

## Empirical Simulation Model for Hydrazine Attitude Control Thrusters

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**A simple model of low-thrust catalytic monopropellant hydrazine thrusters is presented for attitude control simulation purposes. It is derived from test results and represents thrust histories much better than a first-order lag over a wide range of pulse widths, for the thrusters used on the Hermes spacecraft. The structure of the model should be applicable to other cases as well.**

### Introduction

THERE are three principal means of actively controlling the attitude of spacecraft: torques derived from the environment, internal momentum storage devices, and reaction jets or thrusters. The last one is by far the most versatile. Due to the inherently pulsed, and therefore nonlinear, nature of the control logic in this case, computer simulation plays a decisive role in the design process. Hence, it is important that a good model of the thrusters used be incorporated into the simulation. A type of low-thrust engine offering good performance, with relatively low cost and high reliability,<sup>1</sup> is that employing catalytic decomposition of hydrazine monopropellant. Such thrusters were used, for instance, on Hermes (alias Communications Technology Satellite<sup>2</sup>). It is to simulation models of such thrusters that this work is addressed.

The engines in question are monopropellant hydrazine thrusters operating in a pressurized blowdown mode. The physical phenomena occurring in the propellant tank, propellant lines, catalyst chamber, and nozzle are complex, involving chemically reacting fluid flow. The decomposition of hydrazine into nitrogen, hydrogen and ammonia is, on balance, an exothermic reaction where decomposition fractions depend strongly on temperature. For very short pulse widths (7-10 ms), unsteady fluid flow effects are also strong, so thruster performance depends on thermal as well as flow transients. A high-fidelity model of such a complex electro-mechanical-chemical-fluid system is necessarily complicated and difficult. However, for attitude control simulation purposes, moderate accuracy is desired. A simple model for this case is possible if firing data for a specific thruster are taken into account. The model should aim at predicting accurately the total torque impulse imparted to the spacecraft, and the qualitative shape of the thrust-vs-time curve. The prediction of fuel consumed in thruster firing (i.e. the specific impulse) is an equally complex task which is not attempted here.

Some commonly used models are summarized next. The output of the control logic may be thought of as a current pulse  $e(t)$  of width  $\Delta_e$ . This represents the command "turn on for duration  $\Delta_e$ " as a logic state, and is also an idealization of the current input to the solenoid-operated valve. A thruster

model favored by control theorists, and implicit in switching logic-design methods, is a constant gain times  $e(t)$ , i.e., the "electrical" thrust  $F_e(t)$ . The thrust level is chosen to match the impulse, i.e., the area under the curve of "actual thrust" vs time. The actual thrust differs from  $F_e(t)$  in two major respects: it takes a finite time for the thrust to build up, and thrust remains at a significant value even after the pulse width  $\Delta_e$ . The former is largely a mechanical valve-opening effect, but the latter arises from slower decomposition of fuel left in the catalyst chamber after the valve has closed and approximates an exponential decay. For extremely short pulses ( $\Delta_e \approx 8$ -20 ms), most of the thrust is produced by this residual fuel; moreover, thrusting may not begin until after the duration  $\Delta_e$  has expired.

It is seen that for simulation purposes,  $F_e(t)$  is not accurate enough, especially for short pulses. In normal attitude control system operation, however, most of the pulses fired are short.<sup>2</sup> The first-order lag model  $F_l(t)$  for this case is one commonly used. It shows the effect of residual fuel, but suffers from two defects: the lag time constant, chosen to model the decaying part, produces too slow an initial rise; and secondly, the time constant has to be adjusted as a function of pulse width and/or duty cycle, i.e., average thrust over a train of pulses. The first defect may be remedied by using a dual-time-constant lag, but the second one is serious. Therefore, a method was sought in the present study to obtain a better model of the decay, without undue computing burden. A model based on extensive experimental data is presented below, but the structure of the model is more important than actual numbers.

A note on units in this paper—the experimental data was obtained in English units and is reported as such. Empirical curve fits, initially obtained in English units, are given in that form. A translation into metric units is also given, with pressures in megapascals, length in millimeters and time in milliseconds.

