JOURNAL OF GUIDANCE, CONTROL, AND DYNAMICS
Vol. 22, No. 2, March–April 1999

Design of Notch Filters for Structural Responses with Multiaxis Coupling

Animesh Chakravarthy*
Aeronautical Development Agency, Bangalore 560 017, India
and
Girish Deodhare,† Vijay V. Patel,† and Amitabh Saraf†
Centre for Artificial Intelligence and Robotics, Bangalore 560 001, India

A novel method for designing notch filters to attenuate structural responses that show significant cross-coupling (between axes) is presented. This approach differs from the conventional methods in that it is a single-step procedure that enables notch filters of each sensor path to be designed independently of those in other sensor paths while guaranteeing the stability margin requirements. As a result, this approach can result in a less conservative design as compared to the conventional methods. This paper describes the procedure in the context of the coupled lateral-directional axes of an aircraft—the objective being to gain stabilize the structural response uniformly so as to meet the MIL-F-9490D specifications. An example is given to illustrate the power of this method.

Nomenclature

C_{11}	= controller transfer function from roll rate to aileron
C_{21}	= controller transfer function from roll rate to rudder
C_{22}	= controller transfer function from yaw rate to rudder
C_{23}	= controller transfer function from lateral acceleration to rudder
L_i	= loop transfer function
NYENV	= envelope used for design of Ny_F notch filters in Design 1
NYENV_1	= envelope used for design of Ny_F notch filters in first iteration of Design 2
NYENV_2	= envelope used for design of Ny_F notch filters in second iteration of Design 2
Ny	= lateral acceleration, g
Ny_F	= lateral acceleration notch filters
PENV	= envelope used for design of P_F notch filters in Design 1
PENV_1	= envelope used for design of P_F notch filters in first iteration of Design 2
PENV_2	= envelope used for design of P_F notch filters in second iteration of Design 2
P_F	= roll rate notch filters
P_{11}	= plant transfer function from aileron to roll rate
P_{12}	= plant transfer function from rudder to roll rate
P_{21}	= plant transfer function from aileron to yaw rate
P_{22}	= plant transfer function from rudder to yaw rate
P_{31}	= plant transfer function from aileron to lateral acceleration
P_{32}	= plant transfer function from rudder to lateral acceleration
p	= roll rate, deg/s
RENV	= envelope used for design of R_F notch filters in Design 1
RENV_1	= envelope used for design of R_F notch filters in first iteration of Design 2
RENV_2	= envelope used for design of R_F notch filters in second iteration of Design 2
R_F	= yaw rate notch filters
r	= yaw rate, deg/s
$\lvert Z \rvert$	= absolute magnitude of a complex number/array Z
$\lvert Z \rvert_{\mathrm{dB}}$	= magnitude in dB of a complex number/array Z
δ_a	= aileron input
δ_r	= rudder input
ζ_d	= denominator damping coefficient of notch filter
ζ_n	= numerator damping coefficient of notch filter
ω_d	= denominator natural frequency of notch filter
ω_n	= numerator natural frequency of notch filter

I. Introduction

FOR fighter aircraft with feedback control laws, stability margins for any loop need to be determined with other loops closed. This complicates the design of notch filters for structural responses that have coupling between axes, e.g., the lateral and directional axes. Notch filters in multiple sensor paths affect stability margins. A direct design to meet the stability margins in this case leads to an ill-conditioned and complex nonlinear optimization problem especially because each sensor path can have multiple second-order notch filters. In practice, notch filters for coupled-axis responses are designed through ad hoc methods by taking one sensor loop at a time and verifying the margins at each stage. This process not only involves considerable trial and error but can also lead to a very conservative design. In this paper an alternative one-shot procedure is evolved, so that notch filters for each individual sensor path can be designed independently of those in other sensor paths while ensuring that the requirements on the overall stability margins are met. Also, this method turns out to be less conservative than the ad hoc procedures. The notch filters are designed to meet stability margin requirements as stated in MIL-F-9490D.[1] Cheng and Hirner[2] discussed a constrained optimization procedure used in the notch filter design process, but it is limited to design of the filters in a single loop. This paper presents a design process that can be used to design notch filters in each loop for a multi-input/multi-output system; the process guarantees that the gain margins are met. In this paper the design process is illustrated through the design of notch filters for the coupled lateral-directional axes of an aircraft. The paper is organized as follows. The design procedure is evolved in Sec. II. In Sec. III, an example is used to illustrate how the conservativeness in the design can be reduced. Section IV discusses the conclusions.

II. Design Procedure

In this section, we describe a systematic approach to notch filter design. The design procedure for the notch filters of the lateral/directional axes of an aircraft, which is a coupled-axis system, is described. It is assumed that the phase information of the aeroservoelastic transfer functions (in the structural frequency range) is

Presented as Paper 97-3781 at the AIAA Atmospheric Flight Mechanics Conference, New Orleans, LA, Aug. 11–13, 1997; received Dec. 1, 1997; revision received Oct. 10, 1998; accepted for publication Oct. 14, 1998.

*Scientist, Flight Mechanics and Simulation Group.

†Scientist, Controls Division.

unavailable or unreliable, and it is therefore necessary to gain stabilize all of the flexible modes of the aircraft uniformly. It is also assumed that the structural modes are well separated from the rigid body modes so that gain stabilization is feasible. For multiloop systems, MIL-F-9490D[1] suggests that the gain margin for every loop be determined while keeping the other loops closed at their nominal values, and the notch filters be designed so that this margin is in excess of 8 dB. (In this paper the margins are designed to be in excess of 9 dB.) It is standard practice to design the notch filters after the rigid body control laws have been designed or are in a mature stage of their development. These control laws are designed based on an aeroelastically corrected representation of the aircraft rigid body dynamics. The corrections to the rigid body dynamics can be approximated before reliable aeroelastic data are available. Because the structural response data are not available in the early stages of the design, a certain budget in the rigid body stability margins is provided for the introduction of notch filters. The major effect on the rigid body stability margins because of the inclusion of notch filters is on the phase margins. This is because of the phase lag introduced by the notch filters at the rigid body gain crossover region. The filters, however, do not contribute much magnitude at the lower frequencies. Hence, the notch filters have to be designed with a predetermined constraint on their phase lag at the rigid body gain crossover region.

Figure 1 shows the block schematic of the lateral-directional axes of the aircraft with the controller. For the class of aircraft being considered, yaw rate and lateral acceleration are not fed back to the aileron, and therefore C_{12} and C_{13} are zero. However, it is straightforward to modify the analysis presented next to account for these feedback as well.

If the aileron and rudder loops are both broken at the actuator consolidation points, the multivariable loop transfer function can be represented as shown in Fig. 2, where

$$
\begin{aligned}
L_1 &= C_{11}P_{11}P_F, \qquad L_2 = C_{11}P_{12}P_F \\
L_3 &= C_{21}P_{11}P_F + C_{22}P_{21}R_F + C_{23}P_{31}Ny_F \\
L_4 &= C_{21}P_{12}P_F + C_{22}P_{22}R_F + C_{23}P_{32}Ny_F
\end{aligned}
\tag{1}
$$

The transfer function for evaluating the aileron margin with the rudder loop closed is then given by the following expression:

$$T_{y1u1} = L_1 + [L_2L_3/(1 - L_4)] \tag{2}$$

Similarly, the transfer function for evaluating the rudder margin with the aileron loop closed is then given by the following expression:

$$T_{y2u2} = L_4 + [L_2L_3/(1 - L_1)] \tag{3}$$

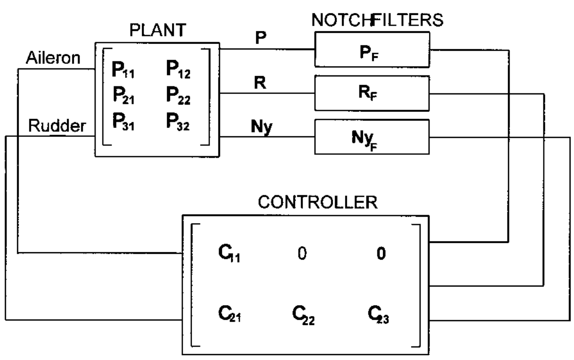

Fig. 1 Schematic of aircraft lateral-directional plant and controller.

