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Predictive Filtering for Attitude Estimation Without Rate Sensors

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A real-time predictive filter is derived for spacecraft attitude estimation without the utilization of angular rate measurements from gyros. The formulation is shown using only attitude sensors (three-axis magnetometers, sun sensors, star trackers, etc.). The new real-time nonlinear filter predicts the required torque modeling error input in order to propagate the spacecraft dynamic model, so that model responses match the measured vector observations. The real-time predictive filter is used to estimate the attitude of the Solar, Anomalous, Magnetospheric Particle Explorer (SAMPEX) spacecraft. Results using this new algorithm indicate that the real-time predictive filter accurately estimates the attitude of an actual spacecraft with the sole use of magnetometer sensor measurements.

Introduction

THE attitude of a spacecraft can be determined by either deterministic methods or by utilizing algorithms that combine dynamic and/or kinematic models with sensor data. Three-axis deterministic methods, such as TRIAD[1] (algebraic method), QUEST,[2] and FOAM,[3] require measurements of at least two vectors to determine the attitude. An advantage of both the QUEST and FOAM algorithms is that the attitude of a spacecraft can be estimated using measurements of more than two vectors. This is accomplished by minimizing a quadratic loss function first posed by Wahba.[4] However, all deterministic methods fail when only one set of vector measurements is available (e.g., magnetometer data only). Estimation algorithms utilize dynamic and/or kinematic models and subsequently can (in theory) estimate the attitude of a spacecraft using a time series of measurements of a single time-varying vector. Although all spacecraft in use today have at least two onboard attitude sensors, estimation techniques can be used to determine the attitude during anomalous periods when only one vector is available, such as solar eclipse and/or vector-measurement coalignment.

The Kalman filter has been proven to be extremely useful for attitude estimation using vector measurements and gyro measurements. The essential feature of the Kalman filter is the utilization of state-space formulations in the system model design. Errors in the dynamic model are usually treated as process noise because system models are not usually improved or updated during the estimation process. The process noise is essentially used to shift the filter confidence from the system model to the measurements. In the Kalman filter, the model error (process noise) is assumed to be modeled by a zero-mean process with known covariance (e.g., ramp noise for gyro drift models). However, in actual practice the determination of the process noise covariance is usually obtained by an ad hoc and/or heuristic approach resulting in suboptimal filter designs.

For spacecraft attitude estimation, the Kalman filter is most applicable to spacecraft equipped with three-axis gyros as well as attitude sensors.[5] However, gyros are generally expensive and are often prone to degradation or failure. Therefore, in recent years gyros often have been omitted [e.g., in Small Explorer spacecraft, such as the Solar, Anomalous, Magnetospheric Particle Explorer (SAMPEX)]. To circumvent the problem of gyro omission or failure, analytical models of rate motion can be used. This approach has been successfully used in a real-time sequential filter (RTSF) algorithm that propagates state estimates and error covariances using a dynamic model.[6,7] The estimation of dynamic rates by the RTSF is accomplished from angular momentum model propagation, and then correcting for these rates by using a gyro-bias component in the filter design. A clear advantage of using dynamic models is shown for the case of near coalignment of the spacecraft-to-sun and magnetic-field vectors. For this case, deterministic algorithms, such as TRIAD and QUEST, show anomalous behaviors with extreme deviations in determined attitudes. Because the RTSF propagates an analytical model of motion, attitude estimates are improved even when data from only one attitude sensor are available. However, the RTSF is essentially a Kalman filter in which the gyro bias (and subsequently the angular momentum correction) is modeled as a Gaussian process with known covariance. Also, fairly accurate models of angular momentum are required to obtain accurate estimates. Subsequently, the design process for choosing the model error covariance becomes difficult. Other approaches to using a Kalman filter with no gyro data are discussed elsewhere.[8]

Although the quaternion representation is the most commonly used for attitude estimation, the problem of maintaining proper normalization exists. This constraint leads to a singularity in the covariance matrix, which in actual practice is difficult to maintain because of linearization and/or computer roundoff error. Three solutions (two of which yield identical results) to this problem are summarized by Lefferts et al.[5] Reference 5 also contains an extensive bibliography of earlier work on quaternion attitude estimation. Bar-Itzhack and Deutschmann[9] show two further approaches to quaternion normalization.

A new approach for performing optimal state estimation in the presence of significant model error has been developed by Mook and Junkins.[10] This algorithm, called the minimum model error (MME) estimator, unlike most filter and smoother algorithms, does not assume that the model error is represented by a Gaussian process. Instead, the model error is determined during the MME estimation process. The algorithm determines the corrections added to the assumed model such that the model and corrections yield an accurate representation of the system behavior. This is accomplished by solving system optimality conditions and an output error covariance constraint. Therefore, accurate state estimates can be determined without the use of precise system representations in the assumed model. This algorithm has been successfully used to estimate the attitude of an actual spacecraft without the utilization of gyro measurements.[8] However, the MME estimator is a batch (offline) estimator that must utilize postexperiment measurements.

