

Laser Satellite Constellations for Strategic Defense—An Analytic Model

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Using mainly geometric reasoning, an analytic model is constructed that predicts the required characteristics of an orbiting constellation of laser battlestations, each of which is designed to destroy ballistic missiles during their boost phase. The model can be applied to ICBM threats located at a single site or distributed over an area. Given either of these distributions, the physical characteristics of the ICBM's, i.e., the required laser fluence and target dwell time, the model predicts the required number of laser satellites, the required individual laser radiant intensities, and the average kill rate per laser. If the laser beamwidth is specified, the required laser power can then be calculated from the required radiant intensity. With the use of high-frequency chemical lasers, one can then estimate the number of Shuttle loads required to transport the chemical fuel into orbit, assuming one knows the efficiency ratio of the primary fuel to the laser power. The effect of target switching time is not incorporated in the calculations, but the predictions of the average kill rate can be used to set a technological goal for this parameter. In order to demonstrate the predictive power of the model, both uniform and point distributions of ICBM's of varied hardness and burn time are considered. The calculations assume that the laser satellites engaging the ICBM's are distributed uniformly on a portion of the sphere associated with the constellation. However, this assumption is not dictated by limitations of the model. One of the main conclusions is that the number of satellites required to destroy all boosters launched simultaneously from a point can be significantly larger than for the same number of boosters launched from an area. Finally, the scaling properties of the laser constellation are discussed with respect to a change in the quantitative nature of the ICBM threat.

I. Introduction

DURING the past several years, the potential for defending the United States against a socially mortal attack using nuclear-armed ballistic missiles has received increasing attention. Since the successful delivery of only a very small number of existing weapons can do much damage, a defense with very low leakage is required. A low-leakage defense is typically viewed as one in which several layers contribute by attacking ICBM's and SLBM's and their warheads during all phases of flight. Attack during the boost phase when the missiles are burning is especially important, not only because the fragile boosters are highly vulnerable and visible, but also because the missiles have not yet had a chance to deploy numerous re-entry vehicles and possibly lightweight decoys. If the boost-phase layer is not effective, then the remaining defensive layers will have to engage a much larger number of objects during the later phases of flight.

Boost-phase attack can, in principle, be accomplished with kinetic energy weapons (space-based rockets and rail guns) or directed energy weapons (high-energy lasers and particle beams). While any of these technologies in a space-based scenario will have to be incorporated into a whole constellation of battlestations for an effective boost-phase layer of defense, we will only discuss the sizing problem of lasers through a simple analytic model. We believe that the qualitative conclusions of the model are correct; however, precise quantitative predictions require computer calculations. Quantitatively speaking, our results are probably good to within a factor of two or so.

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Boost-phase laser systems place the lasers (or the battle mirrors if the energy is relayed from a laser on the ground) in orbit on low altitude (~ 1000 km) satellites that serve as battlestations. Since all satellites except those in geosynchronous orbit move relative to the surface of the Earth, a global constellation is needed if the missiles are to be attacked during their vulnerable boost phase. The actual number of battlestations needed depends upon the capability of the laser weapons themselves, the vulnerability of the missiles to laser radiation, the total time available for engaging each booster (defined to be the target time), and the number and distribution of the ballistic missile threat. In the analysis discussed below, a space-based 25 MW chemical high-frequency laser with a diffraction-limited beam and a 10 m diameter mirror will be used as an example to elucidate the analytic method. Although such a laser has been discussed for a space-based defensive weapon, it should be kept in mind that this is far beyond the state-of-the-art. Such a laser/mirror system can be thought of as the threshold size of a weapon having some military worth.

The principal assumptions in our analytic model are: 1) the local density of satellites for those satellites engaging a launch can be treated as spatially and temporally uniform, 2) the boosters are all engaged at zero altitude, 3) a relationship between satellite altitude and engagement range (this is equal to the horizon range in this paper) is assumed, 4) the time taken by each laser beam to switch from one target to another is negligible, 5) the ICBM's are all launched simultaneously, and 6) it is necessary to kill every missile *before* the end of the boost phase. Assumptions 1–6 result in a rather simple analysis of constellation sizing.

