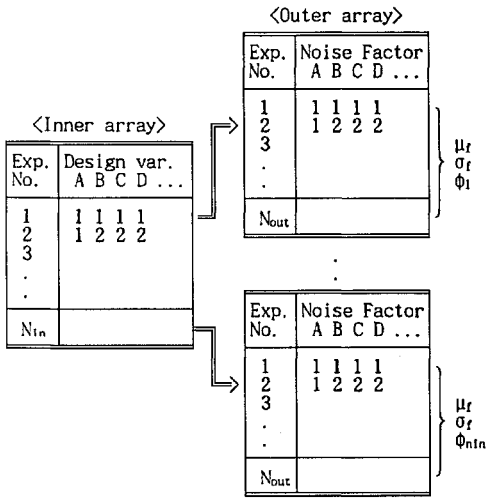

Fig. 2 Arrangement of inner and outer arrays.

high-order interactions. An adequate solution may be derived using fewer function calculations. However, these are extremely expensive in engineering applications.[8] The smallest size of the inner array, for an orthogonal array of design variables, can be obtained by reflecting E and k. For example, if $E = 3$, the orthogonal arrays corresponding to the levels $L_9(3^4)$, $L_{18}(2^1 \times 3^7)$, and $L_{27}(3^{13})$, etc., may be used. The smallest of the orthogonal array is determined by assigning all factors to their columns. If k is less than the number of columns in the array, then the rest of them after assigning columns should be omitted. In this study, the sequential order of assigning factors to columns is from the first to the kth column.

The next step is to determine the amount of the variations in design variables. The number of levels for noise factors is set at three. The size of the outer arrays, which are the orthogonal arrays for noise factors, is selected by the same procedures as the inner arrays. After all the rows are established as shown in Fig. 2, the optimum level of each design variable is determined through the analysis of the pseudo-objective function. From each inner array, a pseudo-objective function is evaluated from μ_f and σ_f. The pseudo-objective functions are utilized in the handling process of the inner array. The pseudo-objective functions from the experiments are added to each level of the design variables. The level with the smallest value is selected for the final design. The process is identical to the Taguchi method, except the pseudo-objective function is used instead of the SN ratio. The suggested method does not need a procedure to distinguish design variables affecting the mean from those affecting the variance, since the pseudo-objective function includes the capability of minimization and robustness.

IV. Example Problems for Robust Design

A. Mathematical Problem

Fenton and Eason's function[10] has an optimum solution $x_1 = 1.743$ and $x_2 = 2.030$, which is obtained from the conventional optimization:

$$f(x_1, x_2) = 12 + x_1^2 + \frac{1 + x_2^2}{x_1^2} + \frac{x_1^2 x_2^2 + 100}{(x_1 x_2)^4} \tag{10}$$

A contour plot of this function is depicted in Fig. 3. In the vicinity of the conventional optimum the graph is very steep. Thus, a different solution is to be selected for a robust design.

First, levels of the two design variables, x_1 and x_2, are chosen at 1, 2, 3, 4, and 5. The sizes of the variation are fixed at $\Delta x = 0.5$ and 1.0. The robust design process is performed with varying weighting factors. The smallest orthogonal arrays, $L_{25}(5^6)$ for the inner array and $L_9(3^4)$ for the outer arrays, are used according to the number of levels. For $\Delta x = 0.5$ and 1.0, the levels of design variables are respectively displayed in the second and third columns of Tables 1 and 2. The values of the pseudo-objective function are in the fourth to sixth columns. The last column shows SN ratios for the nominal-best-type characteristics.

Table 1 $\Phi(x)$ with varying weighting factor α and SN ratio ($\Delta x = 0.5$)

			$\Phi(x)$			
Exp. no.	x_1	x_2	$\alpha = 1.0$	0.50	0.0	SN ratio
1	1	1	116.0	4017	7917	−8.09
2	1	2	24.50	62.65	100.8	−2.78
3	1	3	24.34	25.30	26.27	3.21
4	1	4	30.45	30.22	29.99	3.37
5	1	5	39.2	40.83	42.46	2.73
6	2	1	23.0	60.02	97.05	−3.12
7	2	2	17.7	9.240	0.78	27.05
8	2	3	18.60	9.720	0.84	26.65
9	2	4	20.29	10.74	1.19	24.41
10	2	5	22.52	12.28	2.04	20.75
11	3	1	22.57	17.57	12.57	7.07
12	3	2	21.66	11.93	2.21	19.43
13	3	3	22.14	12.13	2.13	19.90
14	3	4	22.9	12.42	1.94	21.01
15	3	5	23.89	12.79	1.7	22.56
16	4	1	28.58	16.35	4.12	16.99
17	4	2	28.35	15.76	3.18	18.55
18	4	3	28.63	15.88	3.13	18.76
19	4	4	29.06	16.05	3.05	19.13
20	4	5	29.62	16.27	2.93	19.63
21	5	1	37.28	20.63	3.98	19.16
22	5	2	37.22	20.63	4.04	18.81
23	5	3	37.41	20.71	4.01	18.91
24	5	4	37.68	20.83	3.97	19.07
25	5	5	38.04	20.98	3.92	19.28

