Conclusions

In this Note, methods for the determination of reduction indices from the resonance test data of a flexible structure are presented. If the actuators are in a cascade connection with a flexible structure, the reduction indices of the structure are reconstructed from the indices of the actuator-structure joint dynamics.

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Appendix

For a flexible structure in modal coordinates $A = \mathrm{diag}(A_i)$, $B^T = [B_1^T, \ldots, B_n^T]$, $C = [C_1, \ldots, C_n]$, and

$$A_i = \begin{bmatrix} -2\zeta_i\omega_i, & \omega_i \\ \omega_i & 0 \end{bmatrix}, \quad B_i^T = [b_i^T \; 0] \quad C_i = [c_{ri} \; c_{di}/\omega] \tag{A1}$$

where ω_i, ζ_i are modal frequency and damping, respectively; b_i is the row vector of actuator locations; and c_{di}, c_{ri} are column vectors of displacement and rate sensors locations, respectively. For the transfer function $H = C(j\omega I - A)^{-1}B$ one obtains from Eq. (A1)

$$H = \sum_{i=1}^{n} H_i, \qquad H_i = C_i(j\omega I - A_i)^{-1}B_i \tag{A2}$$

Since for $\omega = \omega_i$ one has $||H_k|| \ll ||H_i||$, for $i \neq k$, therefore, one has $H(\omega_i) \cong H_i(\omega_i)$. Next, from Eq. (A1), after some algebra one finds that

$$H_i = a_i c_i b_i \tag{A3}$$

where $a_i = 1/(2\zeta_i\omega_i^2)$, $c_i = \omega_i c_{ri} - jc_{di}$. Define $\cos\Psi = ||c_i b_i|| / (||c_i|| \; ||b_i||)$, which is a measure of collinearity of c_i and b_i; then the norm of H_i, denoted Y_i, is the norm of the output at frequency $\omega = \omega_i$:

$$Y_i = ||H_i(\omega_i)|| = a_i ||b_i|| \cos\Psi\sqrt{(||c_{di}||^2 + ||c_{ri}||^2\omega_i)}$$

$$= a_i ||b_i|| \, ||c_i|| \cos\Psi_i \tag{A4}$$

Note that for orthogonal c_i and b_i one obtains $H_i(\omega_i) = 0$; also, $\cos\Psi_i = 1$ for a single-input as well as for a single-output system and for collocated sensors and actuators.

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Angle-Only Tracking Filter in Modified Spherical Coordinates

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I. Introduction

THE Kalman filter by Hoelzer et al.[1,2] for planar tracking of a nonmaneuvering target appears to be superior to previous approaches because of its use of modified polar coordinates (MPC), which reduce the problems with observability, range bias, and covariance ill-conditioning that are encountered with Cartesian coordinates.

Here, the MPC filter is extended to three dimensions by the use of modified spherical coordinates (MSC).[3] In the MSC filter, the six state variables are two angles, their derivatives, inverse range, and range rate over range, which are transformable into Cartesian position and velocity.

This MSC filter differs from that of Ref. 4, which has five state variables, angle plus range measurements, and is apparently restricted to a surface target.

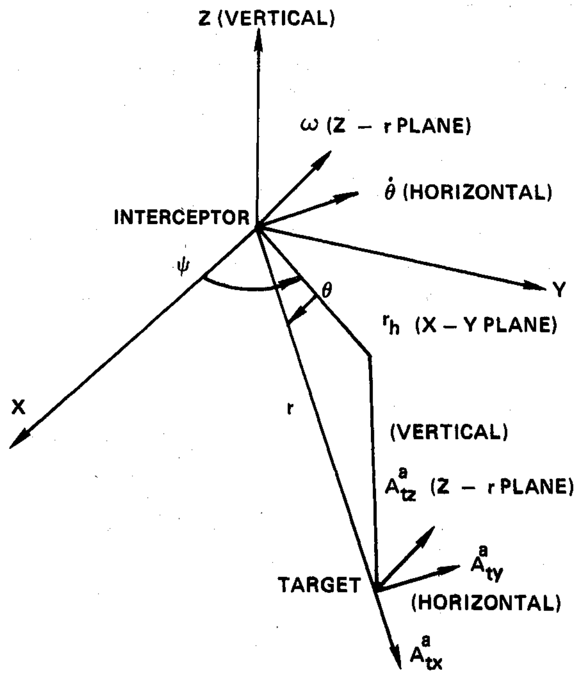

Fig. 1 Basic coordinates for MSC filter.

