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History of Key Technologies

The Kalman Filter: Its Recognition and Development for Aerospace Applications

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Introduction

THE Kalman filter is widely used today for state estimation in aerospace systems. This well-known filter made the transition from a relatively abstract theory to application in many aerospace systems within a very short period during the early 1960's. This paper describes the recognition of its utility and its subsequent development.

The first publicly known Kalman filter application was made during the feasibility studies for the Apollo space program. This paper begins with a review of the events which led to defining the requirements for that filter. Then, a discussion is presented of how the theory came to the author's attention and some of the problems involved in interpreting the theory for potential application to the on-board navigation system for the Apollo mission. Next, a brief review of the dissemination of the findings to the general scientific community is covered. Subsequent sections outline the author's contributions to Kalman filter improvements. The paper thus presents a chronological account of how a mathematical filtering theory came of age and achieved its current wide engineering acceptance and practical utilization.

Preceding Events

In the fall of 1959 Dr. Harry J. Goett, director of NASA's Goddard Space Flight Center (GSFC), invited members of his former division at Ames Research Center (ARC) to visit the Space Task Group located at Langley Research Center to discuss the future manned spacecraft program. (See Ref. 1 for a history of the Apollo program.) Although the manned lunar mission had not yet been selected as a national program, Dr. Goett was anxious to get feasibility studies underway at the NASA research centers. During this trip and the many others made in this early time, the author helped define guidelines for the manned lunar mission and areas where NASA/ARC should concentrate its research work. The outcome of the travels resulted in two problem areas being defined where the author's staff at the ARC Dynamic Analysis Branch might produce useful work. These areas were: 1) midcourse navigation and guidance for the circumlunar mission, and 2) autopilot design for large flexible liquid-fuel boosters.

The branch had about eight analytically oriented personnel so, as Branch Chief, it looked hopeless to try to direct two complex research programs simultaneously. The most logical selection from personnel experience considerations was the booster autopilot problem. However, after considerable branch discussion, it was unanimously decided to focus on solving the midcourse navigation and guidance problem because we felt it presented a greater challenge.

The primary emphasis was to develop the technology for a self-contained system which could be implemented in an on-board digital computer. The system with pilot/navigator input and command was to guide the spacecraft from in-

Stanley F. Schmidt was born in Hollister, Calif., on January 21, 1926. He received the B.E.E. degree from Marquette University in 1946, and M.S. and Ph.D. degrees in electrical engineering from Stanford University in 1952 and 1959, respectively. From 1946 to 1961, he was with NASA's Ames Research Center, where he discovered the utility of the Kalman filter as applied to data processing for the nonlinear navigation equations of the manned lunar mission. Also while at Ames, Dr. Schmidt developed piloted motion simulators, designed nonlinear compensation techniques for saturation effects in control systems, and served as Branch Chief in charge of all analog simulation work. During 1961 and 1962, Dr. Schmidt was with Lockheed Missiles & Space Company. There he applied filter theory and model identification techniques and developed digital computer programs for processing tracking data and giving postflight evaluation of launch vehicle guidance and propulsion systems. From 1962 to 1966, Dr. Schmidt was a senior Staff Scientist with Philco's Western Development Laboratory. There he directed studies of navigation and guidance systems for space vehicle systems and development of digital computer programs for analysis and design of space vehicle systems. Also at Philco, he conceived the fan beam navigation satellite technique and pursued studies to prove the feasibility and accuracy of this concept. In 1966, Dr. Schmidt joined Analytical Mechanics Associates, Inc. where he is Vice President and Technical Director of their Western Division. At AMA, Dr. Schmidt developed a special Kalman filter formulation for the Northrop C-5 navigation system, applied control theory to improve NASA piloted flight simulators, and developed several on-board navigation systems which incorporate square-root formulations of the Kalman filter. Dr. Schmidt is an Associate Fellow of the AIAA and is a member of the IEEE, Tau Beta Pi, and Eta Kappa Nu. He has also served as lecturer at Santa Clara University.

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jection (out of Earth orbit) around the moon and to satisfy reentry corridor requirements on return to Earth.

Once having selected the problem area, there were many things that had to be developed. First, we had no trajectory analysis capability so trajectory programs for lunar missions needed to be developed. Dr. Clarence Gates and others at JPL gave considerable assistance in this area. By mid-1960, we could calculate free-return lunar trajectories and were investigating linear perturbation methods that looked promising for calculating midcourse corrections.