### The Hermes Thrusters

The hydrazine thruster system on Hermes was developed for the Canadian Department of Communications. It is described in detail, with emphasis on design criteria, in Ref. 2. The performance of this system in flight was extensively monitored and is discussed in Ref. 3, in relation to overall mission requirements. The present results are given in a more limited context—that of simulation.

The low-thrust engines (LTE's) are of primary interest here. These are blowdown-type, catalytic-decomposition, monopropellant hydrazine thrusters delivering a nominal steady-state thrust of 0.1 to 0.25 lb<sub>f</sub> (0.445 to 1.112 N). The attitude control system (ACS) fires commanded pulse widths which depend on mission phases, ranging from 7 to 50 ms. The full steady-state thrust is not reached in the majority of

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Index categories: Engine Performance; Spacecraft Dynamics and Control; Spacecraft Simulation.

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ACS pulses, which have durations of less than 20 ms. Figure 1 shows some typical chamber pressure  $P_c$  vs time plots obtained with 7 ms on-time, in space-like thermal vacuum tests. The thrust depends also on the inlet pressure, noted in Fig. 1. Variations between two different engines and two inlet pressures are illustrated. The chamber pressure history for 50-ms pulses is shown for the same two engines in Fig. 2. These are "fully developed" pulses fired at a fixed duty cycle. The influence of time between pulses is shown in Fig. 2. When this time is reduced to 0.95 s, higher steady-state chamber pressure is obtained. This was believed to be due to the catalytic bed temperature, but other data showed that fluid flow effects are dominant. In any case, the difference in the decay rates between Figs. 1 and 2 should be noted.

It is seen from the experimental data presented that a simple gain model is not adequate. This fact is further reinforced by Fig. 3, which shows the "effective gain" (defined in Fig. 3) for a number of duty cycles in a train of pulses. The dependence of gain on on-time is predictably large, and larger than that on off-time. Figure 3 also shows the performance of the model to be given below. The results shown here are but a small part of all the data obtained in tests, but they show the general flavor of this data. Statistics were also collected on total impulse, a fact useful in modelling.

### The Hermes Model

Upon examination of experimental data, several things became clear. The first was that steady-state thrust was a fiction seldom reached in ACS pulses due to their short duration. Secondly, although the catalytic bed temperature went from 300 to 1300°F during a steady-state firing, the thrust changed by less than 4%. This fact is important, for it permits the model to disregard temperature as a variable. Thirdly, it became clear that the decay was almost exponential, but its time constant would have to be modelled as an explicit function of pulse width or a related parameter. Finally, it was found that a more-or-less well-defined maximum pressure exists which depends only on the inlet pressure. Similarly, other parameters of the model could

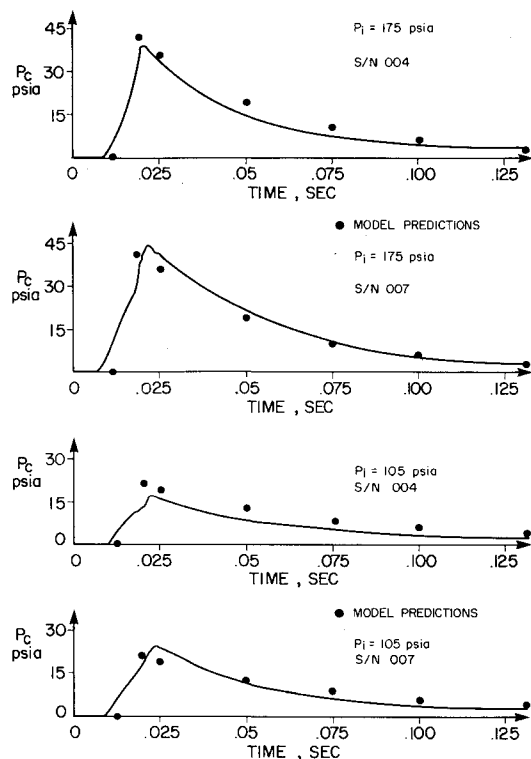


Fig. 1 Experimental chamber-pressure traces for short pulses, compared with model.

perhaps be fitted empirically as functions of the inlet pressure  $P_i$ .