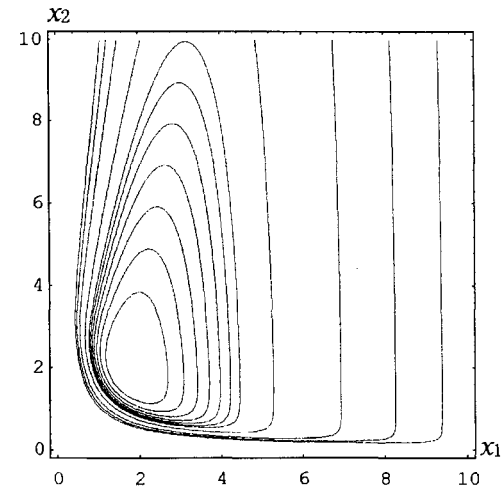

Fig. 3 Contour plot of Eq. (10).

Table 3 represents an outer array, which corresponds to the first row in Table 1. For example, the value of the pseudo-objective function in the first row of Table 1 for $a = 0.50$ is calculated as follows: $\Phi(1, 1) = 0.50(116) + (1 - 0.50)7917.387 = 4017$, where $f(x)$ and σ_f^2 are obtained from Table 3. In Table 1, it is noted that the optimum combination is $x_1 = 2$ and $x_2 = 2$, which is the closest discrete value to the conventional optimum in the case of $\Delta x = 0.5$. On the other hand, when α is equal to zero, the optimum setting is calculated as $x_1 = 3$ and $x_2 = 5$ in Table 2. Therefore, the conventional optimum is useless in the solution for $\Delta x = 1.0$. As shown in Table 2, the SN ratio is considered for comparison with conventional methods. If SN ratios are used, the optimum setting is the same as where only the variance is included. However, the tradeoffs between minimization and robustness cannot be considered. Therefore, the method with a pseudo-objective function is more general.

B. Structural Problems

For the engineering application, the robust design of a three-bar and a ten-bar truss is performed so that the displacement of a specified node is insensitive to noises. The optimum sectional areas are evaluated by conventional optimization, considering constraints.[11] The robust design is applied for the postprocessing. However, it

Table 2 $\Phi(x)$ with varying weighting factor α and SN ratio ($\Delta x = 1.0$)

Exp. no.	x_1	x_2	$\Phi(x)$ $\alpha = 1.0$	0.50	0.0	SN ratio
1	1	1	116.0	3.9E+51	7.9E+50	−9.54
2	1	2	24.50	3.9E+27	7.9E+26	−8.85
3	1	3	24.34	1.5E+02	3.1E+24	−7.42
4	1	4	30.45	6.2E+23	1.2E+23	−6.31
5	1	5	39.2	6.2E+22	1.2E+22	−5.56
6	2	1	23.0	3.9E+27	7.9E+26	−8.85
7	2	2	17.7	441.33	864.97	0.21
8	2	3	18.60	15.62	12.65	15.48
9	2	4	20.29	29.64	38.99	11.98
10	2	5	22.52	53.33	84.14	9.41
11	3	1	22.57	1.5E+25	3.1E+24	−7.42
12	3	2	21.66	18.57	15.49	14.99
13	3	3	22.14	19.60	17.06	14.49
14	3	4	22.9	18.56	14.22	15.62
15	3	5	23.89	17.40	10.92	17.20
16	4	1	28.58	6.3E+23	1.2E+23	−6.31
17	4	2	28.35	33.54	38.73	12.94
18	4	3	28.63	33.72	38.81	12.96
19	4	4	29.06	32.85	36.64	13.35
20	4	5	29.62	31.71	33.81	13.88
21	5	1	37.28	6.2E+22	1.2E+22	−5.56
22	5	2	37.22	50.97	64.72	12.97
23	5	3	37.41	50.83	64.26	13.02
24	5	4	37.68	50.24	62.81	13.19
25	5	5	38.04	49.42	60.81	13.41

