Experimental Studies of Scaling Laws for Plasma Collection at High Voltages

Andrei Konradi,* Bernard McIntyre,† and Andrew E. Potter‡
NASA Johnson Space Center, Houston, Texas

Current-voltage characteristics were determined for stainless steel panels in a laboratory with a simulated ionospheric environment of 5×10^{11} to 2.5×10^{12} electrons/m^3. Panels with areas differing by a factor of up to 100 were biased between 0 and −2300 V. Secondary emission was corrected for using experimental data. The results indicate that the current-voltage characteristics are almost linear and that for voltages above about −500 V the current scales with characteristic panel length as $L^{1.2}$. Comparison with some theoretical predictions is discussed. Because of orbital velocity associated ion ram effects, the results of this study must be used with caution when applied to an orbiting spacecraft.

Introduction

REASONABLE projections of future space developments suggest not only a trend toward larger spacecraft but also use of high voltages for economic reasons. Since many of the low Earth orbits pass through the heart of the ionosphere it becomes apparent that an understanding of high voltage/plasma interactions is essential.

In its simpler forms this interaction can be viewed as the sheath formation around and the current flow to a conductor at some high voltage referenced to the surrounding plasma. Since the work of Langmuir in the beginning of this century, numerous papers have been written on the collection of current by electrically biased conducting bodies immersed in a plasma. This activity was stimulated by the fact that various plasma properties have a direct effect on the current collection by test probes and thus by analysing the current-voltage characteristics it is possible to extract the electron temperature, T_e, and the number density, N_e, of a plasma.

Langmuir et al.[1-4] expressed the probe current in simple form by making two assumptions concerning the potential distribution near a probe. The potential has no maxima or minima in the sheath region and the sheath terminates at a sharp edge where the electric field and potential are zero. The cases treated by Langmuir were then usually categorized as orbit limited or space-charge limited. In the space-charge-limited case, compared to the impact parameter, the sheath is so thin that all of the charges reaching the edge are collected. The probe current is then given in terms of the random thermal current at the sheath edge and of the sheath area. For an infinite plane collector the current saturates, while for a finite plane there is a current increase due to edge effects. For cylindrical and spherical collectors the current equations of Langmuir contain functions α and β, which are functions of the ratio of the sheath thickness to collector radius. For thick sheaths the current is roughly proportional to V^n where $n \simeq 3/4$ for spheres and 0.5 for cylinders. Bohm[5] later established a criterion for the validity of the Langmuir sheath by showing that the cooler of the two plasma species must be accelerated in a presheath region. A weak potential in the range 0 to $kT/2$ exists there so that the cooler species enters the sheath at about one-half of the temperature of the hotter species. On the other hand, if the ion energy $\geq \frac{1}{2}kT$, a presheath will not form. In most cases, if the presheath is large, the plasma ions are the charges accelerated in this presheath region and are thus much more difficult to treat than electrons in a more exact theory.

Langmuir's orbit-limited theory for very large sheaths results in a current which is independent of the sheath size and the shape of the potential. The current is given in terms of the probe size and the potential at the probe. For cylinders $i \propto V^{1/2}$, and for spheres $i \propto V$. Because these orbit-limited equations for current do not contain the sheath parameters they are simpler to use to extract the plasma parameters. A more accurate recent method, which involves integrations in the angular momentum-energy space, was developed by Bernstein and Rabinowitz[6] and must be carried out numerically. This method had been used by a number of workers, but since the object of the calculations is the determination of plasma parameters, the current voltage calculations in general are terminated near $V \simeq 20kT/e$. A detailed description of the work performed to date can be found in several review articles and books.[7-9]

In our case the region of interest is $V \simeq 17{,}000kT/e$. Recently, computer programs have been developed independently by Parker et al.[10,11] and Katz et al.[12] to determine current flow to bodies at high voltages immersed in a collisionless plasma. Parker's approach is to trace particle trajectories to solve the Poisson equation and the particle equations of motion in a self-consistent manner. Katz's approach is embodied in a program called NASCAP and consists of reducing the Poisson-Vlasov sheath problem to a nonlinear equation resembling the Helmholz equation.