The algorithm developed in this paper can be used to estimate the attitude of a spacecraft in real time (as does the Kalman filter). However, the algorithm is not limited to the Gaussian noise characteristics for the model error. Essentially, this new algorithm combines the good qualities of both the Kalman filter (i.e., a real-time estimator) and the MME estimator (i.e., determines actual model error trajectories). The new algorithm is based on a predictive tracking scheme

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introduced by Lu.[11] Although the problem shown by Lu is solved from a control standpoint, the algorithm developed in this paper is reformulated as a filter and estimator with a stochastic measurement process. Therefore, the new algorithm is known as a predictive filter. The advantages of the new algorithm include the following.

1) The model error is assumed unknown and is estimated as part of the solution.

2) The model error may take any form (even nonlinear).

3) The algorithm can be implemented online to both filter noisy measurements and estimate attitude and rate trajectories.

The organization of this paper proceeds as follows. First, a summary of the spacecraft attitude kinematics and sensor models is shown. Then, a brief review of the predictive filter for nonlinear systems is presented. Next, a predictive filter is developed for the purpose of attitude estimation. This approach determines the optimal spacecraft attitude in real time by minimizing a quadratic cost function consisting of a measurement residual term and a model error term. Finally, the predictive filter is used to estimate the attitude of SAMPEX in order to demonstrate the usefulness of this algorithm.

Attitude Kinematics and Dynamics

In this section, a brief review of the kinematic and dynamic equations of motion for a three-axis stabilized spacecraft is shown. The attitude is assumed to be represented by the quaternion, defined as

$$\boldsymbol{q} \equiv \begin{bmatrix} \boldsymbol{q}_{13} \\ q_4 \end{bmatrix} \tag{1}$$

with

$$\boldsymbol{q}_{13} \equiv \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} = \hat{\boldsymbol{n}} \sin\left(\frac{\theta}{2}\right) \tag{2a}$$

$$q_4 = \cos\left(\frac{\theta}{2}\right) \tag{2b}$$

where $\hat{\boldsymbol{n}}$ is a unit vector corresponding to the axis of rotation and θ is the angle of rotation. The quaternion kinematic equations of motion are derived by using the spacecraft's angular velocity ($\boldsymbol{\omega}$), given by

$$\dot{\boldsymbol{q}} = \tfrac{1}{2}\Omega(\boldsymbol{\omega})\boldsymbol{q} = \tfrac{1}{2}\Xi(\boldsymbol{q})\boldsymbol{\omega} \tag{3}$$

where $\Omega(\boldsymbol{\omega})$ and $\Xi(\boldsymbol{q})$ are defined as

$$\Omega(\boldsymbol{\omega}) \equiv \begin{bmatrix} -[\boldsymbol{\omega}\times] & \vdots & \boldsymbol{\omega} \\ \cdots\cdots & \vdots & \cdots\cdots \\ -\boldsymbol{\omega}^T & \vdots & 0 \end{bmatrix} \tag{4a}$$

$$\Xi(\boldsymbol{q}) \equiv \begin{bmatrix} q_4 I_{3\times 3} + [\boldsymbol{q}_{13}\times] \\ \cdots\cdots\cdots\cdots\cdots\cdots \\ -\boldsymbol{q}_{13}^T \end{bmatrix} \tag{4b}$$

where $I_{3\times 3}$ is a 3×3 identity matrix. The 3×3 dimensional matrices $[\boldsymbol{\omega}\times]$ and $[\boldsymbol{q}_{13}\times]$ are referred to as cross-product matrices because $\boldsymbol{a}\times\boldsymbol{b} = [\boldsymbol{a}\times]\boldsymbol{b}$, with

$$[\boldsymbol{a}\times] \equiv \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix} \tag{5}$$

Because a three-degree-of-freedom system is represented by a four-dimensional vector, the quaternions cannot be independent, which is shown by the following normality constraint:

$$\boldsymbol{q}^T\boldsymbol{q} = \boldsymbol{q}_{13}^T\boldsymbol{q}_{13} + q_4^2 = 1 \tag{6}$$

Also, the matrix $\Xi(\boldsymbol{q})$ obeys the following helpful relations:

$$\Xi^T(\boldsymbol{q})\Xi(\boldsymbol{q}) = \boldsymbol{q}^T\boldsymbol{q} I_{3\times 3} \tag{7a}$$

$$\Xi(\boldsymbol{q})\Xi^T(\boldsymbol{q}) = \boldsymbol{q}^T\boldsymbol{q} I_{4\times 4} - \boldsymbol{q}\boldsymbol{q}^T \tag{7b}$$

$$\Xi^T(\boldsymbol{q})\boldsymbol{q} = \boldsymbol{0}_{3\times 1} \tag{7c}$$

$$\Xi^T(\boldsymbol{q})\boldsymbol{\lambda} = -\Xi^T(\boldsymbol{\lambda})\boldsymbol{q} \quad \text{for any} \quad \boldsymbol{\lambda}_{4\times 1} \tag{7d}$$

where $\boldsymbol{0}_{3\times 1}$ is a 3×1 zero vector. The dynamic equations of motion, also known as Euler's equations, for a rotating spacecraft are given by[12]