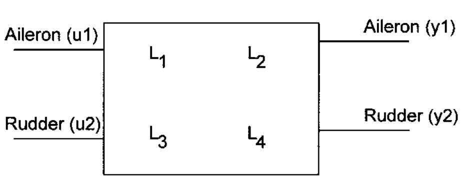

Fig. 2 Schematic of loop transfer functions.

Thus the design requirements are met if

$$|L_1 + [L_2L_3/(1 - L_4)]|_{\mathrm{dB}} \le -9\,\mathrm{dB}$$

$$|L_4 + [L_2L_3/(1 - L_1)]|_{\mathrm{dB}} \le -9\,\mathrm{dB}$$

or equivalently, if

$$
\begin{aligned}
|L_1 + [L_2L_3/(1 - L_4)]| &\le 0.3548 \\
|L_4 + [L_2L_3/(1 - L_1)]| &\le 0.3548
\end{aligned}
\tag{4}
$$

in the structural frequencies range with the constraint on the phase lag of the notch filters at the rigid body gain crossover frequency. Note that the stability margins are evaluated only at the actuator consolidation points.

Now

$$\left|L_1 + \frac{L_2L_3}{1 - L_4}\right| \le |L_1| + \frac{|L_2||L_3|}{|1 - L_4|} \le |L_1| + \frac{|L_2||L_3|}{|1 - |L_4||}$$

Thus Eqs. (4) are satisfied if

$$
\begin{aligned}
|L_1| + \frac{|L_2||L_3|}{|1 - |L_4||} &\le 0.3548 (= m) \\
|L_4| + \frac{|L_2||L_3|}{|1 - |L_1||} &\le 0.3548 (= m)
\end{aligned}
\tag{5}
$$

It can be seen that Eqs. (5) represent a more conservative requirement on the notch filters than Eqs. (4).

Suppose $|L_i| \le a$, where $0 < a < 1$ $(i = 1, 2, 3, 4)$. [Even though the stability of one loop is determined with the other loop closed, this will not lead to a conditional stability situation for the loop under investigation (conditional stability being defined as the result of the loop assumed to be closed being actually opened—e.g., because of a sensor failure), because scalar sums are taken in Eqs. (5). In fact, the margins will be better in such a case.] Then,

$$|L_1| + \frac{|L_2||L_3|}{|1 - |L_4||} \le a + \frac{a * a}{1 - a}$$

$$|L_4| + \frac{|L_2||L_3|}{|1 - |L_1||} \le a + \frac{a * a}{1 - a}$$

or equivalently

$$
\begin{aligned}
|L_1| + \frac{|L_2||L_3|}{|1 - |L_4||} &\le \frac{a}{1 - a} \\
|L_4| + \frac{|L_2||L_3|}{|1 - |L_1||} &\le \frac{a}{1 - a}
\end{aligned}
\tag{6}
$$

Now Eqs. (5) are satisfied if $[a/(1 - a)] \le m$ or

$$a \le [m/(1 + m)] = (0.2619 \approx -11.64\,\mathrm{dB}) \tag{7}$$

Thus, we can translate the requirements in Eqs. (5) to requirements on the loop transfer functions L_i $(i = 1, 2, 3, 4)$:

$$|L_i|_{\mathrm{dB}} \le -11.64\,\mathrm{dB}$$

However, one can note that the terms L_2 and L_3 always occur together as a product term in Eqs. (5). Hence the requirements on L_1, L_2, L_3, and L_4 can be rewritten as

$$
\begin{aligned}
&|L_1|_{\mathrm{dB}} \le -11.64\,\mathrm{dB}, \qquad |L_2|_{\mathrm{dB}} + |L_3|_{\mathrm{dB}} \le -23.28\,\mathrm{dB} \\
&|L_4|_{\mathrm{dB}} \le -11.64\,\mathrm{dB}
\end{aligned}
\tag{8}
$$

The structural filters can now be designed so that the magnitude of L_1, L_2, L_3, and L_4 satisfy Eqs. (8), which in turn implies that Eqs. (4) are satisfied.

Now we determine envelopes for the design of each of the notch filter banks. For each of the plant-controller combinations mentioned in Eqs. (1) (i.e., $C_{11}P_{11}$, $C_{11}P_{12}$, $C_{21}P_{11}$, $C_{22}P_{21}$, $C_{23}P_{31}$, $C_{21}P_{12}$, $C_{22}P_{22}$, and $C_{23}P_{32}$), the maximum envelope, which is the superset for all flight conditions of each plant-controller pair, is determined. (This plant data include aeroelastic effects for all flight conditions.) These are then referred to as $mC_{11}P_{11}$, $mC_{11}P_{12}$, $mC_{21}P_{11}$, $mC_{22}P_{21}$, $mC_{23}P_{31}$, $mC_{21}P_{12}$, $mC_{22}P_{22}$, and $mC_{23}P_{32}$, respectively.

From the first requirement in Eqs. (8), we have [from Eqs. (1)],

$$|mC_{11}P_{11}|_{\mathrm{dB}} + |P_F|_{\mathrm{dB}} \le -11.64\,\mathrm{dB} \tag{9}$$

Similarly, from Eqs. (1), we find that the second requirement in Eqs. (8) leads to

$$\begin{aligned}(|mC_{11}P_{12}P_F||mC_{21}P_{11}P_F + mC_{22}P_{21}R_F\\ + mC_{23}P_{31}Ny_F|)_{\mathrm{dB}} \le -23.28\,\mathrm{dB}\end{aligned} \tag{10}$$

However, because

$$\begin{aligned}&|mC_{11}P_{12}P_F||mC_{21}P_{11}P_F + mC_{22}P_{21}R_F + mC_{23}P_{31}Ny_F|\\ &\quad\le |mC_{11}P_{12}P_F||mC_{21}P_{11}P_F| + |mC_{11}P_{12}P_F||mC_{22}P_{21}R_F|\\ &\quad + |mC_{11}P_{12}P_F||mC_{23}P_{31}Ny_F|\end{aligned} \tag{11}$$

it is possible to satisfy Eq. (10) by satisfying the following inequality:

$$\begin{aligned}&(|mC_{11}P_{12}P_F||mC_{21}P_{11}P_F| + |mC_{11}P_{12}P_F||mC_{22}P_{21}R_F|\\ &\quad + |mC_{11}P_{12}P_F||mC_{23}P_{31}Ny_F|)_{\mathrm{dB}} \le -23.28\,\mathrm{dB}\end{aligned} \tag{12}$$

Distributing equally between all three of the preceding terms, the requirement on L_2 and L_3 is satisfied if the following are true:

$$\begin{aligned}|mC_{11}P_{12}P_F|_{\mathrm{dB}} + |mC_{21}P_{11}P_F|_{\mathrm{dB}} &\le -32.82\,\mathrm{dB}\\ |mC_{11}P_{12}P_F|_{\mathrm{dB}} + |mC_{22}P_{21}R_F|_{\mathrm{dB}} &\le -32.82\,\mathrm{dB}\\ |mC_{11}P_{12}P_F|_{\mathrm{dB}} + |mC_{23}P_{31}Ny_F|_{\mathrm{dB}} &\le -32.82\,\mathrm{dB}\end{aligned} \tag{13}$$

