On-Line Aircraft State and Stability Derivative Estimation Using the Modified-Gain Extended Kalman Filter

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A new on-line state and parameter identification algorithm called the modified-gain extended Kalman filter (MGEKF) is applied to the problem of on-line state estimation and identification of the stability derivatives of a F-111 type of vehicle. The conceptual basis for the MGEKF is the existence of a class of nonlinear functions that allow a universal linearization with respect to the measurement function. This class includes the problem of identification of linear systems. The previous single-output formulation is extended to a multioutput formulation where the only available measurements are acceleration and pitch rate, but not elevator deflection. The filter formulation includes a simplified Dryden wind gust model. The inclusion of the wind gust model results mainly in a slowed response in the estimation of the stability derivatives associated with the acceleration state; estimates of the stability derivatives associated with the pitch rate still respond very quickly. The accuracy of the acceleration stability derivatives depends upon the amplitude and frequency components of the persistently exciting dither signal.

I. Introduction

THE historical development of aircraft parameter identification is given in Refs. 1–3. These studies are designed primarily for off-line use. A very complete study of recursive identification schemes for on-line use is given in Ref. 4. However, the usual assumption that the parameters be constant produces gains that are asymptotically inversely proportional to time and therefore become vanishing small. These schemes are not applicable to aircraft systems that must operate continuously and identify changes in the stability derivatives as the flight conditions change. One motivation for this type of on-line state and parameter estimation scheme is for use in adaptive flight control systems.

In Ref. 5, various schemes for identifying constant system parameters are compared on a common problem. Among these schemes is the extended Kalman filter (EKF) whose performance is shown to be relatively poor. This problem was again analyzed in Refs. 6 and 7, where a new estimation scheme called the modified-gain extended Kalman filter (MGEKF) is used. For a special class of nonlinearities of which state and parameter estimation in linear systems is a member, there exists a universal linearization of these special nonlinearities with respect to the measurement function. In order to obtain nonlinearities in this class, the observability coordinate system rather than the controllability coordinate frame is used for the problem in Ref. 5. The results given in Ref. 7 indicate a remarkable improvement in performance. The MGEKF described in Ref. 7 is applied here to the problem of on-line state estimation and identification of the stability derivatives of an F-111 type of vehicle.

Section II presents the definition of a modifiable nonlinear system function that forms the basis of the MGEKF algorithm. A simple illustration of a modifiable nonlinearity is given before the general form of the MGEKF algorithm is stated. The essential features of this algorithm are then discussed. The dynamic system model for the aircraft is presented in Sec. III. The short-period longitudinal mode of the aircraft is expressed in acceleration and pitch rate states to be consistent with the measurements. In addition, a first-order model for the actuator and a second-order simplified Dryden wind gust model are described. In Sec. IV, the mechanization of MGEKF using this aircraft model is discussed and, in Sec. V, the performance of the MGEKF algorithm using accelerometer and pitch rate gyros is presented. Conclusions and recommendations are given in Sec. VI.

II. The Modified-Gain Extended Kalman Filter Algorithm

The dynamic nonlinear system model used for combined state and parameter estimation is presented first. The definition of a modifiable nonlinear system function, used as the basis of the MGEKF algorithm, is stated. Then the MGEKF algorithm is presented and its properties discussed.

Dynamical System and Modifiable Nonlinearities

The discrete dynamic system model used for combined state and parameters identification is

$$y_{i+1} = A(\theta_i)\, y_i + B(\theta_i)\, u_i + \underline{w}_i \tag{1}$$

$$\theta_{i+1} = \theta_i + \overline{w}_i \tag{2}$$

and the scalar measurement is

$$z_i = Hy_i + v_i = z_i^* + v_i \tag{3}$$

where y_i is an n-dimensional state vector, θ_i is a vector of maximal dimension $2n$ of unknown parameters representing the elements of the matrices $A(\theta_i)$ and $B(\theta_i)$, $A(\theta_i)$ is an $n\times n$ matrix, and $B(\theta_i)$ is an n vector where both contain up to n unknown elements represented by the elements of θ_i, u_i is a known scalar input, z_i^* is the scalar measurement function, H is a known $1\times n$ measurement matrix, and $\underline{w}_i$, $\overline{w}_i$, and v_i are zero-mean white noise sequences with variances $\underline{Q}_i$, $\overline{Q}_i$, and γ_i, respectively. The formulation given here is for a single-input/single-output system consistent with the results of Ref. 2. Although the extension to more than one input is

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trivial, the extension to more than one output takes some innovation. This extension is done in the following sections.

The nonlinearity in this problem is $A(\theta_i)y_i$. For convenience define

$$x_i^T \triangleq [y_i^T, \theta_i^T] \tag{4}$$

and the nonlinearity as

$$f(x_i) \triangleq \begin{bmatrix} A(\theta_i)y_i + B(\theta_i)u_i \\ \theta_i \end{bmatrix} \tag{5}$$

where x_i has maximal dimension of $3n$.

Modifiable Nonlinearities

The notion of a modifiable nonlinearity is that there exists universal linearization of the function $f(x_i)$ with respect to the measurement function z_i^*.

Definition[7]: A function f: $\boldsymbol{R}^p \rightarrow \boldsymbol{R}^p$ is a modifiable nonlinear system function if there exists a $p \times p$ matrix F: $\boldsymbol{R} \times \boldsymbol{R}^p \rightarrow \boldsymbol{R}^{p \times p}$ so that for any state x_i and known estimate of the state $\hat{x}_i$,

$$f(x_i) - f(\hat{x}_i) = F(z_i^*, \hat{x}_i)(x_i - \hat{x}_i) \tag{6}$$

where $z_i^* = Hx_i$.