In Sec. II we discuss the geometry of the constellation configuration and some general aspects of the coverage problem. In the discussion in Sec. III, the determination of the absentee ratio falls into two main categories that depend upon

whether the Soviet ICBM threat is concentrated at a single location or whether it is distributed as it is now. The situation in which a point threat is deployed simultaneously with the currently existing distributed threat will not be discussed because of the complexities involved in optimizing the assignment of satellites that can engage the point and distributed threats at the same time. Therefore, the two models to be considered are a point-threat model in Sec. III and a distributed-threat model in Sec. IV. In Sec. V, the determination of the respective kill rates for these models is discussed and, in Sec. VI, the scaling properties of the laser constellation with respect to a change in the quantitative nature of the two types of ICBM threats. The Appendix provides a rationale for assumption 1, as well as some of the quantitative results used in Secs. III and IV.

II. Geometric Coverage of Key Zones

The location and spatial distribution of land-based ICBM's on Soviet territory are important factors in determining the optimal orbital geometry for the constellation of defensive battlestations. The Soviet Union extends about 9000 km from east to west and about 3500 km from north to south. The angular extent of its territory ranges from 170° to 20° in longitude and from 75° to 35° in latitude. This region is rectangular in shape. Soviet ICBM's are distributed within a region defined by the western and southern borders of the rectangle and a diagonal line between the rectangle's northwest and southeast corners.

In our analysis of a distributed threat, we will assume that ICBM's are distributed uniformly over this region. To ensure the largest density of satellites over the Soviet Union, a satellite's orbital plane should be such that its trajectory reaches a latitude of approximately 60° . By choosing an orbital inclination of 60 deg with respect to the Earth's equatorial plane, we will maximize the fraction of the time each satellite spends over the Soviet Union. There are two ways of achieving this, as shown in Fig. 1, where both circular satellite orbits make angles of 60 deg with respect to the Earth's equatorial plane.¹

Even with many satellites per orbit, the two satellite orbits in Fig. 1 are not sufficient for a space-based defensive system because the Earth rotates. Additional orbits are required, so that during the Earth's rotation some satellites are over Soviet territory at any time. Also, there is a need to provide satellite coverage of the oceans for the purpose of intercepting Soviet SLBM launches. It is clear from Fig. 1 that, except for the poles, the part of the Earth's surface covered the least is the equator. This can be improved by placing additional planes at regular longitudinal intervals along the equator. The spacing can be determined by assuming a horizon range R_{\max} , which will be referred to as the "weapon range" for a defensive weapon on a battlestation as shown in Fig. 1, and then requiring continuous coverage of the equator. We further assume that the altitude of the satellite is chosen so that a line from the satellite tangent to the horizon will be of length R_{\max} . This assumption makes possible the use of geometric arguments leading to good estimates of the area of the Earth's surface that can be covered by each satellite.

An alternative to the requirement of good equatorial coverage is to space the orbital planes such that all points along a particular latitude (for example, 70°) are completely covered.² This configuration would be optimum if a sizable concentrated threat were deployed within the Soviet Union somewhere along this latitude. It would also provide better polar coverage and allow satellites to participate more effectively in the midcourse phase of battle. This criterion results in fewer satellites because the number of orbital planes required to cover the circular perimeter at 70° latitude is less than that for the equator. This choice can produce inadequate ocean coverage against SLBM forces of appropriate size and distribution at equatorial latitudes, while decreasing the inclination of the