$$\dot{\boldsymbol{H}} = \boldsymbol{N} - \boldsymbol{\omega}\times\boldsymbol{H} = J\dot{\boldsymbol{\omega}} \tag{8}$$

where $\boldsymbol{H}$ is the total angular momentum, $\boldsymbol{N}$ is the total external torque (which includes, e.g., control torques, aerodynamic drag torques, and solar pressure torques), and J is the inertia matrix of the spacecraft. If reaction or momentum wheels are used on the spacecraft, the total angular momentum is given by

$$\boldsymbol{H} = J\boldsymbol{\omega} + \boldsymbol{h} \tag{9}$$

where $\boldsymbol{h}$ is the total angular momentum due to the wheels. Thus, Eq. (8) can be rewritten as

$$\dot{\boldsymbol{H}} = \boldsymbol{N} - [J^{-1}(\boldsymbol{H} - \boldsymbol{h})]\times\boldsymbol{H} \tag{10}$$

Also, from Eqs. (8) and (9), the following angular velocity form of Euler's equation can be used:

$$J\dot{\boldsymbol{\omega}} = \boldsymbol{N} - \dot{\boldsymbol{h}} - \boldsymbol{\omega}\times(J\boldsymbol{\omega} + \boldsymbol{h}) \tag{11}$$

which involves the derivative of the wheel angular momentum.

The measurement model is assumed to be of the form given by

$$\boldsymbol{B}_B = A(\boldsymbol{q})\boldsymbol{B}_I \tag{12}$$

where $\boldsymbol{B}_I$ is a 3×1 dimensional vector to some reference object (e.g., a vector to the sun or to a star, or the Earth's magnetic-field vector) in a reference coordinate system, $\boldsymbol{B}_B$ is a 3×1 dimensional vector defining the components of the corresponding reference vector measured in the spacecraft body frame, and $A(\boldsymbol{q})$ is given by

$$A(\boldsymbol{q}) = \left(q_4^2 - \boldsymbol{q}_{13}^T\boldsymbol{q}_{13}\right)I_{3\times 3} + 2\boldsymbol{q}_{13}\boldsymbol{q}_{13}^T - 2q_4[\boldsymbol{q}_{13}\times] \tag{13}$$

which is the 3×3 dimensional (orthogonal) attitude matrix. A simpler form for the attitude matrix in Eq. (13) is given by

$$A(\boldsymbol{q}) = -\Xi^T(\boldsymbol{q})\Psi(\boldsymbol{q}) \tag{14}$$

where

$$\Psi(\boldsymbol{q}) \equiv \begin{bmatrix} -q_4 I_{3\times 3} + [\boldsymbol{q}_{13}\times] \\ \cdots\cdots\cdots\cdots\cdots\cdots \\ \boldsymbol{q}_{13}^T \end{bmatrix} \tag{15}$$

Also, another useful identity is given by

$$\Psi(\boldsymbol{q})\boldsymbol{\omega} = \Gamma(\boldsymbol{\omega})\boldsymbol{q} \quad \text{for any} \quad \boldsymbol{\omega}_{3\times 1} \tag{16}$$

where

$$\Gamma(\boldsymbol{\omega}) \equiv \begin{bmatrix} -[\boldsymbol{\omega}\times] & \vdots & -\boldsymbol{\omega} \\ \cdots\cdots & \vdots & \cdots\cdots \\ \boldsymbol{\omega}^T & \vdots & 0 \end{bmatrix} \tag{17}$$

Nonlinear Predictive Tracking

In this section, a brief review of the nonlinear predictive tracking algorithm first introduced by Lu[11] is shown. Although the problem

shown in Ref. 11 is solved from a control standpoint, the algorithm developed in this section is reformulated as an estimator/filter with a stochastic measurement process. This development is based on the duality that exists between control problems and estimation problems. In the nonlinear predictive filter, it is assumed that the state and output estimates are given by a preliminary model and a to-be-determined model error vector, given by

$$\dot{\hat{\boldsymbol{x}}}(t) = \boldsymbol{f}[\hat{\boldsymbol{x}}(t), t] + G(t)\boldsymbol{d}(t) \tag{18a}$$

$$\hat{\boldsymbol{y}}(t) = \boldsymbol{c}[\hat{\boldsymbol{x}}(t), t] \tag{18b}$$

where $\boldsymbol{f}$ is an $n \times 1$ model vector, $\hat{\boldsymbol{x}}(t)$ is an $n \times 1$ state estimate vector, $\boldsymbol{d}(t)$ is a model error vector of dimension $l \times 1$, $G(t)$ is an $n \times l$ model-error distribution matrix, $\boldsymbol{c}$ is an $m \times 1$ measurement vector, and $\hat{\boldsymbol{y}}(t)$ is an $m \times 1$ estimated output vector. State-observable discrete measurements are assumed for Eq. (18b) in the following form:

$$\tilde{\boldsymbol{y}}(t_k) = \boldsymbol{c}[\boldsymbol{x}(t_k), t_k] + \boldsymbol{v}(t_k) \tag{19}$$

where $\tilde{\boldsymbol{y}}(t_k)$ is an $m \times 1$ measurement vector at time t_k, $\boldsymbol{x}(t_k)$ is the true state vector, and $\boldsymbol{v}(t_k)$ is an $m \times 1$ measurement noise vector that is assumed to be a zero-mean, Gaussian white-noise distributed process with

$$E\{\boldsymbol{v}(t_k)\} = \boldsymbol{0} \tag{20a}$$

$$E\left\{\boldsymbol{v}(t_k)\boldsymbol{v}^T(t_{k'})\right\} = R\delta_{kk'} \tag{20b}$$

where R is an $m \times m$ positive-definite covariance matrix.

