Precision of Mesh-Type Reflectors for Large Space-Borne Antennas

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Knitted mesh reflectors for deployable, large antennas are shown to conform to minimal surfaces, bounded by the mesh supporting structural members. Closed-form expressions and tabulated values are derived for the root mean square deviation of the mesh from the ideal, parabolic reflector. It is shown that for cylindrical antennas with quadrilateral bays and axisymmetric antennas, particularly those with strong offset of the antenna feed, substantial improvements in precision can be obtained by providing curvature to the mesh supports. Expressions are given for the optimal curvatures.

Nomenclature

a = area of bay projected on tangent plane at vertex of ideal parabola (Fig. 1), $=s^2$
$c, c_1, c_2,$
c_{11}, c_{12}, c_{22} = coefficients in expansion to second order of mesh surface coordinate x_3, Eq. (4)
C = constant, defined following Eq. (7)
R_1, R_2 = principal radii of curvature of mesh surface
s = side of square of the bay's projection on tangent plane
x_1, x_2, x_3 = Cartesian coordinates with origin at vertex of ideal parabola (Fig. 1)
X_1, X_2 = Cartesian coordinates of center of bay projected on tangent plane (Fig. 2)
z_1, z_2 = differences, defined in Eqs. (22a) and (22b), of x_3 coordinate for optimal bay boundary segments as compared with ideal parabolic reflector (Fig. 1)
κ_1, κ_2 = principal curvatures of mesh surface
λ = Lagrange multiplier
μ = mean curvature of mesh surface, Eq. (12)
$\bar{\nu}$ = rms error of mesh surface as compared with ideal parabolic reflector
ξ^1, ξ^2 = contravariant coordinates in mesh surface
σ = parameter; $\sigma = 0$ for cylindrical, $\sigma = 1$ for axisymmetric antenna

Superscripts

$(\ )^*$ = ideal parabolic reflector
$(\ )'$ = difference between expansion coefficients for mesh surface as compared to ideal parabolic reflector, Eq. (16)

NOTE: All listed quantities are made nondimensional by dividing linear dimensions by $2F$ and areas by $(2F)^2$, where F is the distance of the focal point of the ideal parabolic reflector from its vertex (Fig. 1); hence, $2F$ is the radius of curvature at the vertex.

I. Introduction

STUDIES of space-borne antennas with apertures that exceed the dimensions of the payload space in the Shuttle Orbiter have been conducted by the National Aeronautics and Space Administration, the Department of Defense, and the aerospace industry. Although several fundamentally different designs are possible, such as deployed, erected, or space-fabricated structures, a common feature of several designs is a microwave reflecting surface consisting of a metallic or metallized mesh, tricot-knitted from low thermal expansion material. Meshes of this type have the important advantage of being capable of being folded with high packing efficiency into a small volume.

The cylindrical parabolic antenna with quadrilateral bays is illustrated in Fig. 1, where P is the ideal reflector, R the mesh reflector, T the tangent plane to P at the vertex, and F the distance to the focal line. The boundaries of the bays are defined such that, when projected on the tangent plane T at the vertex, they form a square grid (Fig. 2).

The geometry of the mesh surface is determined by the classical membrane equations.[1] An additional simplification is possible because a tricot-knitted mesh cannot support appreciable shear forces. It follows that the membrane force (force per unit length, tangential to the surface and normal to a contour on the surface) is isotropic. The mesh assumes the shape of a saddle surface with zero mean curvature; hence, it belongs to the family of surfaces known as minimal surfaces. (The area of the surface bounded by some given contour is a minimum.) The best known examples of minimal surfaces are soap films stretched across wire loops forming a space curve.[2]

The calculation carried out in the following sections proceeds by defining a pair of surface coordinates in the mesh surface, separately for each bay. From this the surface metric tensor and curvature tensor are calculated. Consistently throughout the calculation, terms that result from third or higher-order terms in the expansion of the mesh surface will be neglected.

