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## Pilot Prefilter Design via $H_\infty$ Model-Matching Approach

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### Introduction

IN recent years the development of relaxed static stability airplanes has required that flight control systems provide higher levels of augmentation for rapid and precise response. Providing acceptable pilot-in-the-loop handling qualities is a critical design criteria for these highly augmenting control systems.<sup>1–3</sup> Many modern fighter aircraft utilize pilot prefilter, which compensate the pilot's command to improve handling qualities to a satisfactory level while maintaining the excellent stability properties provided by feedback controller design. The idea of separating feedback design from prefilter design, which has been around for more than a decade, is to simultaneously achieve excellent stability properties and desired pilot-in-the-loop behavior.<sup>4,5</sup>

A conventional optimization approach to pilot prefilter design employs a quadratic optimization technique to minimize the model-matching error between the ideal pilot-aircraft dynamics and the actual dynamics. Techniques other than optimization approaches also have been used in the past for the design of the prefilter.<sup>6</sup> The purpose of this Note is to propose an  $H_\infty$  model-matching method to design a pilot prefilter with feedforward compensation capability. Model-matching via  $H_\infty$  criterion is a worst-case design, which considers the maximum error between the ideal model and the actual model, while for quadratic criterion, the mean-square error is considered. In addition to the numerous successful applications of the  $H_\infty$  control theory to feedback controller design for flight control systems, it is shown here that the standard  $H_\infty$  framework also provides a very simple and efficient approach to pilot prefilter design.

### Problem Formulation

An experimental aircraft developed by Aeronautical Industry Development Center (AIDC) is considered here. The longitudinal mo-

tion of the aircraft at the flight condition of 0.8 Mach number and 10,000-ft altitude is described by the following state-space model:

$$\begin{bmatrix} \dot{\alpha} \\ \dot{u} \\ \dot{\theta} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} -2.191 & -0.0005 & 0 & 0.9940 \\ -5.2400 & -0.0218 & -32.17 & -0.0001 \\ 0 & 0 & 0 & 1 \\ 4.600 & -0.0006 & 0 & -1.0100 \end{bmatrix} \begin{bmatrix} \alpha \\ u \\ \theta \\ q \end{bmatrix} + \begin{bmatrix} -0.2611 & -0.150 & 0.0397 \\ 23.20 & -15.30 & -24.90 \\ 0 & 0 & 0 \\ -31.10 & 215.4 & -4.170 \end{bmatrix} \begin{bmatrix} \delta_H \\ \delta_{LEF} \\ \delta_{TEF} \end{bmatrix} \quad (1)$$

and the measurement equation is

$$\begin{bmatrix} N_z \\ q \\ \alpha \\ \theta \end{bmatrix} = \begin{bmatrix} 60.70 & 0.0010 & 0 & -0.2770 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \alpha \\ u \\ \theta \\ q \end{bmatrix} + \begin{bmatrix} -5.730 & 4.120 & -2.770 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_H \\ \delta_{LEF} \\ \delta_{TEF} \end{bmatrix} \quad (2)$$

where  $\alpha$ ,  $u$ ,  $\theta$ ,  $q$ , and  $N_z$  are the angle of attack (degrees), velocity along the longitudinal axis (feet per second), pitch angle (degrees), pitch rate (degrees per second), and normal acceleration (feet per second squared), respectively. The proposed control configuration is depicted in Fig. 1. There are three control surfaces used in the longitudinal motion, namely, elevator ( $\delta_H$ ), leading-edge flap ( $\delta_{LEF}$ ), and trailing-edge flap ( $\delta_{TEF}$ ). The bandwidth of the three surface actuators is 4 Hz. The elevator deflection signal comes from the output of the  $H_\infty$  feedback controller  $K(s)$ , which was described in Ref. 7. The LEF/TEF deflections are trim controls and are scheduled with Mach number  $M$  and angle of attack  $\alpha$ , providing the optimal lift/drag ratio. In the form of transfer function, the aircraft dynamics can be expressed alternatively as

$$\begin{bmatrix} N_z(s) \\ q(s) \\ \alpha(s) \\ \theta(s) \end{bmatrix} = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix} \begin{bmatrix} \delta_H(s) \\ \delta_{LEF}(s) \\ \delta_{TEF}(s) \end{bmatrix} \quad (3)$$

