Engineering Notes

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Inadequacy of Single-Impulse Transfers for Path Constrained Rendezvous

S.A. Stern*
University of Colorado, Boulder, Colorado
and
K.M. Soileau†
NASA Johnson Space Center, Houston, Texas

Nomenclature

CW = Clohessy-Wiltshire
FZ = Forbidden Zone
LSS = large space structure
$\hat{n}$ = LSS local surface normal unit vector
t = time, s
TAC = tangential arrival condition
TDC = tangential departure condition
$\vec{V}$ = velocity
$\hat{x}$ = CW unit vector antiparallel to the orbital velocity
$\hat{y}$ = CW unit vector parallel to the orbital radius vector
$\hat{z}$ = CW unit vector parallel to the orbital angular momentum vector
τ = transfer time
ω = orbital angular rate

I. Introduction

THE problem of path constrained rendezvous was first introduced by Stern.[1] A simple statement of the general problem can be made as follows: How is it possible to maneuver from point to point about an orbiting large space structure (LSS) of arbitrary geometrical configuration and spin so as to avoid those transfer paths that pass through the structure itself? Because the classical rendezvous guidance equations assume that the path between the departure and arrival points is unconstrained, it becomes necessary to determine how a transfer can be constructed so as to achieve target intercept *and* avoid the constraint imposed by the presence of the LSS. Path constrained rendezvous is important to structures as small as the planned U.S. space station.[2]

Later, Stern and Fowler[3] used numerical simulations to show that only a small fraction (<10%) of the possible single-impulse transfers connecting arbitrary points on sphere-like, plate-like, and straw-like LSS's successfully complete point-to-point transfers without first colliding with the structure. These workers also demonstrated that the most difficult transfers to accomplish were those between two points physically on the LSS surface.

Most recently, Stern and Soileau[4] derived a pair of necessary and apparently sufficient endpoint conditions for the successful completion of *any* path constrained rendezvous between arbitrary points on the surface of certain widely applicable LSS's.

Here, we demonstrate that some single-impulse transfers between points on the *exterior* surface of *any* closed LSS must fail (i.e., collide with the LSS). We further demonstrate that single-impulse transfers can always be successful between *any* arbitrary points in the *interior* region of *any* LSS whose interior is empty and everywhere convex. Finally, the important operational and architectural consequences of these findings will be discussed.

II. Formalism

We begin by introducing the useful concept of a forbidden zone (FZ). A forbidden zone is defined as any large, unpassable orbital volume that can be represented by a constraint surface. Forbidden zones can be open or closed, and may be of positive or negative curvature; the only requirement placed on FZ geometry is that it be a connected region. Defined in this way, an FZ can be either a physical volume (e.g., an LSS, a space station, an asteroid) or a nonphysical volume (e.g., an antenna radiation avoidance zone); it can have holes (e.g., tunnels), and can be open at one end (e.g., a large antenna).

In constrained rendezvous research, the maneuvering vehicle is assumed to be of much smaller dimensions than the FZ itself, so that the transfer problem is reduced to a point mass moving in the vicinity of a constraint surface. One also requires that the mutual attraction of the FZ and the transfer vehicle be insignificant. These restrictions are introduced as a part of the formalism and do not significantly restrict the practicality of our results.

As before, we restrict our attention to transfers with both endpoints on the FZ surface; such transfers are the most difficult to accomplish.[2]

The Clohessy-Wiltshire (CW) equations of relative motion[5] are often used in the solution of path constrained rendezvous problems. We refer the reader to Stern and Soileau[4] for a complete discussion of the applicability of these equations. Without development, we adopt and present them as follows:

$$x(t) = 2\left[2\frac{\dot{x}_0}{\omega} - 3y_0\right]\sin(\omega t) - 2\frac{\dot{y}_0}{\omega}\cos(\omega t) + \left[6y_0 - 3\frac{\dot{x}_0}{\omega}\right]\omega t + 2\frac{\dot{y}_0}{\omega} + x_0 \tag{1a}$$

$$y(t) = \left[2\frac{\dot{x}_0}{\omega} - 3y_0\right]\cos(\omega t) + \frac{\dot{y}_0}{\omega}\sin(\omega t) - 2\frac{\dot{x}_0}{\omega} + 4y_0 \tag{1b}$$

$$z(t) = z_0\cos(\omega t) + \frac{\dot{z}_0}{\omega}\sin(\omega t) \tag{1c}$$

where ω is orbital angular rate and t is elapsed time. The rotating Cartesian coordinate frame employed exhibits $\hat{x}$ in the orbital plane antiparallel to the velocity vector, $\hat{y}$ directed

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*Research Associate, Laboratory for Atmospheric and Space Physics. Member AIAA.
†Aerospace Technologist.

