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Heat-Transfer Optimization of Convergent Section of a Hypersonic Flow Nozzle

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Nomenclature

A	=	area, m^2
a	=	sonic velocity = $(\gamma RT)^{0.5}$, m/sec
c_p,c_n	=	isobar and polytropic specific heats, respectively, J/kg-°K
D	=	diameter, m
g	=	recovery to stagnation temperature ratio, Eq. (10)
k	=	coefficient of heat conduction, W/m-°K
L	=	length of the convergent part of the nozzle, m
M	=	Mach number
$\dot{m}_0$	=	mass flow rate, kg/sec
N	=	a characteristic number, Eq. (15)
Nu	=	Nusselt number
Pr	=	Prandtl number = $\eta c_p/k$
p,p_0	=	static and total pressures, respectively, N/m^2
Q_m	=	specific heat load, joule/kg, Eq. (14)
q_m	=	specific heat addition, joule/kg
q_s	=	heat flux, W/m^2
R	=	gas constant, joule/kg-°K
Re	=	Reynolds number = $\rho VD/\eta$
r	=	recovery factor = $(Pr)^{0.5}$, Eq. (9)
T,T_0	=	static and total temperatures, respectively, °K
T_r	=	recovery temperature, °K
V	=	gas velocity, m/sec
x	=	axial length, m
γ	=	adiabatic exponent
η	=	dynamic viscosity coefficient, kg/m-sec
λ	=	Lagrange undetermined multiplier
ρ	=	density, kg/m^3
θ	=	a dimensionless function, Eq. (15)

Subscripts and superscripts

$(\ )^*$	=	dimensionless quantity
$(\ )'$	=	derivative with respect to axial coordinate
i	=	nozzle inlet condition
t,w	=	throat and wall condition, respectively
∞	=	freestream condition

Introduction

THE exit Mach number in a coverging-diverging nozzle depends on the square root of ratio of two temperatures—the total temperature at the inlet of the nozzle to the static temperature at the exit. Since the exit static temperature cannot be lowered beyond the saturation temperature of the working gas, an increase in T_{0i}, the total temperature of the working gas at the inlet, is the only way in which the exit Mach number may be increased beyond a certain value. At the nozzle inlet the heating requirement of the gas increases as $P_{0i}A_t(T_{0i})^{0.5}$, whereas the heat flux at the throat region increases as $p_{0i}^{0.8}T_{0i}^{0.6}A_t^{-0.1}$. Thus for a given p_{0i} and A_t, an increase in the exit Mach number results in a proportionate increase in the heating requirement of the gas at the nozzle inlet, whereas the heat flux increases slightly more. Hypersonic nozzles in steady-state operation in general are subjected to very intense heat flux levels, and consequently very high energy losses. The performance of a hypersonic nozzle is dependent strongly on the shape of the convergent part of the nozzle. The design of the divergent part of the nozzle is dependent on some other considerations and has been examined by many authors[1] and is not a part of the present investigation.

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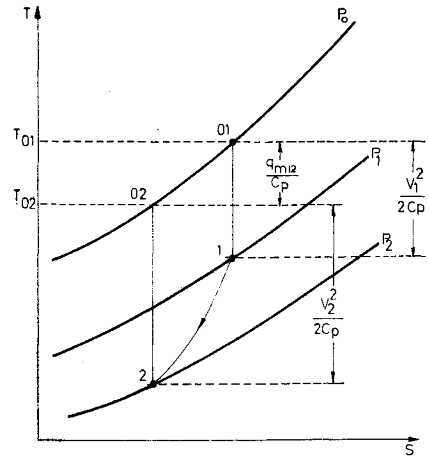

Fig. 1 Effect of heat transfer in an expanding flow on change of state between two points.

The method of the present investigation assumes the nozzle flow to consist of a series of fully developed turbulent pipe flows. Thus each point in the nozzle is assumed to have been preceded by a very long pipe of the local diameter of interest. The method seems to work for nozzles of moderate converging and diverging semiangles. Bartz,[2] for example, conducted his experiments with a nozzle which had semiangles 30° and 15°, respectively. The results are valid only qualitatively at larger semiangles. The method of optimization follows the classical method of Euler using a Lagrange undetermined multiplier. The resulting equations, which are complicated, have been solved numerically by assuming several convergent sections. The results show that a steep concave inlet is the best from the stand-point of heat transfer.