One can convert these to explicit requirements on the three sets of notch filters as follows:

$$\begin{aligned}|mC_{11}P_{12}|_{\mathrm{dB}} + |P_F|_{\mathrm{dB}} + |mC_{21}P_{11}|_{\mathrm{dB}} + |P_F|_{\mathrm{dB}} &\le -32.82\,\mathrm{dB}\\ |mC_{11}P_{12}|_{\mathrm{dB}} + |P_F|_{\mathrm{dB}} + |mC_{22}P_{21}|_{\mathrm{dB}} + |R_F|_{\mathrm{dB}} &\le -32.82\,\mathrm{dB}\\ |mC_{11}P_{12}|_{\mathrm{dB}} + |P_F|_{\mathrm{dB}} + |mC_{23}P_{31}|_{\mathrm{dB}} + |Ny_F|_{\mathrm{dB}} &\le -32.82\,\mathrm{dB}\end{aligned} \tag{14}$$

Again distributing equally and rearranging gives the following conditions:

$$\begin{aligned}(|mC_{11}P_{12}|_{\mathrm{dB}} + |mC_{21}P_{11}|_{\mathrm{dB}}) * 0.5 + |P_F|_{\mathrm{dB}} &\le -16.41\,\mathrm{dB}\\ |mC_{11}P_{12}|_{\mathrm{dB}} + |P_F|_{\mathrm{dB}} &\le -16.41\,\mathrm{dB}\\ |mC_{22}P_{21}|_{\mathrm{dB}} + |R_F|_{\mathrm{dB}} &\le -16.41\,\mathrm{dB}\\ |mC_{23}P_{31}|_{\mathrm{dB}} + |Ny_F|_{\mathrm{dB}} &\le -16.41\,\mathrm{dB}\end{aligned}$$

From the third requirement in Eqs. (8), we have [after substituting from Eqs. (1)],

$$|mC_{21}P_{12}P_F + mC_{22}P_{22}R_F + mC_{23}P_{32}Ny_F|_{\mathrm{dB}} \le -11.64\,\mathrm{dB} \tag{15}$$

Note that

$$\begin{aligned}&|mC_{21}P_{12}P_F + mC_{22}P_{22}R_F + mC_{23}P_{32}Ny_F|\\ &\quad \le |mC_{21}P_{12}P_F| + |mC_{22}P_{22}R_F| + |mC_{23}P_{32}Ny_F|\end{aligned} \tag{16}$$

Thus distributing equally between the three terms, we find that Eq. (15) is satisfied if the following are met:

$$\begin{aligned}|mC_{21}P_{12}|_{\mathrm{dB}} + |P_F|_{\mathrm{dB}} &\le -21.18\,\mathrm{dB}\\ |mC_{22}P_{22}|_{\mathrm{dB}} + |R_F|_{\mathrm{dB}} &\le -21.18\,\mathrm{dB}\\ |mC_{23}P_{32}|_{\mathrm{dB}} + |Ny_F|_{\mathrm{dB}} &\le -21.18\,\mathrm{dB}\end{aligned} \tag{17}$$

Equations (9), (14), and (17) are used to determine envelopes for the p, r, and Ny notch filters. The conditions on the p notch filter can be grouped together as follows:

$$\begin{aligned}|mC_{11}P_{11}|_{\mathrm{dB}} + |P_F|_{\mathrm{dB}} &\le -11.64\,\mathrm{dB}\\ (|mC_{11}P_{12}|_{\mathrm{dB}} + |mC_{21}P_{11}|_{\mathrm{dB}}) * 0.5 + |P_F|_{\mathrm{dB}} &\le -16.41\,\mathrm{dB}\\ |mC_{11}P_{12}|_{\mathrm{dB}} + |P_F|_{\mathrm{dB}} &\le -16.41\,\mathrm{dB}\\ |mC_{21}P_{12}|_{\mathrm{dB}} + |P_F|_{\mathrm{dB}} &\le -21.18\,\mathrm{dB}\end{aligned}$$

From these an envelope PENV for the p notch filter design can be determined such that

$$\mathrm{PENV} + |P_F|_{\mathrm{dB}} \le -11.64\,\mathrm{dB} \tag{18}$$

where

$$\begin{aligned}\mathrm{PENV} = \max\{&|mC_{11}P_{11}|_{\mathrm{dB}}, (|mC_{11}P_{12}|_{\mathrm{dB}} + |mC_{21}P_{11}|_{\mathrm{dB}}) * 0.5\\ &+ 4.77, |mC_{11}P_{12}|_{\mathrm{dB}} + 4.77, |mC_{21}P_{12}|_{\mathrm{dB}} + 9.54\}\end{aligned} \tag{19}$$

Similarly, the requirements on the r notch filter can be grouped together as follows:

$$\begin{aligned}|mC_{22}P_{21}|_{\mathrm{dB}} + |R_F|_{\mathrm{dB}} &\le -16.41\,\mathrm{dB}\\ |mC_{22}P_{22}|_{\mathrm{dB}} + |R_F|_{\mathrm{dB}} &\le -21.18\,\mathrm{dB}\end{aligned} \tag{20}$$

From this one can determine an envelope RENV for the r notch filter design such that

$$\mathrm{RENV} + |R_F|_{\mathrm{dB}} \le -11.64\,\mathrm{dB} \tag{21}$$

where

$$\mathrm{RENV} = \max\{|mC_{22}P_{21}|_{\mathrm{dB}} + 4.77, |mC_{22}P_{22}|_{\mathrm{dB}} + 9.54\} \tag{22}$$

Finally the requirements on the Ny notch filter are grouped together:

$$\begin{aligned}|mC_{23}P_{31}|_{\mathrm{dB}} + |Ny_F|_{\mathrm{dB}} &\le -16.41\,\mathrm{dB}\\ |mC_{23}P_{32}|_{\mathrm{dB}} + |Ny_F|_{\mathrm{dB}} &\le -21.18\,\mathrm{dB}\end{aligned} \tag{23}$$

This results in an envelope NYENV such that the Ny notch filter has to satisfy

$$\mathrm{NYENV} + |Ny_F|_{\mathrm{dB}} \le -11.64\,\mathrm{dB} \tag{24}$$

where

$$\mathrm{NYENV} = \max\{|mC_{23}P_{31}|_{\mathrm{dB}} + 4.77, |mC_{23}P_{32}|_{\mathrm{dB}} + 9.54\} \tag{25}$$

Designing notch filters for the envelopes PENV, RENV, and NYENV guarantees that the design requirements in Eqs. (4) are satisfied. However, the converse is not true. Thus, if one cannot design notch filters within these constraints for the three preceding envelopes, one cannot conclude that no notch filter exists satisfying Eqs. (4). The reason for this is the assumed "equal" distribution between terms while arriving at the envelopes, thus leading to unnecessarily conservative requirements. In the next section, it is illustrated through a design example how the preceding envelopes can be modified to reduce the conservativeness in the design to a large extent.

III. Discussions

A. Example to Illustrate this Design Procedure

Assume an aircraft with a digital controller, the controller operating at 80 Hz. The magnitudes of P_{11}, P_{12}, P_{21}, P_{22}, P_{31}, P_{32}, C_{11}, C_{21}, C_{22}, and C_{23} are all available for various flight conditions, from which the requisite plant-controller maximum envelopes can be determined. These are shown in Figs. 3–10. The maximum magnitude for each of these plant-controller envelopes over the entire frequency range is given in Table 1, along with the filters that have to attenuate these terms. Subsequently, PENV, RENV, and NYENV are determined from Eqs. (19), (22), and (25), respectively. In this example, it is assumed that P_F comprises three digital filters and one analog filter, R_F comprises three digital filters, and Ny_F comprises two digital filters.