Table 3 Outer array for inner array of experiment 1

Exp. no.	x_1	x_2	$f(x)$
1	0.5	0.5	25633.25
2	0.5	1.0	1624.25
3	0.5	1.5	343.08
4	1.0	0.5	1618.25
5	1.0	1.0	116.00
6	1.0	1.5	36.45
7	1.5	0.5	332.63
8	1.5	1.0	35.34
9	1.5	1.5	19.79

is not certain that the robust solution satisfies the constraints, because they are not considered in the robust design process. In the pseudo-objective function, the weighting factor is considered to be zero. The quantity σ_f^2 is the variance of the absolute displacement, $(u_x^2 + u_y^2)^{1/2}$, where u_x is the x-direction displacement and u_y is the y-direction displacement within a specified node. Therefore, the pseudo-objective function is reduced as follows:

$$\Phi(x) = \sigma_f \tag{11}$$

The postprocessing is executed from the optimum point, which gives a feasible objective-function value. Thus, the objective function is deleted from Eq. (11).

1. Three-Bar Truss

The optimization formulation of the three-bar truss, as shown in Fig. 4, is derived from various references.[1] The solution is $A_1 = 16.67$, $A_2 = 1.0$, and $A_3 = 2.533$. The objective is to determine the area of each member so that the displacement of node A in Fig. 4, will be least sensitive to noises.

The levels of design variables are given in Table 4, where each discrete value is obtained from the table of American Standard Steel Beams and Channels.[12] In Table 5, there are three noise factors in the system. The sizes of noises are arbitrarily fixed for this experiment. The $L_{16}(4^5)$ orthogonal array for the inner array and the $L_9(3^4)$ orthogonal array for the outer array can be implemented, since the numbers of levels for design variables and noise factors are respectively set to four and three.

Table 4 Levels of design variables (three-bar-truss)

Level	A_1	A_2	A_3
1	14.7	1.21	2.40
2	17.1	1.67	2.64
3	20.6	1.97	2.79
4	25.9	2.79	2.94

Table 5 Levels of noise factors (three-bar truss)

Level	A_1	A_2	A_3
1	$A_1 - 1.0$	$A_2 - 0.1$	$A_3 - 0.01$
2	A_1	A_2	A_3
3	$A_1 + 1.0$	$A_2 + 0.1$	$A_3 + 0.01$

Table 6 Analysis for pseudo-objective function (three-bar truss)

Area	Sum of $\Phi(x) \times 10^{-6}$ Level 1	Level 2	Level 3	Level 4
A_1	0.506	0.285	0.140	0.058
A_2	0.251	0.249	0.246	0.245
A_3	0.254	0.249	0.245	0.241

Table 7 Comparison of sensitivities between optimum design and robust design ($\times 10^{-4}$, three-bar truss)

Design type	A_1 S_{x1}[a]	S_{y1}[b]	A_2 S_{x2}	S_{y2}	A_3 S_{x3}	S_{y3}
Optimal	2.90	1.86	5.75	7.81	2.90	2.68
Robust	1.35	0.57	1.91	2.39	2.35	2.04

[a] $S_{xi} = |\partial u_x/\partial A_i|$. [b] $S_{yi} = |\partial u_y/\partial A_i|$.

Table 8 Levels of design variables (ten-bar truss)

Level	A_1	A_3	A_4	A_7	A_8	A_9
1	27.7	22.6	14.7	6.16	20.8	20.8
2	35.1	24.1	15.6	7.69	22.6	22.6
3	38.9	25.6	16.8	8.25	24.1	24.1

Table 9 Levels of noise factors (ten-bar truss)

Level	A_1	A_3	A_4	A_7	A_8	A_9
1	$A_1 - 0.5$	$A_3 - 0.5$	$A_4 - 0.5$	$A_7 - 0.5$	$A_8 - 0.5$	$A_9 - 0.5$
2	A_1	A_3	A_4	A_7	A_8	A_9
3	$A_1 + 0.5$	$A_3 + 0.5$	$A_4 + 0.5$	$A_7 + 0.5$	$A_8 + 0.5$	$A_9 + 0.5$

Table 10 Analysis for pseudo-objective function (ten-bar truss)

	Sum of $\Phi(x) \times 10^{-2}$ Level 1	Level 2	Level 3
A_1	0.170	0.156	0.145
A_3	0.161	0.161	0.149
A_4	0.164	0.161	0.146
A_7	0.157	0.154	0.160
A_8	0.168	0.153	0.151
A_9	0.174	0.153	0.145