A cost functional consisting of the weighted sum square of the measurement-minus-estimate residuals plus the weighted sum square of the model correction term is minimized, given by

$$J(t) = \tfrac{1}{2}\{\tilde{\boldsymbol{y}}(t + \Delta t) - \hat{\boldsymbol{y}}(t + \Delta t)\}^T R^{-1} \times \{\tilde{\boldsymbol{y}}(t + \Delta t) - \hat{\boldsymbol{y}}(t + \Delta t)\} + \tfrac{1}{2}\boldsymbol{d}^T(t)W\boldsymbol{d}(t) \tag{21}$$

where Δt is the measurement sampling interval and W is an $l \times l$ weighting matrix. The necessary conditions for the minimization of Eq. (21) lead to the following model error solution:

$$\boldsymbol{d}(t) = -\{[\Lambda(\Delta t)S(\hat{\boldsymbol{x}})]^T R^{-1}\Lambda(\Delta t)S(\hat{\boldsymbol{x}}) + W\}^{-1} \times [\Lambda(\Delta t)S(\hat{\boldsymbol{x}})]^T R^{-1}[\boldsymbol{z}(\hat{\boldsymbol{x}}, \Delta t) - \tilde{\boldsymbol{y}}(t + \Delta t) + \hat{\boldsymbol{y}}(t)] \tag{22}$$

where $S(\hat{\boldsymbol{x}})$ is an $m \times l$ dimensional matrix and $\Lambda(\Delta t)$ is an $m \times m$ diagonal matrix with elements given by

$$\lambda_{ii} = \Delta t^{p_i}/p_i!, \qquad i = 1, 2, \ldots, m \tag{23}$$

where p_i is the lowest order of the derivative of $\hat{y}_i(t)$ in which any component of $\boldsymbol{d}(t)$ first appears in $\hat{\boldsymbol{x}}(t)$. The ith component of $\boldsymbol{z}(\hat{\boldsymbol{x}}, \Delta t)$ is given by

$$z_i(\hat{\boldsymbol{x}}, \Delta t) = \Delta t L_f^1(c_i) + (\Delta t^2/2!)L_f^2(c_i) + \cdots + (\Delta t^{p_i}/p_i!)L_f^{p_i}(c_i), \qquad i = 1, 2, \ldots, m \tag{24}$$

where $L_f^k(c_i)$ is the kth Lie derivative, defined by

$$\begin{aligned} L_f^k(c_i) &= c_i \qquad &&\text{for} \quad k = 1 \\ L_f^k(c_i) &= \frac{\partial L_f^{k-1}(c_i)}{\partial \hat{\boldsymbol{x}}}\boldsymbol{f} \qquad &&\text{for} \quad k > 1 \end{aligned} \tag{25}$$

The ith row of $S(\hat{\boldsymbol{x}})$ is given by

$$s_i = \left\{L_{g_1}\left[L_f^{p_i-1}(c_i)\right], \ldots, L_{g_l}\left[L_f^{p_i-1}(c_i)\right]\right\} \qquad i = 1, 2, \ldots, m \tag{26}$$

where g_j is the jth column of $G(t)$ and the Lie derivative is defined by

$$L_{g_j}\left[L_f^{p_i-1}(c_i)\right] \equiv \frac{\partial L_f^{p_i-1}(c_i)}{\partial \hat{\boldsymbol{x}}}g_j, \qquad j = 1, 2, \ldots, l \tag{27}$$

Therefore, Eq. (22) is used in Eq. (18a) to propagate the state estimates to time t_k, then the measurement is processed at time t_k to find the new $\boldsymbol{d}(t)$ in $[t_k, t_{k+1}]$, and then the state estimates are propagated to time t_{k+1}.

The weighting matrix (W) in Eq. (21) can be determined on the basis that the measurement-minus-estimate error covariance matrix must match the measurement-minus-truth error covariance matrix.[10] This condition is referred to as the covariance constraint, shown as

$$\frac{1}{m_{\text{tot}}}\sum_{k=1}^{m_{\text{tot}}}\{\tilde{\boldsymbol{y}}(t_k) - \hat{\boldsymbol{y}}(t_k)\}\{\tilde{\boldsymbol{y}}(t_k) - \hat{\boldsymbol{y}}(t_k)\}^T \approx R \tag{28}$$

where m_{tot} is the total number of measurement points. The covariance constraint is satisfied when the proper balance between model error and measurement residual has been achieved.