The research work on navigation (that is, estimation of state) was not proceeding very well, and we obviously needed a breakthrough. We had assumed from the beginning that the external measurement would be from a pilot-operated optical sensor. We had also reviewed the method in use by JPL (the iterative weighted least-squares estimator) and had decided it was too complex for state-of-the-art on-board computers. We had considered taking multiple measurements and fitting them to a polynomial. Although this appeared to be a simple enough procedure, the accuracy of state estimation resulting from the assumed optical tracking measurement accuracies was not adequate. For several years, we also had been working with Wiener filter theory, but had not seen any way of utilizing the Wiener theory for this problem.

Recognition and Development of the Extended Kalman Filter

Dr. R.E. Kalman of the Research Institute for Advanced Study (RIAS) and the author had been acquaintances for several years. In the fall of 1960, Dr. Kalman arranged a visit with the author at ARC to discuss topics of mutual interest. He presented his now famous paper[2] to members of the author's staff. Following the presentation, the author decided that the theory might have some merit for our application and assigned staff members to examine the contents of the paper. We had a great deal of difficulty understanding Kalman's paper, because at this date the state space approach to control problems was in its infancy. The notation as well as the concepts used by Kalman were not easy for practicing engineers to grasp. On the author's next trip to NASA/GSFC, a meeting with Kalman took place at RIAS to discuss his paper further. During this meeting, the methodology of applying Kalman's results to a linear system became clear. Also, our guidance study had been based on study of linear perturbations about a nominal trajectory. As a result, Kalman's linear filter theory plus our linear perturbation methods combined to give us a potential solution to the nonlinear navigation problem. This technique also permitted us to complete a digital simulation for validation of this formulation.

The digital simulation program was designed to integrate nonlinear differential equations for the true trajectory, the nominal (or reference) trajectory and the estimated trajectory. The simulated optical measurements were calculated as functions of the true state plus additive biases and noise. The Kalman filter algorithm accepted these measurements and provided incremental changes in the estimated state. The difference between the estimated state and nominal (reference) state was used in the midcourse correction algorithm. This algorithm gave the ΔV vector required to drive the estimated position to the nominal (reference) position at perilune (perigee) at the end of the outbound (inbound) trajectory. To simulate the midcourse correction, the computed ΔV vector was added to the estimated trajectory directly. Random magnitude and pointing errors were added to the computed ΔV vector to simulate control system errors), and the result was added to the true trajectory. The covariance matrix of the estimated state was also increased in a manner corresponding to the statistics of the simulated control system errors. Variational equations were solved about either the reference or the estimated trajectory to provide the transition matrices needed in the filtering and guidance laws.

Studies made with this simulation program established that when we linearized about the current best state estimate, the estimated state converged to the true state for relatively large initial errors. This procedure is currently known as the "extended" Kalman filter. In one particular case, the true trajectory remained in orbit about the Earth where the estimated trajectory started with conditions for the lunar transfer trajectory. The optical measurements were somewhat sparse, but the algorithm succeeded in driving the estimate close to the true trajectory. This occurred even though nonlinear phenomena caused relatively large overshoots during the early measurements.

The simulation program was used extensively to validate the "extended" Kalman filter and the guidance laws we had developed. By early 1961, we had very encouraging results indicating that on-board optical measurements combined with the knowledge of the equations of motion could yield adequate accuracy for the circumlunar navigation/guidance problem. This was the breakthrough we were looking for. At this point in time it was clear that we had achieved a potentially significant result for on-board navigation systems. Our studies indicated that the "extended" Kalman filter would give compatible accuracies to the weighted least-square estimator with a tremendous reduction in requirements for on-board computer memory and computation speed. We had not, however, verified that the "extended" Kalman filter could be mechanized and operate properly with the flight computers available at this time period. This was one of the many problems solved by the MIT Instrumentation Laboratory in their subsequent development of the Apollo system.

The author's recollection of the first person contacted (outside of NASA) and told of our results was Dr. John V. Breakwell of Lockheed Missiles & Space Company (LMSC). Dr. Breakwell and Dr. Charlotte Striebel of LMSC were both sufficiently impressed with the results to begin further research on their part and to explain equivalences to the other trajectory determination methods in use at that time.