Table 1 Maximum magnitude values of the various plant-controller envelopes

Term	Maximum magnitude value, dB	Filter
$mC_{11}P_{11}$	24.26	P_F
$mC_{11}P_{12}$	9.05	P_F
$mC_{21}P_{11}$	34.54	P_F
$mC_{21}P_{12}$	21.42	P_F
$mC_{22}P_{21}$	20.60	R_F
$mC_{22}P_{22}$	16.72	R_F
$mC_{23}P_{31}$	−2.99	Ny_F
$mC_{23}P_{32}$	−14.11	Ny_F

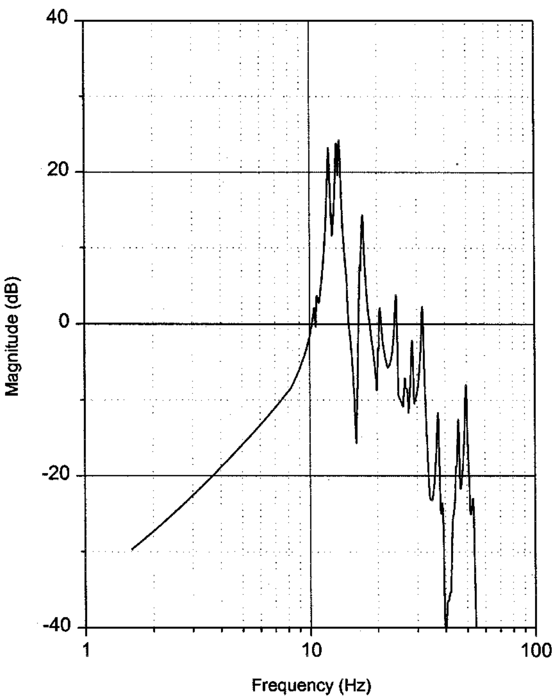

Fig. 3 δ_a-p-δ_a **loop** ($mC_{11}P_{11}$).

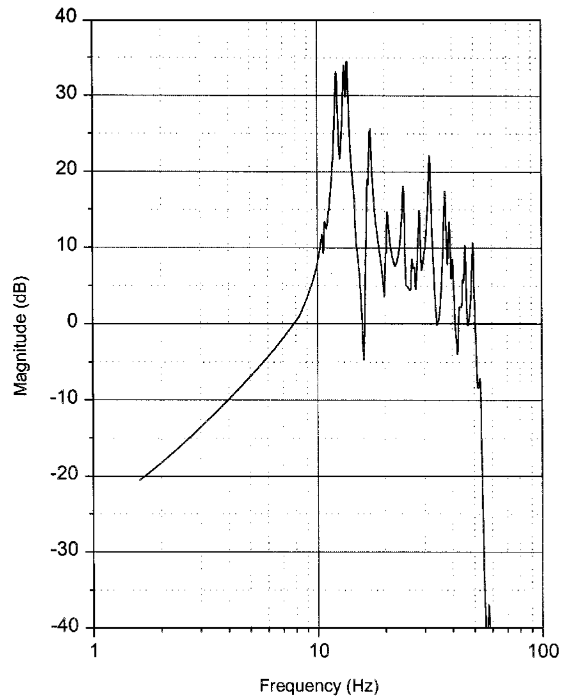

Fig. 5 δ_a-p-δ_r **loop** ($mC_{21}P_{11}$).

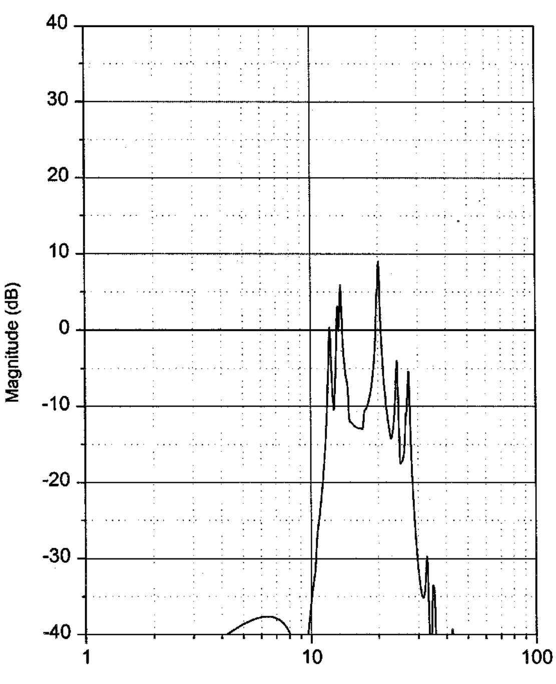

Fig. 4 δ_r-p-δ_a **loop** ($mC_{11}P_{12}$).

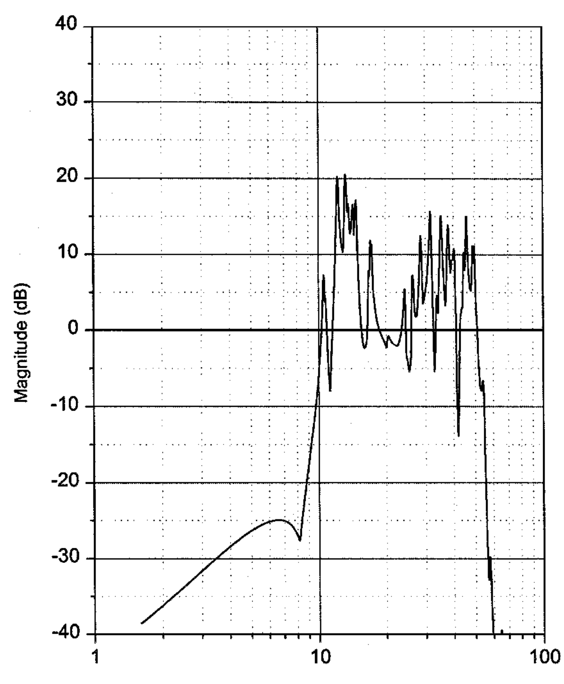

Fig. 6 δ_a-r-δ_r **loop** ($mC_{22}P_{21}$).

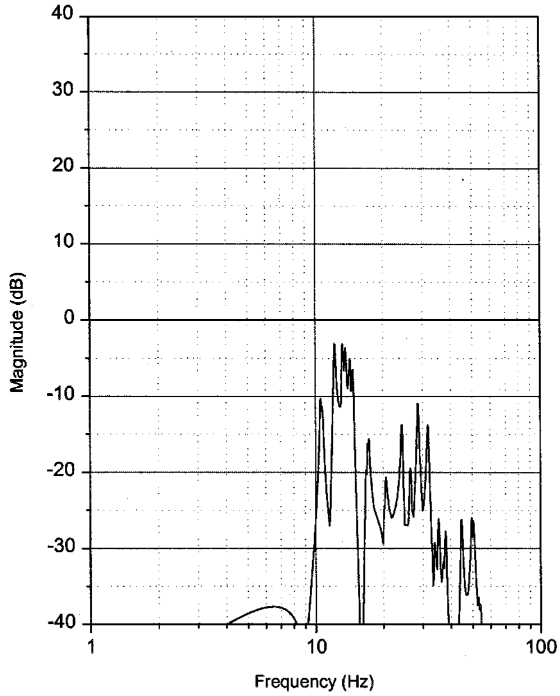

Fig. 7 δ_a-Ny-δ_r loop ($mC_{23}P_{31}$).

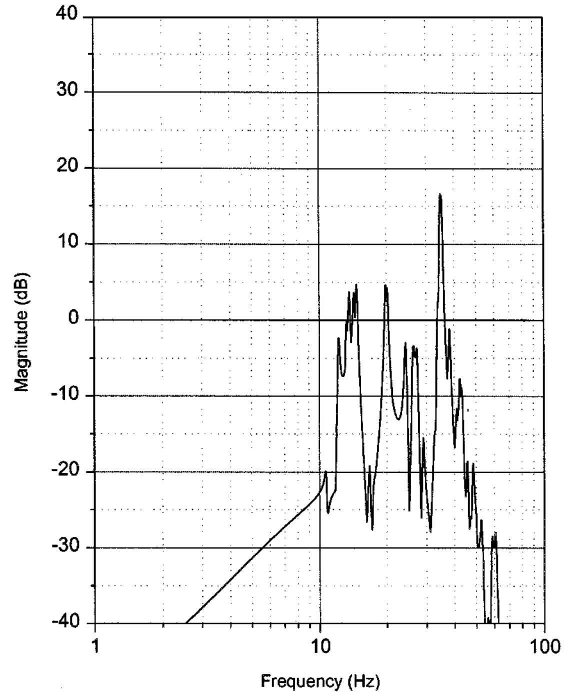

Fig. 9 δ_r-r-δ_r loop ($mC_{22}P_{22}$).