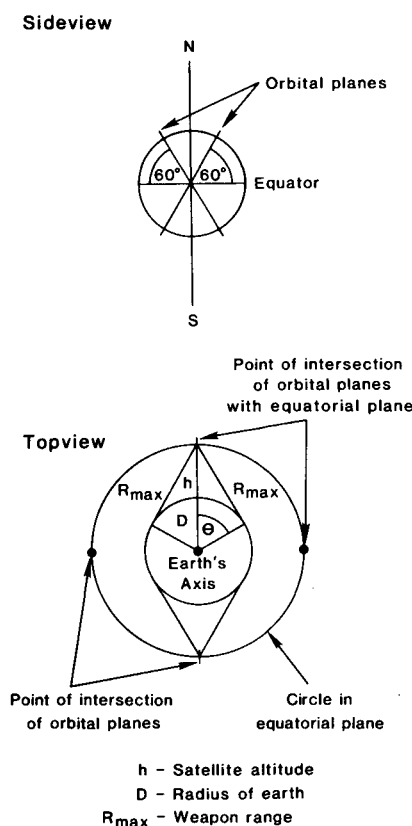


Fig. 1 Circular orbital planes inclined 60 deg with respect to the equatorial plane and spaced at regular longitudinal intervals along the equator for satellite coverage over the Soviet Union as well as the oceans.

orbits can result in lack of coverage in the polar regions. However, it should be noted that the Barents Sea location of Soviet SLBM's can be adequately reached from ± 60 deg inclined orbits. Since we are principally interested in boost-phase interception of land-based ICBM's, we compromise by considering the satellite configuration depicted in Fig. 1.

Dividing the perimeter of the equator by the amount of the equator covered by a single satellite located at the point of intersection of the orbital planes determines the number of orbital planes given by

$$N_{\text{op}} = \pi / \theta \quad (1)$$

where

$$\tan \theta = R_{\max} / D \quad (2)$$

and D is the radius of the Earth, which is approximately 6400 km. According to Fig. 1, the satellite's altitude h is given by

$$h = \sqrt{D^2 + R_{\max}^2} - D \quad (3)$$

Note that the altitude of the satellites in the constellation has been chosen to provide maximum weapon coverage on the ground (horizon to horizon), which is actually optimum only if the target altitude is zero.² If we use the same criterion to determine the number of satellites needed per orbit, then for good equatorial coverage and consequently approximate coverage of the whole Earth, at least between latitudes of 60°N and 60°S , the number of satellites is given by

$$N_{\text{sat}} = \pi^2 / \theta^2 \quad (4)$$

where θ is given by Eq. (2). Note that better coverage of the polar regions is provided as R_{max} increases. Equation (4) determines the constellation size once the weapon range R_{max} is specified. A plot of Eq. (4) is presented in Fig. 2, which also contains other information that will become relevant in later sections. According to Eqs. (2-4), a satellite's altitude is a function of the number of satellites in the constellation. The predictions of Eqs. (2-4) are presented in Table 1. From Table 1, note that the altitude for a very large number of satellites is unrealistically low, since atmospheric drag requires that satellites be above 300 km if their orbits are to be long-lived.

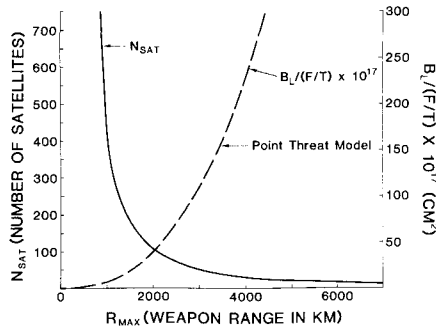


Fig. 2 Plot of the required number of satellites N_{sat} (solid curve) for coverage over the Soviet Union and the oceans vs the weapon range. Also plotted is the required individual laser radiant intensity B_L (dotted curve) for a point threat of 1400 ICBM's divided by the ratio of the target fluence F and the boost time minus the warning time T vs weapon range R_{max} .