Note that $F(z_i^*, \hat{x}_i)(x_i - \hat{x}_i)$ in Eq. (6) is a universal linearization of $f(x_i)$ with respect to the measurement function z_i^* without any approximation. Notice that the known function $F(z_i^*, \hat{x}_i) = F(Hx_i, \hat{x}_i) \neq F(H\hat{x}_i, \hat{x}_i)$, where the latter quantity is the differential of f evaluated at $\hat{x}$, as used in the linearization.

The noiseless case of a simple linear dynamical system with an unknown coefficient illustrates the idea of a modifiable nonlinearity. The system is represented as

$$y_{i+1} \triangleq \theta_i y_i, \qquad \theta_{i+1} = \theta_i, \qquad z_i^* = y_i \tag{7}$$

where y_i is a scalar state at stage i and θ_i the unknown parameter at stage i. The nonlinearity is put into modifiable form by writing

$$\begin{aligned}(x_{i+1} - \hat{x}_{i+1}) &= \begin{bmatrix} \theta_i y_i \\ \theta_i \end{bmatrix} - \begin{bmatrix} \hat{\theta}_i \hat{y}_i \\ \hat{\theta}_i \end{bmatrix} \\ &= \begin{bmatrix} \theta_i \hat{y}_i - \hat{\theta}_i y_i + \hat{\theta}_i y_i - \hat{\theta}_i \hat{y}_i \\ \theta_i - \hat{\theta}_i \end{bmatrix} \\ &= F(z_i^*, \hat{x}_i)(x_i - \hat{x}_i)\end{aligned} \tag{8}$$

where $x_i^T \triangleq [y_i, \theta_i]$, $\hat{y}$ and $\hat{\theta}$ are the estimated values of y and θ, and

$$F(z_i^*, \hat{x}_i) = \begin{bmatrix} \hat{\theta}_i & z_i^* \\ 0 & 1 \end{bmatrix}, (x_i - \hat{x}_i) = \begin{bmatrix} y_i - \hat{y}_i \\ \theta_i - \hat{\theta}_i \end{bmatrix} \tag{9}$$

Note that the estimation error in Eq. (8) is propagated without approximations by a linear equation. Since the measurements are linear, the update formula for the error is also linear. In the noiseless case where this filter reduces to a nonlinear observer, the error of this observer is shown to be exponentially convergent by Lyapunov's second method.[6,7] In the noise-corrupted case where only the noisy measurement is available and not the measurement function, under certain a priori uncheckable conditions the MGEKF is shown to be exponentially bounded in the mean square sense.[6,7]

The MGEKF Algorithm

The discrete formulation of the MGEKF from Ref. 7, based on the dynamic system [Eqs. (1–3)] using Eqs. (4) and (5), is summarized as

$$\bar{x}_{i+1} = f(\hat{x}_i) \tag{10}$$

$$\hat{x}_i = \bar{x}_i + K_i(z_i - H\bar{x}_i) \tag{11}$$

$$K_i = M_i H^T(HM_iH^T + \gamma_i)^{-1} \tag{12}$$

$$M_{i+1} = F(z_i, \hat{x}_i)P_iF(z_i, \hat{x}_i) + Q_i \tag{13}$$

$$P_i = (I - K_iH)M_i(I - K_iH)^T + K_i\gamma_iK_i^T \tag{14}$$

where $\bar{x}_i$ is the propagated state and parameter estimates, K_i the modified Kalman gain calculated by Eqs. (12–14), M_{i+1} the propagated pseudo error-covariance matrix, P_i the updated pseudo error-covariance matrix, Q_i the process noise covariance matrix composed of diagonal matrix elements $\underline{Q}_i$ and $\overline{Q}_i$, and γ_i the measurement noise covariance matrix.

The functions $f(\hat{x}_i)$ and $F(z_i, \hat{x}_i)$ can be expressed in a simple way when the dimension of θ_i is assumed to be $2n$. Note that $B(\theta_i)$ becomes just the last n elements of θ_i. Furthermore, the function $F(z^*, \hat{x}_i)$ obtained from a modifiable nonlinear function $f(x_i)$ becomes

$$F(z^*, \hat{x}_i) = \left[\begin{array}{c|c|c} A(\hat{\theta}_i) & z_i^* I_n & u_i I_n \\ \hline 0_{2n \times n} & \multicolumn{2}{c}{I_{2n}} \end{array}\right] \tag{15}$$

where I_n is an $n \times n$ identity matrix and $0_{2n \times n}$ is a $2n \times n$ matrix of zeros. In Ref. 7, this matrix is obtained in the observability canonical form where the unknown parameters lie in the last column of the A matrix. It should be noted that in the gain algorithm for propagating the pseudo error-covariance matrix [Eq. (13)], the actual measurement z_i is used rather than the measurement function z^* in Eq. (15). Finally, for use later when describing the MGEKF for the aircraft application

$$f(\hat{x}_i) = F(0, \hat{x}_i)\hat{x}_i \tag{16}$$

The key to applying the MGEKF to the parameter estimation problem is to ensure that those unknown parameters being identified enter the dynamic equation so as to multiply the states or controls that are directly measured. As shown in Ref. 7, this means that the coordinate frame must be chosen carefully. Furthermore, the results given in Ref. 7 apply to only a single output problem. The results here give an example of how the MGEKF can be extended to two or more outputs.