The important question of relating chamber pressure to thrust is conventionally subsumed within a dimensionless "nozzle coefficient"  $C_F C_D$ :

$$F = A_t C_F C_D P_c \quad (1)$$

where  $A_t$  is the geometrical nozzle-throat area. The average nozzle coefficient depends on the flow conditions, and hence on the pressure and temperature of the gases in the decomposition chamber. The quantity  $C_F C_D$  is entirely empirical in that a detailed analytical model is very difficult. Experimental values of  $C_F C_D$  are illustrated in Fig. 4, giving the range of values for different bed temperatures vs chamber pressure  $P_c$ . This was obtained in steady-state firings. An empirical curve fit lying in the middle of this range is also shown in Fig. 4. This average curve is given by

$$\begin{aligned} \overline{C_F C_D} &= 1.800 - 0.704 P_c^{-0.383} & (P_c \geq 0.240 \text{ psi}) \\ &= 1.816 P_c^{0.776} & (P_c < 0.240 \text{ psi}) \end{aligned} \quad (2a)$$

or, in metric units

$$\begin{aligned} \overline{C_F C_D} &= 1.800 - 0.1047 P_c^{-0.383} & (P_c \geq 1.655 \text{ KPa}) \\ &= 86.38 P_c^{0.776} & (P_c < 1.655 \text{ KPa}) \end{aligned} \quad (2b)$$

where, in Eq. (2b),  $P_c$  is in megapascals (MPa). The empirical fit is continuous and has continuous first derivative at the junction point of the two expressions, to four significant digits.

If we recognize that most of the impulse delivered by a pulse is given by  $P_c > 5$  psia, an average constant value can be used ( $C_F C_D = 1.64$ ) to greatly simplify the model. The effect

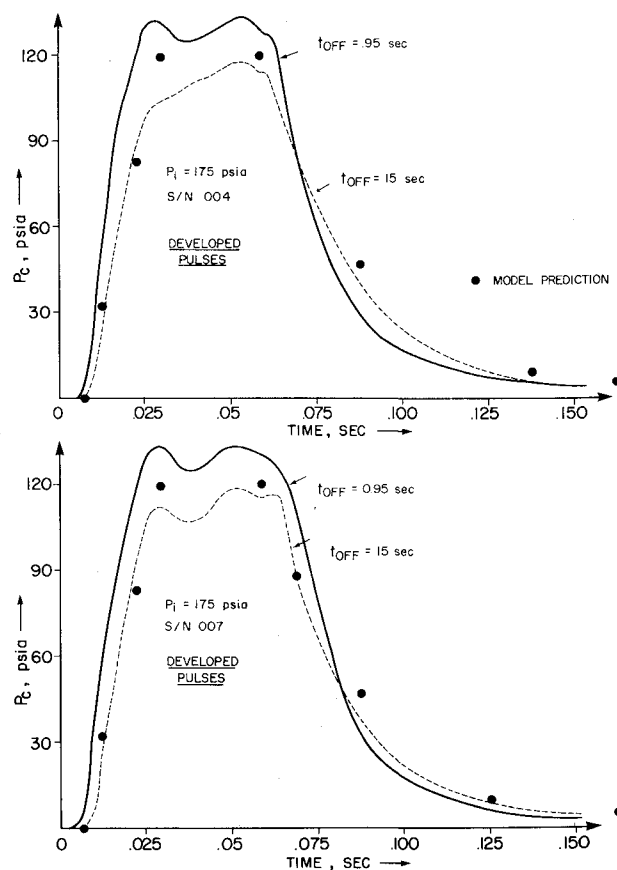


Fig. 2 Experimental pressure traces for 50-ms pulses, compared to model.

