Strapdown Navigation Technology: A Literature Survey

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I. Introduction

SELF-CONTAINED inertial navigation depends on the integration of acceleration with respect to a Newtonian reference frame. Conventional systems physically maintain such a reference frame by means of gimbals. This is termed a stabilized platform, and the system gyros and accelerometers are mounted on it. Many high-performance systems of this type have evolved during the past two decades. The "strapdown" version of this idea, wherein sensors are mounted directly on the vehicle and the transformation from the sensor to inertial reference frame is computed rather than mechanized, has been recognized as long as that of the stabilized platform. Potential advantages of the strapdown concept, compared to stabilized platforms, are often discussed and include lower cost, reduced weight and power consumption, increased reliability, and ease of maintenance, manufacture, and redundant design.

Strapdown system development commenced in the late 1950's. The first flight-operational hardware, built by Honeywell for the PRIME re-entry vehicle, was flown in December 1966. A major milestone was reached in 1969 when the Apollo abort guidance system achieved man-rated operational status. These early strapdown systems, however, operated in a relatively benign dynamic environment. Dealing with rapid maneuvers and vibrations would have required sensors with much greater dynamic range and so much on-board computation as to be prohibitive with the computers available then. In the last decade, major advances in computer technology have prompted extensive development of sensors uniquely suited to strapdown requirements. Thus a trend toward widespread use of strapdown inertial navigation has been discernible for some time. Whether strapdown navigation has "arrived," however, is still a point of lively debate, cf. the 1976 papers by Shaw[1*] and Peterson[2]. The present paper does not enter into this debate.

The benefits of the strapdown approach accrue from more than just the deletion of gimbals. The integration of inertial with other types of navigation information is facilitated, and modular design is encouraged. Since a more powerful computer is used, sophisticated data processing, estimation, and failure processing methods can be employed. Redundancy for increased reliability can be implemented conveniently at the sensor and component level, rather than at the system level, and skewed configurations can be used to minimize the number of sensors. Above all, a strapdown system has the potential to show a considerable reduction in ownership and life-cycle cost over its gimbaled counterpart. These and other advantages have been touted widely for many years; however, the development of truly competitive strapdown systems has been only quite recent.

As strapdown navigation technology has matured over the years, a significant body of literature covering all aspects of the field has become available. Many review papers discuss the development of strapdown navigation in its various stages. The evolution of strapdown navigation from the conceptual stage to the development of early flight hardware is discussed in Refs. 3-11; the maturing of the strapdown approach as hardware limitations were surmounted is discussed in Refs. 12-16; and the dynamic and competitive present-day situation is described in Refs. 1, 2, 17, and 18. To the authors' knowledge, no general literature survey of the field has appeared in the past, however, and the present paper attempts to fill this role.

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*References are grouped according to sections.

Approximately 300 technical reports and papers in the open literature were reviewed in the course of this study, which originally was sponsored by the Communications Research Centre of the Canadian Department of Communications. The aim of the paper is to provide an organized source of references and an overview of the entire field of strapdown navigation technology. It is felt that the bibliography accompanying this paper is comprehensive; however, the authors do not claim that it is complete and regret any major omissions that may have occurred.

II. Overview and Basic Concepts

This section gives an overview of the paper and details further the characteristics of a strapdown system. A schematic of the fundamental elements of a strapdown navigator is shown in Fig. 1. The specific force f^n is supplemented by the computed gravity[2] and possibly "inertial" force field to give the inertial acceleration a^n. This is integrated twice to obtain position and velocity, which are also inputs to navigation coordinate computation (e.g., latitude and longitude) and to the gravity computer. Typically, other sensors, such as barometric altimeters, are used to improve error propagation. Other sensors may include radio and optical aids; the requisite "data mixing" functions also then would be contained in the navigation computer. The functions described so far are also found in gimbaled systems, as discussed in standard texts.[1-3]