III. The Aircraft Dynamical System

The linear longitudinal dynamics representing the short period motion are

$$\dot{\alpha} = Z_\alpha \alpha + Z_q q + Z_e e - Z_\alpha \alpha_G + b_\alpha \tag{17}$$

$$\dot{q} = M_\alpha \alpha + M_q q + M_e e - M_\alpha \alpha_G + b_q \tag{18}$$

where α is the total angle of attack, q the pitch rate, e the elevator deflection, α_G the angle of attack due to wind gust, b_α and b_q the trim biases associated with the steady-state conditions of α and q, respectively, and Z_α, Z_q, Z_e, M_α, M_q, and M_e the aircraft stability derivatives.

Transformation of State Space

Aircraft, such as that in Ref. 8 and the F-111 type, have normal acceleration and pitch rate measurements available from an accelerometer and pitch rate gyro. Therefore, it is advantageous to convert from angle of attack to acceleration in the dynamical representation of the aircraft for MGEKF applications.

The accelerometer measures the combined acceleration of the center of mass and the acceleration relative to the center

of mass due to the moment arm X_{acc}. This relationship between A, q, and α is

$$A = \beta_1(q - \dot{\alpha}) + \beta_2\dot{q} \tag{19}$$

where A is the normal acceleration (in g's) and

$$\beta_1 \triangleq \frac{2\pi u}{360g}, \qquad \beta_2 \triangleq \frac{2\pi X_{acc}}{360g} \tag{20}$$

where u is the aircraft velocity (in ft/s), g the gravitational acceleration, and X_{acc} the X distance from the aircraft c.g. to the accelerometer.

Equations (17–19) can be used to derive a new set of dynamical equations using A and q.[8] The new dynamical system is

$$\dot{A} = D_A A + D_q q + D_e e + D_G \alpha_G + \Omega_e \dot{e} + \Omega_G \dot{\alpha}_G + B_A \tag{21}$$

$$\dot{q} = H_A A + H_q q + H_e e + H_G \alpha_G + B_q \tag{22}$$

where

$$D_A \triangleq Z_\alpha + \frac{M_\alpha \Omega_q}{\Omega_\alpha}$$

$$D_s = Z_s\Omega_\alpha - Z_\alpha\Omega_s + M_s\Omega_q - \frac{M_\alpha\Omega_q\Omega_s}{\Omega_\alpha}, \qquad s = q, e, G$$

$$H_A \triangleq \frac{M_\alpha}{\Omega_\alpha}; \qquad H_s \triangleq M_s - \frac{M_\alpha\Omega_s}{\Omega_\alpha}, \qquad s = q, e, G$$

$$\Omega_s \triangleq \beta_2 M_s - \beta_1 Z_s, \qquad s = \alpha, e, G$$

$$\Omega_q \triangleq \beta_2 M_q + \beta_1(1 - Z_q)$$

$$B_A = \Omega_q - \frac{\Omega_\alpha\beta_2}{\beta_1}B_q, \qquad B_q \triangleq \frac{\beta_1 b_\alpha - \beta_2 b_q}{\Omega_\alpha} + b_q$$

Note that Z_G and M_G are the stability derivatives for acceleration and pitch rate associated with wind gust effects. It can be seen from Eqs. (17) and (18) that $Z_G = -Z_\alpha$ and $M_G = -M_\alpha$; if these relationships hold, they imply that $D_G = 0$ and $H_G = 0$. Interestingly, this results in only the acceleration equation being directly affected by the wind gusts and then only by $\dot{\alpha}_G$. Although not directly affected by wind gusts, the pitch rate is affected by the wind gusts through the acceleration term. The aircraft dynamics can now be written as

$$\dot{A} = D_A A + D_q q + D_e e + \Omega_e \dot{e} - \Omega_\alpha \dot{\alpha}_G + B_A \tag{23}$$

$$\dot{q} = H_A A + H_q q + H_e e + B_q \tag{24}$$

Note that B_A and B_q are biases associated with A and q, respectively. In the next sections, the elevator actuator and wind gust models are described.

Elevator Dynamics

A measurement of the elevator deflection is not available. The first-order elevator actuator dynamics, which determine the actual position of the elevator in response to an elevator command e_c, are assumed to be of the form

$$\dot{e} = -H_1 e + H_2 e_c \tag{25}$$

where H_1 and H_2 reflect the dominant dynamical characteristic of the elevator actuator. As in Ref. 8, the actuator dynamic coefficients are assumed to remain *constant*; therefore, H_1 and H_2 need *not* be estimated. However, since the actual elevator deflection is not measured, it must be estimated on-line.