Next, we find the contours of the bay boundaries (illustrated in Fig. 3) that minimize (in the rms sense) the distance between the mesh surface and the ideal parabolic reflector. The fact that the mean curvature of the mesh surface vanishes, results in an equation of constraint in the minimiza-

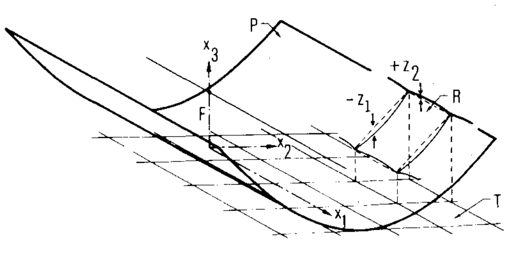

Fig. 1 Cylindrical parabolic antenna.

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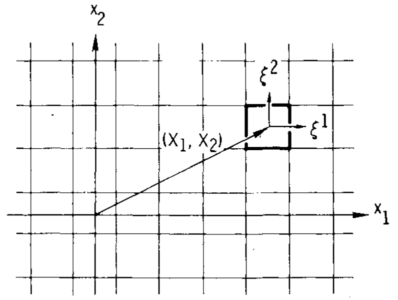

Fig. 2 Projection of the bay boundaries on tangent plane T.

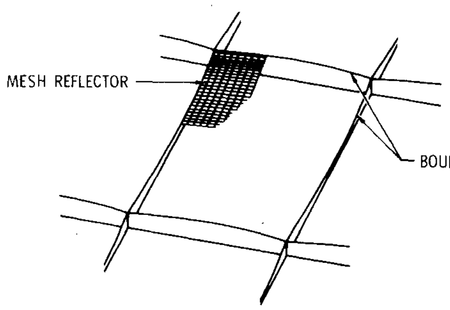

Fig. 3 Quadrilateral bay with curved boundary segments.

tion. The problem is solved by the method of Lagrange multipliers. The reader who is interested primarily in the results, rather than the mathematical detail, will find them stated in Sec. VI.

II. Differential Geometry of Mesh Surface

We designate the principal radii of curvature of the surface using R_1 and R_2, and the corresponding tensile forces, per unit length, tangential to the surface, using N_1 and N_2. It then follows from the classical theory of thin membranes[1] that in the absence of a pressure difference across the surface

$$\frac{N_1}{R_1} + \frac{N_2}{R_2} = 0 \tag{1}$$

Shear forces for a tricot-knitted mesh reflector are negligible in comparison with tensile forces. Hence the force tensor is isotropic, $N_1 = N_2$. With $\mu = (1/2)(1/R_1 + 1/R_2)$ designating the mean curvature, it then follows that the principal radii of curvature are equal and opposite so that the mesh forms a saddle surface with

$$\mu = 0 \tag{2}$$

The equivalence of surfaces of zero mean curvature and minimal surfaces was already known to Meusnier in 1785.[3]

If F is the distance from the vertex of the ideal reflector to its focus, then all lengths will be made nondimensional by division through $2F$, i.e., by the radius of curvature at the vertex of the ideal reflector.

The mesh surface is determined by the nondimensional position vector $\boldsymbol{x}$ with Cartesian coordinates (x_1, x_2, x_3). As illustrated in Fig. 2, the centers of the bays defined through their projection on the tangent plane T are given by the position vector $\boldsymbol{X}$ with Cartesian coordinates (X_1, X_2).

The position vector $\boldsymbol{x}$ for points on the mesh surface is taken as a function of two parameters ξ^1 and ξ^2 which serve as surface coordinates. With a view toward later satisfying the boundary conditions on the boundaries of any specified bay, a convenient choice for the surface coordinates is

$$\xi^\alpha = x_\alpha - X_\alpha, \qquad \alpha = 1, 2 \tag{3}$$

Here, and in what follows, Greek letter suffixes have the range 1, 2. Also defined is a pair of covariant base vectors tangential to the mesh surface

$$\boldsymbol{\epsilon}_\alpha = \partial \boldsymbol{x} / \partial \xi^\alpha$$

The mesh surface will be specified by

$$x_3 = c + c_1\xi^1 + c_2\xi^2 + c_{11}(\xi^1)^2 + 2c_{12}\xi^1\xi^2 + c_{22}(\xi^2)^2 + \mathcal{O}(s^3) \tag{4}$$

where the coefficients c are constants, depending only on the bay specified, and s is the nondimensional length of the side of the bay projected on the tangent plane T. In the applications of interest, the dimensions of the bays are small compared with the focal length of the antenna. Terms of order s^3 will be neglected in this calculation.