Predictive Attitude Estimation

In this section, a nonlinear predictive filter is derived for spacecraft attitude using the quaternion kinematic equations and the angular momentum version of Euler's equation. The nonlinear predictive filter for this case minimizes the following cost function:

$$J = \tfrac{1}{2}\{\tilde{\boldsymbol{B}}_B(t + \Delta t) - A(\hat{\boldsymbol{q}})\boldsymbol{B}_I(t + \Delta t)\}^T R^{-1} \times \{\tilde{\boldsymbol{B}}_B(t + \Delta t) - A(\hat{\boldsymbol{q}})\boldsymbol{B}_I(t + \Delta t)\} + \tfrac{1}{2}\boldsymbol{d}^T(t)W\boldsymbol{d}(t) \tag{29}$$

subject to

$$\dot{\hat{\boldsymbol{q}}} = \tfrac{1}{2}\Omega(\hat{\omega})\hat{\boldsymbol{q}} = \tfrac{1}{2}\Xi(\hat{\boldsymbol{q}})\hat{\omega}, \qquad \hat{\boldsymbol{q}}(t_0) = \boldsymbol{q}_0 \tag{30a}$$

$$\dot{\hat{\boldsymbol{H}}} = \boldsymbol{N} - [J^{-1}(\hat{\boldsymbol{H}} - \boldsymbol{h})] \times \hat{\boldsymbol{H}} + \boldsymbol{d}, \qquad \hat{\boldsymbol{H}}(t_0) = \boldsymbol{H}_0 \tag{30b}$$

$$\hat{\omega} = J^{-1}(\hat{\boldsymbol{H}} - \boldsymbol{h}) \tag{30c}$$

Because the body measurements ($\tilde{\boldsymbol{B}}_B$) are used as the required tracking trajectories, the output vector in Eq. (18b) is given by

$$\boldsymbol{c}(\hat{\boldsymbol{x}}) = A(\hat{\boldsymbol{q}})\boldsymbol{B}_I \tag{31}$$

where

$$\hat{\boldsymbol{x}} \equiv \begin{bmatrix} \hat{\boldsymbol{q}} \\ \cdots \\ \hat{\boldsymbol{H}} \end{bmatrix} \tag{32}$$

Because $\boldsymbol{c}$ depends on $\hat{\boldsymbol{q}}$ and not explicitly on $\hat{\boldsymbol{H}}$, the lowest-order derivative of Eq. (32) in which any component of $\boldsymbol{d}$ first appears in $\hat{\boldsymbol{q}}$ is two, so that $p_i = 2$. Therefore, the Λ and $\boldsymbol{z}$ quantities in Eq. (22) are given by

$$\Lambda = (\Delta t^2/2)I \tag{33a}$$

$$\boldsymbol{z} = \Delta t \boldsymbol{L}_f^1 + (\Delta t^2/2)\boldsymbol{L}_f^2 \tag{33b}$$

where

$$\left[\boldsymbol{L}_f^k\right]_i \equiv L_f^k(c_i) \tag{34}$$

Using the definitions in Eqs. (7), (14), and (16), the derivative of Eq. (31) with respect to $\hat{\boldsymbol{q}}$ can be shown to be

$$\frac{\partial \boldsymbol{c}}{\partial \hat{\boldsymbol{q}}} = -2\Xi^T(\hat{\boldsymbol{q}})\Gamma(\boldsymbol{B}_I) \tag{35}$$

Therefore, $\boldsymbol{L}_f^1$ is given by

$$\boldsymbol{L}_f^1 = -\Xi^T(\hat{\boldsymbol{q}})\Gamma(\boldsymbol{B}_I)\Xi(\hat{\boldsymbol{q}})J^{-1}(\hat{\boldsymbol{H}} - \boldsymbol{h}) \tag{36}$$

The derivative of Eq. (36) with respect to $\hat{\boldsymbol{q}}$ is given by

$$\frac{\partial \boldsymbol{L}_f^1}{\partial \hat{\boldsymbol{q}}} = 2[\hat{\omega}\times]\Xi^T(\hat{\boldsymbol{q}})\Gamma(\boldsymbol{B}_I) \tag{37}$$

The derivative of Eq. (36) with respect to $\hat{\boldsymbol{H}}$ is given by

$$\frac{\partial \boldsymbol{L}_f^1}{\partial \hat{\boldsymbol{H}}} = -\Xi^T(\hat{\boldsymbol{q}})\Gamma(\boldsymbol{B}_I)\Xi(\hat{\boldsymbol{q}})J^{-1} \tag{38}$$

Therefore, L_f^2 is given by

$$L_f^2 = [\hat{\omega}\times]\Xi^T(\hat{q})\Gamma(B_I)\Xi(\hat{q})\hat{\omega} - \Xi^T(\hat{q})\Gamma(B_I)\Xi(\hat{q})J^{-1}(N - [\hat{\omega}\times]\hat{H}) \tag{39}$$