Wind Gust Dynamics

The wind gust model is also included in the formulation. Two commonly accepted wind gust models used in the analysis of aircraft motion are the Dryden and the von Kârmân models.[9] For estimation purposes, a simplified Dryden model[9,10] which compares very well with the Dryden model, is used. The simplified Dryden wind gust model is

$$\ddot{\alpha}_G = K_1\alpha_G + K_2\dot{\alpha}_G + \omega_G(t) \tag{26}$$

where ω_G is a zero-mean white noise process with spectral density $K_3^2 Q_w$ and

$$Q_w = 2\frac{C_2}{C_3}\frac{\sigma^2 L_w}{u} \qquad \underline{C}_2 = \frac{1+3\beta}{2\beta^{4/3}}, \qquad \underline{C}_3 = \frac{(1+\beta)^{4/3}}{\beta^{4/3}}$$

$$\beta = \frac{b}{2L_w}, \qquad L_w = L_\infty\frac{h}{h+h_0}$$

$$K_1 = -\underline{C}_3\frac{u^2}{L_w^2}, \qquad K_2 = -\underline{C}_2\frac{u}{L_w}, \qquad K_3 = \underline{C}_3\frac{u}{L_w^2}$$

where h is the altitude (in ft), $h_0 = 2500$ ft, $L_\infty = 2000$ ft, b the wing span (in ft), u the aircraft velocity (in ft/s), and σ the rms gust velocity (in ft/s). By letting $\alpha_1 = \alpha_G$ and $\alpha_2 = \dot{\alpha}_G$, the following set of linear equations is obtained from Eq. (26):

$$\dot{\alpha}_1 = \alpha_2 \tag{27}$$

$$\dot{\alpha}_2 = K_1\alpha_1 + K_2\alpha_2 + K_3\omega_G(t) \tag{28}$$

Augmented State Variables and Modified Nonlinearities

If Eqs. (27), (28), and (25) are augmented to the dynamics of Eqs. (23) and (24), then the dynamical system is expanded to fifth order in the states A, q, e, α_1, and α_2. If in addition, the constant biases B_A and B_q are augmented as states, then we obtain a seventh-order system.

However, the system dynamics are *not* modifiable since the unmeasured state e multiplies some of the parameters to be estimated. However, the system dynamics can be made modifiable by replacing state e with *two* new states defined as

$$X_1 \triangleq \bar{D}_e e, \quad X_2 \triangleq H_e e \tag{29}$$

where $\bar{D}_e \triangleq D_e - H_1\Omega_e$. Note that e_c, the commanded elevator position, is available and is assumed to be known perfectly. This results in the following modifiable dynamical system:

$$\begin{bmatrix} \dot{A} \\ \dot{q} \\ \dot{X}_1 \\ \dot{X}_2 \\ \dot{\alpha}_1 \\ \dot{\alpha}_2 \\ \dot{B}_A \\ \dot{B}_q \end{bmatrix} = \begin{bmatrix} D_A & D_q & 1 & 0 & 0 & -\Omega_\alpha & 1 & 0 \\ H_A & H_q & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & -H_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -H_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & K_1 & K_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\times \begin{bmatrix} A \\ q \\ X_1 \\ X_2 \\ \alpha_1 \\ \alpha_2 \\ B_A \\ B_q \end{bmatrix} + \begin{bmatrix} \Omega_e \\ 0 \\ H_2\bar{D}_e \\ H_2 H_e \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} e_c + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \omega_G \\ 0 \\ 0 \end{bmatrix} \tag{30}$$

where $\bar{\Omega}_e \triangleq H_2\Omega_e$. Note that by this device the parameters $\bar{D}_e$ and H_e are now multiplied by e_c, which is a known input. Thus, with acceleration and pitch rate measurements, the nonlinearities are modifiable nonlinearities. As stated in Ref. 7, it is important that the system be observable. By using the observability test in Ref. 11, it can be shown that this system is observable.

Transformation from Continuous to Discrete Form

Since the discrete version of the MGEKF will be implemented, the aircraft dynamical equations must be transformed into discrete form. By assuming the sample time Δt to be sufficiently small, the discrete dynamical equations are approximately

$$\begin{bmatrix} A \\ q \\ X_1 \\ X_2 \\ \alpha_1 \\ \alpha_2 \\ B_A \\ B_q \end{bmatrix}_{i+1} = \begin{bmatrix} D_A & D_q & 1 & 0 & 0 & R_3 & \Delta t & 0 \\ H_A & H_q & 0 & 1 & 0 & 0 & 0 & \Delta t \\ 0 & 0 & C_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & C_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & \Delta t & 0 & 0 \\ 0 & 0 & 0 & 0 & R_1 & R_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}_i \times \begin{bmatrix} A \\ q \\ X_1 \\ X_2 \\ \alpha_1 \\ \alpha_2 \\ B_A \\ B_q \end{bmatrix}_i + \begin{bmatrix} S_A \\ S_q \\ C_2 D_e \\ C_2 H_e \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}_i e_{c_i} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \omega_{\alpha 1} \\ \omega_{\alpha 2} \\ 0 \\ 0 \end{bmatrix} \tag{31}$$

$$D_{A_i} \approx D_A \Delta t + 1, \qquad D_{q_i} \approx D_q \Delta t$$

$$D_{e_i} \approx \bar{D}_e \Delta t, \qquad H_{A_i} \approx H_A \Delta t$$

$$H_{q_i} \approx H_q \Delta t + 1, \qquad H_{e_i} \approx H_e \Delta t$$

$$S_{A_i} \approx H_2 [\Omega_e (D_{A_i} + 1) + D_{ei}] \frac{\Delta t}{2}$$

$$S_{q_i} \approx H_2 (\Omega_e H_{A_i} + H_{e_i}) \frac{\Delta t}{2}$$

Note that $x_{1_i} = \bar{D}_{e_i} e_i$ and $x_{2_i} = H_{e_i} e_i$ and that the following parameters are assumed known at each time step i:

$$C_1 = e^{-H_1 \Delta t}, \qquad C_2 = \frac{H_2}{H_1}(1 - C_1)$$

$$\omega_{\alpha 1} = R_4 \eta_G, \qquad \omega_{\alpha 2} = R_5 \eta_G$$

$$R_1 \approx K_1 \Delta t, \qquad R_2 \approx K_2 \Delta t + 1, \qquad R_3 \approx -\Omega_\alpha \Delta t$$

$$R_4 \approx \frac{K_3 (\Delta t)^2}{2}, \qquad R_5 \approx \frac{(2 + K_2 \Delta t) K_3}{2}$$

where η_G is a zero-mean noise sequence with variance $Q_w/\Delta t$. In making the discrete approximation, the exact discrete form is used when convenient; otherwise, the above is the first term of a Taylor series in Δt.