The strapdown system differs from the gimbaled in that f^b is measured (in the body frame), and the attitude transformation to f^n is computed from gyro data, because the strapdown sensors are fixed to the vehicle frame. The form of the attitude computation problem depends on the gyros used. Several types of gyros and accelerometers exist for strapdown application. These and other hardware components are discussed in Sec. III. Strapdown spinning-mass gyros are "caged" electronically to the maneuvering vehicle by servo commands, termed rebalance torques. The laser gyro does not require caging. Strapdown gyros are usually rate-integrating gyros and generate incremental angle pulses, a measure of angular velocity ω in the limit as pulse size approaches zero. The attitude parameters ξ then are found from solving the kinematic equations $\dot{\xi} = Z(\xi, \omega)$, where ω must be corrected for finite pulse size. Various attitude representations and numerical integration methods are reviewed in Sec. IV. The integration of ξ must be done at a high rate and is the chief computational load when rate gyros are used. The electrostatic gyro obviates this problem entirely, since it monitors the parameters ξ directly.

Strapdown sensor outputs contain a number of errors such as gyro bias and torquer scale factor nonlinearities. The errors remaining after systematic compensation contribute to system error. This contribution is different from gimbaled systems in that 1) the dynamic environment is more severe, so that sensors of given relative accuracy yield larger system errors, and 2) the contribution depends strongly on the vehicle mission profile. Error generation and propagation receive attention in Sec. V. Often, some errors are treated as additional state variables to be estimated in flight,[4] and even deterministic calibration models can be rather elaborate.[5] Kalman filters are used ubiquitously in estimation of states,

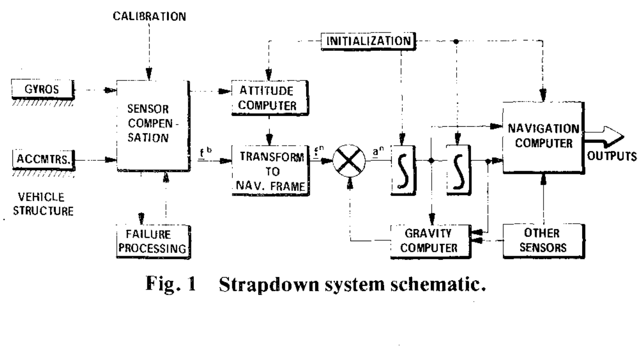

Fig. 1 Strapdown system schematic.

especially when data from various sources are combined. For instance, sun sensors and celestial[6] or terrestrial landmark[7] sightings may be used to aid the attitude and/or navigation functions. The filtering task usually is done at lower iteration rates and is an example of system-level data processing tasks. Other examples are initial condition determination or "alignment" of the strapdown navigator, and periodical orthogonalization of the attitude transformation matrix. System data processing is considered in Sec. VI.

The estimates of error states generally can be used to detect and isolate failures in redundant configurations. System reliability can be enhanced greatly by such methods. Real-time failure correction and system reconfiguration are possible if redundancy is implemented at the sensor level, as is done most conveniently in a strapdown system. The selection of optimal redundant configurations and failure processing methods are important aspects of reliability enhancement, which is reviewed in Sec. VII. Improvements in testing methods and ease of maintenance, leading to overall cost savings, result when a modular approach is adopted in implementing redundant configurations. This is an example of current trends being recognized in strapdown system evolution. A subjective discussion and evaluation of these are given in Sec. VIII.

Certain aspects of this field could not receive attention in this review. Perhaps the most important ones are test methods, procedures, and performance results. These are, to a large extent, applicable to particular systems. Papers dealing with the foregoing topics are grouped together in the general bibliography at the end of the reference section. Computational and hardware aspects of gimbaled systems which are common to strapdown systems are not reviewed in detail.

III. Strapdown Components

In this section, a representative body of literature on the components fundamental to a strapdown system is reviewed. Attention is focused primarily on the major gyros currently available, namely, floated rate-integrating, dry tuned, laser, and free-rotor gyros. Accelerometers are not emphasized, since for the most part their design is similar to those used in gimbaled systems,[47] and the literature dealing explicitly with strapdown versions is sparse. The evolving technology in the strapdown computer field is discussed briefly.

