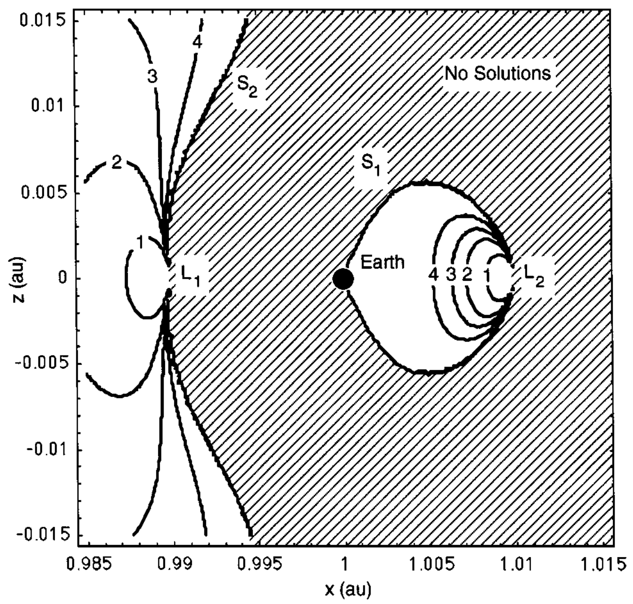

Fig. 3 Contours of sail lightness numbers in the x–z plane with $\eta = 0.9$. Contours: 1—0.02, 2—0.04, 3—0.06, and 4—0.1.

Lastly, using Eq. (11), it is found that the required sail lightness number may be obtained in terms of the lightness number for an ideal solar sail $\tilde{\beta}$ as

$$\beta = \frac{2}{(1+\eta)} \frac{\sqrt{1+\tan^2\phi}}{(1-\tan\theta\tan\phi)^2}\tilde{\beta} \tag{13}$$

where $\tilde{\beta}$ is defined by Eq. (5). Therefore, using Eqs. (9), (12), and (13), one can obtain the sail orientation and lightness number required for an equilibrium solution.

The effect of a nonideal flat solar sail is shown in Fig. 3 for a reflectivity of 0.9, typical of an aluminized sail film. First it can be seen that the volume of space available for equilibrium solutions about L_2 is significantly reduced. This is due to the centerline angle, which limits the direction in which the radiation pressure force vector can be oriented. For solutions near L_1 the main effect of the nonideal sail is to displace the equilibrium solutions toward Earth. This is due to the reduction in the magnitude of the radiation pressure force rather than the centerline angle.

IV. Conclusions

It has been shown that a partially reflecting solar sail can be used to generate artificial Lagrange points in Sun–planet three-body systems. However, the nonperfect reflectivity of the solar sail can have a significant effect on the volume of space in which such equilibrium solutions are possible. The main reason for the sensitivity of the problem to the sail reflectivity is the centerline angle, which limits the direction in which the radiation pressure force vector can be oriented.

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Simple Correction Algorithm of Scanning Horizon Sensor Measurement for Earth Oblateness

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Introduction

THE scanning horizon sensor is widely utilized in the Earth-pointing satellite to estimate its attitude with respect to the local vertical, i.e., roll and pitch angles. It typically uses an infrared detector together with a pencil beam to sense the abrupt change in the infrared radiation intensity as the beam sweeps from cold space across the horizon. The beam scans the horizon either by using an internal mechanism (such as an oscillating mirror, a motor-driven rotating reflective or refractive optic, etc.) or by mounting the sensor on a bias momentum wheel. Although the scan mechanisms are different, the operating principle of the scanning horizon sensors can be generally described as follows: The field of view (FOV) of the infrared detector diverts from the spin axis at a specific angle and traces out a cone (or part of a cone) as the sensor scans. The rising pulse and the falling pulse are generated as the Earth's horizon is encountered going from cold space, across the Earth, and then back into cold space. Given the fixed sensor reference, the phase angles of the horizon crossing points, which define the geometric intersection of the scan cone with the Earth, can be measured from the timing of the pulses, and the roll and pitch angles of the satellite are estimated. Because the Earth is not an exact sphere, but approximately an oblate spheroid relative to the polar axis, the Earth oblateness must be corrected in the attitude estimation, or an attitude error will remain. Several authors (e.g., Ref. 1 and the references therein) have studied the impact of the Earth oblateness on the attitude errors, but most of results have been scattered in the open literature and internal technical documents.