With our choice of the surface coordinates, the Cartesian components (indicated by the second subscript) of the base vectors become

$$\epsilon_{\alpha\beta} = \delta_{\alpha\beta}$$
$$\epsilon_{\alpha 3} = c_\alpha + 2c_{\alpha\gamma}\xi^\gamma + \mathcal{O}(s^2) \qquad \alpha, \beta, \gamma = 1, 2 \tag{5}$$

where $\delta_{\alpha\beta}$ is the Kronecker delta and the summation convention is employed. Without loss of generality we let $c_{12} = c_{21}$.

The surface metric tensor $g_{\alpha\beta} = \boldsymbol{\epsilon}_\alpha \cdot \boldsymbol{\epsilon}_\beta$ becomes[4]

$$g_{\alpha\beta} = \delta_{\alpha\beta} + c_\alpha c_\beta + \mathcal{O}(s) \qquad \alpha, \beta = 1, 2 \tag{6}$$

and the metric

$$g = \begin{vmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{vmatrix} = C^2 + \mathcal{O}(s) \tag{7}$$

where $C^2 = 1 + c_1^2 + c_2^2$. The unit normal vector $\boldsymbol{n} = \boldsymbol{N}/N$ where $\boldsymbol{N} = -(\boldsymbol{\epsilon}_1 \times \boldsymbol{\epsilon}_2)$ becomes, after straightforward calculation,

$$n_\alpha = C^{-1}\left(1 - 2C^{-2}c_\beta c_{\beta\gamma}\xi^\gamma\right)\left(c_\alpha + 2c_{\alpha\beta}\xi^\beta\right) + \mathcal{O}(s^2)$$
$$n_3 = -C^{-1}\left(1 - 2C^{-2}c_\beta c_{\beta\gamma}\xi^\gamma\right) + \mathcal{O}(s^2) \qquad \alpha, \beta, \gamma = 1, 2 \tag{8}$$

The surface curvature tensor $b_{\alpha\beta} = (\partial \boldsymbol{n}/\partial \xi^\alpha) \cdot \boldsymbol{\epsilon}_\beta$ is

$$b_{\alpha\beta} = 2C^{-1}c_{\alpha\beta} + \mathcal{O}(s) \tag{9}$$

and the two principal curvatures κ_1 and κ_2 are obtained as the roots of the determinental equation $|b_{\alpha\beta} - \kappa g_{\alpha\beta}| = 0$. Defining the quantity

$$m = C^{-3}\left[\left(1 + c_1^2\right)c_{22} + \left(1 + c_2^2\right)c_{11} - 2c_1c_2c_{12}\right] \tag{10}$$

it follows that

$$\kappa_1, \kappa_2 = m \pm \sqrt{m^2 + \frac{4\left(c_{12}^2 - c_{11}c_{22}\right)}{\left(1 + c_1^2 + c_2^2\right)^2}} + \mathcal{O}(s) \tag{11}$$

Hence, the (nondimensional) mean curvature is

$$\mu = \tfrac{1}{2}(\kappa_1 + \kappa_2) = m + \mathcal{O}(s) \tag{12}$$

III. Minimization of rms Error

Henceforth the asterisk will refer to the ideal parabolic reflector which, in terms of the already defined Cartesian

coordinates, is given by

$$x_3^* = \tfrac{1}{2}\left(\sigma x_1^2 + x_2^2\right) = \tfrac{1}{2}\left[\sigma\left(X_1 + \xi^1\right)^2 + \left(X_2 + \xi^2\right)^2\right] \quad (13)$$

with $\sigma = 0$ and 1 for the cylindrical and axisymmetric reflectors, respectively. If we expand and write the result to Eq. (4) analogously, the coefficients become

$$c^* = \tfrac{1}{2}\left(\sigma X_1^2 + X_2^2\right), \quad c_1^* = \sigma X_1, \quad c_2^* = X_2$$

$$c_{11}^* = \tfrac{1}{2}\sigma, \quad c_{22}^* = \tfrac{1}{2}, \quad c_{12}^* = c_{21}^* = 0 \quad (14)$$

