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Navigation Filter Estimate Fusion for Enhanced Spacecraft Rendezvous

J. Russell Carpenter*
NASA Johnson Space Center, Houston, Texas 77058
and
Robert H. Bishop†
University of Texas at Austin, Austin, Texas 78712

Techniques for optimally mixing the outputs from a pair of Kalman filters are presented, generalizing previous results. These techniques are derived under the assumption that the designs of the filters are fixed and cannot be modified to support fusion requirements. Conditions for using the optimally fused estimates to periodically reinitialize the Kalman filters are described. The results are applied to an optimal spacecraft rendezvous problem, and simulated performance results indicate that use of the optimally fused data leads to significantly improved robustness to initial target vehicle state errors.

Nomenclature

δ_{jk}	= Kronecker delta
$\delta(t-\tau)$	= Dirac delta function
^	= a posteriori estimates, i.e., estimates immediately following a measurement update
$^-$	= a priori estimates, i.e., estimates immediately prior to the incorporation of a new measurement

Introduction

HISTORICALLY, navigation systems have consisted of arrays of sensors, which provided indirect or partial measurements of position, velocity, and attitude. In such systems, these measurements are passed in raw or minimally smoothed form to a centralized computing facility where they are typically processed by a statistical estimator, such as a Kalman filter. With the advent of modern microprocessors, it has become increasingly possible to produce smart sensors in which the state estimation process is moved inside the navigation sensor box. A typical example is the user segment of the global positioning system (GPS) in which the receiver and navigation software are usually integrated into a single receiver/processor. Decentralizing the navigation process in this fashion has obvious advantages in terms of spreading the overall computational burden among parallel processors and, as a consequence, increasing fault tolerance at the cost of requiring a solution to a potentially complex integration problem.

Although a solution to the problem of optimally fusing the outputs from two or more local estimators was presented at least as long ago as 1976 by Willner et al.,[1] this problem has received considerable attention in the literature in the last 15 years, typically with a focus on minimizing computation and/or communication requirements. Among the early works were those of Hassan et al.[2] and Speyer,[3] who proposed decentralized estimation schemes that produce results identical to a centralized Kalman filter. Speyer's[3] work is notable for compressing all of the information communicated between local processors into a data vector, which has only the dimension of the control vector (if only the estimation problem is being solved, then the data vector has the same dimension as the state vector). Speyer's work was generalized in the works of Willsky et al.[4] Willsky et al.[4] presented necessary and sufficient conditions for estimating a global state from local estimates of arbitrary dimension and expressed in arbitrary coordinate frames. They also predicted that their work could be simplified. One such simplification can be found in the work of Alouani and Birdwell.[5] These authors[5] applied the results of their solution to the nonlinear decentralized estimation problem to the linear data fusion problem. In all of the approaches just cited, a great deal of the data transmitted between the local processors is related to correlations among the processors that arise due to common initial conditions and/or common process noise. One solution that eliminates some of these requirements (at least in comparison to Speyer's work) is the unification collating filter, which has been described by Kerr.[6] In this work, only the information needed to construct a globally optimal estimate at one, rather than all, of the local nodes is presented. Bierman and Belzer[7] presented an approach in which the cross correlations are eliminated by designing the local processors such that the information to be combined does not contain such correlations. More recently, Carlson[8] has developed an approach known as federated filtering, which extends Bierman's approach. Rather than assigning all of the common information to a single one of the local estimators, as in Ref. 7, Carlson[8] designs the local estimators in such a way that the common information is disjointly shared. In contrast, the works of Bar-Shalom,[9] Bar-Shalom and Campo,[10] and Bar-Shalom and Fortmann[11] have indicated how the cross correlations can be advantageously used in the data association problem for which it must be determined whether two estimates that are to be combined actually originate from the same tracked object. Note that the results of Refs. 9–11 do not yield the same results as a centralized Kalman filter, as explained by Bar-Shalom and Li.[12] A related, more general result is that of McReynolds.[13] The problem in which some data sources are raw measurements and others are the outputs of estimation schemes has been examined both in Willsky et al.[4] and in Blackman and Bar-Shalom.[14] Other interesting implementations of decentralized filtering architectures have been presented; the works of Wei and Schwarz[15] and Oshman and Isakow[16] are two recent examples.

In our previous work,[17] a solution to the problem of fusing two Kalman filters operating in parallel is presented in the context of spacecraft navigation. In the approach presented there, the outputs, or state estimates, of the two filters are combined using weights based on the filters' covariance matrices, as well as the cross covariance accounting for any correlation between the filters. In this work, a different point of view is taken in deriving the same result as Refs. 9–11 that explicitly states the cost function for which the fusion is optimal. This approach was motivated by the problem of

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*Aerospace Engineer, Code EG4, Aeroscience and Flight Mechanics Division. Member AIAA.

†Associate Professor, Department of Aerospace Engineering and Engineering Mechanics. Senior Member AIAA.

retrofitting GPS onto the Space Shuttle because it was desired to avoid modifications to existing GPS and Space Shuttle navigation filters. To be a candidate solution for this problem, a data fusion algorithm must efficiently fuse the outputs of two local filters without requiring modifications inside the local filters, e.g., by adjusting the local processors to eliminate cross covariances. We called the approach taken "estimate fusion" to distinguish it from other solution methods to the data fusion problem. This paper extends the approach presented in our previous work by presenting the solution to the problem of fusing two filters with possibly noncommon states, as well as to the problem of how the fused estimate and its covariance can be used to periodically reinitialize the Kalman filters and at what rate this reinitialization should take place. Results from application of the estimate fusion technique to a spacecraft rendezvous scenario are shown, and the technique is found to combine in a complementary way the accuracies of a filter with relative state measurements and a filter with inertial state measurements.

Problem Statement

Consider the case in which two continuous/discrete extended Kalman filters[18] are operating on a system modeled by filter i ($i = 1, 2$) as

$$\dot{\mathsf{x}}_i(t) = \boldsymbol{f}_i[\mathsf{x}_i(t)] + \mathsf{w}_i(t); \qquad \mathsf{w}_i(t) \sim N[0, S_i(t)\delta(t-\tau)] \tag{1}$$

$$\mathsf{y}_{ij} = \boldsymbol{h}_i[\mathsf{x}(t_j)] + \mathsf{v}_{ij}; \qquad \mathsf{v}_{ij} \sim N(0, R_{ij}\delta_{jk}) \tag{2}$$

where filter i processes the discrete sequence of random measurements

$$\mathsf{Y}_i(j) \triangleq \{\mathsf{y}_{i1}, \mathsf{y}_{i2}, \ldots, \mathsf{y}_{ij}\}$$