The purpose of the present study is to provide an experimental basis in the form of limited scaling laws to test and validate theoretical and computational approaches. From an experimental point of view the problem of interest taken here can be viewed as follows: The net current to an isolated body immersed in a plasma is zero. Thus, if a voltage differential is imposed on various parts of a surface, the body will assume a potential distribution with respect to the plasma such as to equalize the positive and negative plasma currents to the surface. Since the electron random current density is usually about two orders of magnitude greater than the ion current density, the positive end of the body will float near and somewhat above the plasma potential, while the negative end will be appreciably below it. This implies that since electrons

Submitted Sept. 3, 1982; revision received May 17, 1983.

*Physicist, Space Science Branch, Planetary and Earth Science Division, Space and Life Sciences Directorate. Member AIAA.

†Physicist; presently Associate Professor, University of Houston, Central Campus, Houston, Texas.

‡Chief, Space Science Branch, Planetary and Earth Sciences Division, Space and Life Sciences Directorate. Member AIAA.

can be collected very readily, the total current of both charges will be determined by processes limiting the collection of positive ions. In view of this the positive current flow to high-voltage conductive panels was measured and dimensional scaling laws governing the current collection were determined.

Plasma Generation and Environment

In experiments involving formation of large high-voltage sheaths it is important to insure that there be no coupling between the plasma sheaths and the walls of the container in which the experiments are performed. To assure this condition, the experiment reported here was performed in a cylindrical vacuum chamber with a working volume which had a diameter of 17 m and a height of 27 m. A Kaufman thruster operating on argon was used to generate the plasma environment. No bias was applied to the accelerating grids of the thruster and thus the energy distribution of the emitted ions was a function of the internal discharge potential only. However, at ambient pressures of 10^{-5} Torr, many of the energetic ions from the thruster underwent charge exchange collisions with the residual gas.

The ion population had two components: One consisted of argon ions emitted from the Kaufman thruster with a mean energy of 16.5 eV and an energy spread of approximately 30 eV. The other consisted of cold ions formed from the residual ambient gas and for the most part was composed of H_2O^+, H_3O^+, N_2^+, and Ar^+. The latter formed the majority of the plasma ions. Thus this experiment does not completely simulate ionospheric conditions. Measurements of the current in the ion attracting region of the Langmuir probe characteristics indicated that as a function of position the ion population closely followed the electron density.

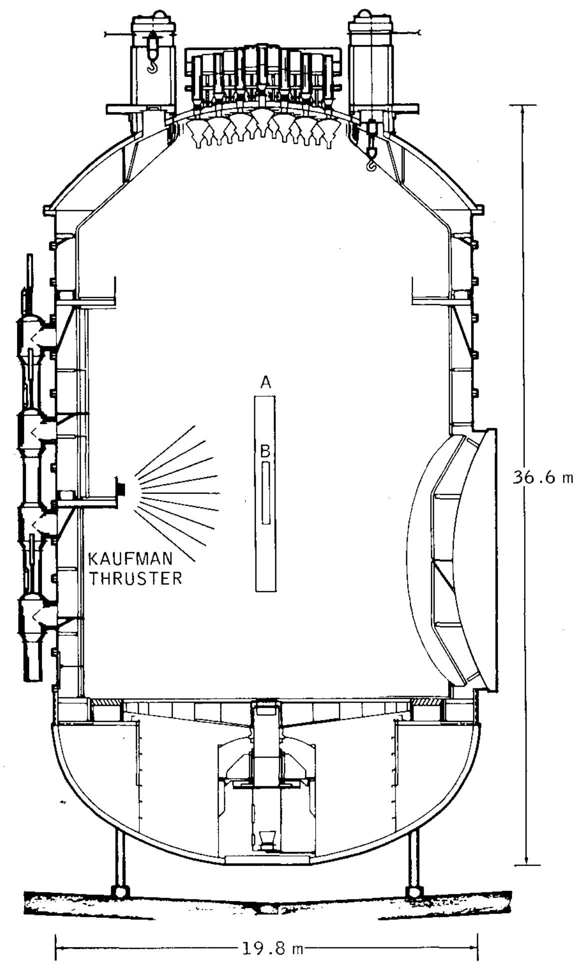

Fig. 1 Experimental setup for obtaining current-voltage characteristics.