The feedback controller  $K(s) = [K_1(s) \ K_2(s)]$  that employs the measurements

$$\begin{bmatrix} N_z \\ q \end{bmatrix}$$

to generate the elevator deflection  $\delta_H = K_1(s)N_z + K_2(s)q$  has been designed<sup>7</sup> to compensate for the effect of c.g. shift and to satisfy the following specifications. 1) Short-period damping ratio  $\zeta_{sp}$

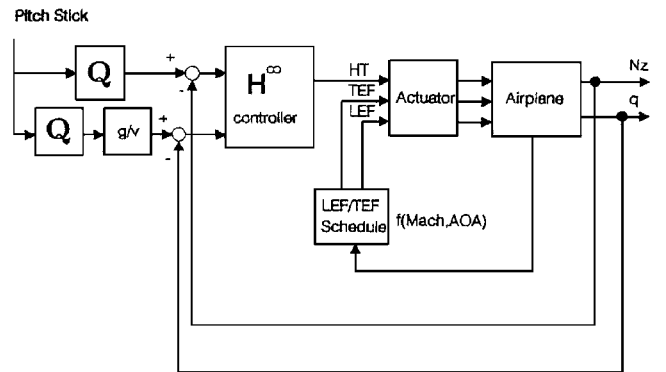


Fig. 1 Longitudinal control configuration.

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should lie between 0.5 and 0.7. 2) Short-period natural frequency should lie between 10 and 12 rad/s. 3) Gain/phase margins should be 6 dB and 45 deg, respectively. 4) The feedback loop gain should be below the loop-gain limit producing the structural coupling effect.

Although the restriction of the feedback loop gain helps avoid structural coupling effects, it may also cause insufficient control energy and degrade the handling qualities of the aircraft. For the present aircraft, we have recognized that feedback control design alone cannot satisfy the flying qualities and the handling qualities, simultaneously. To remedy this defect, pilot prefilter  $Q(s)$  is designed separately to enhance handling qualities, while at the same time, keeping the flying qualities 1–4 satisfied by the feedback controller. Handling qualities will be evaluated by 1) the bandwidth of the pitch control loop,<sup>8</sup> 2) Gibson's frequency domain tracking criteria,<sup>1</sup> and 3) Neal-Smith<sup>3</sup> pilot-in-the-loop criteria.

### $H_\infty$ Prefilter Design

According to MIL-STD-1797,<sup>8</sup> the transfer function from the stick input  $F_s$  to the pitch angle  $\theta$  can be described by an equivalent second-order system

$$\left[ \frac{\theta(s)}{F_s(s)} \right]_{\text{ideal}} = T_1(s) = \frac{k(s + 1/T_{\theta 2})e^{-\tau s}}{s^2 + 2\zeta_{sp}\omega_{sp}s + \omega_{sp}^2} \quad (4)$$

The dynamics contained in this transfer function includes pilot prefilter, feedback flight control system, and the aircraft dynamics. With the proper choice of the free parameters in Eq. (4), an ideal  $\theta(s)/F_s(s)$  function can be found such that the pilot-in-the-loop system can satisfy the handling quality criteria 1–3. On the other hand, the actual transfer function from  $F_s(s)$  to  $\theta(s)$  (see Fig. 1) is described by

$$\left[ \frac{\theta(s)}{F_s(s)} \right]_{\text{actual}} = T(s)Q(s) \quad (5)$$

where

$$T(s) = [0 \quad 1]G_{21}(s)K(s)[I + G_{21}(s)K(s)]^{-1} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

is the transfer function from the prefilter output to  $\theta$ . Once the feedback controller  $K(s)$  has been designed, the design objective is to determine the pilot prefilter  $Q(s)$  such that the actual transfer function  $T(s)Q(s)$  approximates the ideal function  $T_1(s)$  as closely as possible. This approximation can be interpreted as a