In this Note a simple algorithm is presented to correct the scanning horizon sensor measurement for the Earth oblateness in the satellite attitude estimation. Compared with the method described in Ref. 2, the problem is transferred from solving a three-dimensional vector equation of the horizon crossing vector to solving a scalar equation of the phase angle of the horizon crossing point, and the algorithm is simplified. Considering that the flattening coefficient of the oblate Earth is small, a first-order correction algorithm is also derived, which achieves relatively high accuracy with much simpler computation.

Reference Frames

Define several reference frames as follow: 1) the geocentric-equatorial inertial frame $O_I - \boldsymbol{X}_I\boldsymbol{Y}_I\boldsymbol{Z}_I$, where O_I is the center of the Earth, $\boldsymbol{X}_I$ points in the vernal equinox direction, $\boldsymbol{Z}_I$ points to the North Pole, and $\boldsymbol{Y}_I$ completes the right-handed triad; 2) the satellite body frame $O_B - \boldsymbol{X}_B\boldsymbol{Y}_B\boldsymbol{Z}_B$, where O_B is the mass center of the satellite, and $\boldsymbol{X}_B$, $\boldsymbol{Y}_B$, and $\boldsymbol{Z}_B$ are the body-fixed roll, pitch, and yaw axes, respectively; 3) the horizon sensor frame $O_B - \boldsymbol{X}_S\boldsymbol{Y}_S\boldsymbol{Z}_S$, where $\boldsymbol{X}_S$ is opposite to the sensor spin axis, $\boldsymbol{Z}_S$ is orthogonal to the spin axis, the fixed sensor reference is in the plane $O_B - \boldsymbol{Z}_S\boldsymbol{X}_S$, and $\boldsymbol{Y}_S$ completes the right-handed triad; and 4) the auxiliary measurement frame $O_B - \boldsymbol{X}_A\boldsymbol{Y}_A\boldsymbol{Z}_A$, which is defined by the sensor spin axis and the satellite position vector $\boldsymbol{r}$ pointing from O_I to O_B as $\boldsymbol{X}_A = \boldsymbol{X}_S$, $\boldsymbol{Y}_A = (\boldsymbol{X}_A \times \boldsymbol{r}^0)/|\boldsymbol{X}_A \times \boldsymbol{r}^0|$, and $\boldsymbol{Z}_A = \boldsymbol{X}_A \times \boldsymbol{Y}_A$, where $\boldsymbol{r}^0$ is the unit vector of $\boldsymbol{r}$. Figure 1 illustrates the geometry of the reference frames.

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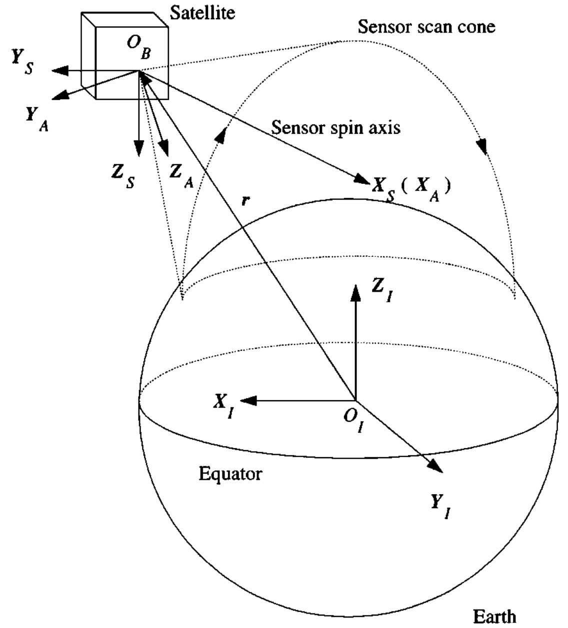

Fig. 1 Geometry of the reference frames.