Sensors

Early strapdown systems employed single-degree-of-freedom (SDF), floated rate-integrating gyros. This has been referred to as the conventional approach. In this implementation, the gyro spinning mass is "caged" to the vehicle; i.e., the gyro is made to follow the vehicle motion by applying a rebalance torque about the gyro output axis, which nullifies the precession torque due to rotations about the input axis. The torque rebalance loops are generally pulsed in nature, with binary, ternary, forced binary, and time-modulated schemes being used. In some applications, a dual-mode torquing scheme is used to permit operation in both a high and low rate mode with automatic mode switching. The development of SDF floated gyros for strapdown use is discussed in Refs. 1-3, including test results.[4-8] Some papers are concerned predominantly with various types of rebalance loops[9,10] and their associated errors.[11,12] These are discussed further in Sec. V of this paper. Although still a prime candidate for space applications, this type of gyro is being eclipsed by "unconventional" instruments for general strapdown use.

Two-degree-of-freedom (TDF) gyros also may be used in a torque rebalance mode, such as the gas-bearing free-rotor gyro in Ref. 13. However, the tuned rotor gyro is the most significant mature TDF strapdown gyro.[14] This device employs a rotor that is essentially a gimbaled universal joint[15] spun about its axis of symmetry. Rotations about axes normal

to the spin axis are resisted by springs within the joint. The spring constants are made to cancel a negative spring constant induced by gyroscopic effects, so that ideally there is zero effective coupling between the rotor and the gyro case about the two axes normal to the spin axis. A two-axis torquer with damping provided electronically is used to "cage" the gyro. The principles of operation of the tuned rotor gyro are clarified in Refs. 15 and 16, and its applications to strapdown systems are discussed in Refs. 14 and 17.

The most successful free-rotor gyro is the electrostatically supported gyro (ESG), which has been developed for both gimbaled and strapdown applications. The ESG employs a lightweight spherical beryllium rotor suspended in an evacuated envelope by an electrostatic field. The design intent is to eliminate all torques on the rotor, which then provides a long-term attitude reference. Displacements of the housing relative to the rotor give attitude and can be sensed either optically or by introducing a known minute mass unbalance into the rotor and observing the resultant modulation of the suspension current input. The development of the ESG is discussed in Refs. 18 and 19 and its strapdown applications in Refs. 20-24.

Besides the tuned rotor gyro and the ESG, the laser gyro[26] is another strapdown gyro competing for the low-cost inertial navigation market. This device (not a gyro in the classical sense) has been developed chiefly for strapdown applications, and, although not yet recognized universally as being "mature," it has reached operational status. The ring laser gyro consists of a resonant cavity around which two oppositely directed laser beams travel in a closed (usually triangular) path. Rotation of the instrument about an axis normal to this path results in an observed frequency difference between the two beams which is proportional to the rate of rotation. This physical phenomenon is predicted by the theory of relativity.[25] Several alternative laser gyro designs are available today, differing mainly in the means used to overcome the lock-in phenomenon (deadband for low rate inputs). The principles of operation of this gyro and design considerations are discussed in Refs. 26-29. New concepts in optical rate sensing still are being proposed.[30] Current capabilities of laser gyros are analyzed in Refs. 31 and 32, and typical applications are considered in Refs. 33-37.

Computers

For many years, progress in strapdown development was impeded by the lack of suitable computing equipment. With the dramatic advances made in computer technology during the past decade and still continuing, computers are no longer an obstacle. A survey of the computational hardware applicable to strapdown systems would be a major task in itself; the related literature is discussed but briefly here.

Early strapdown systems made heavy use of digital differential analyzers (DDA's). Discussions on the use of DDA vs general-purpose computers occur in Refs. 38 and 39. Whole-number computers gradually replaced DDA's, although in some applications hybrid systems were used, in which DDA's handled the high-frequency attitude updates while a general-purpose machine did the rest. Eventually, dedicated strapdown computers were developed,[40,41] and in some applications the strapdown navigation functions were done in a general-purpose computer.[42] Many current strapdown systems are microprocessor-based,[43-45] and the current rapid developments in this technology are yet to be imparted fully to strapdown use. Currently available computers also are outlined in papers describing complete systems. A review of spacecraft computers to about 1974 is given by Aukstikalnis.[46]

IV. Analytical Aspects

We now turn to the analytical and numerical bases of strapdown navigation. Rigid-body kinematics, numerical integration of ordinary differential equations (ODE's), and formulation of navigation frames and equations are the topics considered.