IV. Implementing the MGEKF

The MGEKF algorithm, formulated in Eqs. (10–14), is to be applied to the problem of estimating the aircraft states and parameters. This algorithm is extended from a single output to the output of acceleration and pitch rate. We have already shown that the dynamic nonlinearities are modifiable nonlinearities. The acceleration and pitch rate measurements are

$$z_{A_i} = A_i + v_{A_i}, \qquad z_{q_i} = q_i + v_{q_i} \tag{32}$$

where v_A and v_q are constant standard deviations associated with the measurement noise of A and q, respectively, so that

$$Y_i = \begin{bmatrix} v_A^2 & 0 \\ 0 & v_q^2 \end{bmatrix} \tag{33}$$

The formulation of the MGEKF algorithm requires that the matrices $F(z_i, \hat{x}_i)$, H, and $f(\hat{x}_i)$ be formed from the aircraft dynamic system [Eq. (31)] and measurements [Eq. (32)]. Since we did not include the biases (B_A, B_q) in our linear simulation, they are not included in the state space defined now as

$$x_i^T \triangleq [A, q, X_1, X_2, \alpha_1, \alpha_2, D_A, D_q, H_A, H_q, S_A, S_q, D_e, H_e]_i \tag{34}$$

and the measurement is defined as the two-vector

$$z_i \triangleq [z_A, z_q]_i^T \tag{35}$$

Then

$$H = [I_{2\times 2}, 0_{2\times 12}] \tag{36}$$

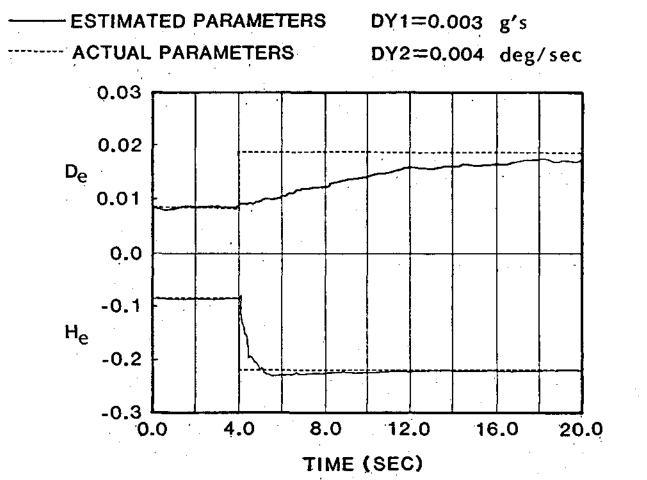

Fig. 1 Parameter tracking with a step change in flight conditions, WG = 1 ft/s, accurate instruments, and low-amplitude dither signal.

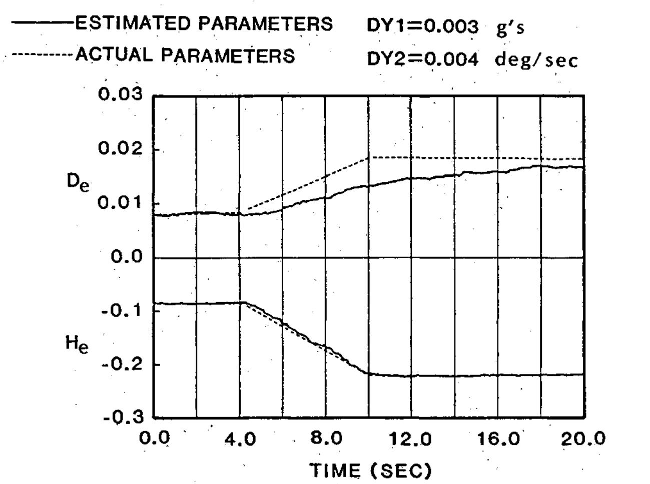

Fig. 2 Parameter tracking with a ramp change in flight conditions, WG = 1 ft/s, accurate instruments, and low-amplitude dither signal.

$$F(z_i, \hat{x}_i) = \begin{bmatrix} \hat{D}_A & \hat{D}_q & 1 & 0 & 0 & R_3 & z_A & z_q & 0 & 0 & e_c & 0 & 0 & 0 \\ \hat{H}_A & \hat{H}_q & 0 & 1 & 0 & 0 & 0 & 0 & z_A & z_q & 0 & e_c & 0 & 0 \\ 0 & 0 & C_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & C_2 e_c & 0 \\ 0 & 0 & 0 & C_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & C_2 e_c \\ 0 & 0 & 0 & 0 & 1 & \Delta t & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & R_1 & R_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & 0_{8\times 6} & & & & & & I_{8\times 8} & & & & & \end{bmatrix}_i \tag{37}$$

$$f(\hat{x}_i) = F(0, \hat{x}_i)\hat{x}_i \tag{38}$$

Note the placement of the measurements in the error dynamics matrix $F(z_i, \hat{x}_i)$ [see Eqs. (6) and (13)]. Each measurement multiplies the corresponding parameter error for that particular state. For example, the acceleration measurement z_A multiplies its corresponding parameter error $(D_A - \hat{D}_A)$. No parameter associated with any of the states except acceleration and pitch rate is identified. Since input e_c is multiplied by C_2 in the rows associated with states X_1 and X_2, the estimated parameters are now D_e and H_e rather than $C_2 D_e$ and $C_2 H_e$.