Let $\nu = \nu(\xi^1, \xi^2)$ be the normal distance in the direction of $\boldsymbol{n}$ between the mesh surface and the ideal reflector, where $\nu = \mathcal{O}(s^2)$. Hence

$$\sigma\left(x_1 + \frac{c_1 \nu}{C}\right)^2 + \left(x_2 + \frac{c_2 \nu}{C}\right)^2 = 2\left(x_3 - \frac{\nu}{C}\right) + \mathcal{O}(s^3) \quad (15)$$

It is convenient to define coefficients $c', c_1', \ldots$, all of $\mathcal{O}(1)$, by letting

$$c = c^* + s^2 c', \quad c_\alpha = c_\alpha^* + s c_\alpha'$$

$$c_{\alpha\beta} = c_{\alpha\beta}^* + c_{\alpha\beta}', \qquad \alpha, \beta = 1, 2 \quad (16)$$

It follows that $c_{\alpha\beta}' = c_{\beta\alpha}'$, and from Eqs. (15) and (3), after some calculation, that

$$\nu = \left(1 + \sigma X_1^2 + X_2^2\right)^{-\frac{1}{2}}\left(c_{\alpha\beta}' \xi^\alpha \xi^\beta + s c_\alpha' \xi^\alpha + s^2 c'\right) + \mathcal{O}(s^3) \quad (17)$$

where again the summation convention is employed.

We wish to minimize, by an optimal choice of the contour bounding each bay, the rms error of the mesh surface, i.e., the quantity

$$\int\!\!\int \nu^2 \sqrt{g}\, \mathrm{d}\xi^1\, \mathrm{d}\xi^2$$

where the integral is extended over the specified bay. Since the metric g, from Eqs. (7), (16), and (14), $g = 1 + \sigma X_1^2 + X_2^2 + \mathcal{O}(s)$, this amounts to minimizing the surface integral

$$\int\!\!\int \left[c_{11}'(\xi^1)^2 + 2c_{12}'\xi^1\xi^2 + c_{22}'(\xi^2)^2 + s c_1'\xi^1 + s c_2'\xi^2 + s^2 c'\right]^2 \mathrm{d}\xi^1\, \mathrm{d}\xi^2 \quad (18)$$

neglecting higher-order terms. The minimization is subject to the constraint $\mu = 0$ or, from Eqs. (10), (16), and (14),

$$\left(1 + \sigma X_1^2\right)\left(\tfrac{1}{2} + c_{22}'\right) + \left(1 + X_2^2\right)\left(\tfrac{1}{2}\sigma + c_{11}'\right) - 2\sigma X_1 X_2 c_{12}' = 0 \quad (19)$$

Since the rms error $\bar{\nu}$ is minimized separately for each bay, in principle a step results where two contiguous bays meet. Whereas $\bar{\nu}$ is $\mathcal{O}(s^2)$, the step is $\mathcal{O}(s^3)$ and, consistent with the earlier approximations, can therefore be neglected.

The variables of integration in Eq. (18) extend from $-s/2$ to $+s/2$. The integration is tedious but straightforward, with the result

$$\bar{\nu}^2 = a^2\left(1 + \sigma X_1^2 + X_2^2\right)^{-1}\left[c'^2 + \frac{1}{12}\left(c_1'^2 + c_2'^2 + 2c'c_{11}' + 2c'c_{22}'\right) + \frac{1}{80}\left(c_{11}'^2 + c_{22}'^2\right) + \frac{1}{72}\left(2c_{12}'^2 + c_{11}'c_{22}'\right)\right] \quad (20)$$

where $a = s^2$ is the nondimensional area of the bay projected on the tangent plane.

IV. Optimally Curved Boundary Segments

If not all derivatives (Jacobians) of the left-hand side of Eq. (19) vanish at the point of the extremum, a constant multiplier (Lagrange multiplier) λ exists such that

$$c' + \tfrac{1}{12}\left(c_{11}' + c_{22}'\right) = c_1' = c_2' = 0$$

$$\tfrac{1}{36}c_{12}' = -\lambda\sigma X_1 X_2$$

$$\tfrac{1}{6}c' + \tfrac{1}{40}c_{11}' + \tfrac{1}{72}c_{22}' = \lambda\left(1 + X_2^2\right)$$

$$\tfrac{1}{6}c' + \tfrac{1}{40}c_{22}' + \tfrac{1}{72}c_{11}' = \lambda\left(1 + \sigma X_1^2\right) \quad (21)$$

The values of the coefficients $c', c_1', \ldots$ and λ for which $\bar{\nu}$ is a minimum will be the solution set of Eqs. (21) together with the equation of constraint, Eq. (19). These equations, which are linear, are readily solved. The result, together with the final result for $\bar{\nu}$, is indicated in Table 1.