Kinematics

Rotations are characterized by the three-dimensional manifold R of orthogonal 3×3 matrices with determinant $+1$. It is known[1,2] that the minimal unique representation of it must employ five parameters. However, there are useful three- and four-parameter representations. Euler angles[3] and the Euler-Rodriguez parameters,[4,5] as well as the "rotation vector" concept of Laning,[5,6] employ three parameters. The last derives from Euler's theorem that any rigid-body orientation can be obtained by a single rotation ϕ about an axis $\boldsymbol{e}$; the rotation vector $\boldsymbol{\phi}$ is simply $\phi\boldsymbol{e}$. Three-vectors defining rotations are considered by Davenport,[7] and two other three-parametrizations in terms of skew-symmetric matrices are described by Stuelpnagel.[1] All of the foregoing are not unique and lead to nonlinear kinematic differential equations with singularities.

Four-parameter formulations include the complex Cayley-Klein parameters[3,8] and Euler symmetric parameters (ESP),[8] which are related intimately to Hamilton's quaternions. The last two, when expressed as column matrices, obey linear differential equations with bounded solutions. These features make the quaternion/ESP favored in comparative studies.[9-11] Its use in strapdown computations has been delineated by several authors,[12-14] including a classification of computational errors. Direct use of quaternion algebra also has been considered.[15,16] On balance, it seems that the ESP formulation is best suited to strapdown computations. Other formulations include the five-parameter scheme of Hopf,[1] the six-parameter scheme, in which two columns of the direction cosine matrix (DCM) are taken, and of course the DCM itself. The last two obey linear equations. Analytical solutions for rigid-body attitude are scarce; examples are single-axis rotation, an approximate scheme based on it due to Kane,[17] and the torque-free Poinsot motion solution due to Morton et al.[8] Finally, we note that the kinematics problem is relevant only to rate gyro systems.

Integration

The numerical integration of ODE's is both central to the strapdown concept and an extensive area of active research. A good current reference on it is the book by Lapidus and Seinfeld.[18] The rudimentary Euler method (rectangular rule) was used almost universally in the early DDA computers. Present computer capacities allow a choice of algorithms. Basically, we have two types: single-step and multistep methods. Single-step methods of the explicit type, typified by Runge-Kutta methods,[18-21] are noniterative, self-starting, allow easy modification of step size, but require n (order of method) evaluations of the ODE per step. Their stability (see Ref. 22 for an introduction to this and related concepts) and roundoff properties are somewhat better than multistep methods. Despite popular practice, local error estimates can be obtained for such methods with some extra computation. The advantages of doing so are demonstrated clearly in Ref. 23. Implicit single-step methods[18] offer higher accuracy for the same order but require iteration; some of these require derivative evaluations at irrational arguments.

Linear multistep methods are also commonly used. Typical of these are predictor-corrector methods (without corrector iteration). These require only two derivative evaluations per step regardless of order, but are not self-starting, require increased storage, and step-size changes are more difficult, although an error estimate per step is obtained easily. Iterating the corrector more than once is not usually beneficial.[24,25] The relatively complete theory of error propagation, including roundoff errors and stability in multistep methods, is given by Henrici.[25]

The interactions of quantization and noncommutativity effects in current gyros with integration error propagation is a

problem not often addressed. Adaptive, variable-step-size integration based on computed local error estimates should prove to be of merit in certain applications. Other relevant areas of numerical analysis include numerical differentiation to obtain ω, approximation of trigonometric functions (in ESG readout schemes), and Riccati equation solution for filtering purposes.

Navigation

The various frames used in navigation equations are described in books on the subject.[26,27] Some of the more exotic ones are described by Fedchin[28] and Zakharin.[29] The error analysis methodology is common to most reference frames and is set up by Britting,[27] including many simulation results. He makes it clear that the strapdown mechanization is a special case of inertial navigation systems which is treated in a straightforward manner. For terrestrial systems, the geographic or local vertical frames are commonly used. The long-term behavior of overall system error, e.g., whether it is oscillatory or unbounded, depends on the choice of the navigation frame.[30,31] Since the navigation function is essentially identical to gimbaled systems, it will not be pursued further.