The Process Noise Covariance Matrix and Filter Tuning

Process noise is assumed for the parameters [see Eq. (2)] in order to keep the filter gains associated with the parameters from going to zero. The values of the process noise variance for the parameters are chosen by tuning the filter to obtain best performance. The discrete stochastic equation is

$$x_{i+1} = F(0, x_i)\, x_i + w_i \tag{39}$$

where w_i is composed of the six-dimensional vector $\underline{w}_i$ associated with the states and the eight-dimensional vector $\overline{w}_i$ associated with the parameters. Note that $w_3(i)$, $w_4(i)$, $w_5(i)$, and $w_6(i)$ are elements of $\overline{w}_i$ that have the state-dependent forms

$$w_3(i) = (C_1 e + C_2 e_c)_i w_{13}(i) + D_e w_e(i)$$

$$w_4(i) = (C_2 e + C_2 e_c)_i w_{14}(i) + H_e w_e(i)$$

$$w_5(i) = \omega_{\alpha 1}(i), \qquad w_6(i) = \omega_{\alpha 2}(i)$$

while the remaining noise processes are assumed independent.

The process noise covariance matrix $Q_i = E[w_i w_i^T]$ has values along its diagonal and the only nonzero off-diagonal elements are

$$Q_{3,4} = Q_{4,3} = D_e H_e Q_{3,3}$$

$$Q_{5,6} = Q_{6,5} = R_4 R_5 Q_w / \Delta t$$

$$Q_{3,13} = Q_{13,3} = \bar{e} Q_{13,13}$$

$$Q_{4,14} = Q_{14,4} = \bar{e} Q_{14,14}$$

$$\bar{e} = C_1 e + C_2 e_c$$

where the dependence on the time stage i has been suppressed. Since some of the off-diagonal elements of Q_i depend on the parameters or states, they must be approximated. The adaptive form of the process noise covariance matrix suggested in Ref. 7 uses the current estimates of the parameters to adapt Q_i.

The first choice for the process noise standard deviations associated with the states other than wind gusts was to set them to zero, since no modeled process noises exist on any of the states except wind gusts. However, to enhance the MGEKF performance in the presence of modeling inaccuracies, small power spectral densities are assumed. Although the same reasoning can be used in choosing the standard deviations associated with the estimated parameters, it was discovered experimentally that setting the standard deviation of the parameters to about the magnitude of the difference between the minimum and maximum values of the parameters over the flight envelope yielded the best results.

V. Computer Simulation and Control System Design Consideration

The aircraft simulation routine provides the MGEKF routine with acceleration A, pitch rate q, commanded elevator position e_c, forward velocity u, and altitude h. The MGEKF uses the measurements of A, q, and e_c directly, while the wind gust model uses u and h to calculate wind gust coefficients in $F(z_i, \hat{x}_i)$.

The simulation uses an exact discrete form of the continuous dynamic equations. The trim biases are not included. The dynamic system is persistently excited by an oscillatory dither of the elevator input in order to estimate the parameters. The dither signal maintains an adequate signal-to-noise ratio in the absence of pilot input, enabling the filter to differentiate between the response of the dynamical system and noise on the system.[4]

Shape, amplitude, and frequency are the three major aspects of the dither signal important to the performance of the MGEKF. A sinusoidal dither signal composed of three frequencies gave good results. Two frequencies are at the high (4.3 rad/s) and low (1.8 rad/s) ends of the expected short-period frequency of the aircraft over the flight envelope. The third is at the frequency of the higher-order actuator (20 rad/s) because experimentation indicated that the parameters associated with the control (S_A, S_q, D_e, and H_e) are more easily identified if a frequency corresponding to the natural frequency of the actuator is included in the dither. Use of these frequencies results in the improved performance of the MGEKF, which allows a decrease in the amplitude of the dither signal while still maintaining performance.

In order to determine the effect of sensor accuracy on the performance of the filter, the accelerometer noise standard deviations are alternately set to 0.003 and 0.03 g. The pitch rate gyro noise standard deviations are alternately set to 0.004 and 0.04 deg/s. Finally, three levels of clear air turbulence were considered: $\sigma = \mathrm{WG} = 1$, 5, and 15 ft/s.

The objective of our numerical experiments are to show how the MGEKF tracks the states and stability deviations through a change in flight conditions from an altitude of 15,000 ft and a Mach number of 0.6 to an altitude of 500 ft and a Mach number of 0.69. In the linear simulation, the transition from one flight condition to the other was formed either as a step or a ramp. The ramp is essentially a linear interpolation between the parameters associated with each flight condition over a 6 s period. Since only the dither signal excites the aircraft during the transition, it is expected that the performance shown here is somewhat conservative, since changes in flight conditions require control inputs that will generate additional acceleration and pitch rate.