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II. Tracking Filter in Modified Spherical Coordinates

This Note is a condensation and revision of Ref. 3. Figure 1 shows the nonrotating, Earth-oriented coordinates on the aircraft; the line of sight (LOS) to the target has two angles and two perpendicular angular-rate vectors, i.e. $\dot{\theta}$ (horizontal) and ω. An extended Kalman filter (EKF)[5] for a nonmaneuvering target has been developed with the following state vector in MSC:

$$x = \left[\theta\ \dot{\theta}\ \psi\ \omega\ \frac{1}{r}\ \frac{\dot{r}}{r}\right]^T \tag{1}$$

The plant equations[3] in scalar form are

$$\frac{\mathrm{d}\theta}{\mathrm{d}t} = \dot{\theta} \tag{2}$$

$$\frac{\mathrm{d}\dot{\theta}}{\mathrm{d}t} = -2\frac{\dot{r}}{r}\dot{\theta} - \omega^2 \tan\theta - \frac{A^a_{tz}}{r} + \frac{A^a_{mz}}{r} \tag{3}$$

$$\frac{\mathrm{d}\psi}{\mathrm{d}t} = \frac{\omega}{\cos\theta} \tag{4}$$

$$\frac{\mathrm{d}\omega}{\mathrm{d}t} = \left[-2\frac{\dot{r}}{r} + \dot{\theta}\tan\theta\right]\omega + \frac{A^a_{ty}}{r} - \frac{A^a_{my}}{r} \tag{5}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{1}{r} = -\frac{1}{r}\frac{\dot{r}}{r} \tag{6}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\dot{r}}{r} = \dot{\theta}^2 + \omega^2 - \left[\frac{\dot{r}}{r}\right]^2 + \frac{A^a_{tx}}{r} - \frac{A^a_{mx}}{r} \tag{7}$$

In Eqs. (3), (5), and (7), the terms in target acceleration (subscript t) are treated as a process noise, although they would be additional state elements in a filter for a maneuvering target.

The following heuristic argument for observability is adapted from Ref. 1. Referring to Eqs. (2-7), the angles θ and ψ are directly measured, albeit with additive measurement noise, and so the derivatives $\dot{\theta}$ and $\dot{\psi}$ are observable. If the latter variables are nonzero, Eqs. (3) and (5) show that $\dot{r}/r$ affects their derivatives and, hence, it also should be observable, even if the ownship accelerations A^a_{mz}, A^a_{my}, and A^a_{mx} in antenna coordinates are all zero. Because the state variable $1/r$ is multiplied by the known A^a_{mz}, A^a_{my}, and A^a_{mx} in Eqs. (3), (5), and (7), respectively, a nonzero value of any acceleration leads to enhanced observability of $1/r$.

In keeping with the choice of x in Eq. (1), Eqs. (2-7) in MSC are used to obtain a transition matrix for covariance propagation, gain calculation, and update. On the other hand, it seems desirable to predict the state estimate forward in Cartesian coordinates, for the sake of simplicity and accuracy. Otherwise, the approach to this nonlinear filtering problem is essentially that of Ref. 5.

Both theory and simulation indicate the superiority of MSC over Cartesian coordinates for the case of no range measurements. It is clear that a major uncertainty in range would cause an EKF in Cartesian coordinates to have strongly coupled covariances in the errors of the three position coordinates, which leads to computational difficulty. This is avoided in MSC, as a consideration of Eqs. (1-7) helps to show.