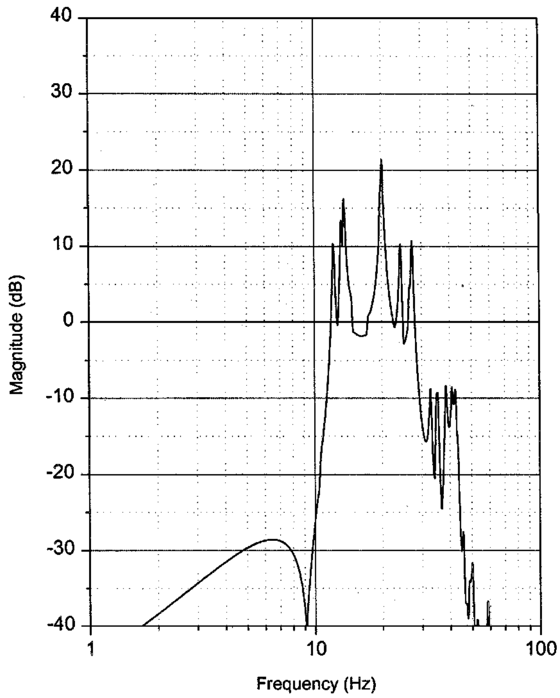

Fig. 8 δ_r-p-δ_r loop ($mC_{21}P_{12}$).

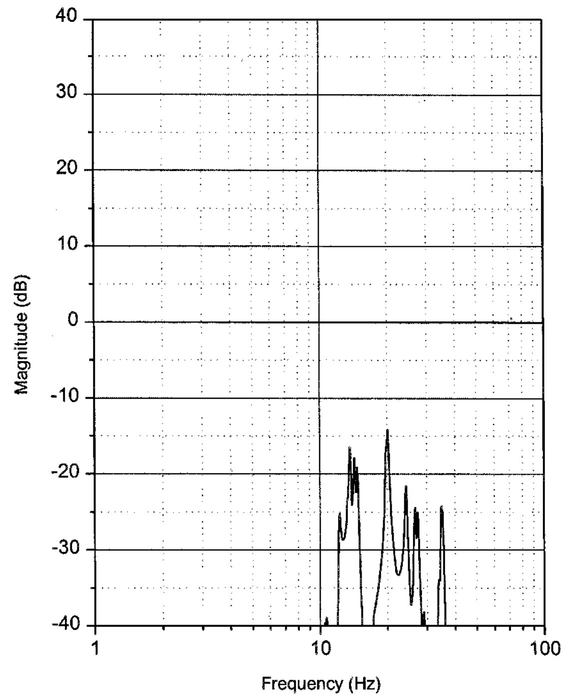

Fig. 10 δ_r-Ny-δ_r loop ($mC_{23}P_{32}$).

It is assumed that the rigid body control laws have already been designed, and while determining the rigid body stability margins, a certain budget had been provided for the phase introduced by the notch filters. Assume the rigid body gain crossover frequency to be 1 Hz and the budget provided for the phase introduced by the notch filters to be 15 deg in each sensor path. Hence it is required that, at 1 Hz, the phase-lag contribution of each bank of notch filters should not exceed 15 deg. This is required to ensure that the designed rigid body stability margins are not violated.

In this example, we perform two designs. The first design is performed to meet the gain margin requirements with the phase lag constraints being satisfied; this is presented in Sec. III.B. The second design (presented in Sec. III.D) is performed to show how the amount of phase lag introduced by the notch filters at the lower frequencies can be controlled.

B. Discussion of Design 1

The design problem is formulated as three independent optimization problems as follows:

1) Determine the coefficients of the set of notch filters comprising P_F so as to minimize the phase lag at 1 Hz contributed by these filters with the constraint that Eq. (18) is satisfied, PENV being defined as in Eq. (19).

2) Determine the coefficients of the set of notch filters comprising R_F so as to minimize the phase lag at 1 Hz contributed by these filters with the constraint that Eq. (21) is satisfied, RENV being defined as in Eq. (22).

3) Determine the coefficients of a set of notch filters comprising Ny_F so as to minimize the phase lag at 1 Hz contributed by these filters with the constraint that Eq. (24) is satisfied, NYENV being defined as in Eq. (25).

Table 2 Notch filter coefficients obtained from Design 1

Filter	Transfer function
P_F notch filter − 1 ($P_F - 1$)	$\dfrac{1+2*(0.046/95.0)s+(s/95.0)^2}{1+2*(0.421/61.8)s+(s/61.8)^2}$
P_F notch filter − 2 ($P_F - 2$)	$\dfrac{1+2*(0.038/84.1)s+(s/84.1)^2}{1+2*(0.369/84.1)s+(s/84.1)^2}$
P_F notch filter − 3 ($P_F - 3$)	$\dfrac{1+2*(0.036/158.4)s+(s/158.4)^2}{1+2*(0.278/116.8)s+(s/116.8)^2}$
P_F notch filter − 4 ($P_F - 4$)	$\dfrac{1+2*(0.079/162.7)s+(s/162.7)^2}{1+2*(0.659/162.7)s+(s/162.7)^2}$
R_F notch filter − 1 ($R_F - 1$)	$\dfrac{1+2*(0.090/93.7)s+(s/93.7)^2}{1+2*(0.251/90.4)s+(s/90.4)^2}$
R_F notch filter − 2 ($R_F - 2$)	$\dfrac{1+2*(0.051/78.3)s+(s/78.3)^2}{1+2*(0.164/52.8)s+(s/52.8)^2}$
R_F notch filter − 3 ($R_F - 3$)	$\dfrac{1+2*(0.802/250.0)s+(s/250.0)^2}{1+2*(0.802/42.8)s+(s/42.8)^2}$
Ny_F notch filter − 1 ($Ny_F - 1$)	$\dfrac{1+2*(0.861/126.4)s+(s/126.4)^2}{1+2*(0.990/119.4)s+(s/119.4)^2}$
Ny_F notch filter − 2 ($Ny_F - 2$)	$\dfrac{1+2*(0.105/84.8)s+(s/84.8)^2}{1+2*(0.302/67.9)s+(s/67.9)^2}$

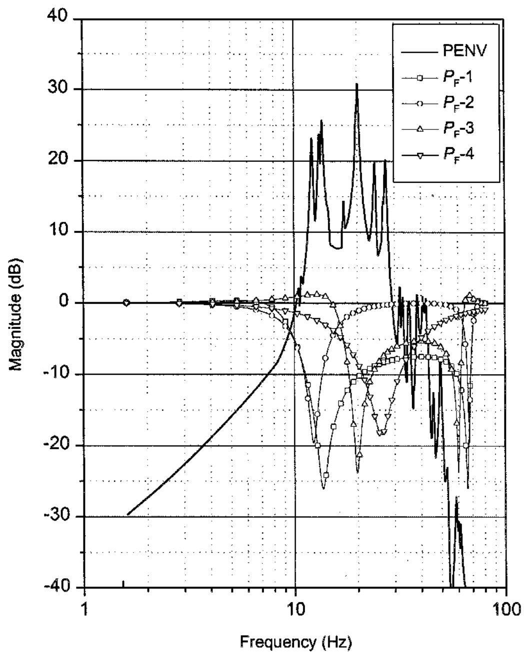

Fig. 11 PENV and individual notch filter sections of P_F—Design 1.