The final conditions of a 10 s convergence run are used as the initial conditions for each run. This represents a *steady-state* starting condition. A ramp change from one aircraft flight condition to another occurs between 4.0 and 10.0 s, with no changes made from 10.0 s to the end of the run at 20.0 s.

In the following figures, solid lines represent the estimated parameter and the dashed lines represent the actual parameter. Also printed on these figures are DY1 and DY2, the standard deviations of the measurement noise on the accelerometer and the pitch rate gyro, respectively. WG is the rms value of the wind gusts, which indicates the process noise on the system. The sample frequency is 100 Hz.

Only the parameters D_e and H_e are used to compare results of variations in system design and model design, because D_e is a good indicator of the tracking characteristics of the other parameters associated with the acceleration equations and H_e is a good indicator of the pitch rate parameters.

A step jump in flight conditions shown in Fig. 1 indicates the step response characteristics of the MGEKF. The slow response in $\hat{D}_e$ is characteristic of the parameters associated with the acceleration state. This is due to the wind acceleration term in the acceleration equation. In contrast, note the rapid response of the estimated parameter $\hat{H}_e$.

The Dryden wind gust model is obtained empirically from many atmospheric studies[10]; therefore, the wind gust characteristics of the real atmosphere will not exactly correspond to the assumed wind gust model in Eqs. (22) and (28). This fact must be considered when analyzing the performance of the MGEKF, since its performance may suffer if the actual

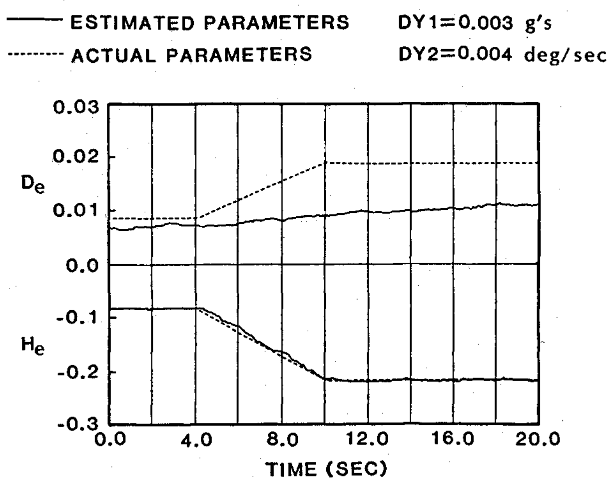

Fig. 3 Parameter tracking with a ramp change in flight conditions, WG = 5 ft/s, accurate instruments, and low-amplitude dither signal.

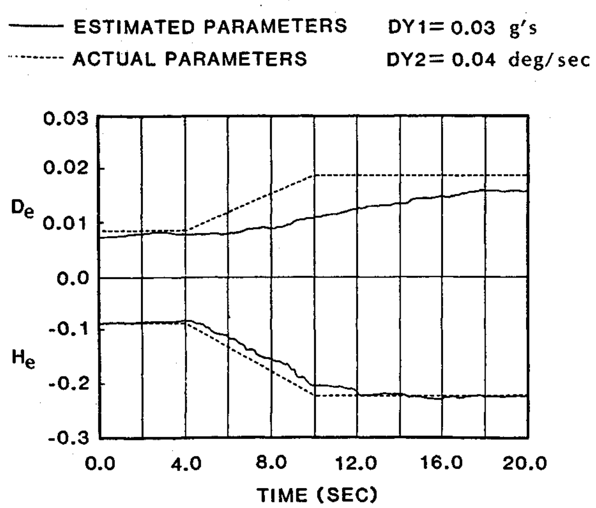

Fig. 4 Parameter tracking with a ramp change in flight conditions, WG = 1 ft/s, reduced accuracy instruments, and low-amplitude dither signal.

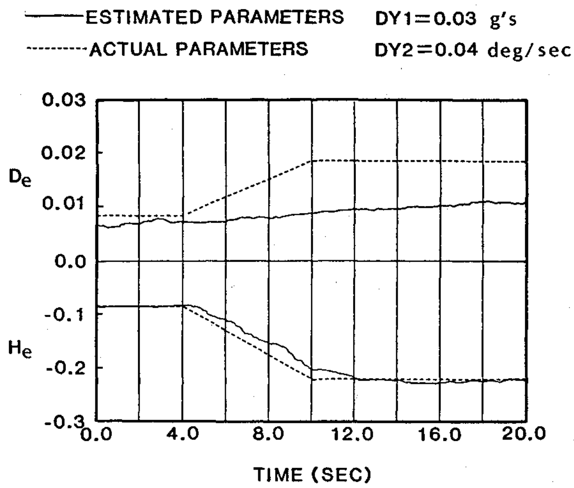

Fig. 5 Parameter tracking with a ramp change in flight conditions, WG = 5 ft/s, reduced accuracy instruments, and low-amplitude dither signal.

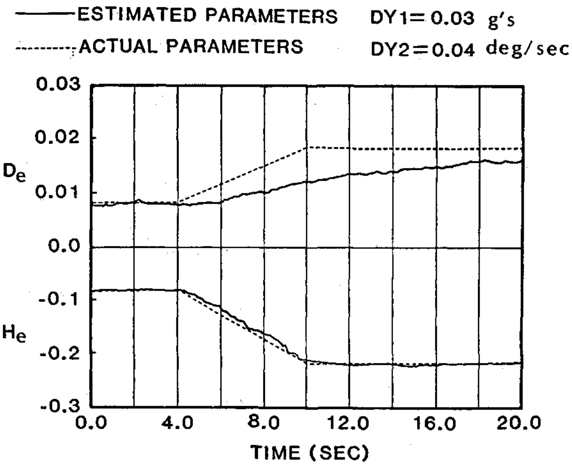

Fig. 6 Parameter tracking with a ramp change in flight conditions, WG = 5 ft/s, reduced accuracy instruments, and increased-amplitude dither signal.