In the correction algorithm of the horizon sensor measurement, the components of $\boldsymbol{X}_S$ and $\boldsymbol{r}$ in the frame $O_I - \boldsymbol{X}_I\boldsymbol{Y}_I\boldsymbol{Z}_I$ are known (donated as X_{SI} and r_I). The term X_{SI} is determined by the transformation matrix from inertial to body frame C_{BI} and the transformation matrix from body to horizon sensor frame C_{SB}, and r_I is determined by the orbit parameters of the satellite, and so the transformation matrix from inertial to auxiliary measurement frame C_{AI} is also determined.

Attitude Estimation with Scanning Horizon Sensor

The surface of the oblate Earth in the frame $O_I - \boldsymbol{X}_I\boldsymbol{Y}_I\boldsymbol{Z}_I$ is described by the following:

$$x^2 + y^2 + z^2/(1-f)^2 = R_E^2 \tag{1}$$

where R_E is the equator radius of the Earth reference spheroid and f is the flattening coefficient. Let $\alpha_{\text{OBL-IN}}$ and $\alpha_{\text{OBL-OUT}}$ be the phase angle measurements of the in-crossing and out-crossing points of the oblate Earth in the frame $O_B - \boldsymbol{X}_S\boldsymbol{Y}_S\boldsymbol{Z}_S$ (the phase angle is defined to be positive in the direction of $\boldsymbol{Y}_S$). Consider the situation of the sensor FOV crossing the horizon of an imaginary sphere with the radius of R_E, and the corresponding in-crossing and out-crossing phase angles are denoted as $\alpha_{\text{SPH-IN}}$ and $\alpha_{\text{SPH-OUT}}$. It is assumed that $\alpha_{\text{SPH-IN}} = \alpha_{\text{OBL-IN}} - \delta\alpha_{\text{IN}}$, and $\alpha_{\text{SPH-OUT}} = \alpha_{\text{OBL-OUT}} - \delta\alpha_{\text{OUT}}$, where $\delta\alpha_{\text{IN}}$ and $\delta\alpha_{\text{OUT}}$ are correction terms of the horizon sensor measurement for the Earth oblateness.

Define a unit vector $\boldsymbol{E}^0 = -\boldsymbol{r}^0$, and then its phase angle in the frame $O_B - \boldsymbol{X}_S\boldsymbol{Y}_S\boldsymbol{Z}_S$ is

$$\theta = \frac{\alpha_{\text{SPH-IN}} + \alpha_{\text{SPH-OUT}}}{2} \tag{2}$$

The term η, the angle between $\boldsymbol{X}_S$ and $\boldsymbol{E}^0$, is solved from the following:

$$\cos\rho = \cos\gamma\cos\eta + \sin\gamma\sin\eta\cos(\Omega_{\text{SPH}}/2) \tag{3}$$

where γ is the half-cone angle of the horizon sensor, $\rho = \sin^{-1}(R_E/r)$ is the Earth angular radius as seen from the satellite, and $\Omega_{\text{SPH}} = \alpha_{\text{SPH-OUT}} - \alpha_{\text{SPH-IN}}$ is the chord width between the in-crossing and out-crossing points. Based on θ, η, and the transformation matrix C_{SB}, the components of $\boldsymbol{E}^0$ in the frame $O_B - \boldsymbol{X}_B\boldsymbol{Y}_B\boldsymbol{Z}_B$ can be determined, and the estimation of the satellite roll and pitch angles is obtained.