Comparing Eqs. (18a) and (30), the G matrix is given by

$$G = \begin{bmatrix} 0_{4\times 3} \\ I_{3\times 3} \end{bmatrix} \tag{40}$$

where $0_{4\times 3}$ is a 4×3 zero matrix and $I_{3\times 3}$ is a 3×3 identity matrix. The S matrix, which is formed using Eq. (26), is given by

$$S = -\Xi^T(\hat{q})\Gamma(B_I)\Xi(\hat{q})J^{-1} = [A(\hat{q})B_I\times]J^{-1} \tag{41}$$

The 3×3 matrix $[A(\hat{q})B_I\times]$ is analogous to the sensitivity matrix used in a Kalman filter.[5] This matrix has, at most, rank 2, which reflects the fact that there is no information about rotations around the current measurement vector. The extension to multiple measurement sets is achieved by stacking these measurements, e.g.,

$$e = \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_{m_{\text{sen}}} \end{bmatrix} \tag{42}$$

where m_{sen} is the total number of vector measurement sets available at time t_k.

An advantage of the nonlinear predictive filter over the conventional Kalman filter for attitude estimation[5] is that quaternion normalization is always preserved. Several methods overcome this difficulty.[5,9] However, because Eq. (30a) is used to determine the quaternion estimate, and because the measurements only enter through the model error (d) in Eq. (30b), normalization is always maintained in the nonlinear predictive filter. Also, the predictive filter determines the model-error trajectory as part of the solution, as opposed to the Kalman filter, which assumes that this error is modeled by a zero-mean Gaussian process with known covariance.

Attitude Estimation of SAMPEX

In this section, the predictive filter is used to estimate the attitude, rate, and input torque trajectories of the SAMPEX spacecraft using vector measurement observations. The SAMPEX general mission is to study energetic particles and various types of rays. The spacecraft is in a 550×675 km elliptical orbit with an 82-deg inclination. The attitude control hardware consists of a magnetic torquer assembly (MTA) and a single reaction wheel. The attitude determination hardware consists of five coarse sun sensors (primarily for sun acquisition), one fine sun sensor (FSS), and a three-axis magnetometer (TAM). Also, no rate-sensing instruments are present on the spacecraft.

The onboard computer routine to determine attitude is based on the TRIAD deterministic method. The spacecraft is controlled by the MTA to maintain the fixed solar arrays perpendicular to the sun line. The reaction wheel is used to point the instrument boresight axis as required by the scientific mission. During eclipse, no sun measurements are available from the FSS. Attitude control is maintained by using a constant sun-line vector as a pseudomeasurement, so that the reaction wheel is still utilized. During vector coalignment, the spacecraft is placed in a coast mode in which neither the MTA nor the reaction wheel is used.[13] The required nominal attitude determination accuracy is ± 2 deg. During anomalous conditions (eclipse and/or measurement vector coalignment) the attitude cannot be determined by deterministic methods, such as TRIAD. The predictive filter presented in this paper can determine the attitude in real time using TAM measurements only, so that normal control procedures using the MTA and the reaction wheel can be maintained.

The inertial field trajectories are obtained by using an eighth-order spherical harmonic model of the Earth's magnetic field with International Geomagnetic Reference Field coefficients.[14] Magnetometer measurements by the TAM are known to be extremely accurate (within 0.3 mG). However, experience has shown that errors in the magnetic-field model have a standard deviation of about 3 mG (Ref. 7). Therefore, 9 mG^2 is chosen for the diagonal elements of the measurement covariance matrix. For the FSS measurements, the primary source of error is the digitization noise of about 0.5 deg. This error is assumed to be uniformly distributed so that the standard deviation of the FSS angle measurements is $0.5/\sqrt{(12)}$ deg.

The angular velocity measurement of the wheel contains a substantial amount of noise (see top part of Fig. 1). To filter this noise, the predictive filter is used as a linear filter with a simple second-order plant that estimates for the angular velocity of the wheel and its derivative, given by

$$\begin{bmatrix} \dot{\hat{h}}(t) \\ \ddot{\hat{h}}(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{h}(t) \\ \dot{\hat{h}}(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} d(t) \tag{43a}$$

$$\hat{h}(t) = [1 \quad 0] \begin{bmatrix} \hat{h}(t) \\ \dot{\hat{h}}(t) \end{bmatrix} \tag{43b}$$

The standard deviation of the wheel noise is about 1 rad/s. The filtered wheel-speed estimates are plotted in the bottom part of Fig. 1. Clearly, the simple linear filter substantially reduces the noise in the wheel-speed measurements.