The results, shown in Table 6, are calculated as the sum of the values of the pseudo-objective function that corresponds to each level of every design variable in the inner array $L_{16}(4^5)$. From the table, the optimum combination is derived at $A_1 = 25.9$ (level 4), $A_2 = 2.79$ (level 4), and $A_3 = 2.94$ (level 4). In Table 7, the sensitivity of the robust solution is compared with that of the conventional optimum. It is important to compare the sensitivities between the two solutions, because this approach is used as a postprocessing of constrained problems, although the robust design is used for

Table 11 Comparison of sensitivities between optimum design and robust design ($\times 10^{-5}$, ten-bar truss)

Design type	A_1		A_3		A_4		A_7		A_8		A_9	
	S_{x1}[a]	S_{y1}[b]	S_{x3}	S_{y3}	S_{x4}	S_{y4}	S_{x7}	S_{y7}	S_{x8}	S_{y8}	S_{x9}	S_{y9}
Optimal	2.818	1599	1308	1342	1543	1531	61.91	405.1	8.100	2272	1.327	2163
Robust	1.534	955	1080	1104	1278	1269	53.21	322.3	5.732	1768	0.969	1744

[a] $S_{xi} = |\partial u_x/\partial A_i|$. [b] $S_{yi} = |\partial u_y/\partial A_i|$.

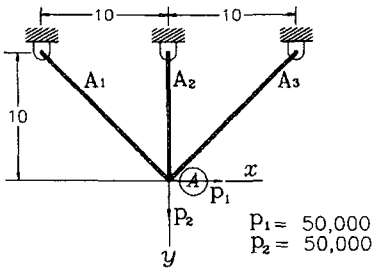

Fig. 4 Three-bar truss.

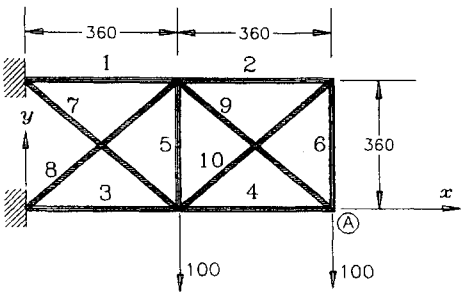

Fig. 5 Ten-bar truss.

unconstrained problems. In Table 7, the sensitivity of robust design is seen to decrease by 14–69% from that of the optimum solution.

2. *Ten-Bar Truss*

In Fig. 5, the optimum solution within the structure is $A_1 = 30.03$, $A_2 = 0.1$, $A_3 = 23.27$, $A_4 = 15.28$, $A_5 = 0.1$, $A_6 = 0.55$, $A_7 = 7.46$, $A_8 = 21.20$, $A_9 = 21.62$, and $A_{10} = 0.1$. The optimization formulation is explained in Ref. 5. A_2, A_5, A_6, and A_{10} are constants when the members have lower bounds or similar values. The robust design is applied in order for the displacement of node A to be insensitive to noises. The levels of design variables and noise factors are respectively listed in Tables 8 and 9. Two $L_{18}(2^1 \times 3^7)$ orthogonal arrays are utilized for the inner and the outer array. The first and last column of arrays can be deleted, since six design variables with three levels are chosen. The optimum combination is $A_1 = 38.9$ (level 3), $A_3 = 25.6$ (level 3), $A_4 = 16.8$ (level 3), $A_7 = 7.69$ (level 2), $A_8 = 24.1$ (level 3), and $A_9 = 24.1$ (level 3). The corresponding sums are presented in Table 10. The comparison of sensitivity between the robust solution and the optimum solution is given in Table 11. The sensitivities are decreased by 14–45%. Therefore, the discrete and robust design are performed simultaneously for the three- and ten-bar trusses.

V. Conclusions

The following statements are concluded from this study.

1) The discrete design is first developed by postprocessing a constrained optimization. Through a robust design procedure, a system is designed to be less sensitive to noises.

2) The robust design is performed considering the nominal values of the design variables and their tolerances. For structural problems, high, medium, and low tolerance limits of members can be considered in the design stage. This approach leads to an economic design.

3) For the mathematical problem, the robust design within a discrete design space yields fairly good results. In the structural problems of the three-bar and ten-bar trusses, which are special examples of robust design, the sensitivity may be reduced. However, the final configuration of the robust design may violate constraints for the constrained problems. That is, the design may be more robust, but unacceptable.

4) In the future, more efforts are needed in the application of robust design to constrained optimizations.

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