Here and henceforth, scalars are denoted by lower case letters set in italic type, e.g., x or α; matrices are denoted by upper case letters set in italic type, e.g., A or Γ; and vectors are denoted by upper or lower case letters set in bold italic type, e.g., $\boldsymbol{y}$, $\boldsymbol{B}$, $\boldsymbol{\beta}$, or $\boldsymbol{\Sigma}$. Random variables are denoted by letters set in sans serif type, e.g., x and realizations of random variables are denoted as ordinary vectors and scalars. A normally distributed random variable r with mean μ and variance σ is denoted by $\mathsf{r} \sim N(\mu, \sigma)$. We allow that the filters' states and measurements may be divided into subsets common to both filters and subsets unique to each. Also, although not explicitly indicated in this work, the common subsets may be expressed in different coordinate frames, in which case the transformation between these frames must be appended to the algorithms shown here. (As noted by Willsky et al.,[4] there does not, in fact, have to be any physical relationship between the subsets viewed as common as long as any assumptions about relationships in the mapping of the states onto the measurements is preserved in both filters' model realizations.) We assume the process noises $\mathsf{w}_1(t)$ and $\mathsf{w}_2(t)$, as well as the measurement noises v_{1j} and v_{2j}, to be correlated, where

$$S_{12}(t)\delta(t-\tau) \triangleq E\left[\mathsf{w}_1(t)\,\mathsf{w}_2^T(\tau)\right] \quad \text{and} \quad R_{12j}\delta_{jk} \triangleq E\left[\mathsf{v}_{1j}\mathsf{v}_{2k}^T\right]$$

However, we require only that both $S_{12}(t)$ and R_{12j} be nonnegative definite to allow that state or measurement subsets unique to one filter could be uncorrelated with subsets unique to the other. Hereafter, the time index subscript j will be suppressed as appropriate for clarity.

Given a sequence ($j = 1, 2, \ldots$) of observations $\boldsymbol{Y}_i(j)$, which are realizations of the random variables $\mathsf{Y}_i(j)$, each filter ($i = 1, 2$) propagates its state between measurements via

$$\dot{\bar{\boldsymbol{x}}}_i(t) = \boldsymbol{f}_i[\bar{\boldsymbol{x}}_i(t)]; \qquad (t_j \le t \le t_{j+1}) \tag{3}$$

with $\hat{\boldsymbol{x}}_i(t_j)$ the estimate from its previous update as its initial condition. The filters propagate their covariances using

$$\bar{P}_i(t) = \Phi_i(t, t_j)\hat{P}_i(t_j)\Phi_i^T(t, t_j) + S_{\Delta i}(t) \tag{4}$$

where

$$\bar{P}_i(t) \triangleq E\left[\{\mathsf{x}_i(t) - \bar{\boldsymbol{x}}_i(t)\}\{\mathsf{x}_i(t) - \bar{\boldsymbol{x}}_i(t)\}^T | \boldsymbol{Y}_i(j)\right]$$

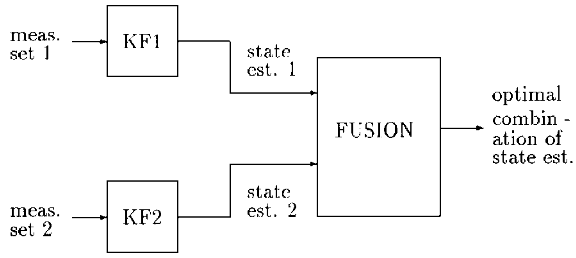


Fig. 1 Overview of estimate fusion.

and

$$\hat{P}_i(t_j) \triangleq E\left[\{\mathsf{x}_i(t_j) - \hat{\boldsymbol{x}}_i(t_j)\}\{\mathsf{x}_i(t_j) - \hat{\boldsymbol{x}}_i(t_j)\}^T | \boldsymbol{Y}_i(j)\right]$$

Here,

$$\dot{\Phi}_i(t, t_j) = F_i[\bar{\boldsymbol{x}}_i(t)]\Phi_i(t, t_j)$$

$$\Phi_i(t_j, t_j) = I, \qquad F_i[\bar{\boldsymbol{x}}_i(t)] = \left.\frac{\partial \boldsymbol{f}_i[\boldsymbol{x}_i(t)]}{\partial \boldsymbol{x}_i(t)}\right|_{\bar{\boldsymbol{x}}_i(t)} \tag{5}$$

and

$$S_{\Delta i}(t) = \int_{t_j}^{t} \Phi_i(t, \tau)S_i(\tau)\Phi_i^T(t, \tau)\,\mathrm{d}\tau$$

Each filter updates its state estimate and state error covariance matrix at time t_j using

$$\hat{\boldsymbol{x}}_i(t_j) = \bar{\boldsymbol{x}}_i(t_j) + K_{ij}\{\boldsymbol{y}_{ij} - \boldsymbol{h}_i[\bar{\boldsymbol{x}}_i(t_j)]\} \tag{6}$$

and

$$\hat{P}_i(t_j) = [I - K_{ij}H_{ij}(\bar{\boldsymbol{x}}_i)]\bar{P}_i(t_j) \tag{7}$$

where K_{ij} is the Kalman gain for filter i at time t_j,

$$K_{ij} = \bar{P}_i(t_j)H_{ij}^T(\bar{\boldsymbol{x}}_i)\left[H_{ij}(\bar{\boldsymbol{x}}_i)\bar{P}_i(t_j)H_{ij}^T(\bar{\boldsymbol{x}}_i) + R_{ij}\right]^{-1}$$
$$H_{ij}(\bar{\boldsymbol{x}}_i) = \left.\frac{\partial \boldsymbol{h}_i[\boldsymbol{x}_i(t_j)]}{\partial \boldsymbol{x}_i(t_j)}\right|_{\bar{\boldsymbol{x}}_i(t_j)} \tag{8}$$

It is assumed that the filters are stable and operating optimally with respect to their own measurements. The problem at hand is to fuse the outputs of the two Kalman filters, as depicted in Fig. 1, in an optimal fashion.

Problem Solution

Let x_1 and x_2, the state vectors of filters 1 and 2, be partitioned according to those states that are common to both filters and those states that are unique to each:

$$\mathsf{x}_1 = \left[\mathsf{x}_\xi^T,\ \mathsf{x}_\eta^T\right]^T, \qquad \mathsf{x}_2 = \left[\mathsf{x}_\xi^T,\ \mathsf{x}_\zeta^T\right]^T \tag{9}$$

where x_ξ are the states common to both filters, x_η are the states unique to filter 1, and x_ζ are the states unique to filter 2.

Optimal Combination

A form for the optimal combination, in which the filters' a posteriori estimates are linearly mixed, is assumed as follows:

$$\hat{\mathsf{x}}_* = W_1\hat{\mathsf{x}}_1 + W_2\hat{\mathsf{x}}_2 \tag{10}$$

where the gain matrices W_i, $i = 1, 2$, are to be determined and the subscript $*$ denotes a quantity resulting from fusing the estimates. The gain matrices are to be chosen such that $\hat{\mathsf{x}}_*$ is an unbiased, minimum variance estimator of the state of the system.