of model-matching error.  $H_\infty$  criterion considers the maximum distance between  $T_1(s)$  and  $T(s)Q(s)$ , i.e.,  $\|T_1(s) - T(s)Q(s)\|_\infty = \sup_\omega |T_1(j\omega) - T(j\omega)Q(j\omega)|$ , and the role of the pilot prefilter  $Q(s)$  is to minimize this maximum distance. Theoretically, we wish to solve the following mathematical problem:

$$\min_{Q \in RH_\infty} \|W(s)[T_1(s) - T(s)Q(s)]\|_\infty \quad (6)$$

where  $RH_\infty$  is the function space containing all rational stable functions. The role of the weighting function  $W(s)$  in Eq. (6) is to penalize the model-matching error, indicating the frequency range over which the model-matching error is of most concern. Recall that  $T_1(s)$  is the ideal transfer function satisfying the handling quality criteria 1–3. The possibility that the actual closed-loop response  $T(s)Q(s)$  also satisfies the handling quality criteria relies on how small the error between  $T_1(s)$  and  $T(s)Q(s)$  can be made.

### Solution of $H_\infty$ Model-Matching Problem

The first step in the  $H_\infty$  prefilter design is to determine the ideal transfer function  $T_1(s)$ . The free parameters in Eq. (4) are estimated by curve fitting to meet the three handling quality criteria 1–3. There are many sets of parameters satisfying these specifications. One of the admissible ideal  $T_1(s)$  is

$$T_1(s) = \frac{10(s + 12)}{s^2 + 2 \times 0.60 \times 11s + 11^2} e^{-0.035s} \quad (7)$$

A minimum state-space realization of  $T_1(s)$  is given by

$$T_1(s) = \left[ \begin{array}{ccc|c} -70.343 & -875.29 & -6914.3 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \hline -10 & 451.43 & 6857.1 & 0 \end{array} \right] \quad (8)$$

where the exponential delay term  $e^{-0.035s}$  has been replaced by the first-order Padé approximation.  $T(s)Q(s)$  is the actual transfer function from the stick command  $F_s(s)$  to the aircraft's pitch angle  $\theta(s)$ , where

$$T(s) = [0 \quad 1]G_{21}(s)K(s)[I + G_{21}(s)K(s)]^{-1} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

can be evaluated using plant  $G_{21}(s)$  from Eq. (1) and using the  $H_\infty$  feedback controller  $K(s)$  from Ref. 7. The state-space realization of the resulting  $T(s)$  is given by

$$T(s) = \left[ \begin{array}{cccccc|c} -25.1 & 31.9 & 37.6 & -502 & -224 & -1005 & 0 & 4.77 \\ 152 & 50.1 & 70.7 & -393 & -141 & -826 & 0 & -26.5 \\ 51.3 & -22.3 & -27.6 & 419 & 189 & 847 & 0 & -7.11 \\ 85.7 & 24.4 & 40.5 & -286 & -143 & -458 & 0 & -1.46 \\ -96.7 & -30.0 & -40.6 & 160 & 30.3 & 420 & 0 & 2.86 \\ -14.1 & -5.04 & -7.14 & 37.2 & 11.6 & 84.9 & 0 & -1.13 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.034 \\ \hline -21.0 & -12.8 & 18.8 & 3.27 & -13.0 & 54.8 & 0.001 & 0 \end{array} \right] \quad (9)$$

model-matching problem where  $T(s)Q(s)$  is designed to match the target model  $T_1(s)$ . In the present case, however, perfect model matching is not possible. For highly augmented unstable aircraft,  $T(s)$  is always nonminimum phase and the direct evaluation of  $Q(s)$  from  $T^{-1}(s)T_1(s)$  will inevitably result in unstable pilot prefilter  $Q(s)$ .