Correction Algorithm of Scanning Horizon Sensor Measurement

To determine the correction terms $\delta\alpha_{\text{IN}}$ and $\delta\alpha_{\text{OUT}}$, the phase angle of the horizon crossing points in the frame $O_B - \boldsymbol{X}_A\boldsymbol{Y}_A\boldsymbol{Z}_A$ is studied. Let $\beta_{\text{OBL-IN}}$ and $\beta_{\text{OBL-OUT}}$ be phase angles of the horizon crossing points of the oblate Earth in the frame $O_B - \boldsymbol{X}_A\boldsymbol{Y}_A\boldsymbol{Z}_A$, and let $\beta_{\text{SPH-IN}}$ and $\beta_{\text{SPH-OUT}}$ be corresponding phase angles of the horizon crossing points of the imaginary sphere. Notice that the frame $O_B - \boldsymbol{X}_A\boldsymbol{Y}_A\boldsymbol{Z}_A$ can be transformed from the frame $O_B - \boldsymbol{X}_S\boldsymbol{Y}_S\boldsymbol{Z}_S$ with the rotation about the axis $\boldsymbol{X}_S$, and it is obvious that

$$\delta\alpha_{\text{IN}} = \delta\beta_{\text{IN}} = \beta_{\text{OBL-IN}} - \beta_{\text{SPH-IN}} \tag{4}$$

$$\delta\alpha_{\text{OUT}} = \delta\beta_{\text{OUT}} = \beta_{\text{OBL-OUT}} - \beta_{\text{SPH-OUT}} \tag{5}$$

where

$$\beta_{\text{SPH-IN}} = -\cos^{-1}\left[\frac{\cos\rho - \cos\gamma\cos\eta}{\sin\gamma\sin\eta}\right]$$

$$\beta_{\text{SPH-OUT}} = +\cos^{-1}\left[\frac{\cos\rho - \cos\gamma\cos\eta}{\sin\gamma\sin\eta}\right]$$

and $\beta_{\text{OBL-IN}}$ and $\beta_{\text{OBL-OUT}}$ are determined by solving the scalar equation of phase angle of the horizon crossing points of the oblate Earth given next.

Equation of Phase Angle of the Horizon Crossing Points

Let a unit vector $\boldsymbol{V}^0$ denote the FOV of the scanning horizon sensor, and $V_I^0 = [V_{IX}^0 \;\; V_{IY}^0 \;\; V_{IZ}^0]^T$ and $r_I^0 = [r_{IX}^0 \;\; r_{IY}^0 \;\; r_{IZ}^0]^T$ are components of $\boldsymbol{V}^0$ and $\boldsymbol{r}^0$ in the frame $O_I - \boldsymbol{X}_I\boldsymbol{Y}_I\boldsymbol{Z}_I$, respectively; then the sensor FOV is described by the linear equation

$$x = p \cdot V_{IX}^0 + r \cdot r_{IX}^0, \qquad y = p \cdot V_{IY}^0 + r \cdot r_{IY}^0$$
$$z = p \cdot V_{IZ}^0 + r \cdot r_{IZ}^0 \tag{6}$$

where p is an auxiliary variable. Combining Eqs. (6) and (1) leads to a second-order equation of (p/r)

$$A(p/r)^2 + 2B(p/r) + C = 0 \tag{7}$$

whose coefficients are given as follows:

$$A = \left(V_{IX}^0\right)^2 + \left(V_{IY}^0\right)^2 + \left(V_{IZ}^0\right)^2\Big/(1-f)^2 = 1 + g \cdot \left(V_{IZ}^0\right)^2 \tag{8}$$

$$\begin{aligned} B &= r_{IX}^0 V_{IX}^0 + r_{IY}^0 V_{IY}^0 + \left(r_{IZ}^0 V_{IZ}^0\right)\Big/(1-f)^2 \\ &= \left(r_I^0\right)^T V_I^0 + g \cdot r_{IZ}^0 \cdot V_{IZ}^0 \end{aligned} \tag{9}$$

$$\begin{aligned} C &= \left(r_{IX}^0\right)^2 + \left(r_{IY}^0\right)^2 + \left(r_{IZ}^0\right)^2\Big/(1-f)^2 - R_E^2\Big/r^2 \\ &= \cos^2\rho + g \cdot \left(r_{IZ}^0\right)^2 \end{aligned} \tag{10}$$

