Conclusions

An explicit function associated with the invertibility of the nonlinear attitude controller with momentum management has been obtained. The function is given in terms of the mass properties of the spacecraft and the attitude of principal body axes relative to the local-vertical, local-horizontal frame. The TEA were shown to occur at orientations that are as far as can be from the singularity surface. In fact, the TEA occur at local minima and maxima of the singularity function. Knowledge of the singularity function may provide the basis for development of an explicit method to avoid singularities in the feedback linearized control law.

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Aerodynamic Parameter Estimation for High-Performance Aircraft Using Extended Kalman Filtering

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Introduction

THE design of flight control laws, the verification of performance predictions, and the implementation of flight simulations are all tasks that require a mathematical model for the dynamics of an aircraft. This dynamic model is typically characterized by coefficients or parameters whose numerical values must be determined for various flight conditions of interest. Among the most important of these are the parameters in the mathematical models for the aerodynamic forces and moments, often referred to as the aerodynamic coefficients.[1] Numerical values for these parameters are often first derived from wind-tunnel test data. However, the wind-tunnel conditions typically do not replicate the actual flight environment. It is desirable then to derive estimates for the aerodynamic coefficients directly from flight test data.

A number of parameter estimation methods, such as maximum likelihood[2] (ML) and linear regression,[3] have been applied to derive aerodynamic coefficients from aircraft flight test data. Filter-based methods, such as the extended Kalman filter[4] (EKF), have also been used with varying degrees of success (see bibliography in Ref. 5). One advantage of the EKF approach relative to most other approaches (including most ML formulations and essentially all least squares methods) is that it places no linearity restrictions on the form in which either the states or the parameters appear in the dynamic equations describing the system. It also does not require the parameters to be time invariant, nor does it require stability of the system. Finally, the EKF produces estimates of the parameters that approximately minimize the mean square error in the parameter estimates themselves, as opposed to minimizing a cost function that is based on matching the output variable behavior given a specific input trajectory, which is what most ML and least squares techniques are designed to do.

The EKF approach, however, requires the designer to select the statistical properties of the process and measurement noises and a model for the parameter dynamics. Some of this information is typically unknown to the user, and this introduces a degree of freedom in the EKF design that can be difficult to resolve. Furthermore, the EKF produces time histories for each of the estimated parameters, which is very useful if the time-varying nature of the parameters is of interest. However, in most aircraft parameter estimation situations, a single numerical value for each parameter is desired, and these must be derived from the EKF time histories.

This Note presents results from the application of the EKF to the estimation of aerodynamic coefficients for both NASA's X-31 drop model and the high-angle-of-attack research vehicle (HARV) from flight test data, which is part of an ongoing effort to develop systematic procedures for the design of EKFs for parameter estimation. The assumption of a fictitious noise process, or pseudonoise, driving the parameter model is shown to improve the EKF parameter estimates for the HARV. In addition, a residual correlation method[6] (RCM), originally derived for the linear Kalman filter,[4] was used to determine the appropriate process noise intensity and measurement noise covariance matrices for this case.

The Note is organized as follows. The next two sections introduce the aircraft dynamic equations and the EKF parameter estimator. Then, the parameter estimation results for the X-31 drop model and the HARV are presented and discussed. Conclusions follow.

Aircraft Dynamic Equations

The equations of motion for a rigid aircraft can be expressed in the general state-space form[1]

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{f}[\boldsymbol{x}(t), \boldsymbol{u}(t), \boldsymbol{\zeta}, t] + \boldsymbol{w}(t) \tag{1}$$

where the state vector $\boldsymbol{x}$ usually consists of the linear velocity components along the body axes u, v, and w; the angular velocity components about the body axes p, q, and r; and two Euler angles (usually pitch and roll, φ and θ) that describe the orientation of the body axes with respect to a fixed frame of reference. See, for instance, Ref. 1 for a detailed version of Eq. (1). Note that among the terms in Eq. (1) are the aerodynamic forces along the body axes X, Y, and Z and the aerodynamic moments about the body axes L, M, and N, which depend on the aerodynamic coefficients of interest. The vector $\boldsymbol{\zeta}$ contains these parameters. The control input $\boldsymbol{u}$ generally includes the control surface deflections and possibly some thrust-related variables. The vector noise process $\boldsymbol{w}$, which is assumed white with intensity $Q(t)$, represents unknown random perturbation inputs driving the plant (process noise), e.g., perturbation forces and moments arising from atmospheric turbulence.