V. Error Generation and Propagation

The overall system errors are related to 1) individual sensor or instrument errors, 2) computational errors, and 3) the interaction of single-channel errors to produce system-level errors. Strapdown systems operate in a harsher dynamic environment and require greater care and understanding to yield high-quality performance.

Instrument Errors

Dynamic errors of a single-degree-of-freedom floated gyro have received much attention.[1-20] A detailed account appears in some reports from a NASA contract.[3,14-16] Basically, errors arise because a gyro is more than a "pure" angular momentum vector; it is a three-dimensional, elastic, geometrically misaligned, massive rotor, enclosed in fluid and torqued to null by complex, pulsed electronics. Errors due to output axis angular acceleration, cross-coupling torques when the output angle is nonzero, misalignment, bias drift, and torquer scale factor asymmetry and nonlinearity are discussed by several authors,[4,6,8-13] including computer simulation results[6] and generation of error models for compensation.[8,9] These error sources are an important subset of the definitive treatment in Ref. 3. Design considerations relating to better performance with different types of torquing schemes appear in Refs. 5 and 7, using describing functions. Similar error sources appear in pendulous accelerometers. Single-axis platforms are considered in Refs. 11, 18, and 22. Back-to-back configurations[21-23] reduce some errors at the expense of more components.

The principles of a dry tuned two-degree-of-freedom gyro are discussed in Refs. 27 and 28 and by Craig.[24,25] Its error sources are considered in Refs. 24-26 and 29. As before, cross-coupling, anisoinertia, misalignment, and torquer scale factor errors exist, but now we also have errors due to nonexact tuning, damping in the spring joints, and disturbances occurring at twice spin frequency. Some other error sources common to these and floated gyros are rectification errors under vibratory excitation: pseudoconing, psuedosculling, undetected coning, and undetected sculling; these arise from combinations of more than one gyro/accelerometer. Furthermore, torquer loop sampling, quantization, and possibly limit cycling errors should be mentioned.[3] For recent work on advanced torquing methods, see Refs. 30-33.

The remaining competitive rate gyro is the laser "gyro," presently under intensive development and test,[34-40] with a promising future in the next decade.[41-44] Lucid discussions of error sources are given in Ref. 33 and by Savage,[38] who also details current capabilities in relation to requirements and to other sensors. Errors due to inertia and torquing are absent, since a laser gyro has neither; thus, excellent scale factor linearity is possible. Overcoming the lock-in phenomenon (it causes a deadband in gyro output) causes new errors,[37-40] depending upon the method used. Another significant source of error is wide-band noise in output rate, as well as random-walk type of behavior, i.e., noise in angle output.[36,37] In addition, the path length of the laser cavity must be controlled closely by servos.

The electrostatic gyro is a whole-angle attitude instrument. Its dominant errors result from geometric, dynamic (e.g., mass unbalance), and electric-field asymmetries and from the readout mechanism.[45-47] Small deviations from spherical symmetry cause residual torques on the rotor whose effect is manifested as attitude drift. Detailed discussion of electric-field errors occurs in Refs. 48-50, including a description of the rotor mass unbalance readout technique used in the MICRON system[49,50] and development of a calibration model.[48] Optical readout errors are detailed in Ref. 46, projected error magnitudes are discussed in Ref. 47, and performance evaluations are made in Refs. 51 and 52. With proper compensation,[48,53] an electrostatic gyro has the potential of significantly lower drift rates and computational demands than massive rate gyros.[47]

Algorithm Errors

The dominant computational errors arise in the integration of kinematic equations, for rate gyros. Some methods typical of early systems, which used directly the incremental angle outputs to update the direction cosine matrix (DCM), are given in Refs. 54-58; recent approaches with a view to improving performance are given in Refs. 59 and 60. Errors in whole-number attitude computers, using DCM integration, are summarized in Refs. 61 and 62 and detailed in Ref. 63, discussing limitations due to bandwidth (i.e., update frequency), truncation, quantization, and roundoff. Over a long time, the dominant secular errors arise from non-commutativity[68] of successive rotations, as noted in Refs. 64-66. A good detailed analysis of this and other error sources is given by Jordan,[66] including an algorithm for DCM integration.[67]