Table 1 Predictions of the point-threat model

| N_{sat} (total number of satellites) | N_{pt} (number of satellites over Soviet Union) | η_{pt} (absentee ratio) | R_{max} , km (weapon range) | h , km (satellite altitude) |
|---|--|---|--|-------------------------------------|
| 1623.6 | 3.696 | 439.24 | 500 | 19.50 |
| 831.59 | 3.692 | 225.24 | 700 | 38.17 |
| 410.82 | 3.683 | 111.56 | 1000 | 77.65 |
| 287.29 | 3.675 | 78.18 | 1200 | 111.53 |
| 186.22 | 3.661 | 50.87 | 1500 | 173.43 |
| 146.42 | 3.644 | 36.03 | 1700 | 221.93 |
| 107.58 | 3.632 | 29.62 | 2000 | 305.22 |
| 90.03 | 3.618 | 24.88 | 2200 | 367.57 |
| 71.17 | 3.597 | 19.79 | 2500 | 470.95 |
| 61.93 | 3.582 | 17.29 | 2700 | 546.22 |
| 51.37 | 3.558 | 14.44 | 3000 | 668.24 |
| 45.91 | 3.542 | 12.96 | 3200 | 755.42 |
| 39.41 | 3.517 | 11.21 | 3500 | 894.52 |
| 35.92 | 3.500 | 10.26 | 3700 | 992.56 |
| 31.63 | 3.474 | 9.11 | 4000 | 1147.2 |
| 29.26 | 3.457 | 8.47 | 4200 | 1255.1 |
| 26.28 | 3.431 | 7.66 | 4500 | 1423.7 |
| 24.60 | 3.413 | 7.21 | 4700 | 1540.4 |
| 22.44 | 3.388 | 6.62 | 5000 | 1721.6 |

A way of characterizing the distribution of satellites over the Soviet Union is to consider the absentee ratio. This is defined as the ratio of the total number of satellites in the constellation to the number of satellites that can engage ICBM targets over the Soviet Union at any instant in time. Even if the total number of satellites is well defined, the

number that can participate in an engagement depends on the density of satellites over the Soviet Union, the distribution of ICBM's over Soviet territory, the range at which the satellites engage the boosters (or, if that is not well defined, the altitude of the satellites), and the ICBM burnout altitude.² A complication in specifying a range for directed-energy weapons like a laser (especially continuous-wave or repetitively pulsed systems) is that the dwell time required to destroy a booster varies inversely with the square of the range. Further complications can arise since distant early warning (DEW) systems, with kill rates that are low relative to less distant systems, may contribute to the destruction of significant numbers of boosters. In our analysis, we determine the region of space in which a satellite can engage boosters by assuming that the weapon range R_{max} is determined by the range to the horizon. In actuality, R_{max} is, of course, determined by the radiant intensity of the laser, the vulnerabilities of the boosters, and their altitude. However, once R_{max} is determined by our weapon range criterion, it can be used as a parameter to estimate the required size of the satellite constellation.

A problem with using the average number of satellites as a means of estimating the constellation size is that the orbital motions of satellites result in local fluctuations in the number of available satellites. In addition, the satellite constellation can be assumed to be a mixture of inclined orbits. At latitudes near the average orbital inclination, the lines of orbit become concentrated, resulting in a local satellite density that is high relative to the densities at lower latitudes. At a precise quantitative level, accounting for such latitude-dependent density and fluctuation effects requires a computer, resulting in a need for extremely complex time-dependent models.² Rather than adopting this approach, we will treat the density as spatially uniform and time independent in the vicinity of the Soviet Union. This is consistent with a statistical analysis that results in average coverage and capability requirements for the satellite constellation (see Appendix). We will assume that the orbits within the constellation have been arranged so that the satellite densities over the Soviet Union are 1.5 times the density of the complementary global region battlestations. A justification for choosing the value of 1.5 is presented in the Appendix. If the orbital geometry can be efficiently matched to the latitude of the threat, this enhancement factor can be larger. For example, Garwin³ uses a factor of 3.1. Furthermore, it is assumed that all of the boosters are engaged at zero altitude, so that geometric reasoning will result in a laser engagement range fixed at the horizon. This assumption means that the potential contributions of faraway battlestations shooting at ICBM's as they come over the horizon² in the latter part of boost phase will be ignored. Given these assumptions, the determination of the required laser kill rates for various concentrated threats and distributed threats is straightforward.