For this optimal case, the four boundary segments of the bays satisfy the equations

For $x_1 = \text{const} = X_1 \pm s/2$:

$$x_3(\xi^2) = x_3^*\left(x_1 = X_1 \pm s/2, \quad x_2 = X_2 + \xi^2\right) + z_1(\xi^2) \quad (22a)$$

For $x_2 = \text{const} = X_2 \pm s/2$:

$$x_3(\xi^1) = x_3^*\left(x_1 = X_1 + \xi^1, \quad x_2 = X_2 \pm s/2\right) + z_2(\xi^1) \quad (22b)$$

Table 1 Formulas for quadrilateral bays

Optimally curved boundary segments	$\lambda = -\dfrac{1}{36}\dfrac{1 + \sigma\left(1 + X_1^2 + X_2^2\right)}{5\left(1 + \sigma X_1^2\right)^2 + 5\left(1 + X_2^2\right)^2 + 4\sigma X_1^2 X_2^2}$ $c_{11}' = 90(1 + X_2^2)\lambda$, $c_{22}' = 90(1 + \sigma X_1^2)\lambda$ $c_{12}' = -36\sigma X_1 X_2 \lambda$, $c' = -\frac{1}{12}(c_{11}' + c_{22}')$	$\bar{\nu} = \dfrac{a}{\left(1 + \sigma X_1^2 + X_2^2\right)^{\frac{1}{2}}}\left[-c'^2 + \frac{1}{80}\left(c_{11}'^2 + c_{22}'^2\right) + \frac{1}{72}\left(2c_{12}'^2 + c_{11}'c_{22}'\right)\right]^{\frac{1}{2}}$ For $x_1 = \text{const} = X_1 \pm s/2$: $x_3 = \frac{1}{2}(\sigma x_1^2 + x_2^2) + z_1$
Optimal straight line boundary segments	$c_{11}' = -\sigma/2$, $c_{22}' = -\frac{1}{2}$ $c_{12}' = 0$, $c' = \frac{1}{24}(1 + \sigma)$	For $x_2 = \text{const} = X_2 \pm s/2$: $x_3 = \frac{1}{2}(\sigma x_1^2 + x_2^2) + z_2$ where $z_1 = \left(c' + \dfrac{c_{11}'}{4}\right)s^2 \pm c_{12}'s(x_2 - X_2) + c_{22}'(x_2 - X_2)^2$ $z_2 = \left(c' + \dfrac{c_{22}'}{4}\right)s^2 \pm c_{12}'s(x_1 - X_1) + c_{11}'(x_1 - X_1)^2$

where z_1 and z_2 are correction terms corresponding to the difference of the x_3 coordinates of the optimal boundary and the ideal reflector, respectively. Terms z_1 and z_2 are of second order and are obtained from Eqs. (4), (13), (14), and (16). The result is again given in Table 1.

It follows easily from Eq. (11), and from $m=0$, that the two principal curvatures of the mesh surface are

$$\kappa = \pm \frac{2}{1+\sigma X_1^2 + X_2^2}\left\{(36\sigma X_1 X_2 \lambda)^2 - \left[\tfrac{1}{2} + 90\lambda(1+\sigma X_1^2)\right]\right.$$
$$\left.\times\left[(\sigma/2) + 90\lambda(1+X_2^2)\right]\right\}^{1/2} \tag{23}$$

where λ is obtained from the expression given in Table 1. Although the proof is not given here, in the case of the cylindrical antenna ($\sigma = 0$), it is not difficult to show that the two solutions for κ are real for all values of X_1 and X_2. No such general proof was obtained for the axisymmetric case ($\sigma = 1$); but numerical calculations show that the optimal curvatures are real also in the latter case for all values of X_1 and X_2 that are of interest in applications. Corresponding remarks also apply to the expression for $\bar{\nu}$ in Table 1.

