



Fig. 1 Geometric interpretation of projection matrices rr^T and $I - rr^T$.

Three-Dimensional Ranging and Multiple-Axes Measurements

These equations inherently apply to the full three-dimensional, bearings-only ranging filter. Multiple measurement axes are incorporated into the derivations by letting $a = [a_1, a_2]$, where a_1 and a_2 are the noncolinear measurement axes.

Geometric Interpretation

Examination of the matrices rr^T and $I - rr^T$ provide insight to the fundamental geometry of the target location estimation problem. These matrices are symmetric, idempotent orthogonal projection matrices.⁴ The matrix rr^T projects all vectors onto the space spanned by r . The matrix $I - rr^T$ is the orthogonal complement of rr^T and projects all vectors onto the space orthogonal to r (the null space rr^T). This relationship is readily seen by noting that $(I - rr^T)r = 0$. This geometrical significance is illustrated in Fig. 1.

The expressions $rr^T V$ and $(I - rr^T)V$ are understood to be the radial and circumferential velocities of the platform about the target, respectively. The expression $(1/|R|)(I - rr^T)V$ represents the angular velocity of the platform about the target. The similarity of Eqs. (3) and (6) is a consequence of the fact that Eq. (3) describes the effect of the relative motion of the platform moving about the target and Eq. (6) describes the effect of the relative motion of the target moving about the platform.

Conclusions

By defining the measurement equation for a bearings-only estimation filter in terms of the directional cosine to a known axis rather than the angle to the axis, the resulting filter equations are algebraic rather than transcendental. The three-dimensional problem and the multiple-axes problem is inherently solved, and additional insight to the geometry of the problem is achieved.

References

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- ²Aidala, V. J., and Hammel, S. E., "Utilization of Modified Polar Coordinates for Bearings-Only Tracking," *IEEE Transactions on Automotive Control*, Vol. AC-28, No. 3, 1983, pp. 283-294.
- ³Springarn, K., "Passive Position Location Estimation Using the Extended Kalman Filter," *IEEE Transactions on Aerospace and Electronic Systems*, Vol. AES-23, No. 4, 1987, pp. 558-567.
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Book Announcements

VARDULAKIS, A. I. G., *Linear Multivariable Control: Algebraic Analysis and Synthesis Methods*, Wiley, Chichester, England, U.K., 1991, 369 pages, \$125.00

Purpose: This book presents a detailed account of polynomial matrix descriptions (PMDs) of linear multivariable control systems. It is meant for graduate students and researchers.

Contents: Real rational vector spaces and rational matrices; polynomial matrix models; pole and zero structure of rational matrices at infinity; dynamics of polynomial matrix models; proper and Ω -stable rational functions and matrices; feedback system stability and stabilization; some algebraic design problems.

HABETS, L. C. G. J. M., *Robust Stabilization in the Gap-Topology*, Lecture Notes in Control and Information Sciences, Vol. 150, Springer-Verlag, Berlin, 1991, 126 pages, \$25.00.

Purpose: This monograph deals with robust stabilization based on the notion of the gap between two closed subspaces.

Contents: Robust stabilization; the gap-topology; sufficient conditions for robust stabilization; optimally robust control and controller design; order reduction of the compensator.

AHMED, A., *Semigroup Theory with Applications to Systems and Control*, Longman Scientific and Technical, U.K., Copublished in the U.S. with Wiley, New York, 1991, 282 pages, \$40.00.

Purpose: This monograph is an introduction to semigroup theory, a unified theory for the study of differential equations

on Banach space. Applications to control and stability are treated.

Contents: Basic properties of semigroups; generation theorems for semigroups; special properties; perturbation theory for semigroups; differential equations on Banach space; stochastic differential equations; applications to systems and control.

APLEVICH, J. D., *Implicit Linear Systems*, Lecture Notes in Control and Information Sciences, Vol. 152, Springer-Verlag, Berlin, 1991, 176 pages, \$31.00.

Purpose: This monograph treats the implicit model of linear dynamical systems and its applications to analysis and design.

Contents: System models; the Kronecker form; analysis of singularities; systems of minimal dimension; canonical representations; algebraic design applications; optimization with quadratic cost; system identification; large-scale systems; extensions.

KAMP, Y. and HASLER, M., *Recursive Neural Networks for Associative Memory*, Wiley, Chichester, England, U.K., 1990, New York, 1991, 194 pages.

Purpose: This book discusses the different problems which arise in the analysis and design of discrete time, discrete valued recursive networks.

Contents: Principles, problems, and approaches; deterministic and statistical approaches; thermodynamic extension; higher order networks; network design.