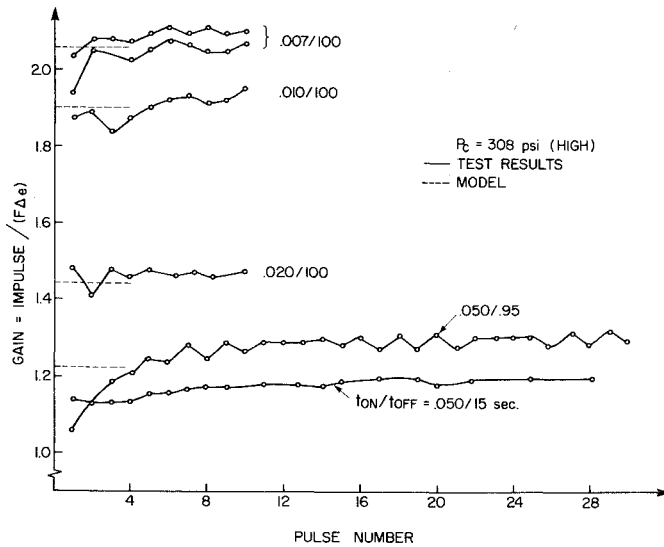


Fig. 3 Average impulse gains (based on steady-state thrust) for several duty cycles.

is to make the model thrust smaller than actual for low  $P_c$ . This is compensated for somewhat by the fact that the actual thrust for low  $P_c$  decays slower than an exponential. Hence, we simplify the model by setting:

$$\overline{C_F C_D} = 1.64 \quad A_t = 7.306 \times 10^{-4} \text{ in.}^2 \quad (3a)$$

$$\overline{C_F C_D} = 1.64 \quad A_t = 0.4714 \text{ mm}^2 \quad (3b)$$

It remains now to model the chamber pressure history  $P_c(t)$ . The major criterion is obtaining the correct impulse. Data for three inlet pressures were examined and an average value in each case was obtained for the maximum pressure  $P_{c_{\max}}$ . The initial rise of  $P_c(t)$  was found to be fast enough so that it could be approximated by a ramp, which is easier to compute than an exponential. The slope  $S$  of this ramp was also deduced from experimental data. An initial valve-opening delay  $\Delta$  also exists, which is best modelled as a pure time delay. The parameters  $P_{c_{\max}}$ ,  $S$ , and  $\Delta$  are given for three inlet pressures in Table 1.

It was decided to model the decaying portion of the pressure history by a negative exponential. As seen in Figs. 1 and 2, this is a satisfactory approximation at least for the initial decay. It was found that the rate of decay is reduced as the pressure  $P_c$  falls, especially for short pulses (7-20 ms). For these pulses, a small residual thrust may exist for as long as  $50\tau$ , where  $\tau$  is the initial decay time constant. Although this residual thrust is neglected in an exponential approximation, the choice of  $\overline{C_F C_D}$  in Eq. (3) does overestimate the thrust for low  $P_c$ , relative to the exponential model. It is also noteworthy that for these short pulses, the pressure limit  $P_{c_{\max}}$  may not actually be reached. The maximum pressure obtained is denoted  $P_m$ .

Table 1 Parameters of Hermes model

Inlet pressure, psi (MPa)	Initial delay, ms	Maximum pressure, psi(MPa)	Initial slope, psi(MPa)/s
305 (2.110)	10.2	202 (1.397)	$10.0 E + 3$ (68.95)
175 (1.207)	11.2	120 (0.828)	$5.6 E + 3$ (38.61)
105 (0.724)	12.5	76 (0.524)	$2.8 E + 3$ (19.31)

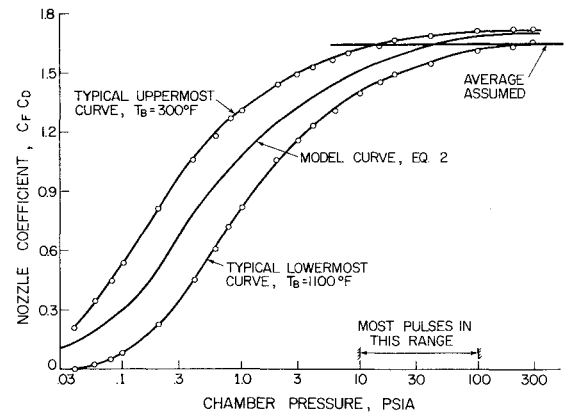


Fig. 4 Summary of nozzle-coefficient data, with empirical curve fit of mean values.