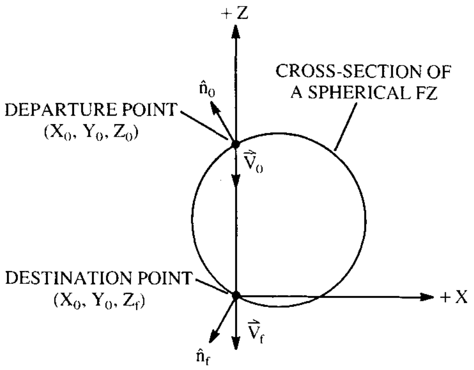

Fig. 1 A spherical FZ.

radially outward from the Earth's center through the coordinate origin, and $\hat{z}$ completing the right-hand triad.

Equations (1) can be used to target a rendezvous or intercept by locating the origin at the transfer endpoint, and then forcing the x, y, and z separations to zero (i.e., an intercept) after some time τ (the transfer time), and solving for the initial velocities $\dot{x}_0$, $\dot{y}_0$, and $\dot{z}_0$ required to effect intercept. We again reference Stern and Soileau[4] and Dunning.[5]

III. Analysis

Successful transfers between arbitrarily chosen points on an FZ surface must *necessarily* satisfy at least two point-conditions[6]; such transfers must: 1) depart along a path away from the FZ surface and 2) intercept the target from above the FZ surface.

We call the first of these two conditions the tangential departure condition (TDC), since it forces the maneuvering transfer vehicle to depart at least tangentially from the FZ surface; for analogous reasons, we call the second condition the tangential arrival condition (TAC). In effect, these two conditions, which hold for any FZ geometry, require the transfer to at least begin and end in a manner consistent with FZ avoidance.

Analytically, the tangential departure and arrival conditions may be stated as inequality constraints on the dot product of the departure and arrival velocity vectors and the local FZ surface normal:

$$\hat{n}_0 \cdot \vec{V}_0 > 0 \qquad \text{(TDC)}, \quad \hat{n}_f \cdot \vec{V}_f < 0 \qquad \text{(TAC)} \tag{2}$$

Given the tangential departure and arrival conditions, and an understanding of the physics contained in the equations of motion, it is possible to prove the following two observations:

Observation I: Regardless of how one designs an LSS, there will always be certain *external* transfers that cannot be successfully accomplished using single-impulse techniques.

Observation II: If an empty region exists in the *interior* of an LSS, and if the empty region is everywhere convex, then single-impulse techniques can be used to reach any interior point from any other interior point.

These results are previously unanticipated properties of the TDC and TAC conditions first derived by Stern and Soileau.[4] In the following discussion, we will employ geometrical arguments to demonstrate the validity of observations I and II. For brevity's sake, we rely heavily on the results and formal discussions given in the references cited above.

We consider first observation I. Clearly, any physically realizable FZ (i.e., LSS) must be three-dimensional; therefore, there must be points on the FZ's exterior surface separated only in their out-of-plane coordinate z. As demonstrated by the solutions to the C-W equations of motion, however, purely out-of-plane transfers will lie entirely along a path parallel to the z axis. Therefore, any purely out-of-plane transfer must traverse the line segment connecting the departure and arrival states. Since, regardless of the FZ's design, there will always be points separated only in z whose connecting line segment pierces the forbidden zone, such transfers will fail the tangential conditions at either the departure or arrival point, or both, and will therefore be unsuccessful. Figure 1 illustrates such a case for a spherical FZ. Since many purely out-of-plane departure/arrival point pairs exist for any LSS, single-impulse transfers cannot be used to travel from any arbitrary point on an LSS to any other point. Therefore, *single-impulse techniques are inadequate as a general scheme for transferring about the exterior of any LSS, regardless of its design.*

We now justify observation II. Consider the class of all internally convex FZ's; by *internally* convex, we mean that the forbidden zone encloses an empty convex region. Convexity itself means that any line segment connecting two points in the FZ's interior must lie entirely inside the FZ; i.e., any line segment connecting arbitrary points in the empty region must not pierce the forbidden zone. Examples of such FZ's include the interior of spheres, triaxial ellipsoids, cylinders of all types, and boxes. Because Stern[2] rigorously demonstrated that the CW equations reduce to rectilinear transfers in the limit as $\tau \to 0$, one can always find a sufficiently rectilinear transfer between any two points in the interior of an everywhere convex FZ that avoids collisions with the FZ surface. In fact, for any departure/arrival point combination in such a region, there exists a continuous span of transfer times $0 < \tau < \tau_{max}$, all of which will result in successful transfers between the desired points, where a τ_{max} transfer forms the limiting case in which a transfer just avoids collision. We thus conclude that single-impulse transfers will always suffice in the interior of *any* internally convex LSS; one specific and useful application of this result is the accessibility of points on the interior of any large parabolic antenna in space.

Summarizing, single-impulse transfers cannot, under any circumstance of LSS design or transfer time, guide a vehicle from any given departure point on the exterior of a closed LSS to all other points on that LSS. *Because they can never be generally applicable, we conclude that single-impulse transfers are inadequate for path constrained rendezvous guidance.*

In closing, we note that multiple-impulse transfers, though more complicated, have been shown by Stern[1] to render feasible any transfer about an LSS of arbitrary geometrical complexity. In particular, Stern demonstrated that two-impulse transfers provide very high success rates. We recommend that the community now attack the formal and rigorous optimization of the two-impulse transfer problem for large space structures.