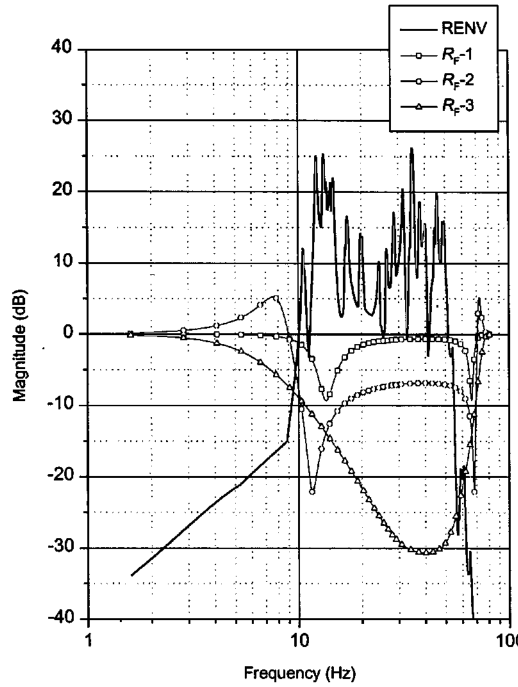

Fig. 12 RENV and individual notch filter sections of R_F—Design 1.

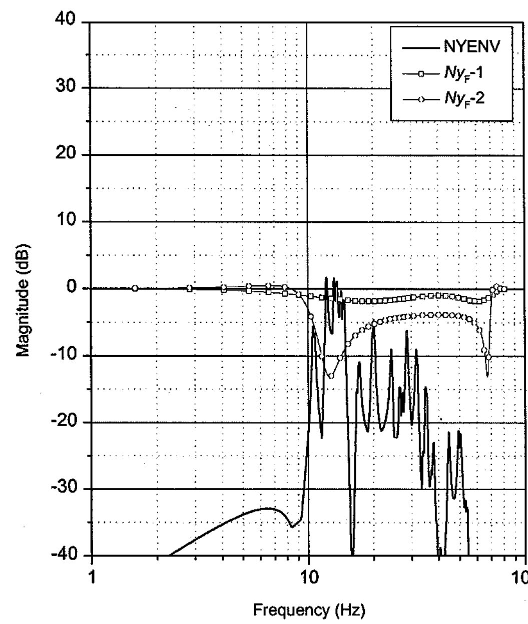

Fig. 13 NYENV and individual notch filter sections of Ny_F—Design 1.

The notch filters are designed using a constrained optimization program available in a MATLAB™ toolbox,[3] viz. "constr.m." Each notch filter is designed to possess the structure $[(s/\omega_n)^2 + (2\zeta_n s/\omega_n) + 1]/[(s/\omega_d)^2 + (2\zeta_d s/\omega_d) + 1]$. The digital filters are represented in the s domain; however, the design of the digital filters is done in the digital domain. The digital filters in the z domain are obtained by a Tustin transformation (no prewarping). Thus each filter has four unknowns ζ_n, ζ_d, ω_n, and ω_d (all of which are determined by the optimization program) with the constraints $\zeta_n < \zeta_d$; $\omega_d < \omega_n$. (These are essential constraints to ensure the notchlike structure of the filter). Additionally, bounds are imposed so that $\zeta_n, \zeta_d \in [0.01, 1]$ and $\omega_n, \omega_d \in [40, 250 \text{ rad/s}]$. Finally, the notch filters are constrained such that the overall magnitude contribution of all of the notch filters in a sensor path at any frequency is less than 0 dB, although individual notch filter sections are allowed to have magnitudes in excess of 0 dB.

After a suitable set of initial conditions is chosen, the notch filters are obtained from the optimization program. These are represented in Table 2. As previously mentioned, the filters are designed in the digital domain, with the z-domain representation being obtained through a Tustin transformation with no prewarping; e.g., P_F notch filter − 1 has the equivalent z-domain representation $(0.404z^2 - 0.372z + 0.372)/(z^2 - 1.154z + 0.559)$. Figures 11–13 show the envelopes PENV, RENV, and NYENV, respectively. Also plotted on Figs. 11–13 are the magnitude plots of the individual notch filter sections comprising the P_F, R_F, and Ny_F banks, respectively. With these notch filters, the aileron and rudder loop margins are calculated, as in Eqs. (2) and (3). Note that the margins incorporate the foldback due to the digital controller. The plots of these margins are shown in Figs. 14 and 15, respectively. From these figures, it is evident that in both the cases the loop magnitudes are attenuated by at least 9 dB at all frequencies in the structural range, thus satisfying the gain margin requirements. The phase lag contributed by the P_F, R_F, and Ny_F banks at the rigid body gain crossover frequency of 1 Hz is found to be 11.56, 14.32, and 3.45 deg, respectively. It is evident that each of these are within the 15-deg phase lag budget.

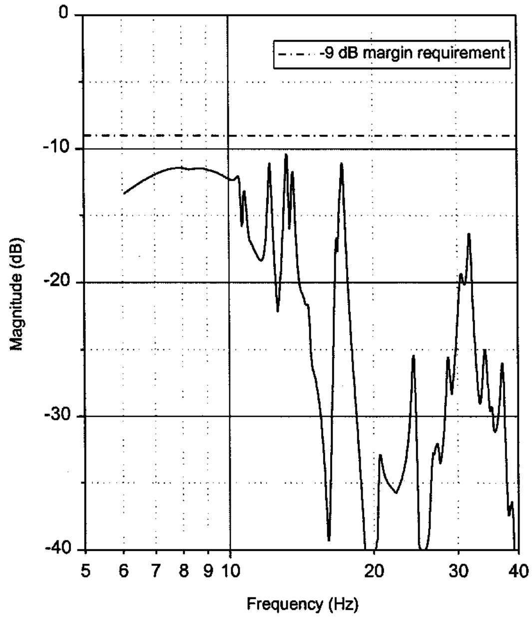

Fig. 14 Aileron loop margin—Design 1.

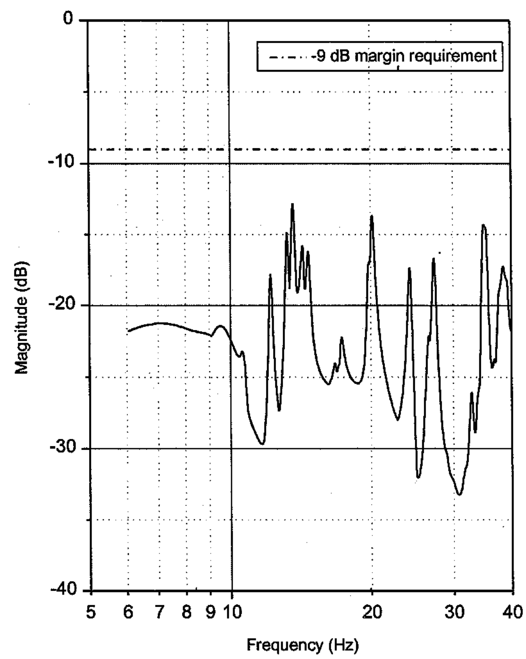

Fig. 15 Rudder loop margin—Design 1.

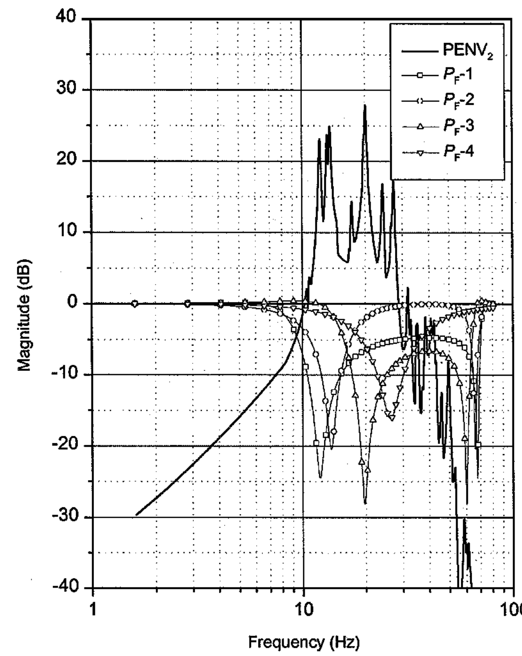

Fig. 16 PENV$_2$ and individual notch filter sections of P_F—Design 2.