Analysis

The assumptions made herein are 1) the flow is "one-dimensional" and in steady state; 2) the nozzle is axisymmetric; 3) there is no axial heat conduction; 4) ideal gas with frozen composition; 5) Pr = 1; and 6) T_w = const.

The equation for change of enthalpy per unit mass along the nozzle is[3,4]

$$dq_m = c_p dT + VdV = c_n dT \tag{1}$$

where

$$c_n = c_p(n - \gamma)/[\gamma(n - 1)] \tag{2}$$

By introducing a definition of a local total temperature

$$T_0 = T + V^2/(2c_p) \tag{3}$$

one gets from Eq. (1)

$$dq_m = c_p dT_0 = c_n dT = c_p dT + VdV \tag{4}$$

which follows $dp_0 = 0$.

Thus in case of a heat transfer the total pressure remains constant while the total temperature changes, which has been explained in Fig. 1 with the help of a Temperature—Entropy (T,s) diagram for an expanding flow.

Between q_m and q_s, the heat flux, the relation for an axisymmetric nozzle is

$$dq_m = -(\pi D q_s/\dot{m}_0)ds \quad (5)$$

in which the relation between the line element s along the surface and the axial coordinate is

$$ds = (1 + 0.25D'^2)^{0.5}dx \quad (6)$$

Equation (5) has a negative sign in the right side, because heat flux to the wall, considered positive, is due to heat removal from the gas, which is usually considered negative.

Close form approximation of Nusselt number due to Bartz[2] is

$$Nu = 0.026Re^{0.8}Pr^{0.4} \quad (7)$$

The heat flux is related to Nusselt number by the relation

$$q_s = kNu(T_r - T_w)/D \quad (8)$$

T_r is obtained from the usual definition of a recovery factor

$$r = (T_r - T_\infty)/(T_{0\infty} - T_\infty) \quad (9)$$

and from the relation[5] $r = (Pr)^{0.5}$. From Eqs. (3) and (9), and from the sonic velocity relation, one gets the recovery to total temperature ratio

$$g = T_r/T_0 = [1 + \{0.5(\gamma - 1)M^2r\}]/[1 + \{0.5(\gamma - 1)M^2\}] \quad (10)$$

Results show that $g = 1$ for either $Pr = 1.0$, or $M \rightarrow 0$, whereas for $M \rightarrow \infty$, $g = Pr^{0.5}$. Thus for most of the gases in subsonic region the assumption $T_r = T_0$ is a valid approximation. In Eq. (8), therefore, one can replace T_r by T_0, the symbol for a freestream being dropped in an one-dimensional nozzle analysis.

From Eqs. (5–8)

$$q_m' = c_pT_0' = -[\pi kNu(T_0 - T_w)/\dot{m}_0][1 + 0.25D'^2]^{0.5} \quad (11)$$

Integrating Eq. (11) with the initial boundary condition, $T_0 = T_{0i}$, at $x = 0$, one obtains the relation

$$(T_0 - T_w)/(T_{0i} - T_w) = \exp[-\textstyle\int\{\pi kNu(1 + 0.25D'^2)^{0.5}/(\dot{m}_0c_p)\}dx] \quad (12)$$

For a constant diameter tube, Eq. (12) is reduced to the well-known exponential relation for temperature change. Even for a converging-diverging nozzle T_0 decreases somewhat exponentially.

Substituting Eq. (12) into Eq. (11), one gets

$$q_m' = -(\pi kNu/\dot{m}_0)(1 + 0.25D'^2)^{0.5}(T_{0i} - T_w) \times \exp\left[-\int_0^x\{\pi kNu(1 + 0.25D'^2)^{0.5}/(\dot{m}_0c_p)\}dx\right] \quad (13)$$

in which the relation for Nu, Eq. (7), can be substituted to get an exact distribution of q_m'.