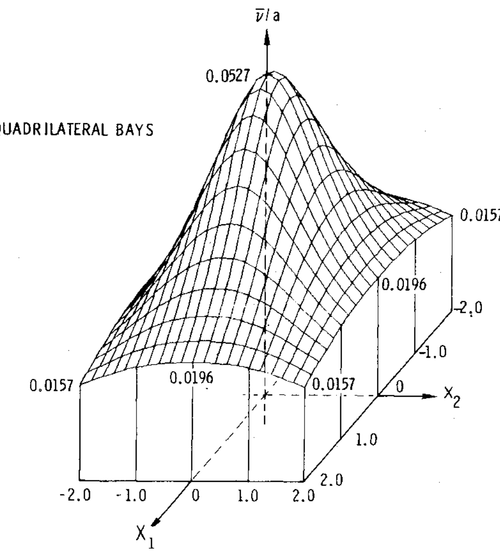

Fig. 4 Function $\bar{\nu}/a$ for axisymmetric antenna with optimally curved boundary segments.

V. Optimal Straight Line Boundary Segments

It is of interest to compare the improvement in the rms error that can be achieved with optimal curved boundary segments with the case of optimal straight line segments. The latter, of course, allow a simpler construction.

In this case, $c_{11} = c_{22} = 0$ and the method of Lagrange multipliers yields the relations

$$c' - \tfrac{1}{24}(1+\sigma) = c'_1 = c'_2 = c'_{12} = 0$$
$$c'_{11} = -\frac{\sigma}{2}, \qquad c'_{22} = -\tfrac{1}{2} \tag{24}$$

It follows that $c_{12} = 0$, showing that the boundary segments are coplanar.

Expressions needed to determine the coordinates of the boundary segments and the rms error $\bar{\nu}$ of the mesh surface are given in Table 1. The resulting expression for $\bar{\nu}$ can be further simplified to

$$\bar{\nu} = \frac{a(1+\sigma)^{\frac{1}{2}}}{12\sqrt{5}\left(1+\sigma X_1^2 + X_2^2\right)^{\frac{1}{2}}} \tag{25}$$

This shows that $\bar{\nu}$ in this instance is independent of the polar angle $\tan^{-1}(X_2/X_1)$ when $\sigma = 1$. (In the case considered in Sec. IV this would not be valid.)

VI. Results and Discussion

The results derived in the preceding sections are collected in Table 1. The coefficients $c'_{11}, c'_{22}, \ldots$ and λ are calculated from the expressions given there, where X_1, X_2 are Cartesian coordinates defining the center of the specified bay. Expressions for the coefficients are given in the case of optimally curved boundary segments (cf., Fig. 3) and also for the more restrictive case of straight line boundary segments, in both cases optimally adjusted to minimize $\bar{\nu}$, the rms error of the mesh surface. With the coefficients evaluated, it is then possible to calculate $\bar{\nu}$ and z_1, z_2. These latter quantities determine the height x_3 of the optimal boundary segments above the tangent plane T (Fig. 1). The quantities z_1 and z_2 are correction terms applied to the ideal reflector. The actual height x_3 of the boundary segments is obtained from substituting the correction terms into the four formulas for x_3, each giving the height above the tangent plane of one of the four boundary segments.