Because $\hat{\mathsf{x}}_1$ and $\hat{\mathsf{x}}_2$ are Kalman filter estimates, these quantities may be assumed to be expressible as

$$\hat{\mathsf{x}}_i = [I - K_iH_i(\bar{\mathsf{x}}_i)]\bar{\mathsf{x}}_i + K_i[H_i(\bar{\mathsf{x}}_i)\mathsf{x}_i + \mathsf{v}_i] \tag{11}$$

for $i = 1, 2$. The a posteriori estimation error is defined as $\hat{\mathsf{e}}_i \triangleq \mathsf{x}_i - \hat{\mathsf{x}}_i$, $i = 1, 2, *$; it follows that

$$\hat{\mathsf{e}}_* = \mathsf{x}_* - W_1(\mathsf{x}_1 - \hat{\mathsf{e}}_1) - W_2(\mathsf{x}_2 - \hat{\mathsf{e}}_2) \tag{12}$$

By assuming that $E[\hat{e}_i|Y_i] = 0$ and that the filters are operating optimally, the expectation of the fused estimation error, conditioned on the measurements of the filters, is found to be

$$E[\hat{e}_*|(Y_1, Y_2)] = E\Big\{\Big(\big[x_\xi^T,\ x_\eta^T,\ x_\zeta^T\big]^T - W_1\big[x_\xi^T,\ x_\eta^T\big]^T - W_2\big[x_\xi^T,\ x_\zeta^T\big]^T\Big)\Big|(Y_1, Y_2)\Big\} \tag{13}$$

Choosing W_1 and W_2 to be complementary as

$$W_1 = \begin{bmatrix} (I - W_\xi) & 0 \\ 0 & I \\ 0 & 0 \end{bmatrix}, \qquad W_2 = \begin{bmatrix} W_\xi & 0 \\ 0 & 0 \\ 0 & I \end{bmatrix} \tag{14}$$

implies that $E[\hat{e}_*|(Y_1, Y_2)] = 0$.

Next, in pursuit of a minimum variance fusion of the estimates, the covariance of the fused estimate is found:

$$\hat{P}_* = E\big[\hat{e}_*\hat{e}_*^T\big|(Y_1, Y_2)\big] = W_1\hat{P}_1W_1^T + W_2\hat{P}_2W_2^T + W_1\hat{Q}\,W_2^T + W_2\hat{Q}^T W_1^T \tag{15}$$

where

$$\hat{P}_1 = E\big[\hat{e}_1\hat{e}_1^T\big|Y_1\big], \qquad \hat{P}_2 = E\big[\hat{e}_2\hat{e}_2^T\big|Y_2\big]$$

and

$$\hat{Q} = E\big[\hat{e}_1\hat{e}_2^T\big|(Y_1, Y_2)\big]$$

The latter, $\hat{Q}$, represents the cross covariance of filter 1 and filter 2 and is updated via

$$\hat{Q} = [I - K_1H_1(\bar{x}_1)]\bar{Q}[I - K_2H_2(\bar{x}_2)]^T + K_1R_{12}K_2^T \tag{16}$$

where $\bar{Q}$ has been propagated from the last update interval.[17] The issue of how $\bar{Q}$ is propagated will be visited in the sequel. Note that, in general, Q is neither symmetric nor square.

Now, $\hat{P}_1$, $\hat{P}_2$, and $\hat{Q}$ are partitioned into blocks corresponding to common and unique states:

$$\hat{P}_1 = \begin{bmatrix} \hat{P}_{1\xi\xi} & \hat{P}_{1\xi\eta} \\ \hat{P}_{1\xi\eta}^T & \hat{P}_{1\eta\eta} \end{bmatrix}, \qquad \hat{P}_2 = \begin{bmatrix} \hat{P}_{2\xi\xi} & \hat{P}_{2\xi\zeta} \\ \hat{P}_{2\xi\zeta}^T & \hat{P}_{2\zeta\zeta} \end{bmatrix}$$
$$\hat{Q} = \begin{bmatrix} \hat{Q}_{\xi\xi} & \hat{Q}_{\xi\zeta} \\ \hat{Q}_{\eta\xi} & \hat{Q}_{\eta\zeta} \end{bmatrix} \tag{17}$$

Then

$$W_1\hat{P}_1W_1^T = \begin{bmatrix} (I - W_\xi)\hat{P}_{1\xi\xi}(I - W_\xi)^T & (I - W_\xi)\hat{P}_{1\xi\eta} & 0 \\ \hat{P}_{1\xi\eta}^T(I - W_\xi)^T & \hat{P}_{1\eta\eta} & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{18}$$

$$W_1\hat{Q}\,W_2^T = \begin{bmatrix} (I - W_\xi)\hat{Q}_{\xi\xi}W_\xi^T & 0 & (I - W_\xi)\hat{Q}_{\xi\zeta} \\ \hat{Q}_{\eta\xi}W_\xi^T & 0 & \hat{Q}_{\eta\zeta} \\ 0 & 0 & 0 \end{bmatrix} \tag{19}$$

$$W_2\hat{Q}^TW_1^T = \big(W_1\hat{Q}\,W_2^T\big)^T = \begin{bmatrix} W_\xi\hat{Q}_{\xi\xi}^T(I - W_\xi)^T & W_\xi\hat{Q}_{\eta\xi}^T & 0 \\ 0 & 0 & 0 \\ \hat{Q}_{\xi\zeta}^T(I - W_\xi)^T & \hat{Q}_{\eta\zeta}^T & 0 \end{bmatrix} \tag{20}$$

and

$$W_2\hat{P}_2W_2^T = \begin{bmatrix} W_\xi\hat{P}_{2\xi\xi}W_\xi^T & 0 & W_\xi\hat{P}_{2\xi\zeta} \\ 0 & 0 & 0 \\ \hat{P}_{2\xi\zeta}^TW_\xi^T & 0 & \hat{P}_{2\zeta\zeta} \end{bmatrix} \tag{21}$$

Substituting Eqs. (18–21) into Eq. (15) yields

$$\hat{P}_{\xi\xi*} = (I - W_\xi)\,\hat{P}_{1\xi\xi}\,(I - W_\xi)^T + (I - W_\xi)\,\hat{Q}_{\xi\xi}\,W_\xi^T + W_\xi\,\hat{Q}_{\xi\xi}^T\,(I - W_\xi)^T + W_\xi\,\hat{P}_{2\xi\xi}\,W_\xi^T \tag{22}$$

$$\hat{P}_{\xi\eta*} = \hat{P}_{\eta\xi*}^T = (I - W_\xi)\hat{P}_{1\xi\eta} + W_\xi\hat{Q}_{\eta\xi}^T \tag{23}$$

$$\hat{P}_{\xi\zeta*} = \hat{P}_{\zeta\xi*}^T = (I - W_\xi)\hat{Q}_{\xi\zeta} + W_\xi\hat{P}_{2\xi\zeta}^T \tag{24}$$

$$\hat{P}_{\eta\eta*} = \hat{P}_{1\eta\eta}, \qquad \hat{P}_{\eta\zeta*} = \hat{P}_{\zeta\eta*}^T = \hat{Q}_{\eta\zeta}, \quad \text{and} \quad \hat{P}_{\zeta\zeta*} = \hat{P}_{2\zeta\zeta} \tag{25}$$

where

$$\hat{P}_* = \begin{bmatrix} \hat{P}_{\xi\xi*} & \hat{P}_{\xi\eta*} & \hat{P}_{\xi\zeta*} \\ \hat{P}_{\xi\eta*}^T & \hat{P}_{\eta\eta*} & \hat{P}_{\eta\zeta*} \\ \hat{P}_{\xi\zeta*}^T & \hat{P}_{\eta\zeta*}^T & \hat{P}_{\zeta\zeta*} \end{bmatrix} \tag{26}$$

Interestingly, even though only those states common to both filters are fused, the correlations between these states and those that are unique to both filters are updated.