Figure 1 shows the relative locations of the thruster and panels. Panels A and B are shown simultaneously; however, during the measurements only one panel at a time was deployed.

Measurements made with a pulsed Langmuir probe indicated that the plasma electrons had temperatures on the order of 0.15 eV and that the maximum achievable density at the center was about 7×10^{12} m^{-3}. The vertical and horizontal electron distributions at the center of the chamber are shown in Fig. 2. The origin in both cases is the horizontal line connecting the center of the thruster and the center of the panels used in this experiment. The vertical distribution is symmetric with respect to the origin. The three lines labeled A, B, and C represent the length of the three panels. While panel C was exposed to a more or less uniform electron density, there clearly existed an electron density gradient across panels A and B. The horizontal electron distribution is not symmetric with respect to the origin. However, the panels were positioned edge-on to the thruster so that the horizontal gradient had little effect on the plasma distribution along the width of the panel. When the vertical plasma density gradient was taken into account the average electron density for panels A, B, and C was found to have been 82, 97, and 100% of the peak value measured at the center of the chamber.

All of the data reported in this paper have been corrected for this effect. Electron temperatures and densities were measured using a pulsed plasma probe system similar to the one developed by Szuszczewicz.[13] The measurements were made in the presence of the Earth's field which was 0.28 *g* within the cnamber and had an inclination of about 63 deg to the horizontal. All measured potentials were referred to ground which was also the chamber wall. The plasma potential was about 2.3-3.0 V negative with respect to the ground. In relation to the high applied voltages the difference is small and is neglected except when the applied voltage is of the same order.

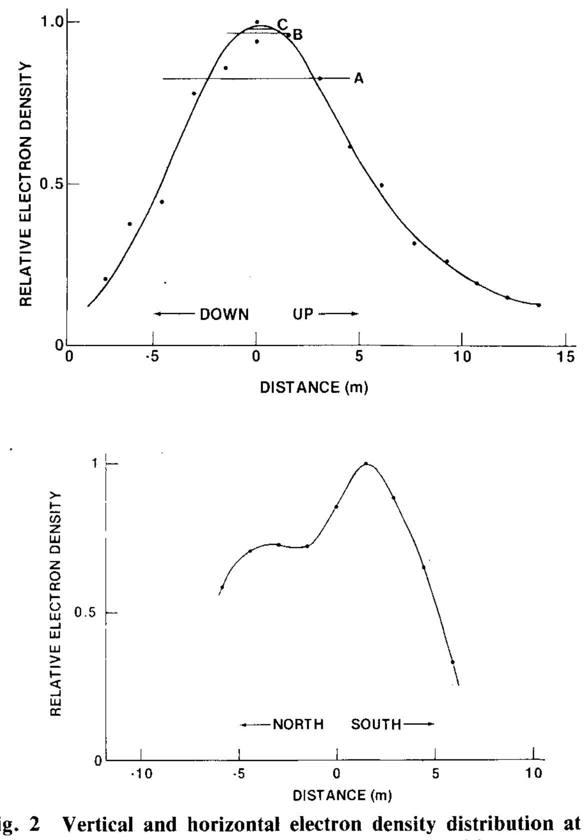

Fig. 2 Vertical and horizontal electron density distribution at the position of the panels and the average electron densities surrounding the panels.

Experiment

This experiment was concerned with the measurement of the high voltage *I-V* characteristics of conductive panels immersed in a simulated ionospheric plasma. Three flat, 0.0001-m-thick stainless steel panels with area ratios of 1:10:100 were used.

The panel dimensions are given in Table 1.

All of the results reported here are based on current-voltage measurements between −1 V and approximately −2500 V bias applied to each panel. The characteristics were obtained for the three panels at plasma densities roughly between 5×10^{11} and 2×10^{12} m^{-3}. In practice, the maximum bias applied to the panels was limited by unipolar arcs which seemed to occur when for a given plasma density the voltage exceeded some critical value. No special effort has been made to study the nature of the arc discharges. However, two general observations can be mentioned: The critical voltage seems to increase with an increase in the exposure time of the panels to the vacuum and a decrease in the ambient plasma density. Typically the critical voltage was in the the range between −1000 and −2600 V.