The parameter g in the preceding equations is $g = (1-f)^{-2} - 1$. The unit vector $\boldsymbol{V}^0$ can also be described by its phase angle β in the frame $O_B - \boldsymbol{X}_A\boldsymbol{Y}_A\boldsymbol{Z}_A$. The components of $\boldsymbol{V}^0$ and $\boldsymbol{r}^0$ in the frame $O_B - \boldsymbol{X}_A\boldsymbol{Y}_A\boldsymbol{Z}_A$ have the form as $V_A^0 = [\cos\gamma \;\; \sin\gamma\sin\beta \;\; \sin\gamma\cos\beta]^T$ and $r_A^0 = [-\cos\eta \;\; 0 \;\; -\sin\eta]^T$, and so we have

$$B = -\cos\gamma\cos\eta - \sin\gamma\sin\eta\cos\beta + g \cdot r_{IZ}^0 \cdot V_{IZ}^0(\beta) \tag{11}$$

$$V_{IZ}^0(\beta) = [0 \quad 0 \quad 1]C_{AI}^T \begin{bmatrix} \cos\gamma \\ \sin\gamma\sin\beta \\ \sin\gamma\cos\beta \end{bmatrix} \tag{12}$$

The geometric position of the horizon sensor FOV crossing the oblate Earth corresponds to the situation that Eq. (7) has the two same roots. Define a function of β as $\Delta(\beta) = B^2 - CA$, and then $\Delta(\beta) = 0$ gives the scalar equation of phase angle of the horizon crossing point of the oblate Earth, whose roots are $\beta_{\text{OBL-IN}}$ and $\beta_{\text{OBL-OUT}}$.

Iterative Correction Algorithm of the Horizon Sensor Measurement

The derivative of the function $\Delta(\beta)$ with respect to β is

$$\Delta'(\beta) = 2B \cdot B' - C \cdot A' \tag{13}$$

where

$$A' = 2g \cdot V_{IZ}^{0}(\beta) \cdot \left[V_{IZ}^{0}(\beta)\right]' \tag{14}$$

$$B' = \sin\gamma \sin\eta \sin\beta + g \cdot r_{IZ}^{0} \cdot \left[V_{IZ}^{0}(\beta)\right]' \tag{15}$$

$$\left[V_{IZ}^{0}(\beta)\right]' = [0 \quad 0 \quad 1]C_{AI}^{T} \begin{bmatrix} 0 \\ \sin\gamma\cos\beta \\ -\sin\gamma\sin\beta \end{bmatrix} \tag{16}$$

Based on the Newton–Raphson technique, an iterative algorithm is derived as follows:

$$\beta_{k+1} = \beta_k - \Delta(\beta_k)/\Delta'(\beta_k) \tag{17}$$

The initial value of the iteration is chosen as $\beta_{\text{SPH-IN}}$ and $\beta_{\text{SPH-OUT}}$, respectively, and $\beta_{\text{OBL-IN}}$ and $\beta_{\text{OBL-OUT}}$ can be obtained with the