To minimize the variance, an optimal W_ξ is chosen to minimize the trace of $\hat{P}_*$. (The centralized Kalman filter minimizes a different cost function.) Note that

$$\operatorname{tr}\hat{P}_* = \operatorname{tr}\Big[(I - W_\xi)\hat{P}_{1\xi\xi}(I - W_\xi)^T + (I - W_\xi)\hat{Q}_{\xi\xi}W_\xi^T + W_\xi\hat{Q}_{\xi\xi}^T(I - W_\xi)^T + W_\xi\hat{P}_{2\xi\xi}W_\xi^T + \hat{P}_{1\eta\eta} + \hat{P}_{2\zeta\zeta}\Big] \tag{27}$$

i.e., the off-diagonal blocks of $\hat{P}_*$ do not contribute to $\operatorname{tr}\hat{P}_*$. Therefore, because $\partial\hat{P}_{1\eta\eta}/\partial W_\xi = 0$ and $\partial\hat{P}_{2\zeta\zeta}/\partial W_\xi = 0$, the problem of determining the optimal weighting matrix $W_{\xi_{\text{opt}}}$ is equivalent to the problem in which the filters have identical process models, which was solved in the authors' previous work.[17] The optimal gain $W_{\xi_{\text{opt}}}$ is determined by setting $\partial\operatorname{tr}\hat{P}_*/\partial W_\xi$ to zero, yielding

$$W_{\xi_{\text{opt}}} = (\hat{P}_{1\xi\xi} - \hat{Q}_{\xi\xi})\big(\hat{P}_{1\xi\xi} + \hat{P}_{2\xi\xi} - \hat{Q}_{\xi\xi} - \hat{Q}_{\xi\xi}^T\big)^{-1} \tag{28}$$

Use of the optimal gain simplifies the expression for $\hat{P}_{\xi\xi*}$, viz.,

$$\hat{P}_{\xi\xi*} = \hat{P}_{1\xi\xi} - W_{\xi_{\text{opt}}}\big(\hat{P}_{1\xi\xi} - \hat{Q}_{\xi\xi}^T\big) \tag{29}$$

Propagation of the Cross Covariance

The cross covariances explicitly contain the shared memory of the two filters, which originates from common initial conditions and/or common process noise models. The shared memory is maintained in the fusion algorithm's propagation stage. For disjoint measurement sets, it cannot be created through the updates but only modified. Although we assume that the initial conditions and process noise models associated with the states unique to one filter are uncorrelated with those of the other filter, we allow that states unique to a given filter may be correlated (through initial conditions or process noise models) with the states common to both filters, allowing for a significant degree of information sharing between the filters.

As with the extended Kalman filter covariance matrices, propagation of the cross covariances may be expressed in the notation of a Riccati equation or via state transition matrices. Because the latter is generally viewed as computationally superior, we report this form only. The derivation closely parallels that of the Kalman filter's covariance propagation (e.g., Ref. 18), and so we only sketch certain unique aspects. By definition,

$$\bar{Q}(t_j) \triangleq E\Big\{\hat{e}_1(t_j)\hat{e}_2(t_j)^T\Big|\big[Y_1(j-1), Y_2(j-1)\big]\Big\} = E\Big[\{x_1(t_j) - \bar{x}_1(t_j)\}\{x_2(t_j) - \bar{x}_2(t_j)\}^T\Big] \tag{30}$$

By expressing the continuous-time process models as equivalent discrete-time difference equations, Eq. (30) may be expanded and the expectation carried out so that we arrive at

$$\bar{Q}(t_j) = \Phi_1(t_j, t_{j-1})\hat{Q}(t_{j-1})\Phi_2^T(t_j, t_{j-1}) + S_{\Delta 12}(t_j) \tag{31}$$

in which

$$S_{\Delta 12}(t) = \int_{t_{j-1}}^{t} \Phi_1(t, \tau)S_{12}(\tau)\Phi_2^T(t, \tau)\,d\tau \tag{32}$$

and

$$S_{12}(t) = E\left\{\begin{pmatrix} \mathsf{w}_{1\xi}(t) \\ \mathsf{w}_{1\eta}(t)\end{pmatrix}\begin{bmatrix} \mathsf{w}_{2\xi}^T(t) & \mathsf{w}_{2\zeta}^T(t)\end{bmatrix}\right\} \tag{33}$$

It is assumed that $E[\mathsf{w}_{1\eta}(t)\;\; \mathsf{w}_{2\zeta}^T(t)] = 0$, because these noise terms are applied to the states unique to each filter. Then

$$S_{12}(t) = \begin{bmatrix} S_{12\xi\xi}(t) & S_{12\xi\zeta}(t) \\ S_{12\eta\xi}(t) & 0 \end{bmatrix} \tag{34}$$

This matrix, like the filter process noise spectral density matrices S_1 and S_2, is determined as part of the navigation system tuning process.

Reducing Data Transmission Requirements

Assuming the fusion processor has internally stored or computed the filter state transition matrices and process noise matrices, transmission of the local filters' states, covariances, Kalman gains, and measurement geometry matrices are required whenever a measurement is processed. In this section, some methods for achieving reductions are discussed.

Updating the Cross Covariance Using the A Priori Covariances

The optimal Kalman filter covariance update [Eq. (7)] can be rearranged as $\hat{P}_i \bar{P}_i^{-1} = (I - K_i H_i)$, $i = 1, 2$, and hence an alternate form for the cross-covariance update is $\hat{Q} = \hat{P}_1 \bar{P}_1^{-1} \bar{Q} \bar{P}_2^{-1} \hat{P}_2$. This update formula may be employed for situations in which the Kalman filters transmit estimates and covariances only. Because two additional matrix inversions are required, however, speed and numerical accuracy may be compromised. Also if the local filters are suboptimal, Eq. (7) will be inaccurate. Careful consideration must be given to the issue of whether or not these disadvantages offset the decrease in data transmission requirements for a particular application.