Figure 3 shows a typical *I-V* characteristic for panel A obtained at a density of 6.7×10^{11} electrons/m^3. This curve illustrates the morphology of all *I-V* curves analyzed in this study.

The curve rises rapidly between 0 and −50 V and after about −100 V flattens out to the point where it has an almost constant slope. An excellent fit to this particular curve can be obtained at negative biases in excess of 100 V using the relationship $I=I_0+AV^b$ where $b=1.05\pm0.03$. We noticed from the data that for the same panel and different plasma densities the *I-V* curves differ from each other only by a constant factor. This fact seems to hold true for all three panels individually and is illustrated in Fig. 4, where a least-squares fit was drawn to the panel current vs measured plasma density for the three panels at −1000 V bias. The straight line was constrained to go through the origin. The lines on either side of the fitted line represent the standard deviation of the fit. Thus for panel A at −1000 V and 10 mA the plasma density is $(1.91\pm0.06)\times10^{12}$ m^{-3}. This procedure made it possible to reduce the measurement for each panel to only one curve, arbitrarily normalized to a plasma density of 1×10^{12} m^{-3}. The three curves are shown in Fig. 5. These curves represent the total panel current which has two components: One is the ion current originating at the sheath boundary and the other is the secondary electron current produced by the impinging primary energetic ions. The second component becomes important when the incident ions have an energy above several hundred volts. To determine the contribution of the secondary electrons it is necessary to know the secondary emission coefficient. It is though that the main constituents of the plasma are H_2O^+, H_3O^+, $N_2{}^+$, and Ar^+. To the authors' knowledge, no published data on their secondary production from stainless dteel exist. The secondary emission coefficient was therefore measured in the ambient environment. For ion energies below 600 eV the measurements were done using a retarding potential analyzer (RPA) whose collector plate was made of material identical to the stainless steel used for panels A, B, and C. The RPA was suspended directly in the plasma environment and its housing was biased at various negative voltages to accelerate the ambient plasma ions. The secondary yield was obtained by varying the bias on the suppressor grid. This set of measurements had to be terminated at −600 V because of unipolar arcing. For ion energies greater than 3000 eV measurements of the secondary emission coefficient were made separately in a vacuum tank by accelerating Ar^+ ions electrostatically into a RPA. The combined data are shown in Fig. 6. The displayed trend is almost linear with voltage and reaches a value of about 25% at −2500 V. Thus, for higher voltages, the panel current has a substantial component due to the secondary electron current. Using the data shown in Fig. 6 the secondary electron current was subtracted from the total panel current and the resulting curves are shown as dashed lines in Fig. 5. In view of the impossibility of reproducing exactly the actual ion composition prevalent in the ambient plasma during the current flow measurements, there may be some question of the validity of the measurements of γ at energies above 3000 eV. Therefore in spite of the self consistency of the data no claim is made to an accuracy of better than 50%.

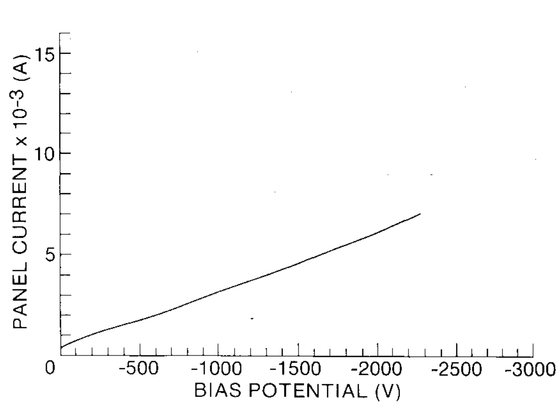

Fig. 3 Typical experimental current-high voltage characteristics for panel A.

Table 1 Dimensions of panels

Panel	Length, m	Width, m	Area, m^2
A	9.144	0.914	2×8.3613
B	2.892	0.289	2×0.8361
C	0.914	0.091	2×0.0836

Secondary emission, as a significant contributor to the panel current, becomes particularly important at zero or low magnetic fields when the size of the cycloidal trajectory of secondary electrons in the crossed electric and magnetic fields becomes comparable to the thickness of the sheath surrounding the panel. Assuming that a secondary electron starts off with zero energy, the maximum distance from the panel that it can reach (nonrelativistically) is[14] $d=2mE/qB^2$, where m is the electron mass, q the electron charge, and E and B the electric and magnetic fields. The work reported here was performed in a field of 2.9×10^{-5} T. The typical sheath size[15] is about 1 m for a −1000 V bias. Thus it is found that $d=13$ m. This means that secondary electrons will escape the sheath field before they are turned around by the magnetic field.