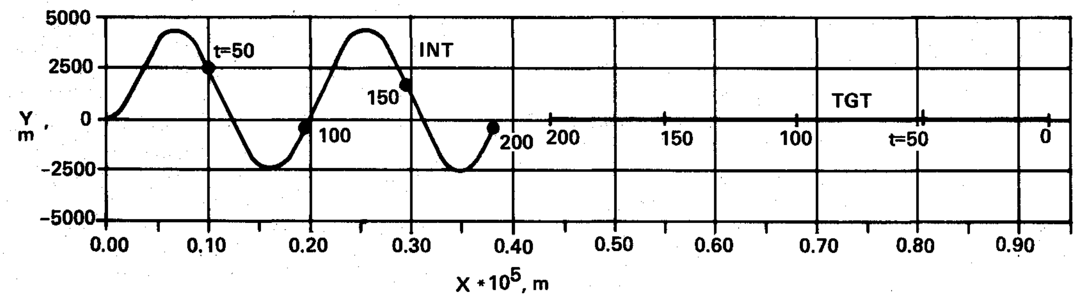

Fig. 2 Trajectories of interceptor aircraft and target in horizontal plane.

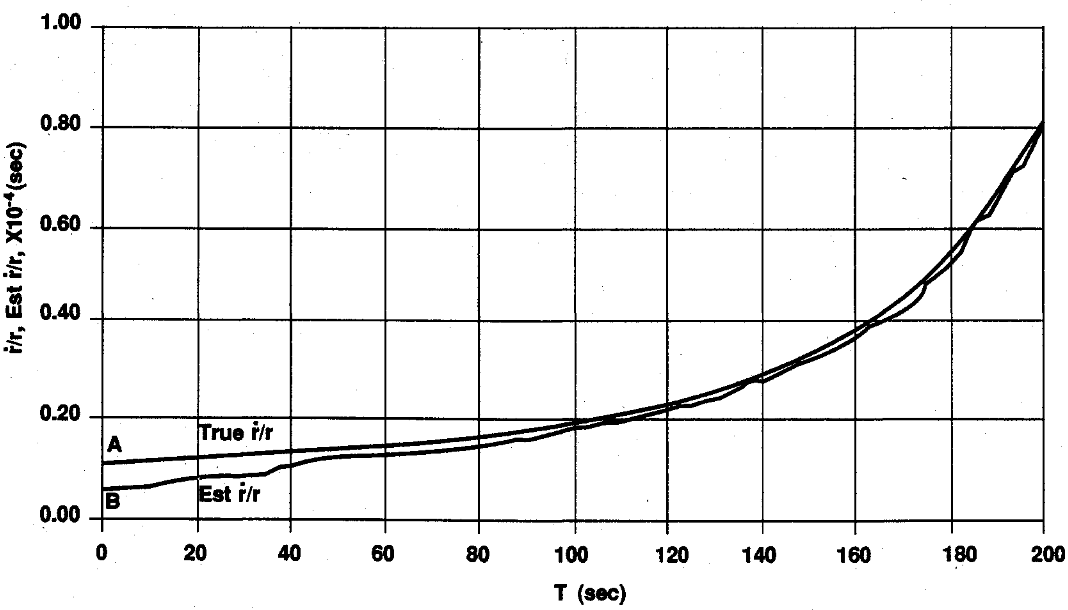

Fig. 3 True and estimated $\dot{r}/r$ vs time.