Table 2 rms error of mesh surface for some selected values of X_1 and X_2

	Minimal rms error when boundary segments are: Curved	Straight line
	Cylindrical antenna ($\sigma = 0$)	
$X_2 = 0$	$\frac{a}{12\sqrt{10}} = 2.635 \times 10^{-2}a$	$\frac{a}{12\sqrt{5}} = 3.726 \times 10^{-2}a$
$X_2 = \pm 1$	$\frac{a}{60\sqrt{2}} = 1.179 \times 10^{-2}a$	$\frac{a}{12\sqrt{10}} = 2.635 \times 10^{-2}a$
	Axisymmetric antenna ($\sigma = 1$)	
$X_1 = X_2 = 0$	$\frac{a}{6\sqrt{10}} = 5.270 \times 10^{-2}a$	$\frac{a}{6\sqrt{10}} = 5.270 \times 10^{-2}a$
$X_1 = \pm 1,\ X_2 = 0$; $X_1 = 0,\ X_2 = \pm 1$	$\frac{a}{20\sqrt{2}} = 3.536 \times 10^{-2}a$	$\frac{a}{12\sqrt{5}} = 3.727 \times 10^{-2}a$
$X_1 = \pm 1/\sqrt{2},\ X_2 = \pm 1/\sqrt{2}$	$\frac{a}{4\sqrt{47}} = 3.647 \times 10^{-2}a$	$\frac{a}{12\sqrt{5}} = 3.727 \times 10^{-2}a$
$X_1 = \pm 2,\ X_2 = 0$; $X_1 = 0,\ X_2 = \pm 2$	$\frac{a}{10\sqrt{26}} = 1.961 \times 10^{-2}a$	$\frac{a}{30\sqrt{2}} = 2.357 \times 10^{-2}a$

Table 3 Function $\bar{\nu}/a$ for axisymmetric antenna with optimal quadrilateral bays[a]

$X_2 =$	0	±0.2	±0.4	±0.6	±0.8	±1.0	±1.2	±1.4	±1.6	±1.8	±2.0
$X_1 = 0$	0.05270	0.05167	0.04880	0.04468	0.04000	0.03535	0.03112	0.02745	0.02436	0.02177	0.01961
±0.2		0.05070	0.04799	0.04407	0.03957	0.03508	0.03095	0.02734	0.02429	0.02173	0.01958
±0.4			0.04570	0.04231	0.03833	0.03425	0.03041	0.02701	0.02408	0.02159	0.01950
±0.6				0.03963	0.03637	0.03290	0.02952	0.02643	0.02371	0.02136	0.01935
±0.8					0.03389	0.03111	0.02829	0.02560	0.02316	0.02099	0.01911
±1.0						0.02901	0.02677	0.02454	0.02243	0.02049	0.01877
±1.2							0.02507	0.02329	0.02153	0.01986	0.01832
±1.4								0.02192	0.02050	0.01910	0.01777
±1.6									0.01939	0.01825	0.01713
±1.8										0.01735	0.01642
±2.0											0.01567

[a]The function is symmetric with respect to interchanging X_1 and X_2.

Table 2 gives values of $\bar{\nu} = \bar{\nu}(X_1, X_2)$ for several selected cases. A comparison of the table entries shows that for *cylindrical* antennas the rms error can be reduced substantially by optimally curving the bays' boundary segments (by a factor of $1/\sqrt{2}$ at the vertex, and by $1/\sqrt{5}$ at an offset of $X_2 = \pm 1$). For *axisymmetric* antennas, the reduction is generally small: there is no reduction at the vertex (where the optimal curvature is zero); only at large offsets from the vertex does the reduction become appreciable. From comparing the fourth and fifth row in the table it is seen that the optimal approximation to the ideal axisymmetric antenna is not axisymmetric, although for distances from the vertex comparable to the focal length it is nearly symmetric. The function $\bar{\nu}/a$ for optimally curved boundary segments is plotted in Fig. 4 and tabulated in Table 3.

The principal curvatures of the mesh surface are easily obtained from Eq. (11), setting $m = 0$. For the cylindrical antenna, at its vertex, the principal curvatures are $\pm\frac{1}{2}$, hence half of the curvature of the ideal parabolic reflector (cf., Fig. 1).

VII. Conclusions

It has been shown that, by proper choice of the curvature of the supports, woven wire mesh reflectors for antennas can be made to approximate optimally the ideal parabolic surface. The improvement in the reflector's rms error that can be obtained is greater for cylindrical than for axisymmetric antennas and greater for off-axis points than for points close to the vertex. (No improvement is obtained at the vertex of the axisymmetric antenna.) When compared with straight line supports, the improvement is a factor of $\sqrt{2}$ at the vertex of cylindrical antennas, and $\sqrt{5}$ for points at a vertex distance twice the focal length.

Explicit formulas, applicable to both cylindrical and axisymmetric reflectors, are given in Table 1. These formulas permit one to calculate the coordinates of optimally curved mesh supports, as well as those of optimal straight line supports. An equation is given, also in Table 1, to calculate the rms error of the mesh surface for both of these cases.

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