Among non-DCM attitude parametrizations, quaternion-ESP errors are analyzed in Refs. 69-70, and time savings of 50% or more are claimed in Ref. 71. Quaternions ideally yield zero skew error (preservation of angles) but retain scale errors (preservation of length). An optimum rescaling method is given in Ref. 72 and corrected in Ref. 73. Attitude algorithm errors using Euler axis/angle variables are discussed and compared with quaternions in Ref. 74. Theoretical upper bounds on computational errors can be obtained at length[75] but are too conservative. A method of retaining transformation orthogonality even with roundoff errors is discussed in Ref. 76. The generation of attitude transformations with different parametrizations is considered in Refs. 77-80, whereas Ref. 81 discusses algorithms for whole-number computers.

System Errors

In a strapdown system, the system-level errors resulting from an interaction of multiple sensor errors and computational errors depend not only on the navigation frame but also strongly on the mission profile, that is, on the vehicle trajectory in the six-dimensional position/orientation space. An alternative to the perturbation error analysis methods of Britting[88] is the so-called psi angle approach for local-level navigation.[83] A typical system error analysis, e.g., for coning motion[84] or spacecraft navigation,[85] is insufficient unless it includes realistic trajectory effects. The latter type of analysis is made in Ref. 86 for different aircraft trajectories, using pulse-rebalanced instruments, a third-order quaternion algorithm, and a Monte Carlo simulation. Similar simulations for fighter aircraft including Kalman filtering in an aided

strapdown configuration are reported in Ref. 87. The significance of trajectory effects is seen clearly.

VI. System Data Processing

This section is devoted to system-level data-processing tasks done at low iteration rates. Specifically, we consider initialization of the navigator state, orthogonalization of attitude transformations, and estimation of the system output and instrument error states.

Alignment

The initial values of position and velocity can be inserted similarly to gimbaled systems, once attitude "alignment" has been performed. This means computation of the body-to-navigation frame matrix $\mathbf{C}_n^b$. Coarse alignment can be performed by sensing two vectors; a fine phase may be added later, along with filtering methods. Sensing the gravity $\boldsymbol{g}$ leads to leveling,[1] whereas the Earth rate ω_e is used in gyrocompassing.[2,3] In this mode, azimuth alignment is very slow.[2] Optical means such as Porro prism[4] and theodolite sightings, possibly coupled with physical azimuth rotation,[5] have been suggested. In noisy situations such as a launch vehicle, ω_e is a poor choice[6]; also, some filtering is necessary.[6-9] Alignment methods using nonrecursive filtering are compared by Farrell,[10] favoring least-squares (LS).

A natural, recursive alternative is the Kalman filter (KF), as summarized in Ref. 7. Comparisons between KF and LS methods[8,9] favor KF when correlated noise is present. An excellent reference on KF theory and practical applications is Ref. 11, with attention to navigation.[12] A rather high-order KF is needed if instrument biases also are to be estimated.[6] The KF is a natural choice in rapid in-flight alignment, as in transfer alignment of a missile.[13] Better accuracy is achievable by physical rotations of the vehicle.[13] Optimum maneuvers in both horizontal and vertical planes, which minimize the final covariance in the KF, are considered in Ref. 14. A performance improvement results when angular rates also are measured.[15] A simple approach employing only three states appears in Ref. 16. For alignment in space, sun and star sightings are useful.[17,18] Much improvement is possible by allowing limited sensor motion.[19,20] Direct initialization of attitude parameters, rather than that of the DCM, is considered rarely.[21]

Orthonormalization

This operation is useful periodically, starting with the initial transformation matrix. For a quaternion parametrization, which has just one extra parameter, an optimum scheme is derived easily.[22,23] For the DCM case, an optimum scheme exists[24,25] which involves a matrix square root. Simple iterative approximations to it are available,[3,5] as well as elaborate ones based on gradient projection,[25] series expansions,[26] polar decomposition of a matrix,[27] and an algorithm dual to it.[28] The last three papers include convergence analyses. Three intuitive, nonoptimal algorithms were proposed in Ref. 29, without convergence criteria. The profusion of DCM orthogonalization schemes is evaluated in Ref. 42 of the bibliography.