The reader should be cautioned that the quantitative connection between this approximate model and the more accurate model involving a nonuniform distribution of satellites everywhere will not be presented. However, the cruder model can be discussed analytically and, in our opinion, provides useful qualitative and quantitative insights into the estimation of kill rates, total system radiant intensities, average dwell times, etc., for the entire satellite constellation.

III. Point-Threat Model

For a point threat, all satellites lying on a cap as depicted in Fig. 3 can participate in the defensive engagement. Mathematically, the absentee ratio is determined by

example, with a 4000 km choice of weapon range, the number of satellites that can engage boosters from a distributed launch area is seven. When the launch distribution area is collapsed to a point, the number of satellites drops to three or four. However, some of these additional satellites in the distributed-threat case are located at the maximum range, so that the nearest targets accessible to them are on the edges of the ICBM deployment region depicted in Fig. 4. This feature will be discussed further in Sec. V.B.

Table 2 Predictions of the distributed-threat model

| N_{sat} (total number of satellites) | N_{pt} (number of satellites over Soviet Union) | η_{pt} (absentee ratio) | R_{max} , km (weapon range) | h , km (satellite altitude) |
|---|--|---|--|-------------------------------------|
| 1623.6 | 68.632 | 23.66 | 500 | 19.50 |
| 831.59 | 41.667 | 19.96 | 700 | 38.17 |
| 410.82 | 25.825 | 15.91 | 1000 | 77.65 |
| 287.29 | 20.676 | 13.90 | 1200 | 111.53 |
| 186.22 | 16.089 | 11.57 | 1500 | 173.43 |
| 146.42 | 14.124 | 10.37 | 1700 | 221.93 |
| 107.58 | 12.058 | 8.92 | 2000 | 305.22 |
| 90.03 | 11.053 | 8.15 | 2200 | 367.57 |
| 71.17 | 9.898 | 7.19 | 2500 | 470.95 |
| 61.93 | 9.293 | 6.66 | 2700 | 546.22 |
| 51.37 | 8.560 | 6.00 | 3000 | 668.24 |
| 45.91 | 8.157 | 5.63 | 3200 | 755.42 |
| 39.41 | 7.649 | 5.15 | 3500 | 894.52 |
| 35.92 | 7.362 | 4.88 | 3700 | 992.56 |
| 31.63 | 6.990 | 4.53 | 4000 | 1147.2 |
| 29.26 | 6.774 | 4.32 | 4200 | 1255.1 |
| 26.28 | 6.489 | 4.05 | 4500 | 1423.7 |
| 24.60 | 6.320 | 3.89 | 4700 | 1540.4 |
| 22.44 | 6.094 | 3.68 | 5000 | 1721.6 |

V. Kill Rates

The rate at which a constellation of laser satellites can engage boosters will depend on many factors. For example, if each booster can be instantly destroyed when illuminated by a laser, the rate at which the boosters can be engaged will be determined by the average time taken to switch the laser beams from target to target. If, instead, a finite illumination time is required, but the beams can still be switched from target to target instantly, then the rate of kill will be determined by the average dwell time. When the time scale of each of these two effects is comparable, they can both be expected to determine the kill rate of laser battlestations. In addition, it is generally assumed that, in order to destroy a booster, a given energy per unit area must be deposited on it. Since the power per unit area from a laser will vary inversely as the square of the range,[‡] the kill rate will vary in the same way.

The technical analysis presented herein will assume the target switching time is small enough to be ignored.³ Using this assumption, we will calculate the average kill rates for a distribution of ranges between the satellites and boosters. Unless a switching time can be achieved that is negligible compared to the inverse of the average kill rate, the kill rates can be expected to be substantially smaller than those we will predict. This criterion will, therefore, allow us to estimate switching time goals for constellations of different assumed size and characteristics.

[‡]We assume boosters are always engaged when their surfaces are perpendicular to the laser beams. Obviously, if this is not the case, more laser battlestations or a higher power per battlestation will be needed to engage the launchers postulated in this paper.