Optimal Scalar Gain Approximation

In some cases, even the covariances from the Kalman filters may not be readily available. An example is the typical GPS receiver, which often provides as output only a state estimate and a figure of merit. This figure of merit is typically derived from the trace of the GPS filter's covariance matrix or some portion thereof. In our previous work,[17] a formulation of the fusion filter was derived that utilizes a figure of merit based on the covariance traces. In summary, we let $\hat{\mathsf{x}}_\sigma = (1 - w)\hat{\mathsf{x}}_1 + w\hat{\mathsf{x}}_2$, where w is restricted to be a scalar, and the subscript σ indicates an optimal combination using the scalar gain. As before, the gain is determined by minimizing the cost function $J \triangleq \operatorname{tr}\hat{P}_\sigma$, where

$$\operatorname{tr}\hat{P}_\sigma = (1 - 2w + w^2)\operatorname{tr}\hat{P}_1 + w^2 \operatorname{tr}\hat{P}_2 + 2(w - w^2)\operatorname{tr}\hat{Q} \tag{35}$$

The resulting optimal scalar gain is

$$w_{\text{opt}} = \frac{\operatorname{tr}\hat{P}_1 - \operatorname{tr}\hat{Q}}{\operatorname{tr}\hat{P}_1 + \operatorname{tr}\hat{P}_2 - 2\operatorname{tr}\hat{Q}} \tag{36}$$

The problem inherent in this approach is the calculation of $\operatorname{tr}\hat{Q}$ when $\hat{P}_1$ and $\hat{P}_2$ are not available. Efficient implementations of time-series methods for covariance calculation appropriate for real-time use could be employed. A somewhat more appealing possibility is to precompute the cross covariances based on a reference trajectory and store their traces. However, because the use of a scalar gain will also introduce inaccuracies, simply neglecting the cross covariance when using the scalar gain approximation may often be appropriate. An alternative may be to treat the trace of the cross covariance as a tuning parameter whose size is chosen to force estimates calculated using the scalar gain to more accurately track estimates determined optimally.

Reinitializing the Kalman Filters

It is possible to use the fused estimate and its covariance to periodically reinitialize the Kalman filters via a feedback configuration, as shown in Fig. 2. In this procedure, the main jobs of the block labeled fusion in Fig. 2 are to propagate the cross-covariance matrix between measurement updates and to update it each time either of the filters performs an update. Then, at some frequency less than or equal to the slower filter's update frequency, a fusion of the filters' state estimates and covariances is performed, with the filters restarted with the fused state and covariance as initial conditions.

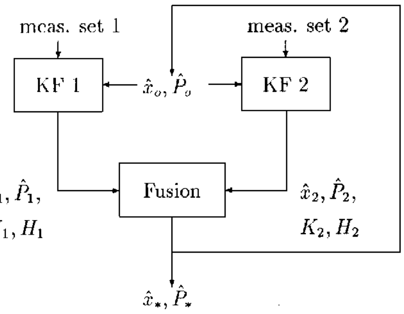


Fig. 2 Overview of estimate fusion feedback.

When such a reinitialization is performed, the cross covariance must also be reinitialized. Denote quantities posterior to such a reinitialization with $\check{}$. Then, the estimate and covariance of filter i are $\check{\mathsf{x}}_i = \hat{\mathsf{x}}_*$ and $\check{P}_i = \hat{P}_*$, and

$$\check{Q} = E\left[\check{\mathsf{e}}_1\check{\mathsf{e}}_2^T \middle| (\boldsymbol{Y}_1, \boldsymbol{Y}_2)\right] = E\left[\check{\mathsf{e}}_*\check{\mathsf{e}}_*^T \middle| (\boldsymbol{Y}_1, \boldsymbol{Y}_2)\right] = \hat{P}_* = \check{P}_1 = \check{P}_2 \tag{37}$$

For the case of common process models and common filter update rates, $\Phi_1 = \Phi_2$ and $S_{12} = S_1 = S_2$, so that

$$\begin{aligned}\bar{Q} &= \Phi_1 \check{Q} \Phi_2 + S_{\Delta 12} = \Phi_1 \check{P}_1 \Phi_1 + S_{\Delta 1} \\ &= \Phi_2 \check{P}_2 \Phi_2 + S_{\Delta 2} = \bar{P}_1 = \bar{P}_2\end{aligned} \tag{38}$$

i.e., no propagation of the cross covariance is required.

Care must be taken, however, to ensure that the filters' common states are statistically independent before reinitialization. To see this, consider the difference between the filters' state estimates, defined as $\hat{\mathsf{d}}_\xi \triangleq \hat{\mathsf{x}}_{1\xi} - \hat{\mathsf{x}}_{2\xi}$, and the difference covariance, defined as

$$\begin{aligned}P_{d\xi\xi} &\triangleq E\left[(\mathsf{e}_{1\xi} - \mathsf{e}_{2\xi})(\mathsf{e}_{1\xi} - \mathsf{e}_{2\xi})^T \middle| (\boldsymbol{Y}_1, \boldsymbol{Y}_2)\right] \\ &= P_{1\xi\xi} + P_{2\xi\xi} - Q_{\xi\xi} - Q_{\xi\xi}^T\end{aligned} \tag{39}$$

Just after a reinitialization $\check{\boldsymbol{d}}_\xi = 0$ and $\check{P}_{d\xi\xi} = 0$. The filters must be allowed to operate long enough between reinitializations for $\hat{P}_{d\xi\xi}$ to become invertible so that $W_{\xi\text{opt}} = (\hat{P}_{1\xi\xi} - \hat{Q}_{\xi\xi})\hat{P}_{d\xi\xi}^{-1}$ can be computed. If $\hat{P}_{d\xi\xi}$ is not invertible, then there exists some $\boldsymbol{\alpha}$ that has at least one nonzero component such that $\boldsymbol{\alpha}^T \hat{P}_{d\xi\xi}\boldsymbol{\alpha} = 0$ implies $\boldsymbol{\alpha}^T \mathsf{d}_\xi = 0$, i.e., the components of $\hat{\mathsf{d}}_\xi$ are linearly dependent.[19] Reference 20 shows that for $\hat{P}_{d\xi\xi}$ to be invertible when the filters share a common process model, a sufficient number of measurements must be processed by the filters prior to each fusion, such that a matrix with the filters' observability grammians along its diagonal has at least the rank of the common state dimension. The observability grammian is given by

$$\Theta_m = \sum_{j=1}^{m} \Phi(t_j, t_m)^T H_j^T R_j^{-1} H_j \Phi(t_j, t_m)$$

where m is the number of measurements processed. Note that the appearance of this singularity is solely a consequence of reinitializing the filters with exactly the same initial conditions and, therefore, it does not appear if the feedback scheme is not used. It has been suggested that maintaining the filters and cross covariances in information form could possibly avoid this singularity. We concur that this is an interesting research topic, but for present purposes it would violate the condition of our approach that the existing subfilters not be modified.

Summary

In summary, for the limited case in which the filters have common process models, have complete state observability from a single update cycle, and do not process any common measurements, the estimate fusion feedback algorithm can be implemented in such a way as to only require transmission of states and covariances, as long the fusion processor has access to both prior and posterior covariance

matrices. A promising alternative is the optimal scalar gain formulation in which the estimates are fused using a scalar weighting factor computed using only the traces of the covariances and cross covariance. If only estimates and covariances can be transmitted to the fusion processor, the a priori covariances can be used to compute the cross-covariance update. For many cases (as in the application considered in the sequel), in which there are frequent and accurate measurements, the cross covariance may often be suboptimally ignored without significantly affecting performance. In such cases, a great deal of the computation and transmission requirements of the estimate fusion algorithm are relieved. In other cases, ad hoc approaches to modeling the effect of the cross covariance, such as that suggested by Blackman and Bar-Shalom,[14] may be employed successfully.