While it is difficult to say what the ratio of the three panel currents is at low voltage, it is clear that the ratio is not that of the areas or 1:10:100. In order to determine the scaling of the panel currents with panel size it was assumed that at any bias voltage the panel current is proportional to the characteristic length of the panel raised to some power. Thus,

$$I_i=K(L_i)^n$$

where i is the index referring to panels A, B, and C; L the length of the panel; and n an exponent.

If the ratios of the currents in the three panels are taken, the two values of the power law exponent n can be determined.

Thus,

$$\frac{I_A}{I_B}=\left[\frac{L_A}{L_B}\right]^{n_{AB}}=10^{(n_{AB}/2)}$$

and

$$\frac{I_B}{I_C}=\left[\frac{L_B}{L_C}\right]^{n_{BC}}=10^{(n_{BC}/2)}$$

from which the values of n_{AB} and n_{BC} can be obtained as a function of the applied voltage. The plot of these values is shown in Fig. 7. Intuitively one would expect that at low bias voltages when the sheath around the panels is very thin, the plasma current should be proportional to the area of the panels. As the applied bias increases, one would expect the sheath to grow and the plasma current to be proportional to the sheath boundary surface. As can be seen from the figure, this occurs at quite low bias voltages and by the time the bias is about -700 V both values of n coverage to about 1.2 and stay at that value all the way to above -2000 V.

Discussion

From measurements described in the preceding section it is clear that at negative voltages greater than about -500 V, secondary electrons make a significant contribution to the total panel current. Thus, it seems that the linearity of the *I-V* characteristic may be due mainly to a fortuitious combination of the primary ion current and the secondary electron current. Unfortunately, because of the difficulty of determining the secondary emission coefficient for the material used, a determination of the true ion current is impossible. The reduced data will, therefore, be compared with the results of several model calculations in order to assess the reasonableness of the interpretation used. An exact solution of the problem involves the simultaneous solutions of the Boltzmann-Vlasov equation and the Poisson equation. Even for very simple shapes of conductors, such as spheres or circular cylinders immersed in a homogeneous two-component Maxwellian plasma, no exact analytic solutions exist and closed-form solutions are obtained only with the help of approximations. Examples of these are the space-charge-limited or orbit-limited current flow equations derived by Langmuir and Blodgett.[1,2] For the geometry of conductors (flat, rectangular plates) used in this experiment, no analytical or numerical solutions are available. Since current flow for bias voltages $V >> E_i/e$ is of particular interest, where E_i is the kinetic energy of ambient plasma ions at infinity, the current collection is essentially space-charge limited. While numerous theoretical calculations of *I-V* characteristics have been produced for use in plasma probe applications, none of them have been made for values of the bias potential which are much greater than the characteristic energy of the plasma ions. Thus, for purposes of comparison, analytic expressions for space-charge-limited current flow derived by Langmuir and Blodgett will be used. In the following it is important to remember that comparisons are being made between experimentally obtained current flowing to finite *rectangular panels* and approximate theoretical predictions of current flowing to *cylinders* and *spheres*. Thus good quantitative agreement should not be expected and, at best, only some qualitative comparisons can be made.

Figure 8 shows the experimentally obtained *I-V* characteristics of the three panels and *I-V* characteristics for spheres and cylinders calculated using the Langmuir-Blodgett relationships for space charge-limited current flow. In each case the surface area of the sphere or cylinder was equal to that of the panel with which it was compared. The single point represents the result of computer simulation of current flow to panel A using Parker's two-dimensional model[11] and assuming an argon plasma.