As a result of the tremendous interest in the Apollo program, the word about our work spread extremely rapidly. At ARC, we had visitors from all parts of the country during the summer and fall of 1961. Two conference papers[3,4] were presented at an AAS meeting in San Francisco in the summer of 1961 on our navigation and guidance studies for the circumlunar mission. These papers were shortened versions of the NASA reports[5,6] published in 1962. These were also the first formal presentations of our efforts to be attended by many scientists. Following the session, several additional hours were spent at the conference discussing details with many government and industry representatives.

The contributions to the Kalman filter applications technology made at NASA/ARC by the fall of 1961 were as follows:

1) The filter's potential for use in on-board systems was demonstrated by digital simulation.

2) Linearization about the current best estimate of state (the "extended" Kalman filter) was developed.

3) Decomposition of the filter algorithm into the time update and the measurement update was designed so that measurements occurring at arbitrary intervals could be handled.

4) The information on this development was widely disseminated to the aerospace community.

No problems had been experienced in the digital simulations due to truncation errors or modeling errors during this feasibility study.

In 1961, the Apollo on-board guidance development project was assigned to the Instrumentation Laboratory of Massachusetts Institute of Technology. On one of my many trips to Washington, I met Dr. R.H. Battin and associates of MIT who were to be involved in the subsequent development of the Apollo system. We spent some time one evening

discussing the Kalman filter and results we had found at ARC. After subsequent development on his own,[7,8] Battin pursued using the filter for its application to Apollo. Later, Potter[8] of MIT devised the first square-root filter implementation which was used for the Apollo system. This implementation could not handle random forcing functions, but it was useful for the small on-board computer of the Apollo system.

Real Data Tests

The first opportunity to test the extended Kalman filter we had developed at ARC on real data came while the author was with LMSC in 1961 and 1962. The basic problem was to validate the performance of the Agena upper stage using Earth-based tracking data from the downrange stations and telemetry data from the vehicle. A general-purpose postflight analysis program was developed which combined tracking data from several stations with the model of the equations of motion for the vehicle. The error state included tracking measurement biases and location errors as well as coefficients of a propulsion model for the thrust of the Agena upper stage.

The procedure developed for the operation was to use the tracking data during coast phases to estimate position and velocity and the covariance matrix of errors at the initiation and termination of the thrust phase. The thrust phase was then handled by starting the program with the initial state and covariance matrix (from coast phase data) with the error state expanded to include coefficients of the thrust model. Tracking data during the thrust phase was processed in the normal manner. At the termination of the thrust, the state estimate from the postburn tracking data was used as a measurement of the six-component state vector, with its covariance matrix used to characterize the accuracy.

This development added the following techniques to the applications technology of the Kalman filter:

1) A bad-data rejection technique was developed which compared the measurement residual magnitude to its standard deviation as computed from the Kalman filter measurement update algorithm. If the residual magnitude exceeded three times the standard deviation the measurement was rejected.

2) The Kalman filter was used as a data compression algorithm to form an equivalent measurement and covariance matrix from the multitude of measurements taken during the coast phases of the vehicle.

3) An iterative approach using backwards integration and forward filtering was developed to remove effects of nonlinearities from the estimate. This was not data smoothing but simply an equivalent procedure to the weighted least-squares estimators in use at that time.

4) The Kalman filter was used to estimate parameters in the measurement and equation of motion models.

Error Analysis Usages and the Kalman-Schmidt Filter

In 1962, the author initiated an effort at LMSC to develop general-purpose error analysis programs for lunar and planetary missions.[9] The author continued this work at Philco-Ford's Western Development Laboratory (1962-1966) under sponsorship by NASA/GSFC. These programs assumed a reduced-state Kalman filter for the state estimation. Error states allowed included measurement biases as well as parameters in the equations of motion.

Due largely to the encouragement of Dr. F.O. VonBun of NASA/GSFC, the Kalman-Schmidt* filter[11] was developed. This filter was included as one of the options provided in the general-purpose error analysis program delivered to GSFC.[12]

In addition to the Kalman-Schmidt filter, the developments at Philco-Ford added the following techniques to recursive filtering technology:

1) General computational techniques for saving machine time by taking account of sparsity of the transition matrix and symmetry of the covariance matrix.

2) Mathematical formulation for combining on-board inertial sensor measurements with ground tracking data.

3) Ad hoc technique for adding pseudo-random forcing functions to minimize the effects of numerical errors.