Optimization Procedure

The optimization problem is formulated with respect to the heat transfer for a given nozzle length and given inlet and throat diameter. The equations which are used for optimization are for total heat load and from length consideration

$$Q_m = \int_0^L q_m'dx; \; D_t - D_i = \int_0^L D'dx \quad (14)$$

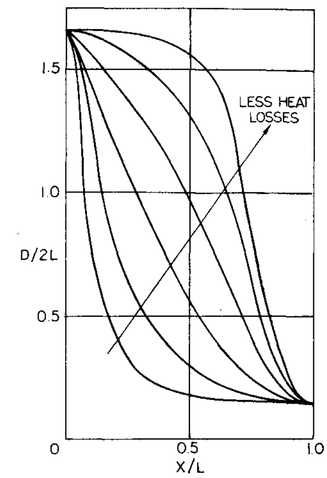

Fig. 2 Chosen subsonic section nozzle contours.

The following dimensionless quantities are introduced:

$$D^* = D/L; \; x^* = x/L$$
$$N = 0.0981\, kPr\dot{m}_0^{-0.2}L^{0.2}/(c_p\eta^{0.8}) \quad (15)$$
$$\theta = ND^{*-0.8}(1 + 0.25D^{*\prime 2})^{0.5}$$

Equations (14) are now transformed into the following set of equations:

$$Q_m = c_p(T_{0i} - T_w)\int_0^1 \theta\exp - \int_0^{x^*}\theta dx^*dx^* \quad (16a)$$

$$D_t^* - D_i^* = \int_0^{x^*} D^{*\prime}dx^* \quad (16b)$$

Introducing a new function

$$F = \theta \exp\left(-\int_0^{x^*}\theta dx^*\right) + \lambda D^{*\prime} \quad (17)$$

in which λ is a constant Lagrange undetermined multiplier, and from Euler's equation for optimization one obtains the

$$[0.95\theta D^{*\prime}/(1 + 0.25D^{*\prime 2})] - [(0.8/D^*) + (\theta/D^{*\prime}) + (\theta^2/D^{*\prime\prime})] = 0 \quad (18)$$

Equation (18) is too complicated for solution. Thus a numerical approach is taken, in which Eq. (16a) is solved for a given subsonic nozzle contour. A set of nozzle contours was chosen, as given in Fig. 2, and numerical analysis was done under the following set of conditions: $T_{0i} = 1773°K$, $D_t = 0.023m$, $L = 0.08m$, $N_i = 0.006903$, Gas = air. Results show that N changes from 0.006903 to 0.00675 over the length. Since N is small with respect to one for applications at temperatures of the order of magnitude of the above given temperature, one can drop the exponential term in Eqs. (16a) and (17). It is found numerically that the heat losses reduce for a given subsonic section length, if the contour is more steep concave, as seen from the inlet side to the throat. Further, a reduction in subsonic section length reduces the heat losses. In the absence of experimental results, these numerical results cannot be verified at present.

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Stability of a Dual-Spin Satellite in Circular Orbit

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Nomenclature

a	= distance from pendulum damper mass to 0
A_0	= moment of inertia of undeformed configuration about $0'x'$ or $0'y'$
$A(\tau),B(\tau),C(\tau),D(\tau)$	= variables of motion defined in Eqs. (8)
a_{11},a_{21}	= constants defined in Eqs. (12)
b_1,b_2	= damping constants of platform and rotor dampers, respectively
c_1,c_2	= dimensionless damping constants, $b_1/m\Omega$ and $b_2/I_s\Omega$, respectively
d_{11},d_{21}	= constants defined in Eqs. (13)
C_0	= moment of inertia of undeformed configuration about $0'z'$
C_R	= moment of inertia of rotor (including sphere) about $0'z'$
G_1,G_2	= spring constants of platform and rotor dampers, respectively
I_s	= moment of inertia of rotor damper about its center of mass
I	= dimensionless inertia, I_s/A_0
J	= C_R/A_0
K_1,K_2	= constants defined in Eq. (6)
k_1,k_2	= dimensionless spring constants $(G_1/m\Omega^2) - 3(1-\mu)$ and $G_2/I_s\Omega^2$, respectively
m	= mass of platform damper
0	= instantaneous mass center of configuration
$0'$	= mass center of undeformed configuration
$0'x'y'z'$	= reference frame associated with undeformed configuration, with $0'x'$ and $0'y'$ aligned along principal axes of the platform, and $0'z'$ aligned along the common principal axes of the two bodies
$0xyz$	= reference frame associated with deformed configuration, constrained to be parallel to $0'x'y'z'$
p,p_1,p_2	= natural frequencies
R	= dimensionless inertia, ma^2/A_0
t	= time
W_1,W_2	= dimensionless variables, ω_x/Ω and ω_y/Ω, respectively
α	= dimensionless rotation rate, $\dot{\gamma}_0/\Omega$
β	= angular deflection of rotor damper
γ	= rotation angle of rotor with respect to platform
$\dot{\gamma}_0$	= angular speed of rotor with respect to platform
Δ	= $(C_0 - A_0)/A_0$
ϵ	= small parameter
ξ	= dimensionless displacement, χ/a
Ξ	= determinant defined after Eq. (10)
Λ_1,Λ_2	= expressions defined after Eqs. (4)
τ	= dimensionless time, Ωt
μ	= ratio of m to satellite mass
Ω	= angular rate of circular orbit
$\omega_x,\omega_y,\omega_z$	= component absolute angular velocities of $0xyz$ reference frame
ψ,θ,ϕ	= Euler orientation angles, depicted in Fig. 1
χ	= linear deflection of platform damper
$(\cdot)$	= denotes differiation with respect to time
$(\ )'$	= denotes differentiation with respect to dimensionless time, τ
$\langle\ \rangle$	= denote "averaged" variables