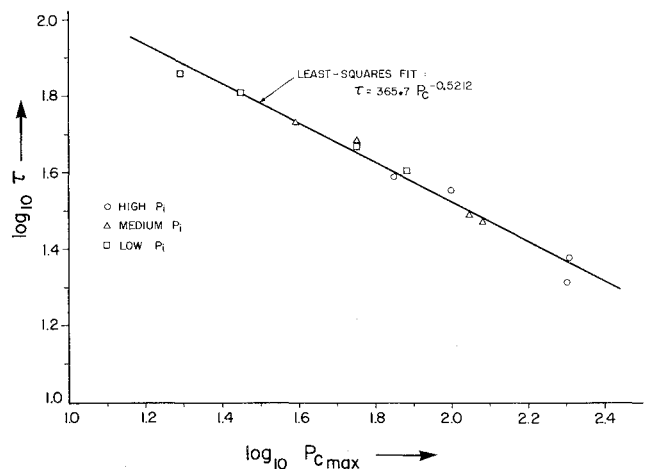


Fig. 5 Least-squares fit of decay time constants vs maximum pressure.

The time constant  $\tau$  was modeled as follows. For a given pulse width  $\Delta_e$ , the decay begins as soon as  $\Delta_e$  has elapsed. The impulse delivered up to the turnoff point is known from the previous parameters:

$$I_1 = (P_m \Delta_e - P_m^2 / 2S) \overline{C_F C_D} A_t \quad (4)$$

where

$$P_m = \min(P_{c_{\max}}, S \Delta_e) \quad (5)$$

The impulse from the exponential part is

$$I_2 = \int_0^\infty P_m e^{-t/\tau} dt \overline{C_F C_D} A_t \\ = \tau P_m \overline{C_F C_D} A_t \quad (6)$$

Since extensive data are available on the total impulse  $I (=I_1 + I_2)$ , one can infer an equivalent  $\tau$  from these equations easily. The mean value of  $\tau$ , derived from  $I$  is plotted vs  $P_m$  in Fig. 5. This plot includes data from five duty cycles, three different inlet pressures, and two different engines. A least-squares straight-line fit of this data is also shown and is found to be good. The resulting equation for  $\tau$  is

$$\tau = 365.7 P_m^{-0.5212} \quad \text{psia} \rightarrow \text{ms} \quad (7a)$$

$$\tau = 27.33 P_m^{-0.5212} \quad \text{MPa} \rightarrow \text{ms} \quad (7b)$$

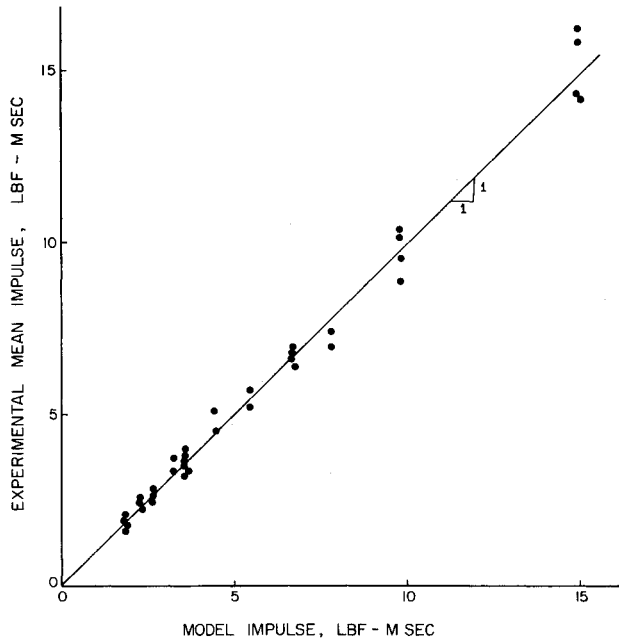


Fig. 6 Performance of the Hermes model on total impulse delivered.

Since  $P_m$  is normally 0.5–1.0 MPa, time constants of 20–40 ms are obtained. This equation, unlike Table 1, applies to all values of  $P_i$ .