The C5A Aircraft Navigation System

In 1966, the opportunity finally arose for the author to participate in the development of a Kalman filter for a real-time on-board application. The author had hoped to proceed with this task at ARC five years earlier. By this date, the Kalman filter had been well recognized throughout the aerospace industry and was in the specifications established by Lockheed for the C5A aircraft navigation system. Northrop won the C5A navigation system competition, and the author was hired as a consultant to develop a filter which combined inertial data with that from many different external navigation aids.

During this development, the real problems of selection of appropriate error states, adding error estimates to the system outputs, working with a modest word size, and making the whole computational burden work within the time frames of small computers were brought to focus. This development put many of the model and numerical compensation techniques[13-15] the author had developed to a real test. Also, due primarily to John Weinberg[15] of Northrop, data compression methods, using measurement averaging for saving computational time at a small expense of accuracy, were perfected. This development also definitely pointed out the need for a general square-root filter method which could work properly with the fixed-point arithmetic of the available on-board computers. (That is, the filter's error covariance would be computed and propagated in square-root form. This would require less computational precision to maintain filter stability.) However, because such a formulation was not available for this system, we used the conventional algorithm with the ad hoc epsilon technique[13,17] for controlling numerical problems.

General Square-Root Filter Method

After hearing of Potter's development, the author made several attempts to find an efficient square-root filter method which would allow use of random forcing functions in the time update algorithm. In early 1968, the author developed a method[16,17] which looked promising for application on a small fixed-point on-board computer. (The reader should consult Ref. 18 for a good summary of the history of the square-root filter development during this time period.) In 1970, NASA/ARC initiated the development of the RAINPAL precision landing navigation system[19-21] where square-root algorithms were tested in a real-time fixed-point on-board system. This is believed to be the first application of the complete square-root technology (including process noise), although as mentioned earlier, Potter's algorithm for the square root was used in the Apollo system. The methods used in this system were Potter's algorithm for the measurement update and Householder's algorithm for the time update.[18]

Since the RAINPAL system was flight tested, the author has designed several other square-root applications for aircraft systems.[22] All have been successful, and no problems have occurred from numerical operations. Today, the engineer has several choices of square-root-type implementations[23] and a large repertoire of ad hoc methods for compensating for modeling errors. The problems of any

*During the 1960's a number of authors[10] referred to Kalman filters as Kalman-Schmidt filters. This was likely due to the fact that credit for the applications technology which produced the extended filter was given to this author. The Kalman-Schmidt filter includes effects of but does not solve for selected error states. This provides a means for optimal compensation for modeling errors provided one knows what the significant model errors are in the filter equations.

Kalman filter development have been fairly well categorized and the task of defining appropriate models for the application is fairly systematic.

Some Problem Areas

The application of the Kalman filter to real data systems is not without some problem areas. A few are summarized here:

1) The filter requires statistical models of all the measurement and model uncertainties. Invariably the most one can hope for from hardware manufacturers is specifications on tolerances. The character of the errors in terms of standard deviations, correlation time constants and so forth are seldom available.

2) Real measurements seldom behave in the analytically desired manner. For example, normal functioning of a TACAN receiver (measurements of range and bearing) can yield: a) good quality data with errors well within specification, b) good quality data interspaced with small segments of hangups (data do not change) and wild points, or c) a few good data points interspaced with large amounts of useless measurements. The development of data rejection algorithms for taking care of the bad measurements which are not flagged invalid by the electronic hardware is frequently a difficult problem.

3) In navigation systems we could frequently use the covariance matrix of errors in the estimated state as well as the estimated state itself. However, because of the lack of valid statistical information, the linearization approximations, modeling errors, and the fact that practical filters are of a reduced state type, the covariance matrix of the on-board filter bears little relationship to the true statistical covariance matrix of errors in the estimated state.

Concluding Remarks

It is perhaps pure luck, and may occur only once in a lifetime, when a practicing engineer has the opportunity to play a significant role in a major technology development. In the case of the Kalman filter, decisions made in 1959 and 1960 set up an engineering requirement for a particular filtering theory. At that same time, such a theory was under development by an excellent theoretician. Fortunately the practicing engineer and the theoretician were good acquaintances, or the applications technology could have been delayed many years. Probably the most important role this author played in the Kalman filter development was recognizing, demonstrating, and publicizing the theory's usefulness. The Apollo program deserves the real credit for this technology since it established the need, provided resources for the application development, and pulled the scientific community together working towards a common objective.

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