The output equation representing the available discrete-time measurements is

$$\boldsymbol{z}(k) = \boldsymbol{h}[\boldsymbol{x}(t_k), \boldsymbol{u}(t_k), \zeta, t_k] + \boldsymbol{v}(k) \tag{2}$$

where the output vector $\boldsymbol{z}$ typically includes some subset of the following quantities: the total airspeed $V\,[=\sqrt{(u^2+v^2+w^2)}]$, the angle of attack $\alpha\,[=\tan^{-1}(w/u)]$, the angle of sideslip $\beta\,[=\sin^{-1}(v/V)]$, the angular velocity components, the Euler

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angles, and the linear accelerations along the body axes (which depend on the states and the current inputs). Sample index k corresponds to time t_k. The vector noise sequence $\boldsymbol{v}$, assumed white with covariance $R(k)$, represents noise corrupting the measurements (measurement noise).

EKF for Parameter Estimation

The EKF is a recursive algorithm that uses input information and output measurements with the assumed system and output models [Eqs. (1) and (2)] and assumed values for the process and measurement noise covariances Q and R to produce estimates of the states and a predicted state estimation error covariance. When the model given by Eqs. (1) and (2) is linear, the EKF reduces to the well-known Kalman filter.[4] The detailed equations describing the EKF algorithm can be found in Ref. 4.

When estimates of the system parameters are desired, the model of Eqs. (1) and (2) is modified by augmenting the state vector with the parameters to be estimated.[4] A state equation for the propagation of the estimated parameters $\boldsymbol{\zeta}_e$ must then be assumed. Typically, the parameters are assumed to be constant or driven by a fictitious random noise process, as

$$\dot{\boldsymbol{\zeta}}_e = \boldsymbol{w}_{\mathrm{PN}} \tag{3}$$

where $\boldsymbol{w}_{\mathrm{PN}}$ is a vector of zero-mean, white noise processes of intensity Q_{PN}, known as pseudonoise. This assumption is useful for modeling time-varying parameters when the parameter variation model is unknown or to account for modeling errors by allowing the parameter time variation to compensate for the unmodeled dynamics. The effect of the pseudonoise assumption is typically to speed up the convergence of the parameter estimates at the expense of increased parameter error variance by keeping the filter gains high.[5]

The values for Q and R (and Q_{PN} if parameter pseudonoise is assumed) are needed for the EKF computation of the state and parameter estimation error covariance. For a filter that is optimal in the mean square estimation error sense, the true values of Q and R should be used. However, different values for Q and R can be selected by the EKF designer to improve the filter performance, e.g., to compensate unknown system dynamics. The need to specify appropriate values for Q and R is probably the single most troublesome characteristic of EKF-based parameter estimation. R can often be obtained in a straightforward manner directly from the measurements using a Fourier decomposition approach[7] or from the characteristics of the instrumentation used. Q, however, is more difficult to determine.

Methods to identify Q have been derived for the standard Kalman filter.[8] One of these, namely, the RCM,[6] has been used here to determine Q for the HARV flight data. The RCM employs the residual sequence generated by the EKF, possibly under inappropriate initial assumptions for Q and R, to produce improved estimates of Q and R, which are then used in subsequent runs of the EKF.