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Radiation Environment Models and the Atmospheric Cutoff

Andrei Konradi* and Alva C. Hardy†
NASA Johnson Space Center
Houston, Texas
and
William Atwell‡
Rockwell International, Houston, Texas

Introduction

RADIATION environment models are widely used to predict radiation doses expected on a variety of space flight missions. In these models, omnidirectional electron and proton fluxes are stored in computer codes as functions of particle energies and the parameters B and L, where B is the local magnetic field intensity in Gauss and L is the McIlwain drift shell parameter.[1]

While trying to predict radiation doses expected at the space station orbit toward the end of this century, we discovered an artifact of the model that necessitates modifications in the standard method of their use. The following discussion explains this artifact and makes suggestions about how it can be avoided.

The most recent models are the AP-8 for protons[2] and the AE-8 for electrons. Each of these models comes in two variants that incorporate different radiation intensities observed in the atmospheric cutoff region about the 1970 solar maximum (MAX) and about the 1964 solar minimum (MIN).

During magnetically quiet periods, magnetic field models, based on the multipole expansion of the internal sources only, represent the Earth's field quiet well up to L values of about 4. Experimental evidence indicates that the Earth's magnetic field is changing: the strength of the dipole is decreasing at about 0.09%/year and geomagnetic surface features are drifting westward[3] at a rate of 0.27 deg/year. Thus, to allow a certain amount of temporal extrapolation, the models contain first-order and sometimes second-order time derivatives.

Analysis

In trying to predict the space station radiation environment using the radiation environment models, we encountered an almost exponential temporal increase of the radiation intensity. Such an unexpected and rather unrealistic result forced us to review the methodology of the calculations and the structuring of the models proper. This increase is especially noticeable in the region of the South Atlantic anomaly (SAA), which we will use here in order to illustrate the problem.

The calculations presented here will be based on the International Geomagnetic Reference Field for 1975 (IGRF 1975)[4] and fluxes of protons with energies above 30 MeV as represented by AP-8 MIN.

Figure 1 shows B and L contours in the SAA at 500 km altitude for the epochs 1965 (a), 1995 (b), and 2025 (c). In each of the plots, the minimum field region stands out clearly. Note a general westward drift of the minimum at a rate of about 0.27 deg/year and a temporal decrease of the magnetic field intensity at the center of the anomaly. This decrease is associated with an observed decay of the Earth's magnetic dipole, which currently proceeds at a rate of about 27 nT/year.[3] In addition the crosses show the location of the flux maximum for protons with energies above 30 MeV. Due to the dependence of the particle energy spectrum on L, the position of the flux maxima is somewhat offset from the magnetic field minima.

Figure 2 shows flux/altitude profiles for the years 1965, 1995, and 2025. These profiles were calculated at the locations of the flux maxima indicated in Fig. 1. Not only does the flux increase with time, but it also increases disproportionately at low altitudes and even below the Earth's surface! If real, such an increase would have implications far beyond anyone's imagination.

Discussion

Our interpretation of this phenomenon rests simply on the fact that the particle flux contained in the model as a function of B and L has built into it the atmospheric cutoff, also as a function of B and L, as of 1964 for solar minimum and as of 1970 for solar maximum. In other words, within experimental uncertainties, AP-8 MIN and MAX represent the average particle distribution as it existed for those two epochs, both deep within the magnetosphere and within the atmospheric cutoff region.

As a result of the temporal decrease in the magnitude of the geomagnetic dipole moment, locations of a fixed magnetic field intensity are found at progressively closer distances to the Earth. Consequently, flux values associated with fixed B values move to lower altitudes[5] and the models artificially move the atmospheric cutoff to lower altitudes also. Since the flux gradient within the geomagnetic cutoff is very steep, within a short time period the order of magnitude flux increases and even subterranean fluxes are predicted. Obviously, this is an artifact of the model and has no physical significance.

The above-described use of the trapped radiation environment models is also based on the assumption that B and L, being derived from the first and second adiabatic invariant, are themselves invariant. In a static magnetic field, this is certainly true and this coordinate system is very successful in ordering particle data and is, therefore, incorporated into models of the Earth's trapped radiation environment. However, since the Earth's dipole is decreasing with a characteristic time of about 1000 years, when calculating particle flux transformations, all three adiabatic invariants have to be taken into account properly.[6,7] When this is done, it is found that for a fixed point in the SAA the flux increases even more rapidly with time! The explanation for this is that, in addition to lowering of the whole B-L coordinate system, which is the cause of the flux increase when B and L are considered constant, we now also have a secular drift in the B-L space.[6]

Thus, unless the trapped radiation models are effectively decoupled from the atmospheric absorption effects and the two applied in conjunction, straightforward calculations will lead to erroneous results.

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*Physicist, Space Science Branch, Solar System Exploration Division, Space and Life Sciences Directorate. Member AIAA.

†Physicist, Space Science Branch, Solar System Exploration Division, Space and Life Sciences Directorate.

‡Member of the Technical Staff, Houston Operations.