The first test case involves using both TAM and FSS measurements. A plot of the finite differenced angular rates using TRIAD determined attitudes is shown in Fig. 2. These rates are extremely noisy, which is due to the large digitization noise associated with the FSS measurements. A plot of the predictive-filter estimated rates

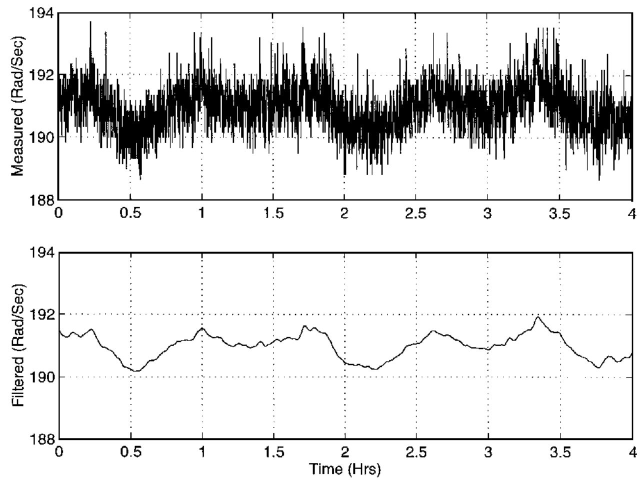

Fig. 1 Plot of measured and filtered wheel speeds.

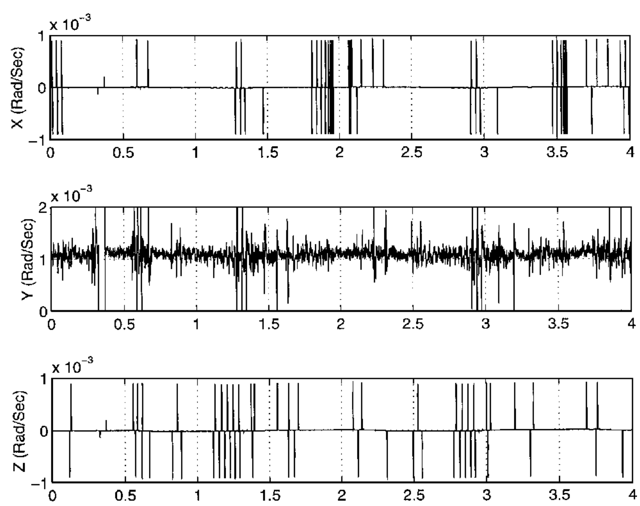

Fig. 2 Plot of TRIAD-determined angular velocities.

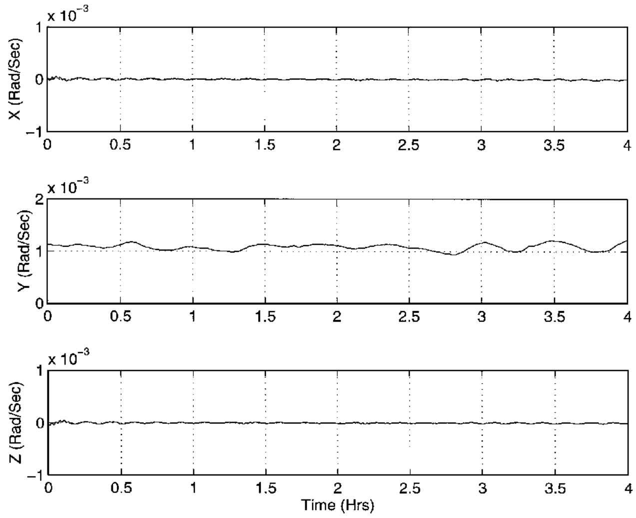

Fig. 3 Plot of angular velocity estimates.

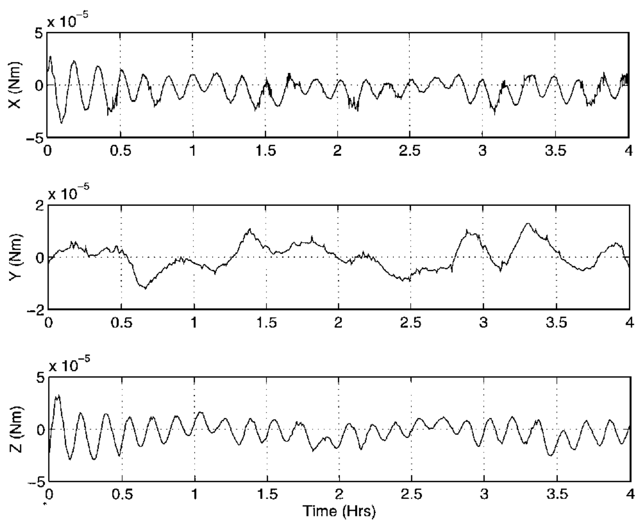

Fig. 4 Plot of determined model error trajectories.

using the same TAM and FSS vector measurements as in the TRIAD solution is shown in Fig. 3. Clearly, these rates are smoother than the TRIAD-determined rates. A plot of the determined input torques from the predictive filter is shown in Fig. 4. These torques correspond to a correction to the dynamic model, so that the model responses match the vector measurement observations.