The EKF satisfies the assumptions of the RCM only approximately, mainly because the augmented linearized system matrix F is time varying due to its dependence on the input and state variables. The time-varying partition of F, however, asymptotically has little influence on the residual covariance equations. This can be shown by writing the residual covariance equations in partitioned form. The time-varying partition of F always appears multiplied either by the parameter partition of the Kalman gain or by the parameter partition of the estimation error covariance matrix. Both the gain and the error covariance partitions asymptotically tend to zero (when Q_{PN} is zero),[9] thereby reducing the influence of any time variation in F on the residual covariance.

Results and Discussion

The EKF algorithm outlined in the preceding section was applied to the estimation of aerodynamic coefficients from flight data for the X-31 drop model and for the HARV.

X-31 Drop Model

The X-31 drop model is a fully instrumented scaled version of an experimental supermaneuverable fighter. It is carried by helicopter to a prescribed altitude and flown by remote control.

The objective for the X-31 was to estimate the aerodynamic coefficients in the expressions for the lateral force Y and the roll and yaw moments L and M from lateral maneuver flight data. Based on previous analyses of the flight data, linear expansions were considered sufficient to model the force and moments[10] as

$$
\begin{aligned}
Y &= S\bar{q}\left(c_{y_0} + c_{y_\beta}\beta + c_{y_p}\bar{b}p/2V + c_{y_r}\bar{b}r/2V + c_{y_{\delta a}}\delta a + c_{y_{\delta r}}\delta r\right)\\
L &= S\bar{q}\bar{b}\left(c_{l_0} + c_{l_\beta}\beta + c_{l_p}\bar{b}p/2V + c_{l_r}\bar{b}r/2V + c_{l_{\delta a}}\delta a + c_{l_{\delta r}}\delta r\right)\\
N &= S\bar{q}\bar{b}\left(c_{n_0} + c_{n_\beta}\beta + c_{n_p}\bar{b}p/2V + c_{n_r}\bar{b}r/2V + c_{n_{\delta a}}\delta a + c_{n_{\delta r}}\delta r\right)
\end{aligned}
\tag{4}
$$

where S is the wing surface area, $\bar{q}$ is the dynamic pressure, $\bar{b}$ is the wingspan, and δa and δr are the aileron and rudder deflections, respectively. In Eq. (4), the c_{ij} are the aerodynamic coefficients of interest.

The EKF estimation results presented subsequently correspond to one flight of the X-31 drop model at an average angle of attack of approximately 36 deg. A Fourier series decomposition method[7] was used to determine the measurement noise covariance R adopted for the EKF. For this case, Q and Q_{PN} were assumed to be zero.

The EKF estimates of the lateral state variables (v, p, r, and φ) showed good agreement with the measurements. The parameter estimate behavior, however, was mixed in character. The estimates of the lateral force parameters generally showed poor convergence characteristics or converged to unrealistic values. However, the calculated standard deviations of the estimation errors (square root of the diagonal elements of the predicted error covariance matrix) decreased little from their assumed original values, showing that little information on these parameters was present in the measurements.

On the other hand, the estimates of the parameters in the roll and yaw moment expressions were good, as indicated by their convergence characteristics together with a considerable decrease in the computed error standard deviation. The results for the roll and yaw moments also agree very well with estimates from wind-tunnel tests and the results of using another parameter identification technique, modified stepwise regression (MSR), on the same flight data.[10] Table 1 shows a comparison of results from these three sources. For the EKF results, a single value for the parameter estimates and the predicted error standard deviations (Std. dev.) was determined by time averaging the time histories after convergence had occurred.