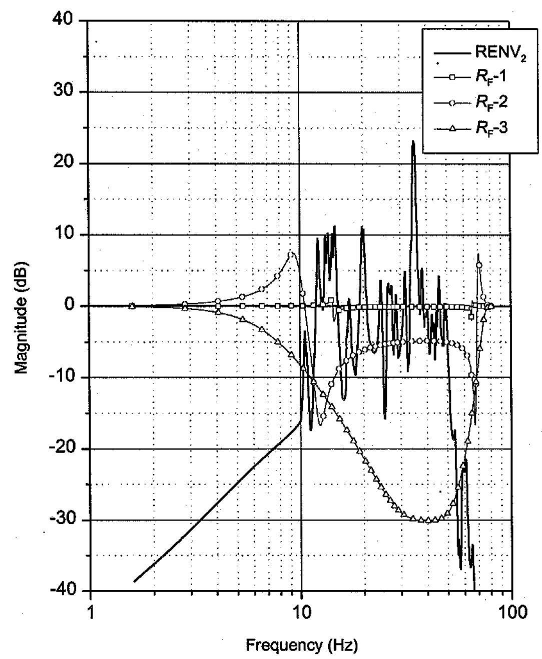

Fig. 17 RENV$_2$ and individual notch filter sections of R_F—Design 2.

Thus the design requirements have been met, and they have been met in a single step.

C. Comparison of this Design Procedure with Other Ad Hoc Design Methods

The ad hoc notch filter design methods, however, may require several iterations before they meet the MIL margin requirements. A typical ad hoc method would adopt the following procedure:

1) Break the roll rate sensor loop at the sensor point and, keeping the yaw rate and Ny sensor loops open, design the P_F bank so as to meet the MIL margin requirements for the roll rate sensor loop.

2) Break the rudder loop at the actuator point and, keeping the aileron loop open while using the P_F bank designed from Eqs. (1), design the R_F and Ny_F banks so as to meet the MIL margin requirements for the rudder loop.

It is evident that in the preceding ad hoc procedure it is not possible to design the R_F and Ny_F banks independently of the P_F bank. Thus one could encounter a situation wherein, after performing step 1 and designing a P_F bank, step 2 shows that no design exists for the R_F and Ny_F banks (which meet the MIL margin requirements) using the P_F bank obtained from step 1. In that case, step 1 would have to be redone, and an alternative P_F bank obtained and then used in step 2. In this way, several iterations would have to be performed before a set of P_F, R_F, and Ny_F banks are obtained that ensure the MIL margins are met.

The advantage of the method presented in this paper (and explained in Sec. II) over the ad hoc method is that it enables the three notch filter banks to be designed independently of each other and thus meet the MIL margins in a single step.

However, as mentioned in the last section, the construction of PENV, RENV, and NYENV, which forms the basis for the design method presented in this paper, is conservative. If it is not possible to design notch filter banks using these envelopes, it does not necessarily mean that no notch filter design exists that meets the margin requirements while also satisfying the constraints on phase lag. We

subsequently show how it is possible to modify the construction of the PENV, RENV, and NYENV envelopes. This can serve the following purposes:

1) If it is found that no notch filter design exists (satisfying the constraints on phase lag) for the original (conservative) envelopes, then using the modified envelopes and performing the design procedure with these envelopes, enhance the possibility of obtaining a successful design.

2) If a successful design has been obtained using the original (conservative) envelopes, then by using the modified envelopes and performing the design procedure with these envelopes, it is possible to have some control over the amount of phase lag introduced by the filters at the lower frequencies (i.e., it is possible to reduce the phase lag of one filter bank, at the cost of increasing the phase lag in some other filter bank).

D. Discussion of Design 2

The manner in which to modify the PENV, RENV, and NYENV envelopes depends on the specific plant-controller data. We shall continue to use the data presented in Figs. 3–10. Because we have already achieved a successful design for these data, we shall illustrate how to modify the envelopes to achieve the second objective discussed in the preceding section.

A study of Figs. 3–10 and Eq. (19) shows that the third term in Eq. (19) does not contribute to PENV at all, and is in fact well below PENV. It is found that this term is at least 14 dB less than PENV at all frequencies. Therefore, if we add 14 dB to this term, it becomes possible to subtract 14 dB from each of the last two equations in Eqs. (14), thus reducing RENV and NYENV. The new envelopes become

$$\mathrm{PENV}_1 = \max\{|mC_{11}P_{11}|_{\mathrm{dB}}, (|mC_{11}P_{12}|_{\mathrm{dB}} + |mC_{21}P_{11}|_{\mathrm{dB}}) * 0.5 + 4.77, |mC_{11}P_{12}|_{\mathrm{dB}} + 18.77, |mC_{21}P_{12}|_{\mathrm{dB}} + 9.54\} \tag{26}$$

$$\mathrm{RENV}_1 = \max\{|mC_{22}P_{21}|_{\mathrm{dB}} - 10.77, |mC_{22}P_{22}|_{\mathrm{dB}} + 9.54\} \tag{27}$$

$$\mathrm{NYENV}_1 = \max\{|mC_{23}P_{31}|_{\mathrm{dB}} - 10.77, |mC_{23}P_{32}|_{\mathrm{dB}} + 9.54\} \tag{28}$$

The design steps 1, 2, and 3 are then redone using PENV_1, RENV_1, and NYENV_1 in lieu of PENV, RENV, and NYENV, respectively. Doing so, we get the phase lag contribution (at 1 Hz) of the notch filter banks P_F, R_F, and Ny_F to be 11.56, 11.17, and 0.33 deg, respectively.

Thus, R_F now has a lower phase lag than before. The phase lag contribution of the Ny_F bank is quite low and has in fact decreased further from the original design (that used NYENV). As an additional modification, we could further reduce the phase lag contribution of both the P_F and the R_F banks by increasing the phase lag of the Ny_F bank. In this example, we allow the phase lag contribution (at 1 Hz) of the Ny_F bank to be approximately 7 deg. To ensure that this happens, we solve the following optimization problem.

Determine a set of notch filters in the Ny_F bank that minimize

$$\mathrm{NYENV}_1 + |Ny_F|_{\mathrm{dB}}$$

with the constraint that the phase lag (at 1 Hz) contributed by Ny_F is at most 7 deg. A set of notch filters is determined, and with this set, it is found that

$$\max\{\mathrm{NYENV}_1 + |Ny_F|_{\mathrm{dB}}\} = -26.7\ \mathrm{dB}$$

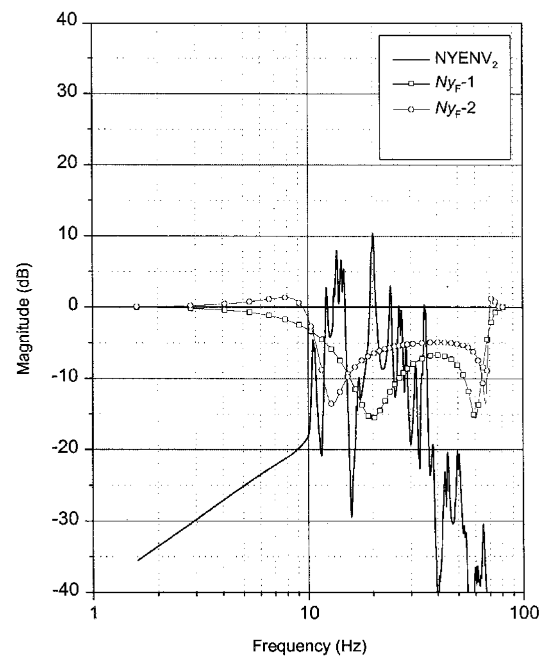

Fig. 18 NYENV_2 and individual notch filter sections of Ny_F—Design 2.