The second case involves using TAM measurements only to estimate the attitude and angular rates. Because only one vector observation is used (i.e., TAM measurements only), a TRIAD solution is not possible. However, attitude estimation is possible using the predictive filter, because the filter utilizes Euler's equation for propagation. A plot of the estimated angular rate trajectories is shown in Fig. 5. These angular rate estimates clearly show a rotation about the spacecraft's y axis, which is the desired motion. A plot of the determined input torques from the predictive filter using TAM data only is shown in Fig. 6. This again corresponds to a torque correction in Euler's equation. A plot of the error between the estimated attitudes using both TAM and FSS data in the predictive filter and the estimated attitudes using TAM-only data in the predictive filter is shown in Fig. 7. Although a true attitude is not known, this comparison can provide some insight into the accuracy of using a TAM-only solution. From Fig. 7, a slight hangoff is seen in the pitch axis. This may be due to nonlinear effects in the magnetic-field model.[15] However, the predictive filter is able to determine attitudes to within 1 deg using TAM data only. This can be useful in determining the attitude when deterministic methods fail. Also, a Kalman filter has been developed for this spacecraft but only after extensive modeling of disturbance torques (used in $\boldsymbol{N}$) and intensive tuning of filter parameters.[7] The predictive filter has been able to estimate the attitude using no modeling of disturbance torques (i.e., by setting $\boldsymbol{N} = \boldsymbol{0}$) and thus is more robust with respect to modeling errors than the Kalman filter.

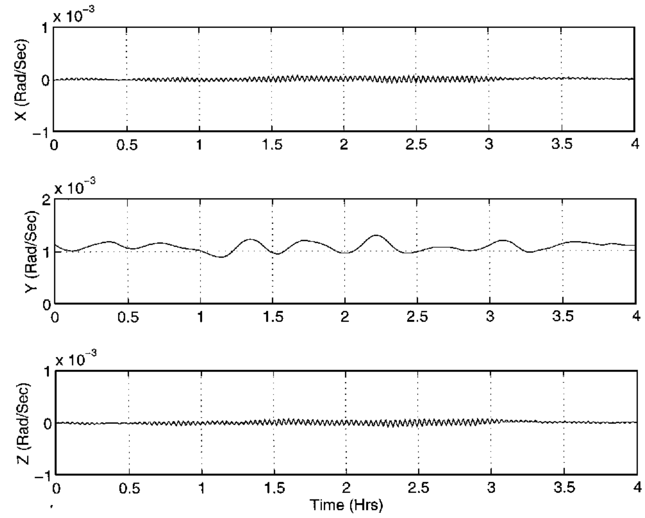

Fig. 5 Plot of angular velocity estimates using TAM only.

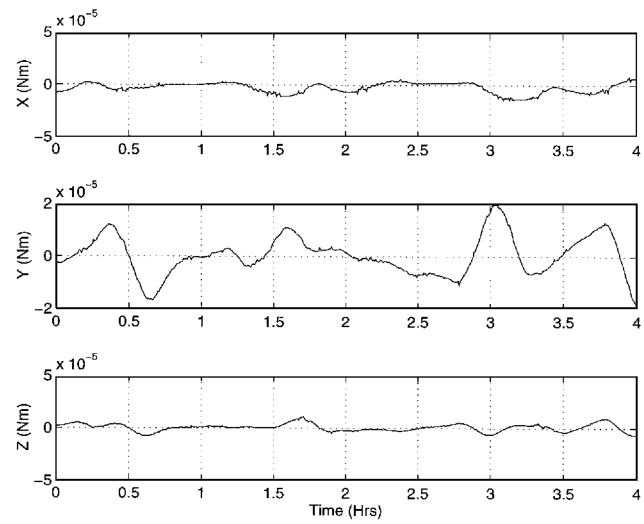

Fig. 6 Plot of determined model error trajectories using TAM only.

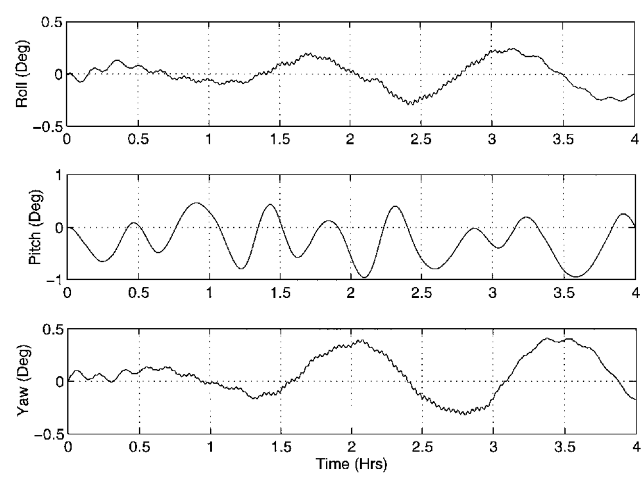

Fig. 7 Plot of attitude errors between TAM/FSS and TAM only.

Conclusions

In this paper, a predictive filter is presented for use in real-time attitude estimation. This algorithm was specifically developed for spacecraft that lack angular rate-sensing equipment. The advantages

of the new algorithm over the Kalman filter for attitude estimation include the following.

1) Quaternion normalization is always preserved throughout the estimation process.

2) Actual model error trajectories are determined as part of the filter solution.

3) No covariance propagation is required, which leads to a computationally less-demanding algorithm.

The predictive filter was used to estimate the attitude of SAMPEX in order to demonstrate the usefulness of this algorithm. Results using SAMPEX data indicate that the predictive filter provides a robust algorithm that can be used to determine the attitude of a spacecraft from magnetometer measurements only.

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