A conventional approach to imperfect model-matching problem employs a linear quadratic optimization technique (least-square method) to minimize the model-matching error  $\|T_1 - TQ\|_2$ . We use the  $H_\infty$  criterion, instead of the quadratic criterion, as a measure

The prefilter  $Q(s)$  is designed to minimize the distance between  $T_1(s)$  and  $T(s)Q(s)$  in the sense of the  $H_\infty$  norm. This distance is actually a function of frequency and must be relatively small over the frequency range from 0.1 to 20 rad/s for the present aircraft. Hence, the problem is equivalent to minimizing the  $H_\infty$  norm  $\|W(s)[T_1(s) - T(s)Q(s)]\|_\infty$ , where the frequency-dependent weight  $W(s)$  is chosen as

$$W(s) = \frac{0.01s + 1}{0.1s + 1} \quad (10)$$

Following the standard  $H_\infty$  model-matching procedures in Ref. 9, we obtain the optimal pilot prefilter  $Q(s)$  as

$$Q_{\text{opt}}(s) = \begin{bmatrix} -116 & 136 & -2.47 & -47.9 & -982 & -2190 & -1.07 \\ 130 & -236 & -50.6 & 49.8 & 1200 & 2760 & 1.41 \\ -7.70 & -50.8 & -40.1 & -5.49 & -17.1 & 23.8 & 0.067 \\ 83.6 & -40.3 & 46.1 & 34.1 & 786 & 1670 & 0.698 \\ -14.2 & 32.3 & 2.31 & -0.847 & -265 & -417 & -0.239 \\ -0.984 & -16.3 & -8.70 & -2.99 & 54.2 & 93.3 & 0.058 \\ -346 & 3310 & 1610 & 64.1 & -10500 & -25900 & -12.0 \end{bmatrix} \quad (11)$$

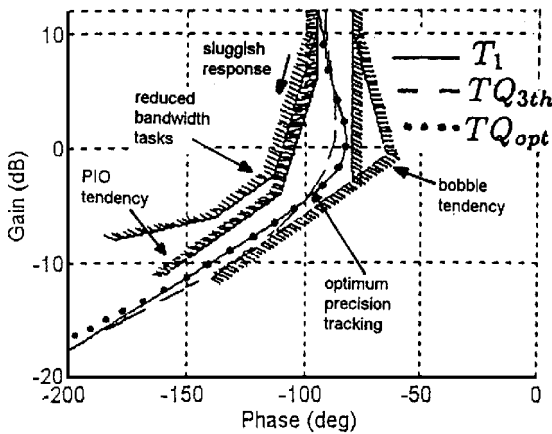


Fig. 2 Gibson's frequency response.

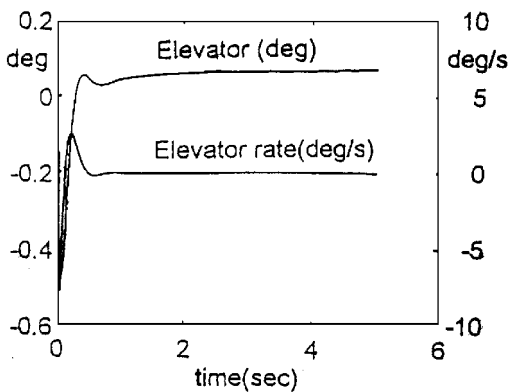
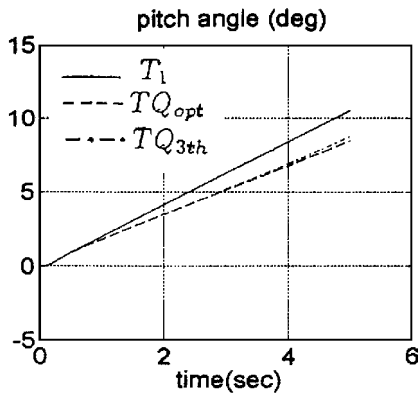


Fig. 3 Time response of pitch angle and elevator angle.