It should be remembered that the length-to-width ratio of each panel was 10. Thus, with increased bias voltage, the growing sheath first would resemble a cylinder surrounding the panel and as the bias voltage continued increasing the expanding sheath would take on spherical characteristics. This qualitative behavior can be seen clearly in Fig. 8. Also in agreement with expectations, the resemblance to cylindrical geometry occurs at a lower voltage for the smaller panel than for the larger ones. All theoretical curves were calculated assuming a sharp sheath edge and some current density of the attracted species at that edge. In these calculations the value

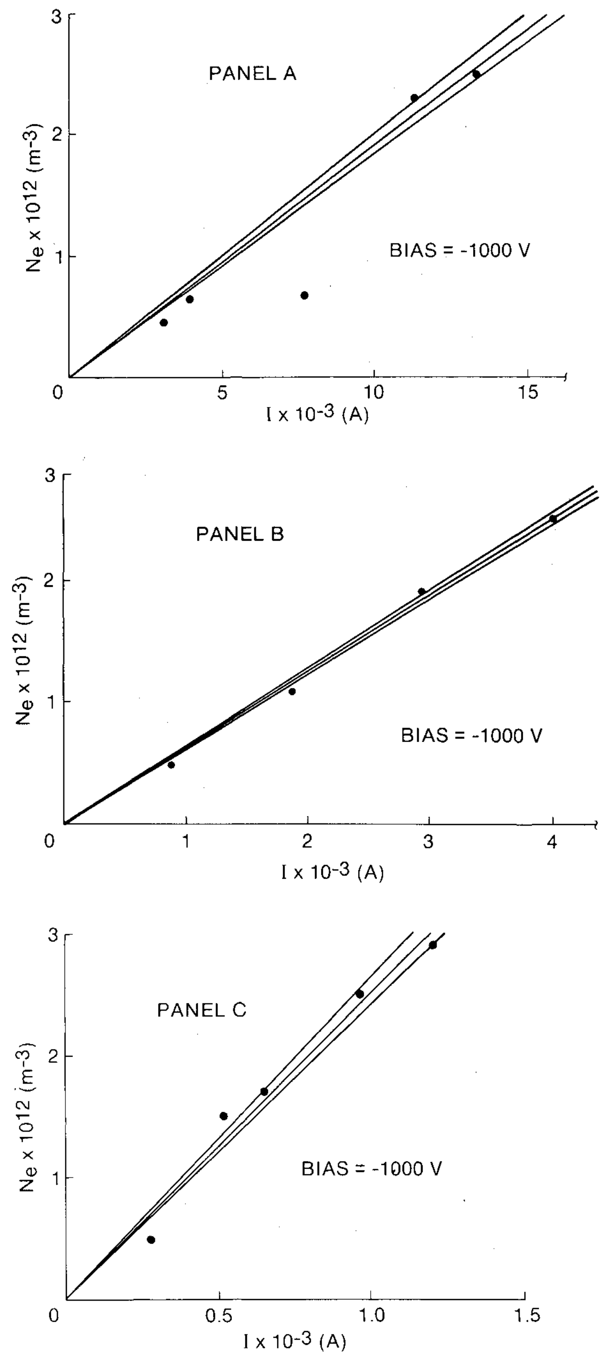

Fig. 4 Panel currents at a fixed bias voltage.

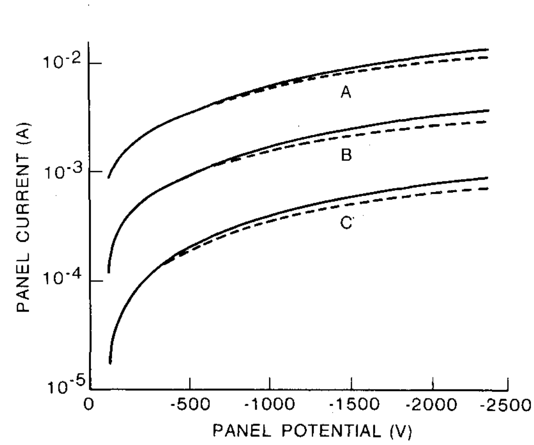

Fig. 5 Current-high voltage characteristics for panels A, B, and C normalized to an electron density, $N_e = 10^{12}$ m^{-3}.