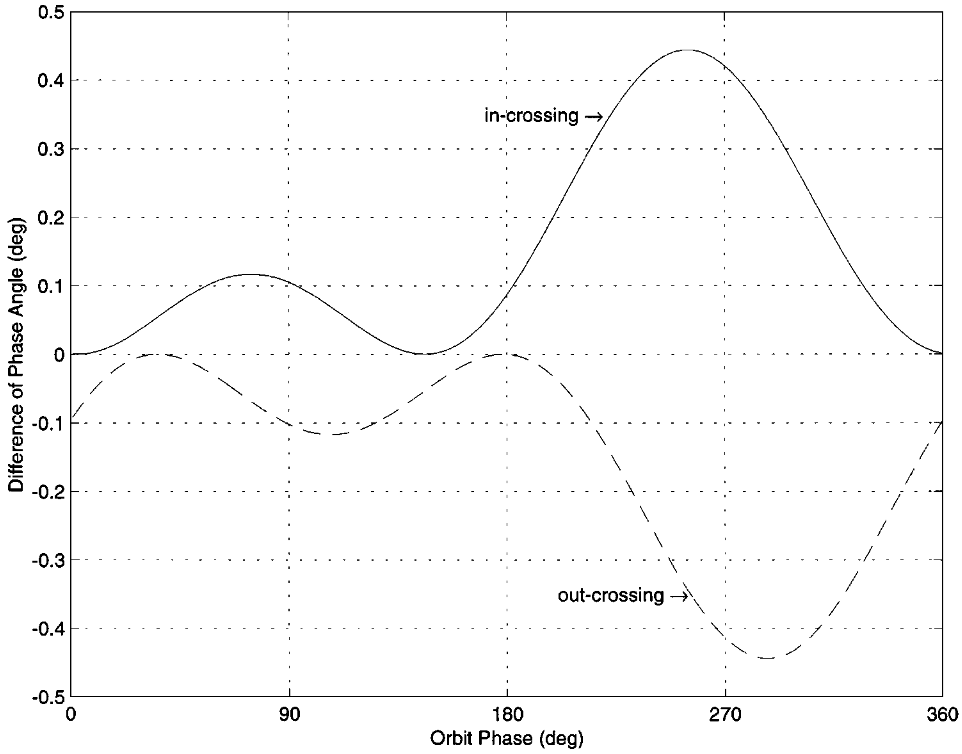

Fig. 2 Phase angle difference of the horizon crossing points between the oblate and spherical Earth models.

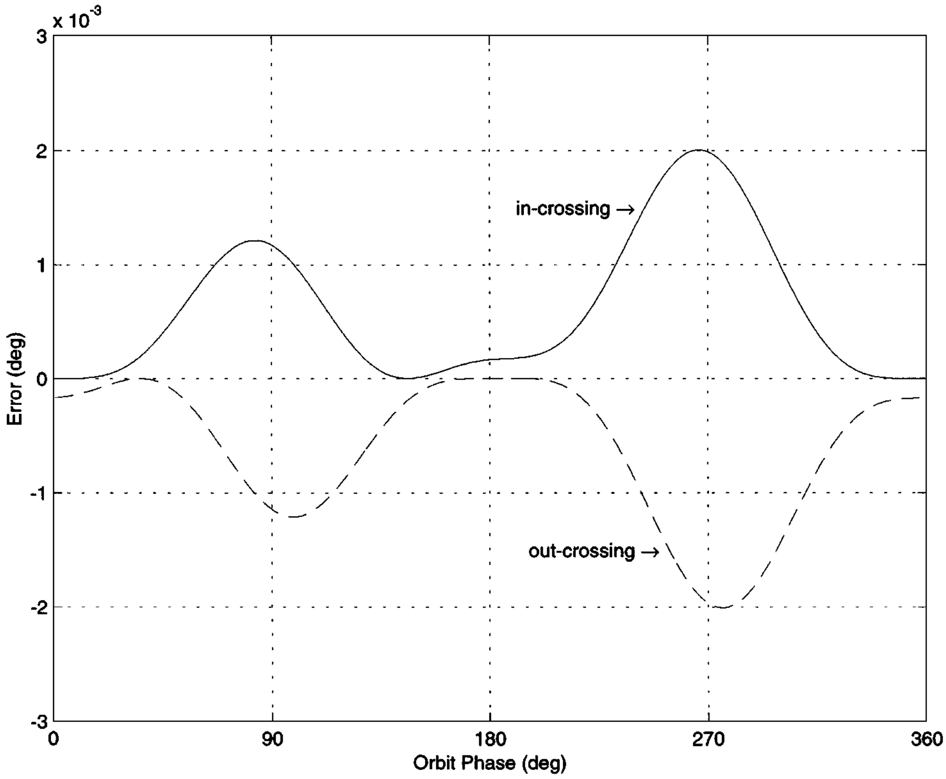

Fig. 3 Error of the first-order correction algorithm of the scanning horizon sensor measurement for the Earth oblateness.

required accuracy. From Eqs. (4) and (5) the correction terms $\delta\alpha_{\mathrm{IN}}$ and $\delta\alpha_{\mathrm{OUT}}$ are determined.