The parameters  $S$ ,  $P_{c_{max}}$ , and  $\Delta$  listed in Table 1 can also be empirically fit as functions of inlet pressure  $P_i$ . This yields

$$\Delta = 12 \text{ ms} \quad (8)$$

$$P_{c_{max}} = 76.0 + 0.63(P_i - 105) \text{ psi} \rightarrow \text{psi} \quad (9a)$$

$$= 0.524 + 0.63(P_i - 0.724) \text{ MPa} \rightarrow \text{MPa} \quad (9b)$$

$$S = 48.6(P_i - 40.4) - .031 P_i^2 \text{ psi} \rightarrow \text{psi/s} \quad (10a)$$

$$= 48.6(P_i - 0.2786) - 4.5 P_i^2 \text{ MPa} \rightarrow \text{MPa/s} \quad (10b)$$

The model is now completed by taking Eqs. (3) and (7) and Table 1 or Eqs. (8–10). Once the parameters  $P_{c_{max}}$ ,  $S$ ,  $\Delta$  are known,  $\tau$  and the thrust history  $F(t)$  can be computed for a given  $\Delta_e$  as follows:

$$F(t) = C_F C_D A_t P_c(t)$$

$$\begin{aligned} P_c(t) &= 0 & (0 \leq t < \Delta) \\ &= \min[P_{c_{max}}, S(t - \Delta)] & (\Delta \leq t < \Delta + \Delta_e) \\ &= P_m e^{-t/\tau(P_m)} & (t \geq \Delta + \Delta_e) \end{aligned} \quad (11)$$

where  $P_m$  is given by Eq. (5), and  $t$ ,  $\Delta$ ,  $\Delta_e$ ,  $\tau$  are all in ms, so that  $S$  from Eq. (10) must be converted to psi/ms or MPa/ms.

### Model Performance

In comparing the predictions of this model with experiment, the first quantity of interest is the total impulse  $I$ . The value obtained from the model is plotted vs the actual mean impulse in Fig. 6, for four duty cycles, three inlet pressures, and two engines. It is evident that the model predicts impulse very well. This is not too surprising in view of the fact that  $\tau$  was chosen to match impulse; however, the dependence  $\tau(P_m)$  is an approximation, validated by Fig. 6. Excellent impulse matching is also evident in Fig 3, which compares the model and experimental “gains” under several different conditions.

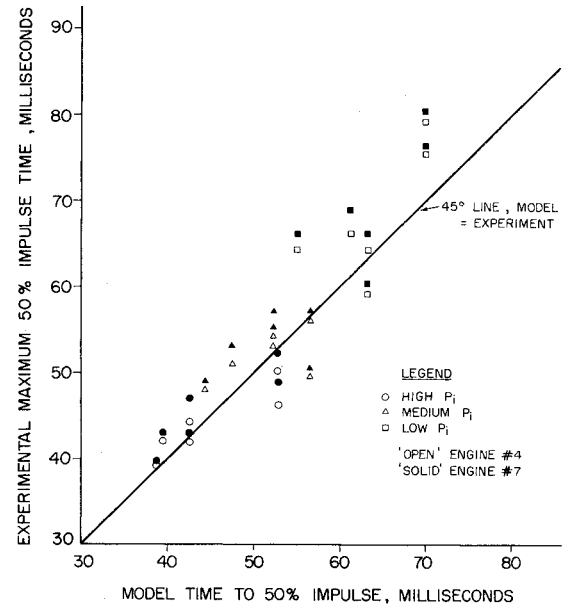


Fig. 7 Performance of Hermes model on time to deliver half impulse.