Performance degradation due to order reduction of the optimal prefilter is also evaluated here. Three-order approximant of the optimal six-order prefilter has been obtained by the weighted Hankel-norm model reduction technique as

$$Q_{3th}(s) = \frac{1.345s^3 + 18.14s^2 + 76.93s - 8.599}{s^3 + 15.27s^2 + 119.0s - 17.75} \quad (12)$$

Figure 2 shows the Gibson's frequency responses for the reduced-order prefilter and for the optimal prefilter.  $TQ_{\text{opt}}$  matches the ideal model  $T_1$  very well and renders the integrated pilot-aircraft behavior to satisfy the handling quality criteria 1–3. The resulting equivalent second-order pilot-aircraft dynamics has damping ratio 0.7, natural frequency 11.0 rad/s, and gain/phase margin =  $\infty/120$  deg. The bandwidth criterion of the pitch-control loop and the Neal-Smith closed-loop requirement of integrated pilot-aircraft also have been tested to yield promising results, but due to limited space, these tests are not shown here. For the three-order prefilter, although the actual  $\theta(s)/F_s(s)$  response does not track the ideal function  $T_1(s)$  well, the gain-phase response still meets the military level-1 requirements. The time responses of pitch angle, elevator deflection, and elevator rate due to 1-g step command input are shown in Fig. 3. Since the maximum allowable elevator position is  $\pm 30$  deg, and the rate limit is 64 deg/s for the present aircraft, it can be deduced from Fig. 3 that the response to 9-g stick input is still safely under the limits of surface deflection and surface rate.

### Conclusions

The  $H_\infty$  model-matching technique has been introduced to design pilot prefilter to enhance handling qualities for highly augmented unstable aircraft. The  $H_\infty$  pilot prefilter has the capability to minimize the worst-case model-matching error between the actual pilot-aircraft response, and the ideal pilot-aircraft response, which satisfies the desired handling qualities. This design philosophy has been realized in an experimental aircraft developed by AIDC in Taiwan, and promising results are achieved.

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# Flight Control System Design by Self-Organizing Fuzzy Logic Controller

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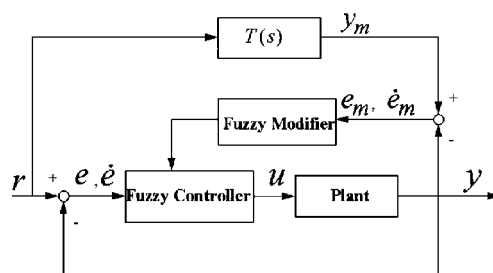
**F**UZZY logic control has been proven to be a powerful tool when it is applied to various control problems.<sup>1-4</sup> In general, fuzzy logic control needs to establish fuzzy inference rules, which are preconstructed by an expert. When the rule base, which represents the experience and intuition of human experts, is not available, an efficient control cannot be expected. To tackle this problem, self-organizing fuzzy logic controllers have been proposed.<sup>5,6</sup> This kind of controller has a learning algorithm and is capable of generating and modifying control rules based on an evaluation of the system's performance. The modification of control rules is achieved by assigning a credit to the control action based on the present performance. However, the self-organizing fuzzy control proposed in Refs. 5 and 6 has some problems. Its control rules are sensitive to set-point changes. And the learning algorithm may generate unreliable credit value and lead to incorrect rule modification. Also, the convergent time of the control action is tedious because only the fired rule is modified each time, and finally, the convergence of the control action is not guaranteed.

This Note proposes a new type of design method of self-organizing fuzzy controller and illustrates its application for flight system control. The proposed model reference self-organizing fuzzy controller (MR-SOFC) has two suites of fuzzy logic; one is for control and the other is for learning. The output of the reference model is used as a reference for rule modification instead of a set point, and so incorrect modification caused by change of set point can be avoided. Also, the learning algorithm will modify the control rules according to the fuzzy inference of the reference model output error and its derivative instead of by the fixed value, so that the learning algorithm can proceed more reasonably and the learned rules can converge more quickly and accurately. The MR-SOFC can start to work even from an empty rule base and can achieve satisfactory control performance after several learning runs.

By applying the proposed MR-SOFC to a flight control system, the simulations illustrate that this MR-SOFC can achieve satisfactory performance and robustness when the flight system is subjected to plant variations arising from different flight conditions.