First-Order Correction Algorithm of the Horizon Sensor Measurement

The correction terms $\delta\alpha_{\mathrm{IN}}$ and $\delta\alpha_{\mathrm{OUT}}$ can be viewed as the function of the flattening coefficient f. Suppose that f were zero; the oblate Earth would be an exact sphere, and $\delta\alpha_{\mathrm{IN}}$ and $\delta\alpha_{\mathrm{OUT}}$ would also be zero. Because f is small, it is reasonable to replace $\delta\alpha_{\mathrm{IN}}$ and $\delta\alpha_{\mathrm{OUT}}$ with their first-order terms $(\delta\alpha_{\mathrm{IN}})_{\text{1st-order}}$ and $(\delta\alpha_{\mathrm{OUT}})_{\text{1st-order}}$ in the correction algorithm of the horizon sensor measurement for the Earth oblateness. Evaluating $\Delta(\beta)$ and its derivative $\Delta'(\beta)$ at $\beta_{\text{SPH-IN}}$ and $\beta_{\text{SPH-OUT}}$ leads to

$$(\delta\alpha_{\mathrm{IN}})_{\text{1st-order}} = (\delta\beta_{\mathrm{IN}})_{\text{1st-order}} = -\frac{\Delta(\beta_{\text{SPH-IN}})}{\Delta'(\beta_{\text{SPH-IN}})} \approx -\frac{g\left[\cos\rho \cdot V_{IZ}^{0}(\beta_{\text{SPH-IN}}) + r_{IZ}^{0}\right]^2}{2\cos\rho\sin\gamma\sin\eta\sin\beta_{\text{SPH-IN}}} \tag{18}$$

$$(\delta\alpha_{\mathrm{OUT}})_{\text{1st-order}} = (\delta\beta_{\mathrm{OUT}})_{\text{1st-order}} = -\frac{\Delta(\beta_{\text{SPH-OUT}})}{\Delta'(\beta_{\text{SPH-OUT}})} \approx -\frac{g\left[\cos\rho \cdot V_{IZ}^{0}(\beta_{\text{SPH-OUT}}) + r_{IZ}^{0}\right]^2}{2\cos\rho\sin\gamma\sin\eta\sin\beta_{\text{SPH-OUT}}} \tag{19}$$

Simulation Results

Digital simulations are carried out to study the effect of the Earth oblateness on the scanning horizon sensor measurement and the accuracy of the first-order correction algorithm. Given an Earth-pointing satellite whose nominal attitude is zero, the altitude is at 500 km, and the orbit inclination is 40 deg. The spin axis of the scanning horizon sensor is aligned with the pitch axis of the satellite. The half-cone angle of the sensor is 50 deg. Figure 2 shows the phase angle differences of the horizon crossing points between the oblate and spherical Earth models, in which the maximum phase angle difference is about 0.5 deg. Figure 3 shows that the error of the first-order correction algorithm of the scanning horizon sensor measurement for the Earth oblateness is less than 0.01 deg.

Conclusions

In this Note the correction of the scanning horizon sensor measurement for the Earth oblateness is discussed. An iterative correction algorithm is presented by solving a scalar equation of the phase angle of the horizon crossing point. Based on the fact that the flattening coefficient of the oblate Earth is small, a first-order correction algorithm is also derived. Compared with the methods in the open literature, the algorithms in this Note achieve relatively high accuracy with simple computation. Simulation results show that the accuracy of the first-order correction algorithm is better than 0.01 deg for a low-Earth-orbit satellite.

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