HARV

The HARV is a twin-engine, single-seat, fighter aircraft, with twin vertical tails and thrust vectoring capability. For this aircraft, estimates of the aerodynamic coefficients characterizing the longitudinal and vertical forces X and Z and the pitching moment M were sought. Linear expansions were used to represent X, Z, and M, based on the analysis of Ref. 3:

$$
\begin{aligned}
X &= S\bar{q}\left(c_{x_0} + c_{x_\alpha}\alpha + c_{x_q}\bar{c}q/2V + c_{x_{\delta h}}\delta h\right)\\
Z &= S\bar{q}\left(c_{z_0} + c_{z_\alpha}\alpha + c_{z_q}\bar{c}q/2V + c_{z_{\delta h}}\delta h\right)\\
M &= S\bar{q}\bar{c}\left(c_{m_0} + c_{m_\alpha}\alpha + c_{m_q}\bar{c}q/2V + c_{m_{\delta h}}\delta h\right)
\end{aligned}
\tag{5}
$$

where $\bar{c}$ is the mean aerodynamic chord length and δh the horizontal tail deflection.

The EKF parameter estimation results presented here correspond to a longitudinal maneuver at angle-of-attack values ranging from 23 to 30 deg. Similar to the X-31 case, the measurement noise covariance R assumed for the filter was estimated from the measurements using Fourier series decomposition.[7]

Preliminary EKF results using the linear aerodynamic model (5) and assuming zero for both Q and Q_{PN} exhibited two problems. First, the residual sequence was not white, as it should be for a properly operating filter. Second, the convergence of the parameter estimates was poor even though their predicted error standard deviations decreased significantly, which ordinarily indicates good estimates. These problems can be caused by modeling errors coupled with incorrect assumptions for the process noise intensity and/or pseudonoise intensity.

Table 1 Estimates of the X-31 lateral aerodynamic coefficients

Source	c_{l_β}	c_{l_p}	c_{l_r}	$c_{l_{\delta a}}$	$c_{l_{\delta r}}$	c_{n_β}	c_{n_p}	c_{n_r}	$c_{n_{\delta a}}$	$c_{n_{\delta r}}$
WT	$-2.8E-1$	$6.4E-1$	$-6.5E-1$	$-8.2E-2$	$1E-2$	$4.5E-2$	$-9E-1$	$5E-1$	$-3.7E-2$	$-7.5E-2$
MSR	$-1.37E-1$	$3.1E-1$	$-4.5E-1$	$-3.52E-2$	$2.1E-3$	$6.6E-2$	$-6.5E-1$	$5.9E-1$	$-2.7E-3$	$-7.8E-2$
Std. dev.	$2.6E-3$	$1.5E-2$	$3.8E-2$	$7.8E-4$	$5.9E-4$	$6.1E-3$	$3.5E-2$	$8.9E-2$	$1.8E-3$	$1.4E-3$
EKF	$-1.464E-1$	$3.55E-1$	$-5.98E-1$	$-4.48E-2$	$1.014E-2$	$4.2E-2$	$-2.7E-1$	$2.1E-1$	$-2.89E-2$	$-8.83E-2$
Std. dev.	$7.4E-4$	$5.6E-3$	$9.3E-3$	$3.1E-4$	$1.8E-4$	$2.0E-3$	$1.6E-2$	$2.2E-2$	$8.7E-4$	$4.3E-4$

Table 2 Estimates of the HARV longitudinal aerodynamic coefficients

Source	c_{z_α}	c_{z_q}	$c_{z_{\delta h}}$	c_{m_α}	c_{m_q}	$c_{m_{\delta h}}$
WT, $\alpha = 27$ deg	-3.23	-3.1	-0.75	-0.31	-6.47	-0.98
min/max	$-3.24/-3.06$	$-3.1/-3.1$	$-1.03/-0.6$	$-0.31/0.12$	$-7.05/-5.88$	$-0.98/-0.96$
MSR, $\alpha = 27$ deg	-2.59	-23.1	-0.85	-0.19	-11.7	-0.94
min/max	$-3.0/-2.0$	$-32.3/-17.7$	$-0.91/-0.79$	$-0.27/-0.12$	$-12.4/-10.0$	$-0.94/-0.90$
EKF PN[a], $\alpha = 27$ deg	-2.52	48	-0.69	-0.273	-15.7	-0.92
Std. dev.	0.92	5	0.05	0.005	0.3	0.009
EKF Q[b], $\alpha = 27$ deg	-2.48	-5.42	-0.34	-0.25	-17	-0.83
Std. dev.	0.22	3.8	0.05	0.09	15	0.2

[a]Pseudonoise assumed in the filter. [b]RCM estimate of Q was used.