Application to Spacecraft Rendezvous

The problem of lunar rendezvous was studied in Ref. 17. It was shown that estimate fusion techniques could be used to improve the performance of a relative navigation filter by fusing its states with the state estimates from an inertial Kalman filter. However, due to the limitation of the estimate fusion algorithm presented in Ref. 17 to common state dimensionality, a perfect target assumption had to be made so that both filters only estimated the chaser vehicle states. It was mentioned that if significant target errors were present, degraded state estimation and possibly filter divergence could occur. With the new results presented in this work, this problem can now be addressed. Further issues associated with implementing the estimate fusion algorithm for the Space Shuttle are addressed in Ref. 21, and flight data results from Shuttle mission STS-69 are presented in Ref. 22.

A brief description of the scenario is presented. The reader is referred to Ref. 17 for details. The navigation system is a distributed system consisting of two Kalman filters. One filter, referred to as the rendezvous filter, processes discrete measurements derived from a radar system of range and elevation angle to the target vehicle, ρ_{T_j} and θ_{T_j}, viz.,

$$\rho_{T_j} = \sqrt{[\bar{\boldsymbol{r}}_T(t_j) - \bar{\boldsymbol{r}}_C(t_j)]^T[\bar{\boldsymbol{r}}_T(t_j) - \bar{\boldsymbol{r}}_C(t_j)]} + v_{(\rho_T)_j} \tag{40}$$

$$\theta_{T_j} = \arctan\frac{\bar{r}_{T_y}(t_j) - \bar{r}_{C_y}(t_j)}{\bar{r}_{T_x}(t_j) - \bar{r}_{C_x}(t_j)} + v_{(\theta_T)_j} \tag{41}$$

where $v_{(\rho_T)_j} \sim N(0, V_{(\rho_T)_j}\delta_{jk})$, $v_{(\theta_T)_j} \sim N(0, V_{(\theta_T)_j}\delta_{jk})$, $\boldsymbol{r}_C(t_j)$ is the active vehicle (chaser) position, $\boldsymbol{r}_T(t_j)$ is the target vehicle position, and $j = 1, 2, \ldots$. Note that updates from these measurements are used to estimate both the target and chaser vehicle inertial states.

The other filter, referred to as the ground beacon filter, processes discrete measurements of the range from two beacons on the lunar surface, ρ_{B1_j} and ρ_{B2_j}. The beacon positions lie on the vehicles' common ground track and have been previously surveyed to high precision. These measurements are derived from the transit time of a signal broadcast by the beacon and are modeled as

$$\rho_{Bi_j} = \sqrt{[\boldsymbol{r}_{Bi} - \bar{\boldsymbol{r}}_C(t_j)]^T[\boldsymbol{r}_{Bi} - \bar{\boldsymbol{r}}_C(t_j)]} + v_{(\rho_{Bi})_j} \tag{42}$$

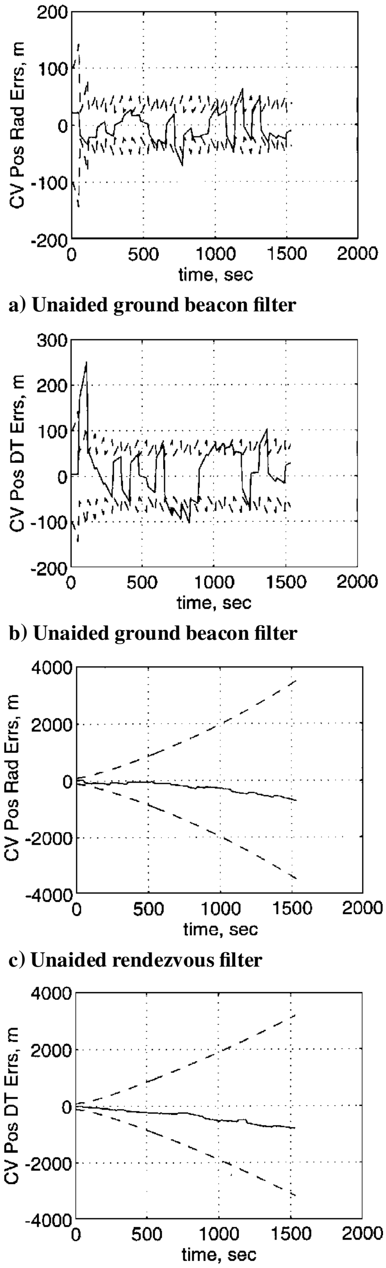
where $i = 1, 2$, $v_{(\rho_{Bi})_j} \sim N(0, V_{(\rho_{Bi})_j}\delta_{jk})$, and $j = 1, 2, \ldots$.

Both filters model the spacecraft dynamics using a Keplerian gravity model, which is augmented by stochastic process noise. A somewhat different model is used for the environment dynamics, which consists of a Keplerian gravity model, augmented by stochastic process noise and a bias term. Thus, the dynamics (for both vehicles) are given by

$$\ddot{\boldsymbol{r}}_{(\cdot)}(t) = -(\mu + \delta\mu)\boldsymbol{r}_{(\cdot)}(t)\Big/\left\|\boldsymbol{r}_{(\cdot)}(t)\right\|^3 - \boldsymbol{w}_{\ddot{r}} \tag{43}$$

where $\delta\mu = 0$ and $\boldsymbol{w}_{\ddot{r}} \sim N[0, W_{\ddot{r}}\delta(t-\tau)]$ for the filter models and $\boldsymbol{w}_{\ddot{r}} \sim N[0, W'_{\ddot{r}}\delta(t-\tau)]$ for the environment. The filters compensate for their imperfect knowledge of the gravity field by choosing $W_{\ddot{r}}$ conservatively.

The passive vehicle's orbit has a radius of two lunar radii. The active vehicle begins its maneuver 100 km behind and 50 km below the passive vehicle, as measured in a curvilinear target-fixed coordinate frame. The transfer is constrained to occur over a 30-deg arc, beginning at longitude 345 deg and ending at longitude 15 deg.



a) Unaided ground beacon filter

b) Unaided ground beacon filter

c) Unaided rendezvous filter

d) Unaided rendezvous filter

Fig. 3 Stand-alone Kalman filters' estimation errors for inertial chaser vehicle position.

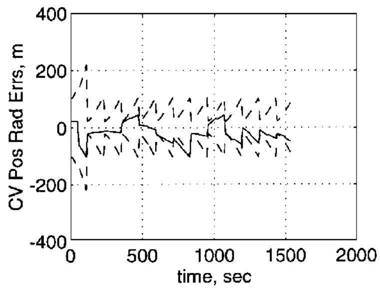
The ground beacons are located at longitudes 330 and 30 deg and remain visible during the entire maneuver. The selenographic frame to which the ground stations are fixed is assumed to be nonrotating, an approximation due to the short length of the maneuver. The orbit transfer takes approximately 25 min, and the midcourse correction maneuver occurs approximately halfway through the transfer, near longitude 0 deg. The initial and midcourse burns are computed using Hill's equations. The design parameters used in tuning the filters are shown in Table 1. Note also that $\delta\mu = 10^{-4}\mu$ and the rms acceleration noise for the environment is $0.001\mu/\|\boldsymbol{r}_C(t_o)\|^2$.