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Introduction

RECENTLY, a "method of averaging" formulation has been used to solve linearized equations describing the motion of a dual-spin satellite rotating in a force-free environment.[1] In the present Note, the formulation and results are extended to account for the influence of the Earth's gravitational field when the satellite is in circular orbit with spin axis approximately normal to the orbit plane.

Equations of Motion

Paralleling Ref. 1, the dual spin configuration chosen for study is composed of a platform which contains a pendulum damper and a rotor which contains a spherical damper, as in Fig. 1. The rotor rotates with respect to the platform about $0'z'$ with angle γ. The satellite axes $(0xyz)$ are referenced to orbital axes $(0A_1A_2A_3)$ by a set of Euler angles (ψ,θ,ϕ) generated by the right-hand rotation scheme in Fig. 2: 1) ψ about $0A_1$, leading to axes $(0B_1B_2B_3)$; 2) θ about $0B_2$ leading to axes $(0C_1C_2C_3)$; 3) ϕ about $0C_3$ leading to axes $0xyz$. Linearized kinematic equations relating the spacecraft angular rates to the Euler angles are

$$\dot{\theta} = \omega_x - \Omega\psi; \quad \dot{\psi} = \omega_y + \Omega\theta; \quad \dot{\phi} = \omega_z - \Omega \tag{1}$$

Both rotor and platform are assumed to be symmetric about $0'z'$ when the configuration is undeformed, and consequently the linearized expression for gravitational torque about axes $0z$ is identically zero. We assume that the rotation angle γ is controlled by supplying a torque with an internal motor in the manner outlined in Ref. 1, which results in the rotation rate being a constant $\dot{\gamma}_0$ and ϕ being zero, under steady-state conditions. Consequently Eqs. (4, 5, and 8) of Ref. 1 are preserved.

The equilibrium solution of interest corresponds to pure rotation of the satellite about the $0A_3$ axis (and hence about $0z$) with the platform pointing at the Earth, i.e.,

$$\phi = 0; \quad \gamma = \dot{\gamma}_0 t; \quad \omega_z = \Omega \tag{2a}$$

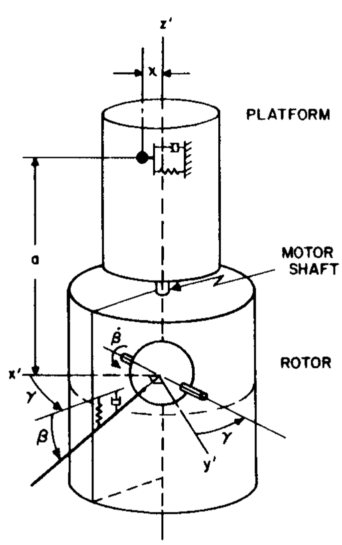

Fig. 1 Dual-spin satellite configuration.