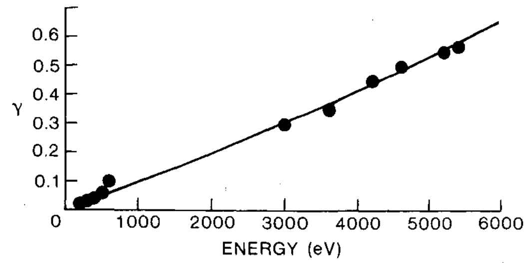

Fig. 6 Secondary electron emission coefficient for ions impacting on uncleaned stainless steel.

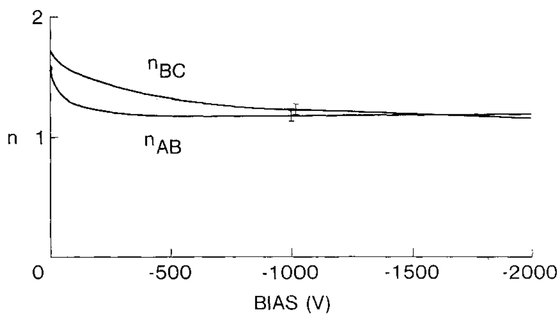

Fig. 7 Characteristic length scaling exponent for a current flow of the form $i \propto L^n$.

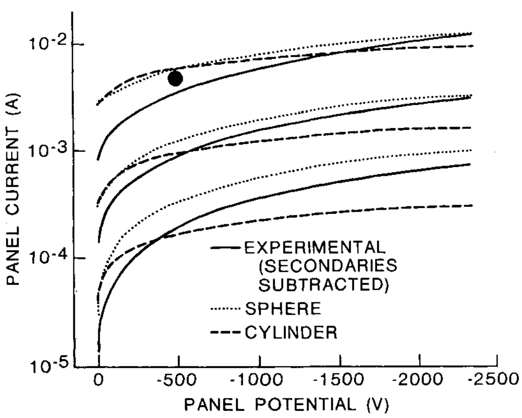

Fig. 8 Current-high voltage characteristics and computed characteristics for space-charge-limited current flow to spheres and cylinders.

for the current density was 1.44×10^{-4} A/m^2. This value is difficult to determine because of the existence of a presheath in which ions are accelerated. Thus the set of theoretical curves possibly could be displaced by as much as a factor of 2.

In all calculations, the single-particle approach was adopted and no collective plasma instabilities which may occur in the sheath especially at very high voltages were considered.

Conclusions

Measurements of plasma ion currents to conductive panels biased at high voltages and immersed in a collisionless plasma with densities resembling those of the F2 region of the ionosphere were presented. The currents measured have been corrected for ion-induced secondary electron emission, which at the higher voltages reached 25%. It was found that for all sizes of the panels used, the current at any voltage is directly proportional to the density of the ambient plasma. After an initial steep rise at panel potentials from 0 to -100 V, the *I-V* characteristics become almost linear with voltage. Measurements with similarly shaped panels whose areas differed by two orders of magnitude show that for a given plasma density and negative voltage above -500 V the current increases as $L^{1.2}$, where L is a characteristic length of the panel. Since no numerical predictions of ion current flow to biased rectangular panels are currently available, the results were compared with analytical models of space-charge-limited current flow to cylinders and spheres as derived by Langmuir and Blodgett. The data indicate that, because of the length to width ratio of the panels and because of sheath growth, for low voltages the current-voltage characteristic resembles that of a cylinder, while at higher voltages it resembles that of a sphere.

The reader should be warned about applying the data directly to an ionospheric orbital situation. The vehicle ram effect and sheath and wake effects[16,17] were not simulated in the present study and could significantly alter the amount of collected current.

Acknowledgments

The authors are pleased to acknowledge the help of the Space Environment Test Division under J. C. McLane Jr. in the construction of some experimental apparatus, the setting up of experiments, and providing the low-pressure environment in the large vacuum chamber A. Stimulating discussions with Dr. L. W. Parker were also appreciated. The authors also thank NASA Lewis Research Center for providing the Kaufman thruster used to create the simulated ionospheric environment.

References

[1]Langmuir, I. and Blodgett, K. B., "Current Limited by Space Charge Between Coaxial Cylinders," *Physical Review, Vol. 22, 1923, pp. 347-356;* also, *Collected Works of Irving Langmuir,* edited by G. Suits, Vol. 3, Pergamon Press, New York, 1961, pp. 115-124.