An indication of the performance of stand-alone versions of the filters can be seen in Fig. 3, which presents simulated data. The filters' performance in estimating the chaser vehicle inertial position states is shown. Figure 3a shows the unaided ground beacon filter's estimation errors for the radial component of the chaser vehicle inertial position, and Fig. 3b shows this filter's performance for the downtrack component. Similarly, Figs. 3c and 3d show the unaided rendezvous filter's estimation errors for the radial and downtrack component of the chaser vehicle's inertial position, respectively. In this and subsequent plots, solid traces represent estimation errors and dashed traces represent the corresponding root mean square

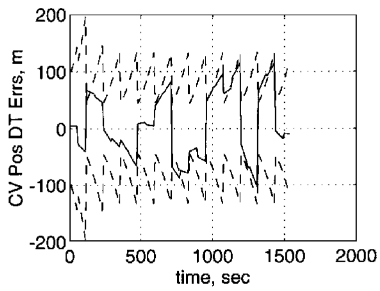
Table 1 Filter design parameters

	Rendezvous filter	Ground beacon filter
Initial rms position error, m	100	100
Initial rms velocity error,[a] m/s	2	2
rms acceleration noise,[b] m/s^2	0.1	0.1
rms range measurement error, m	30	30
rms angle measurement error, deg	0.15	——
Measurement interval, s	60	60

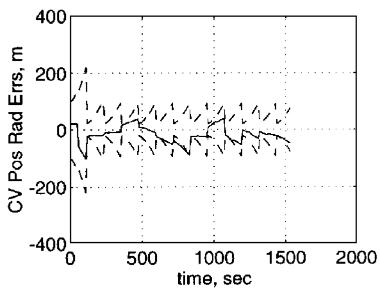
[a]Initial errors uncorrelated; applied equally in all channels, with no correlation.
[b]Noise assumed to be uncorrelated and equal in all channels, with no correlation.



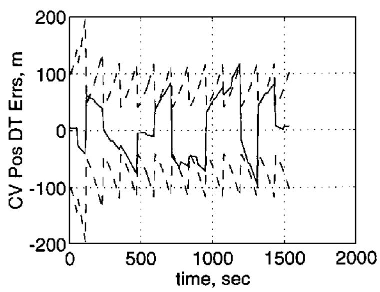
a) Rendezvous filter with optimal fusion



b) Rendezvous filter with optimal fusion



c) Rendezvous filter with suboptimal fusion



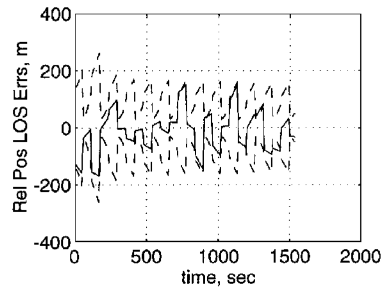
d) Rendezvous filter with suboptimal fusion

Fig. 4 Reinitialized Kalman filters' estimation errors for inertial chaser vehicle position.

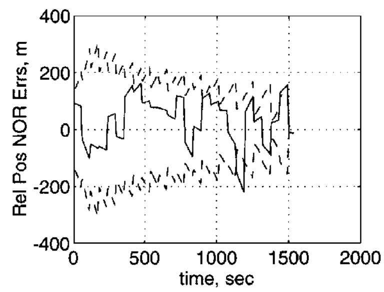
uncertainties of these errors as derived from the error covariance matrices. Note that the error covariance of the rendezvous filter grows quite large.

When these same filters are reinitialized every other measurement pass using the estimate fusion feedback scheme, the large uncertainty in the chaser vehicle's inertial position exhibited by the rendezvous filter is removed by the information provided by the ground beacon filter, as seen in Fig. 4. Here the performance of an optimal configuration is shown in Figs. 4a and 4b, with errors in the radial and downtrack components of inertial position shown in Fig. 4a and Fig. 4b, respectively. In Figs. 4c and 4d, the performance of a suboptimal estimate fusion feedback scheme in which the correlations between the two filters, modeled by the cross-covariance matrix, are ignored by assuming that $Q = 0$. In these subplots as well, errors in the radial and downtrack components of the chaser vehicle inertial position are shown in Fig. 4c and Fig. 4d, respectively.

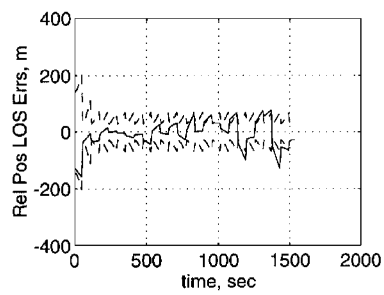
Finally, Fig. 5 shows the simulated relative state estimation performance of the reinitialized and unaided rendezvous filters in Figs. 5a and 5b and Figs. 5c and 5d, respectively. Relative position errors along the line-of-sight from the chaser to the target are shown in Figs. 5a and 5c, while errors in relative position normal to the line of sight are shown in Figs. 5b and 5d. We see that the reinitialized filter approaches the relative state accuracy of the unaided filter only on update cycles in which estimate fusion is not performed. Apparently, the uncertainties in the ground beacon filter's state estimates marginally corrupt the relative state estimates, although with the benefit of substantially improving inertial state estimation performance, as seen from the comparison of Figs. 3 and 4.



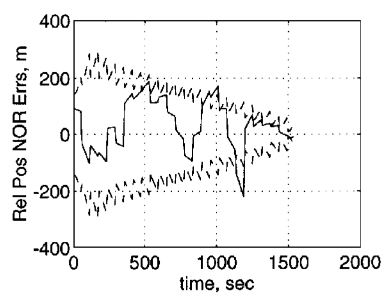
a) Rendezvous filter with optimal fusion



b) Rendezvous filter with optimal fusion

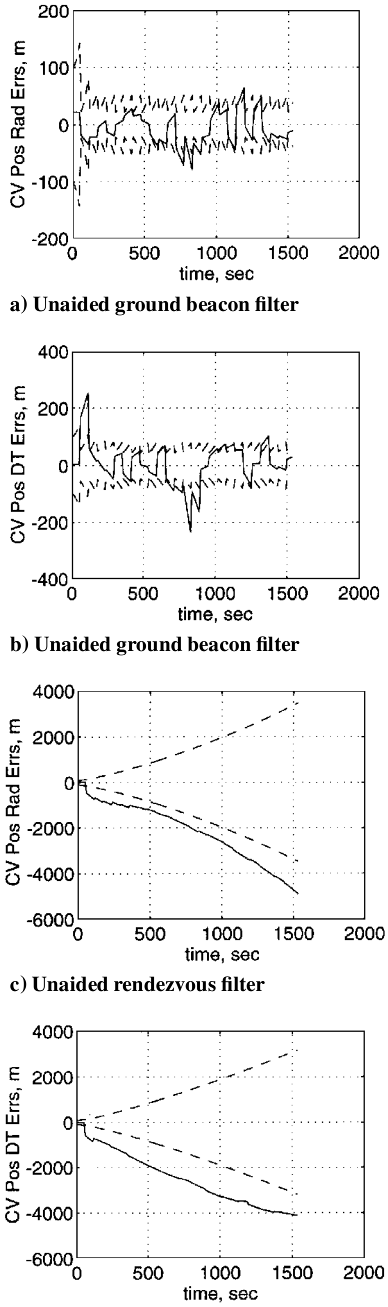


c) Unaided rendezvous filter



d) Unaided rendezvous filter

Fig. 5 Estimation errors for relative position.