To correct this problem, a nonzero value for Q_{PN} was assumed for the filter. Rigorous methods to determine the appropriate values for Q_{PN} are not currently available. For this application, based on previous experience with the EKF,[5] the diagonal elements of Q_{PN} were set to the square of 0.1% of the nominal parameter value. Q was set to zero when nonzero Q_{PN} was assumed.

In addition, the residual correlation method[6] (RCM) was used to obtain estimates of Q and R. The RCM estimate for R was within an order of magnitude of the estimate obtained by Fourier decomposition. When Q and R were set to the RCM estimates, parameter pseudonoise was not assumed.

The state estimates produced by the EKF with either pseudonoise assumed or using the RCM estimates for the noise statistics improved substantially, as indicated by the residuals, which were much closer to a white sequence. The convergence characteristics of the parameter estimates also improved, with predicted error standard deviations that reflected more accurately the parameter estimate behavior.

Table 2 shows a comparison of wind-tunnel (WT) values and MSR and EKF estimates for the HARV aerodynamic parameters. As before, single EKF estimate values were calculated by averaging the last segment of the parameter estimate time histories after convergence occurs. The label $\alpha = 27$ deg indicates the average angle of attack during this flight. The wind-tunnel and MSR values are from Ref. 3. Error standard deviation information was not specified in this source. The wind-tunnel and MSR parameter estimates in Ref. 3 are given as a function of the angle of attack. Table 2 shows the values corresponding to $\alpha = 27$ deg. Additionally, Table 2 shows the maximum and minimum parameter values for the α range achieved during the analyzed flight, to give an idea of the strong variation of the HARV aerodynamic parameters with α.

The EKF estimates shown in Table 2 exhibit a varying degree of agreement with the wind-tunnel values. The agreement is better for the tail effectiveness and the angle-of-attack coefficients (especially when pseudonoise is assumed) and worse for c_{m_q}. When pseudonoise is assumed, the magnitude and the sign of the EKF estimate of c_{z_q} are wrong relative to the wind-tunnel value. Notice that, for c_{m_q} and c_{z_q}, the MSR estimates are also questionable. These results indicate an identifiability problem for the pitch rate coefficients, a problem that has also been discussed in Ref. 3.

Notice also that the predicted error standard deviation for the pitch-moment aerodynamic coefficients is considerably larger when the estimated Q is used. This is due to the relatively large estimate of the intensity for the plant noise corresponding to the pitch-rate state equation.

Conclusions

The EKF has been used to estimate the coefficients in models for the aerodynamic forces and moments for two high-performance aircraft from flight data. The estimates of the lateral moment parameters for the X-31 drop model produced by the EKF are of quality similar to those obtained with linear regression methods in terms of the parameter estimate values and their indicated uncertainty, as given by the corresponding predicted error standard deviations. The EKF parameter estimates are also close to wind-tunnel estimates. For the lateral force parameters of the X-31, however, the EKF estimates were unreasonable, especially when compared to MSR or wind-tunnel values. However, the calculated error covariance for the EKF gave a clear indication of the poor quality of the lateral force estimates.

For the HARV, the results for the longitudinal parameters show that a parameter pseudonoise assumption can substantially improve the effectiveness of the filter, provided that an appropriate value is chosen for the pseudonoise intensity matrix. A residual correlation method was used to estimate the process noise intensity matrix. The use of the estimated process noise intensity also improved the behavior of the EKF estimates. For the HARV, the quality of the estimates varied considerably among the parameters.

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