Table 3 Notch filter coefficients obtained from Design 2

Filter	Transfer function
P_F notch filter − 1 ($P_F - 1$)	$\dfrac{1 + 2 * (0.036/82.2)s + (s/82.2)^2}{1 + 2 * (0.373/62.6)s + (s/62.6)^2}$
P_F notch filter − 2 ($P_F - 2$)	$\dfrac{1 + 2 * (0.039/95.5)s + (s/95.5)^2}{1 + 2 * (0.411/95.3)s + (s/95.3)^2}$
P_F notch filter − 3 ($P_F - 3$)	$\dfrac{1 + 2 * (0.032/156.6)s + (s/156.6)^2}{1 + 2 * (0.393/106.8)s + (s/106.8)^2}$
P_F notch filter − 4 ($P_F - 4$)	$\dfrac{1 + 2 * (0.077/164.7)s + (s/164.7)^2}{1 + 2 * (0.496/164.7)s + (s/164.7)^2}$
R_F notch filter − 1 ($R_F - 1$)	$\dfrac{1 + 2 * (0.014/102.1)s + (s/102.1)^2}{1 + 2 * (0.016/101.5)s + (s/101.5)^2}$
R_F notch filter − 2 ($R_F - 2$)	$\dfrac{1 + 2 * (0.058/83.8)s + (s/83.8)^2}{1 + 2 * (0.098/63.5)s + (s/63.5)^2}$
R_F notch filter − 3 ($R_F - 3$)	$\dfrac{1 + 2 * (0.759/249.9)s + (s/249.9)^2}{1 + 2 * (0.762/44.1)s + (s/44.1)^2}$
Ny_F notch filter − 1 ($Ny_F - 1$)	$\dfrac{1 + 2 * (0.223/154.1)s + (s/154.1)^2}{1 + 2 * (0.818/104.6)s + (s/104.6)^2}$
Ny_F notch filter − 2 ($Ny_F - 2$)	$\dfrac{1 + 2 * (0.111/87.3)s + (s/87.3)^2}{1 + 2 * (0.267/65.7)s + (s/65.7)^2}$

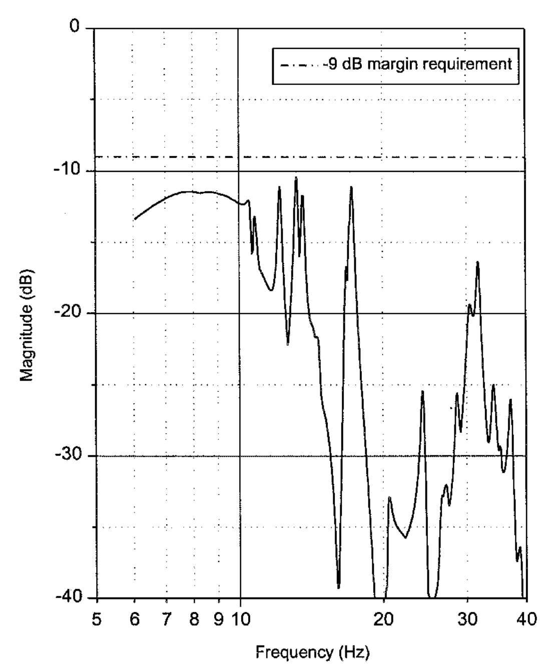

Fig. 19 Aileron loop margin—Design 2.

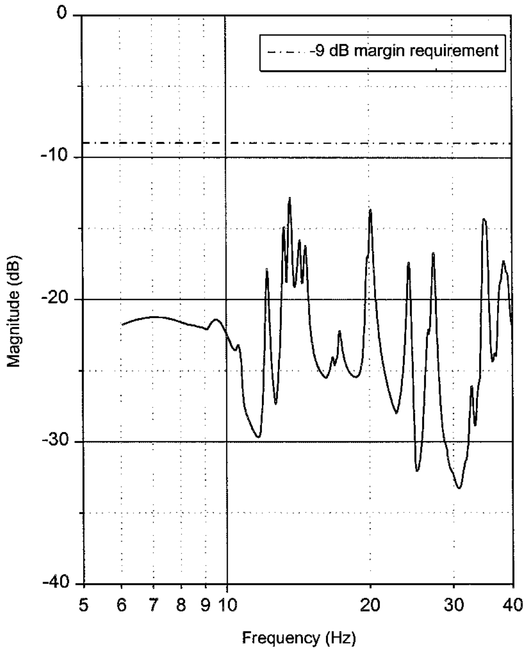

Fig. 20 Rudder loop margin—Design 2.

from which it can be inferred that approximately 15 dB can be added to each of the two terms constituting NYENV_1 in Eq. (28). As a result [from Eqs. (16) and (17)] 3 dB can be subtracted from each of the terms involving $mC_{21}P_{12}$ in PENV_1 and $mC_{22}P_{22}$ in RENV_1. Consequently, the final envelopes become

$$\mathrm{PENV}_2 = \max\{|mC_{11}P_{11}|_{\mathrm{dB}}, (|mC_{11}P_{12}|_{\mathrm{dB}} + |mC_{21}P_{11}|_{\mathrm{dB}}) * 0.5 + 4.77, |mC_{11}P_{12}|_{\mathrm{dB}} + 18.77, |mC_{21}P_{12}|_{\mathrm{dB}} + 6.54\} \tag{29}$$

$$\mathrm{RENV}_2 = \max\{|mC_{22}P_{21}|_{\mathrm{dB}} - 10.77, |mC_{22}P_{22}|_{\mathrm{dB}} + 6.54\} \tag{30}$$

$$\mathrm{NYENV}_2 = \max\{|mC_{23}P_{31}|_{\mathrm{dB}} + 5.77, |mC_{23}P_{32}|_{\mathrm{dB}} + 24.54\} \tag{31}$$

The design steps 1, 2, and 3 are then redone using PENV_2, RENV_2, and NYENV_2. The phase lag contribution (at 1 Hz) of the notch filter banks for this design are 11.17, 10.93, and 6.62 deg in the P_F, R_F, and Ny_F banks, respectively. Comparing this with the phase lag contributions obtained at the end of Design 1 (which were 11.56, 14.32, and 3.45 deg, respectively), we find that the phase lag of both the P_F and R_F banks has decreased further, with a subsequent increase in the phase lag of the Ny_F bank. The new set of notch filters is defined in Table 3. Figures 16, 17, and 18 show the envelopes PENV_2, RENV_2, and NYENV_2, respectively. Also plotted on these figures are the magnitude plots of the individual notch filter sections comprising the P_F, R_F, and Ny_F banks, respectively. With these notch filters, the aileron and rudder loop margins are calculated, as in Eqs. (2) and (3), and their plots shown in Figs. 19 and 20, respectively. From Figs. 19 and 20, it is evident that in both the cases the loop magnitudes are attenuated by at least 9 dB at all frequencies in the structural range, thus satisfying the gain margin requirements.

IV. Conclusions

This paper describes a new approach to the design of notch filters. This method is useful when the structural response (that needs to be attenuated) shows multiaxis cross coupling. It is possible to design the notch filters (to satisfy the gain margin requirements) in such a situation in a single-step procedure by reducing a complex optimization problem to several independent optimization problems. This can be done because it is possible to convert the gain margin requirements on each loop into equivalent gain margin requirements on each of the individual notch filter banks. Subsequent design iterations can then be done (if required) to redistribute the amount of phase lag injected by the notch filters among the different sensor paths at the lower frequencies (i.e., the low-frequency phase lag in one or more sensor paths can be reduced at the cost of increasing the same in the other sensor paths).

Acknowledgments

The authors are grateful to the Structural Dynamics and Aeroelasticity Group, Aeronautical Development Agency, for providing the aircraft structural response database, on which the present notch filter design technique was developed.

References

[1] *Military Specification Flight Control Systems—Design, Installation and Test of Piloted Aircraft General Specification*, Dept. of Defense, MIL-F-9490D, Washington, DC, Oct. 5, 1992.

[2] Cheng, P. Y., and Hirner, T. J., "Automated Procedures for Aircraft Aeroservoelastic Compensation," AIAA Paper 92-4606-CP, Aug. 1992.

[3] Branch, M. A., and Grace, A., *User's Guide to the Optimation Toolbox for MATLAB*, MathWorks, Natick, MA, 1996.