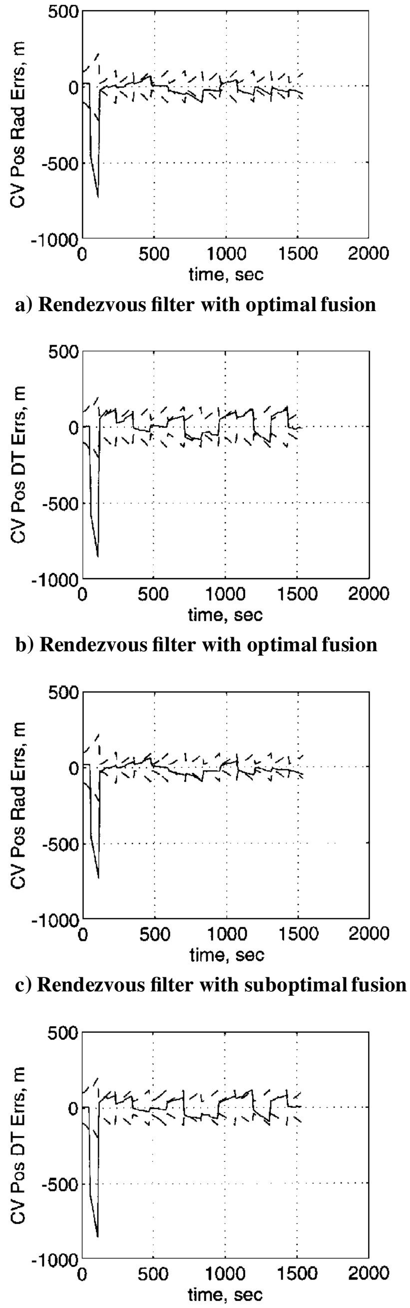
a) Unaided ground beacon filter

b) Unaided ground beacon filter

c) Unaided rendezvous filter

d) Unaided rendezvous filter

Fig. 6 Stand-alone Kalman filters' estimation errors for inertial chaser vehicle position in the presence of significant initial errors in target vehicle states.



a) Rendezvous filter with optimal fusion

b) Rendezvous filter with optimal fusion

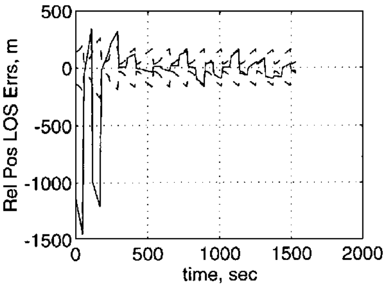
c) Rendezvous filter with suboptimal fusion

d) Rendezvous filter with suboptimal fusion

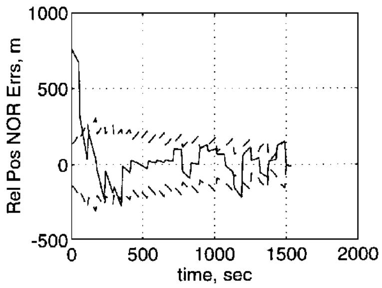
Fig. 7 Reinitialized Kalman filters' estimation errors for inertial chaser vehicle position in the presence of significant initial errors in target vehicle states.

One of the benefits of having good inertial state estimates in a rendezvous scenario is demonstrated in the next sequence of plots in which initial errors having 10 times the standard deviation expected by the filters were introduced into the target vehicle inertial states as a stress case. In Figs. 6 and 7, the inertial position performance of the unaided filters is compared to the two versions of the reinitialized filter (with and without cross-covariance modeling). As seen in Fig. 6, the unaided rendezvous filter's performance in estimating the chaser vehicle inertial states for this case is poor. The arrangement of subplots is the same as that of Fig. 3, with the errors of the unaided ground beacon filter in Figs. 6a and 6b, those of the unaided rendezvous filter in Figs. 6c and 6d, and radial and downtrack components of the chaser vehicle inertial position errors in Figs. 6a and 6c and Figs. 6b and 6d, respectively. By contrast, the reinitialized rendezvous filter, which makes use of very accurate inertial state estimates from the ground beacon filter, is not significantly degraded by the stress case in its ability to estimate the chaser vehicle states, as shown by Fig. 7. The arrangement of subplots is the same as that of Fig. 4, with the errors of the optimal reinitialized rendezvous filter in Figs. 7a and 7b, those of the suboptimal reinitialized rendezvous filter in Figs. 7c and 7d, and radial and downtrack components of the chaser vehicle inertial position errors in Figs. 7a, 7c, 7b, and 7d, respectively.

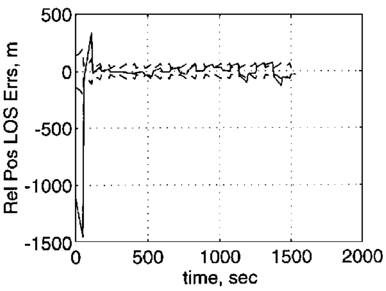
The relative state estimation errors of the reinitialized and stand-alone rendezvous filters for the stress case are shown in Figs. 8a and 8b and Figs. 8c and 8d, respectively. Here, as in Fig. 5, relative position errors along the line-of-sight from the chaser to the target are shown in Figs. 8a and 8c, while errors in relative position normal to the line-of-sight are shown in Figs. 8b and 8d. As in the nominal case, it can be seen that relative state estimates of the reinitialized rendezvous filter are marginally less accurate than those of the stand-alone rendezvous filter, but in light of the stand-alone filter's poor inertial state estimation, the marginal improvement in relative state accuracy seems dubious.



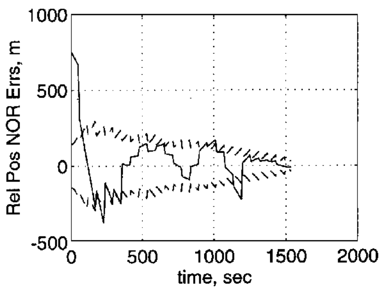
a) Rendezvous filter with optimal fusion



b) Rendezvous filter with optimal fusion



c) Unaided rendezvous filter



d) Unaided rendezvous filter

Fig. 8 Estimation errors for relative position in the presence of significant initial errors in target vehicle states.

Conclusion

Estimate fusion techniques for combining the outputs from Kalman filters operating in parallel have been presented that do not require any restrictive assumptions regarding the process models or state dimension used in the filters being fused. Additionally, it has been shown how the state and covariance resulting from the estimate fusion can be used to periodically reinitialize the Kalman filters in a feedback configuration.

Application of the estimate fusion technique to spacecraft rendezvous has been described. A lunar rendezvous mission previously studied has been reconsidered in light of the new developments presented herein, and it has been shown how to incorporate target vehicle state estimation into this scenario. Simulated performance data show that although relative state accuracy is somewhat degraded under nominal conditions by the inclusion of target state uncertainties, the navigation system is more robust to target errors.

Acknowledgments

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