State Estimation

In most cases, the KF is used for filtering and estimation. Applications to alignment have been mentioned already. The KF is used to estimate angular velocity from angle increment data in Ref. 31. Its use is almost mandatory when data from various sources are combined. Perhaps the first actual implementation occurred in the SPARS system[32]; the algorithms are reviewed in Ref. 33. A key feature was the use of gyro bias drifts as additional state variables, to be estimated using star sightings. The principles are similar to those for platform systems.[34] Many contemporary systems incorporate some form of recursive filtering, to estimate instrument errors in operation. One of the problems of concern is the possible divergence due to unmodeled digitization errors.[41] Another difficulty, the roundoff problem, can be alleviated by using a square-root formulation.[42]

The application of nonlinear filtering techniques is not nearly so widespread. A concise introduction to it, and a comparsion with KF on simple problems relevant to strapdown systems, appears in Ref. 35. The computational burden is severe and the benefits uncertain[35,36]; it turns out that, from the computational viewpoint, the DCM parametrization is simpler than quaternions.[36] An off-line nonlinear filtering procedure is explored in Ref. 37. A simplified situation involving parameter estimation is addressed using nonlinear filtering methods in Refs. 38 and 39; a comment on this work by Farrell is illuminating.[40]

Landmarks

A special data-processing task arises if landmark sightings are added to a strapdown attitude reference, forming a different type of autonomous navigation system. The idea is not new,[43] and both known[44] and unknown[45] landmarks may be used, the chief application being to low-altitude satellites.[46,47] Two recent extensions are possibility of using linear landmarks (rivers, highways, etc.)[48] and an interferometric landmark tracker.[49] Nonlinear filtering techniques are applied to a landmark or "autonomous navigation" system in Ref. 50. These papers are merely highlights of this growing field.

VII. Reliability Enhancement

The most effective means of increasing the reliability of a system is by redundancy. In gimbaled systems, entire systems are duplicated, but, in the strapdown case, sensor-level redundancy offers a decided advantage.[1] This section deals with redundant configuration selection and failure processing concepts and algorithms.

Configurations

Most papers in this area concentrate on gyros (they are usually the critical components[1]) that sense angular velocity. Linear combinations of any three noncoplanar gyros will suffice for ω. Two skewed triads can suffice for ultrahigh reliability if a supply of spare parts and in-flight repair facilities are available.[2] This is not usually the case. In skewed configurations, the gyro error, expressed as a fraction of input rate sensed, increases. The implications of this in systems using four, five, and six SDF gyros are discussed in Ref. 3.

The optimum six-gyro configuration is the dodecahedron configuration.[4,5] It minimizes the average output error covariance for given covariance of rate input and simplifies failure processing. Optimum configurations using TDF gyros, assuming that both axes fail simultaneously, are given in Refs. 6 and 7. The rationale for and the derivation of optimum configurations for this case are detailed in Ref. 8. If we assume that one axis can function even when the other has failed, the optimum gyro mounting configurations change significantly.[9,10] Optimum TDF sensor configurations are detailed further in Refs. 10 and 11. Studies of redundant configurations are made in Refs. 12 and 13, while recent descriptions of systems designed on a redundant basis are given in Refs. 14-16. There seems to be no information on configuration selection and optimization when attitude sensors, such as the electrostatic gyro, are used.

Failure Processing

Failures may be catastrophic, hard (rapid deterioration of performance), or soft (slow changes, difficult to detect). The identification of a failure and of the particular instrument can be based on the internal states of the instrument, outputs of calibration filters, etc., or information from external sources. Failure detection relies upon comparing the outputs of

redundant sensors. A deterministic approach to this problem is simpler and easier to visualize, but a statistical approach is likely to result in fewer false alarms or missed alarms. Deterministic parity-checking concepts are used in the dodecahedron system[5,17,18] and for TDF configurations discussed previously.[6-11] The number of checks ranges from 6 to 15. Identification of the failed sensor is possible in all of these cases for the first failure. The method of Ref. 18 allows each gyro to have a different threshold for parity checks. A useful idea in Ref. 17 is to use prefilters to reduce uncertainties due to quantization and sampling.