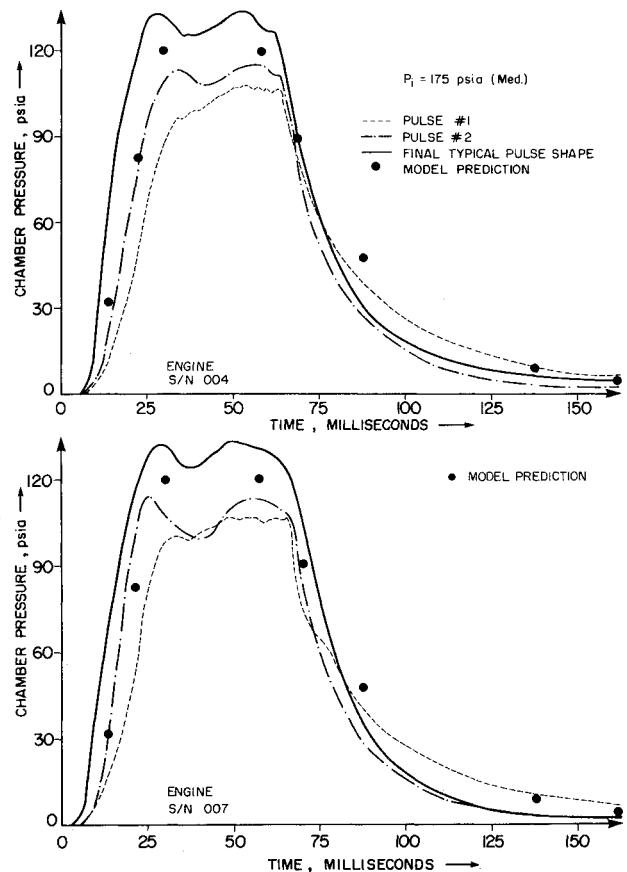


Fig. 8 Typical development of pressure histories in a pulse train.

The model is also quite good in predicting the general shape of  $F(t)$ , as is clear from the points plotted in Figs. 1 and 2, again for a variety of conditions. Since the model is deterministic, the errors from actual thrust will be random, i.e., variable from one pulse to the next. An attempt to model the random variation by randomizing ( $S$ ,  $P_{c_{max}}$ , and  $\Delta$ ) was made successfully, but is not reported here. A good test of fidelity is the time required to reach 50% of the total impulse. This is compared with experiment in Fig. 7. Although a significant amount of “scatter” is seen, the average per-

formance of the model is good for a variety of conditions shown. Generally, the model predicts a slightly lower half-impulse point, i.e., a faster rate of decay. Another comparison point is the time required to deliver 90% of the total impulse. For pulse widths large enough to cause saturation ( $P_c = P_{c_{\max}}$ ), the model performs similarly to Fig. 7. However, for very short pulses (7-10 ms), the 90% time predicted by the model may be up to half that found experimentally. This is because some of these pulses were found to have a very long "tail." The thrust (although not the impulse) in these tails is negligibly small, so this shortcoming is not serious; of course, the model still gives the correct total impulse.

There remain many details of experimental data which this model, being aimed at average values, does not account for. An example is the changes in thrust output during the first few pulses of a sequence. Figure 8 illustrates this phenomenon for  $\Delta_e = 50$  ms and medium inlet pressure. The model predictions lie between the startup and steady-state thrust characteristics. The first few pulses also have a lower impulse, as is evident in Fig 3. Once again, the model gives good average characteristics. Once the engines are "warmed up," the impulse repeatability is within 3%, with the model giving a mean value.

### Conclusion

The empirical equations developed here for the Hermes thrusters represent a thruster model ideal for digital simulation. It covers a wide range of pulse widths, operates over a wide range of inlet pressures, and gives excellent matching of impulse. The detailed shape  $F(t)$  is also modelled very well, on the average.

Two main simplifications were found useful in this model which may be useful in other cases. One is the representation of the nozzle aerothermodynamics by an empirical curve fit for a nozzle coefficient, and its further simplification to a constant. The second is the finding that a simple empirical relationship exists between the decay time constant and the maximum chamber pressure obtained. The author believes that other situations may find these assumptions valid.

### Acknowledgments

The empirical tests on which this paper is based were conducted by the Hamilton Standard Division of United Technologies Corporation. The author wishes to thank the Communications Research Center, Government of Canada, design authority for Hermes, for making test data available to him. Financial support for this work was provided by Spar Aerospace Ltd. and by the Natural Sciences and Engineering Research Council of Canada.

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