In Fig. 1, the fuzzy controller produces an output by using the fuzzy control rules as well as the system output error and its derivative ( $e$  and  $\dot{e}$ ). The rules define the control strategy and correspond to linguistic statements implemented by using fuzzy sets. The rules of a fuzzy controller take the following form:

- $$\begin{array}{ll}
R_1: & \text{If } e \text{ is } A_1, \text{ and } \dot{e} \text{ is } B_1, \text{ then } u \text{ is } r(1) \\
R_2: & \text{If } e \text{ is } A_2, \text{ and } \dot{e} \text{ is } B_2, \text{ then } u \text{ is } r(2) \\
\vdots & \vdots \\
R_k: & \text{If } e \text{ is } A_k, \text{ and } \dot{e} \text{ is } B_k, \text{ then } u \text{ is } r(k)
\end{array} \tag{1}$$



where  $r(i), i = 1, 2, \dots, k$  are the singleton control actions, which are learned from the fuzzy modifier, and  $k$  is the number of the fuzzy rules of the fuzzy controller.

The defuzzification of the controller output is accomplished by the method of center of gravity:

$$u = \frac{\sum_{i=1}^k w(i) \times r(i)}{\sum_{i=1}^k w(i)} \quad (2)$$

where  $w(i)$  is the fired weight of the  $i$ th rule. The defuzzified value  $u$  in Eq. (2) represents the desired control force.

The fuzzy modifier is the essential part of the process of learning. In Fig. 1, the MR-SOFC modifies the control rules with the reference model output error  $e_m$  and its derivative  $\dot{e}_m$ . The rule modifier is a fuzzy system, and the fuzzy modification rules are as follows:

- $$\begin{array}{ll}
R_{m1}: & \text{If } e_m \text{ is } C_1, \text{ and } \dot{e}_m \text{ is } D_1, \text{ then } \delta u \text{ is } p(1) \\
R_{m2}: & \text{If } e_m \text{ is } C_2, \text{ and } \dot{e}_m \text{ is } D_2, \text{ then } \delta u \text{ is } p(2) \\
\vdots & \vdots \\
R_{mn}: & \text{If } e_m \text{ is } C_n, \text{ and } \dot{e}_m \text{ is } D_n, \text{ then } \delta u \text{ is } p(n)
\end{array} \quad (3)$$

where the inference outputs  $p(j)$ ,  $j = 1, 2, \dots, n$  are singletons. The defuzzification output  $\delta u$  is also accomplished by the method of center of gravity.

For the modification of the control rules in Eq. (1), the modification credit value  $\Delta r(i)$  of each rule must be appropriate. In Refs. 5 and 6, only the fired rule is modified each time. Here we propose the rules modification criterion by fuzzy inference values, i.e., the modification credit value  $\Delta r(i)$  of each rule is based on its fired weight  $w(i)$  in Eq. (2). Thus, the learning algorithm can proceed more reasonably, and the learned rules can converge more quickly and accurately. The self-organizing modification algorithm is proposed as follows:

$$\Delta r(i) = \delta u \times \frac{w(i)}{\sum_{i=1}^k w(i)} \quad (4)$$

$$r(i) = r(i) + \Delta r(i) \quad (5)$$

Equation (4) means that the credit value of each control rule is proportional to its fired weight of fuzzy inference. The modified control action  $r(i)$  in Eq. (5) is then used for the next step fuzzy control in Eq. (1).

The fuzzy sets  $A_j$  and  $B_j$  in Eq. (1) are given with 11 triangular membership functions, and  $C_i$  and  $D_i$  in Eq. (3) are given with 7 triangular membership functions.

In the following, the short period longitudinal mode of the F-4E flight control system is considered as the design example.<sup>7</sup> Using Ref. 8, the transfer function forms of  $q(s)/\delta_e(s)$  (pitch rate/deviation of elevator deflection) for four typical flight conditions, where  $s$  is the Laplace operator, are given as follows.

Flight condition 1:

$$P_1(s) = \frac{q(s)}{\delta_e(s)} \bigg|_{\text{FCI}} = \frac{-13.239(s + 0.884)}{(s + 3.068)(s - 1.228)} \quad (6)$$

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