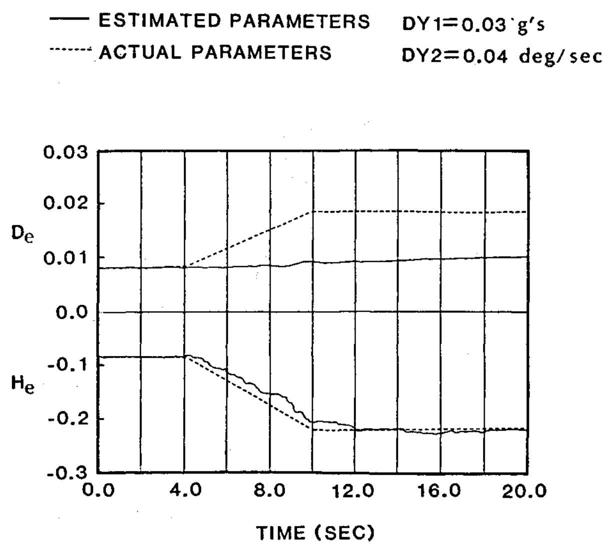

Fig. 7 Parameter tracking with a ramp change in flight conditions, WG = 5 ft/s in filter, WG = 1 ft/s in simulation, reduced accuracy instruments, and low-amplitude dither signal.

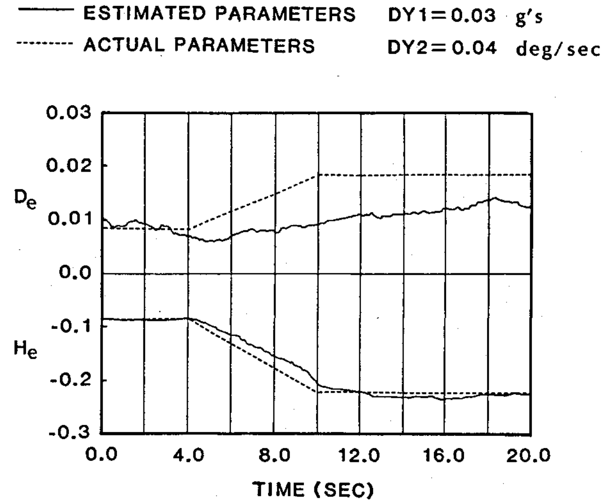

Fig. 8 Parameter tracking with a ramp change in flight conditions, WG = 5 ft/s in filter, WG = 15 ft/s in simulation, reduced-accuracy instruments, and low-amplitude dither signal.

wind gust characteristics are mismatched. The effects are negligible at WG = 5 ft/s when a run having a 15,000 ft mismatch between the actual wind gust dynamics and the wind gust model is compared with the matched case with all other factors being the same.

For the same dither signal and instrument accuracy, Figs. 2 and 3 show the effect of changing clear air turbulene from WG = 1 to 5 ft/s, where the corresponding rms acceleration over the 20-s flight for each profile is 0.0926 and 0.1195 g, respectively. The effect of reduced instrument accuracy on the low-amplitude dither signal is shown in Figs. 4 and 5. Note that, with clear air turbulance of WG = 5 ft/s, comparing Figs. 3 and 5 shows that the effect of an improved measurement device has little effect on D_e but has some effect on H_e. The effect of an increase in dither signal amplitude is shown in Fig. 6, where the rms acceleration over the 20-s flight is 0.4 g. Note that now Figs. 2 and 6 have about the same profile even with reduced instrument accuracy. However, large accelerations and pitch rates produce pilot and passenger discomfort. Finally, Figs. 7 and 8 show the mismatched performance of the filter of Fig. 5 (WG = 5 ft/s) with clear air turbulence in the simulation is lower (WG = 1 ft/s) and higher (WG = 15 ft/s) than WG = 5 ft/s, respectively. Note that R_{3_i} in Eq. (31), which can be determined from the estimated parameters, need be chosen only at some nominal value since WG is never known. That is, if $R_{3_i}\alpha_{2_i}$ is chosen as a state variable, then the wind intensity becomes R_{3_i}WG. The results shown are for a Monte Carlo average of 10 runs.

VI. Conclusion

Although additional investigation of the modified-gain extended Kalman filter (MGEKF) is needed, the results from this study indicate that the MGEKF displays reasonable performance in the areas of convergence characteristics, disturbance rejection, and response to system changes. The slow response of the parameter estimation error associated with the acceleration equation is due mostly to the inclusion of the high-frequency gust acceleration term. Improvements might be made by adding an angle-of-attack meter that measures the relative motion between the aircraft and the air mass. Therefore, the MGEKF seems well suited to application in an adaptive control scheme. The MGEKF can provide state and parameter estimates to a set of control laws that use these estimates to adapt to changing flight conditions. The controller should be designed to rely most heavily on the parameters associated with pitch rate. This is not unreasonable since H_e, which is essentially the change in moment due to elevator deflection, is estimated well and is important in designing responsive flight control systems.

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