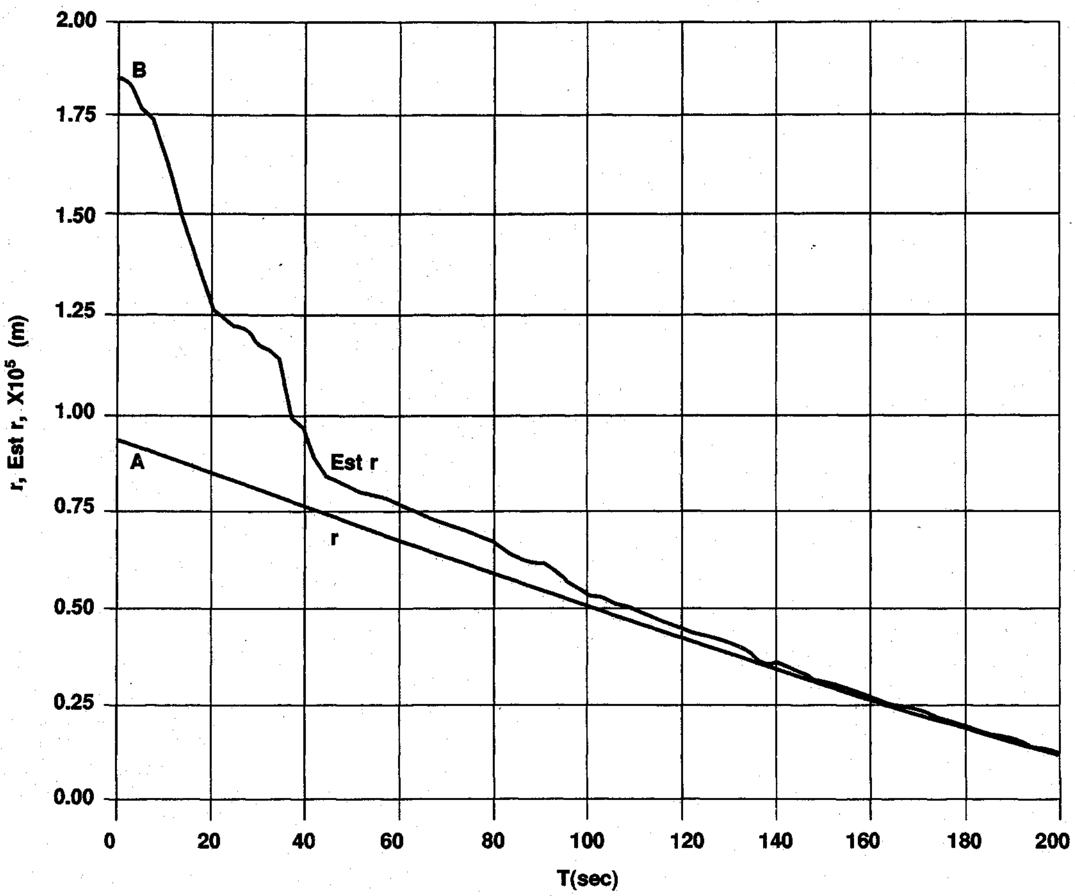

Fig. 4 True and estimated relative range vs time.

III. Simulation Results

An interceptor aircraft, a bomber target, and the six-state filter have been simulated in Advanced Continuous Simulation Language (ACSL).[6] The interceptor flies initially north (parallel to the inertial X axis) at 242.6 m/s (Mach 0.8) at an altitude of 10.668 km (35 kft), while the target flies south with a velocity of 242.6 m/s at an altitude of 305 m (1 kft) without any maneuver.

At an initial X separation of 92.659 km (50 N.mi.), the interceptor detects the jamming target (which denies range measurement) and begins a weaving maneuver with a horizontal acceleration of 2 g to a heading 45-deg west of due north, followed by a short straight leg, a 90-deg turn to the right, a 30-s straight leg, a 90-deg left turn, etc. See the trajectory in Fig. 2.

Once every scan interval of 2.5 s, the interceptor measures the angles θ and ψ in Fig. 1. These measurements, with range-independent noise having a standard deviation of 1 mrad, are utilized in the six-state filter as described in the previous subsection and Ref. 3. The initial range estimate is twice the true value, and the initial estimate of target velocity is 20% high.

A simulation run with noise has shown that the estimates of the first four state variables in Eq. (1) are quite satisfactory. Figure 3 indicates that the error in the estimate $\hat{x}_6$ is initially reduced to a low value at 44 s and stays satisfactorily low thereafter. Figure 4 shows that the error in the estimated range is dramatically reduced by 44 s, but does not converge to a very small value until about 100 s. Qualitatively, this slower convergence for $\hat{x}_5$ than for $\hat{x}_6$ would be expected from Ref. 1. Results for a simulation with much less altitude difference in Ref. 3 were similar.

IV. Conclusions

An examination[3] of previous work in angle-only tracking filters has shown the importance of choosing a good coordinate system, such as modified polar coordinates.[1,2] The work herein extends this approach to three dimensions with modified spherical coordinates.[3] The six-state filter successfully estimates the equivalents of position and velocity in three dimensions for a nonmaneuvering target, using only moderate accelerations of the tracking aircraft. Although the particular application was for "track-while-scan" by an aircraft, the algorithm could also be used by a homing missile with a faster data rate, probably in antenna-based coordinates.

Questions remain as to a complete theory, design refinements, desirable trajectories of the measuring vehicle, extension to a maneuvering target, and limitations of the new approach.

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