[2]Langmuir, I. and Blodgett, K. B., "Current Limited by Space Charge Between Cocentric Spheres," *Physical Review,* Vol. 23, 1924; also, *Collected Works of Irving Langmuir,* edited by G. Suits, Vol. 3, Pergamon Press, New York, 1961, pp. 125-135.

[3]Langmuir, I. and Mott-Smith, H. M. "Studies of Electric Discharges in Gases at Low Pressures," *General Electric Review,* Vol. 27, 1924, pp. 449-453; also, *Collected Works of Irving Langmuir,* edited by G. Suits, Vol. 4, Pergamon Press, New York, 1961, pp. 23-98.

[4]Langmuir I. and Mott-Smith, H. M., "The Theory of Collectors in Gaseous Discharges," *Physical Review,* Vol. 28, 1926, pp. 727-763; also, *Collected Works of Irving Langmuir,* edited by G. Suits, Vol. 4, Pergamon Press, New York, 1961, pp. 99-132.

[5]Bohm, D., *Characteristics of Electrical Discharge in Magnetic Fields,* edited by A. Guthrie and A. K. Wakerling, McGraw-Hill, New York, 1949.

[6]Bernstein, I. B. and Rabinowitz, I. N., "Theory of Electrostatic Probes in a Low Density Plasma," *Physics of Fluids,* Vol. 2, 1959, p. 112.

[7]Chen, F. F., "Electric Probes," *Plasma Diagnostic Techniques,* edited by R. H. Huddlestone and S. L. Leonard, Academic Press, New York, 1965, pp. 113-200.

[8]Schott, L., "Electrical Probes," *Plasma Diagnostics,* edited by W. Lochte-Holtgreven, North-Holland Publishing Co., Amsterdam, Wiley Interscience Division, John Wiley & Sons, Inc., New York, 1968, pp. 668-731.

[9]Swift, J. D. and Schwar, M. J. R., *Electrical Probes for Plasma Diagnostics,* London Iliffe Books; American Elsevier Publishing Co., Inc., New York, 1969.

[10]Parker, L.W., "Power Loss Calculation for High Voltage Solar Arrays." Final Report for NASA Contract NAS3-2085, 1977.

[11]Cooke, D., Parker, L. W., and McCoy, J. E., "Three-Dimensional Space Charge Model for Large High-Voltage Satellites," *Spacecraft Charging Technology 1980,* edited by N. J. Stevens and C. P. Pike, NASA CP 2182, 1981.

[12]Katz, I., Mandell, M. J., Schnuelle, G. W., Parks, D. E., and Steen, P. G., "Plasma Collection by High Voltage Spacecraft at Low Earth Orbit," *Journal of Spacecraft and Rockets,* Vol. 18, Jan.-Feb. 1981, p. 79.

[13]Szuszczewicz, E. P. and Holmes, J. C. "Surface Contamination of Active Electrodes in Plasmas: Distortion of Conventional Langmuir Probe Measurements," *Journal of Applied Physics*, Vol. 46, 1975, p. 5134.

[14]Langmuir, I. and Compton, K. T., "Electrical Dischanges in Gases, Part II. Fundamental Phenomena in Electrical Discharges," *Review of Modern Physics*, Vol. 3, 1931, p. 191.

[15]Konradi, A., McCoy, J. E., and Garriott, O. K., "Current Leakage for Low Altitude Satellites: Modeling Applications," *Quantitative Modeling of Magnetospheric Processes*, Geophysical Monograph 21, edited by W. P. Olson, American Geophysical Union, Washington, D.C., 1979, pp. 617-633.

[16]Reiff, P. H. and Freeman, J. W., "Environmental Protection of the Solar Power Satellite," *Space Systems and Their Interactions with Earth's Space Environment; Progress in Astronautics and Aeronautics, Vol. 71*, edited by H. B. Garrett and C. P. Pike, AIAA, New York, 1979, pp. 554-576.

[17]Parker, L. W., "Plasma Sheath Effects and Voltage Distributions of Large High-Power Satellite Solar Arrays," *Spacecraft Charging Technology 1978*, edited by R. C. Finke and C. P. Pike, NASA CP 2071, pp. 341-357.