A statistical approach to failure detection and identification (FDI) is taken in Refs. 19-22 for the dodecahedron configuration. A maximum likelihood criterion is used in Ref. 19; the criterion of Ref. 20 is derived statistically but applied deterministically. The method of Chien[21,22] is based on Wald's sequential probability ratio test. Although the test implementation is complicated, it bases the decisions on a past measurement history, not just on the current measurement.

A comparison of several FDI algorithm by means of a realistic simulation program has been given by Wilcox.[23] A space shuttle boost trajectory was used, with a step failure introduced halfway through the boost. On the basis of the error at the end of the boost, it appears that deterministic FDI algorithms perform better. Both linear and angular vibration inputs were considered. Finally, we mention briefly the subject of failure correction. It is straightforward to reconfigure the system once a faulty instrument has been identified; this is merely a software change. However, the degradation of performance during the period before identification usually is accepted as a necessary evil. An alternative is considered by Wilcox,[24] but the computational requirements are tremendous.

VIII. Concluding Remarks

The foregoing sections have attempted to convey the present status of strapdown navigation technology, based on work reported in the open literature. This last section is intended to highlight some current areas of development and trends that are expected to continue in the near future. This is, of necessity, a subjective judgement and is limited further by the proprietary nature of development activites.

The performance of strapdown systems has improved dramatically in the past five years. System accuracies during flight-test demonstrations have gone from early 1970 performance of 3 to 4 naut.mile/h position error[8] to quite respectable values of less than 1 naut.mile/h and about 2 ft/s velocity error.[9] The latter values have been attained in development models and are a reasonable goal for production systems.[4,9,10] Further advances are to be expected in sensor, especially accelerometer, performance[10] and testing/calibration methods. The velocity error may be reduced to the 1- to 2-ft/s range in aided strapdown systems. Since the best gimbaled systems can give 0.25 naut.mile/h position and 1 ft/s velocity error,[11] and production models deliver 1 naut.mile/h and 3-4 ft/s, respectively, the performance of strapdown systems has become competitive indeed. One may expect rapid progress toward production models achieving the 1-naut.mile/h goal[10] for position error and further improvements in velocity error performance.

The microprocessor explosion continues to be a driving force in strapdown system development. Certain computing tasks, such as sensor calibration, can be done by distributed microprocessors.[1] This encourages modularization with its attendant standardization and flexibility,[2] which is another current theme of development. This concept can be applied to sensors, power supplies, signal converters, etc., yielding significant cost reduction.[3] Modularity also leads to operational savings due to ease of maintenance, checkout, and testing, and simplifies redundant design.

Low-cost inertial navigation is a benefit long expected to follow from strapdown systems. Apart from the cost reductions possible with modularity and microprocessors, savings can be effected also by integrating the strapdown navigator with other existing equipment. This is an important consideration, favoring integrated navigation/air-data systems for aircraft[4] and hybrid or aided strapdown systems using data from radio sources.[5] Another potential source of information is landmark tracking.[6] Integrated navigation systems also can incorporate data from satellite systems, such as the Global Positioning System.[7] Numerous new applications of strapdown technology can be expected to follow significant cost reductions.

In conclusion, it is perhaps of interest to indicate some possible future subjects for publications in this field. These remarks necessarily refer only to publications in the open literature. Accelerometers and multisensors explicitly for strapdown use have not been described often. The development of detailed sensor error models for use with high-order filters (expected to be feasible with tomorrow's computers) is of interest. Further applications of microprocessors, life-cycle cost forecasting, and fast-developing new concepts in redundancy management and failure processing no doubt will be reported. Comparison of the capabilities of different types of sensors, in relation to required navigation accuracies, is of interest to designers. An evaluation of current and projected microcomputer capabilities in relation to strapdown system errors (using realistic trajectory models and disturbances), applications of nonlinear filtering techniques, algorithms for retroactive failure correction, and comparisons of available algorithms for attitude computation, orthonormalization, failure detection, etc., are some areas deserving further analytical/computational studies.

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