VI. Model Kill Rates

A. Point-Threat Model

The model is depicted in Fig. 3, where R is the weapon range bounded from below by the satellite's altitude and from above by the weapon range R_{max} . The kill rate of a laser at a distance R from the target is given by

$$K = 1/\tau = \beta/R^2 \quad (15)$$

where τ is the dwell time and β is given by

$$\beta = B_L/F \quad (16)$$

with B_L the individual laser radiant intensity given by

$$B_L = \frac{P}{\pi\alpha^2/4} \quad (17)$$

and F (in J/cm²) is the fluence required to kill the booster, P the laser power, and α the beam width (full angle). From the geometry of Fig. 3, it follows that the total kill rate is the sum of the individual kill rates for satellites on the cap, which is given by

$$K_T = \beta \int_0^{2\pi} d\Phi \int_0^\theta \sin\theta' d\theta' \rho r^2 [1/(D^2 + r^2 - 2Dr\cos\theta')] \quad (18)$$

where ρ is the satellite density on the cap discussed in the Appendix and r^2 is given by

$$r^2 = D^2 + R_{\text{max}}^2 \quad (19)$$

For a uniform satellite density ρ , Eq. (18) becomes

$$K_T = N_{\text{pt}} \cdot \langle K \rangle \quad (20)$$

where, according to Table 1, N_{pt} is slightly less than four with the average kill rate per laser $\langle K \rangle$ given by

$$\langle K \rangle = \beta \langle 1/R^2 \rangle \quad (21)$$

and where, from Eq. (18), $\langle 1/R^2 \rangle$ is given by

$$\left\langle \frac{1}{R^2} \right\rangle = \frac{1}{4Dr \sin^2\theta/2} \ell_n \left(\frac{D^2 + r^2 - 2Dr\cos\theta}{D^2 + r^2 - 2Dr} \right) \quad (22)$$

with θ defined by Eq. (8). Of course, we have assumed that during a boost-phase engagement all of the ICBM's are launched simultaneously. The average number of ICBM's killed by a laser on the cap is given by

$$\langle N' \rangle = \langle K \rangle \cdot T \quad (23)$$

where T is the target time, which is the time the ICBM boosters burn minus the warning time. The total number of ICBM's killed is given by

$$N_{\text{ICBM}} = K_T \cdot T \quad (24)$$

From Eqs. (20), (23), and (24), it follows that $\langle N' \rangle$, the average number of ICBM's killed, is given by

$$\langle N' \rangle = N_{\text{ICBM}}/N_{\text{pt}} \quad (25)$$

so that, for $N_{\text{ICBM}} = 1400$ and $N_{\text{pt}} \approx 3.65$ (see Table 1), $\langle N' \rangle \approx 385$. By specifying the total number of ICBM's that must be killed, the fluence, target time, and weapon range (i.e., the number of satellites), Eqs. (23) and (25) determine the re-

quired laser radiant intensity per satellite B_L . (Note that B_L can be made up from several lasers on a single satellite and that it also depends upon the enhancement factor defined by Eq. (A7), which has been chosen to be 1.5.) Since a booster's vulnerability is determined by both the fluence and the total time available for engaging it, it is useful to consider the quantity

$$\frac{B_L}{(F/T)} = \frac{\langle N' \rangle}{\langle 1/R^2 \rangle} \quad (26)$$

where Eq. (26) is plotted in Fig. 2 as a function of R_{\max} along with N_{sat} from Eq. (4). Estimates for the F and T values from a government source¹ are presented in Table 3 for the liquid-fuel SS-18, a solid-fuel booster, and a fast-burn solid-fuel booster. Since $\langle N' \rangle$ is approximately constant for a given size threat, Eq. (26) implies that as R_{\max} increases (i.e., the number of satellites gets smaller), the laser radiant intensity must increase. For example, at short range or low altitude, B_L increases roughly with R_{\max}^2 according to Eq. (26).

The total system radiant intensity is given by

Table 3 Fluences and target times for ICBM's

| ICBM | F , MJ/cm ² | T , s | F/T |
|--------------------|--------------------------|---------|-----------------------|
| SS-18 | 1×10^{-3} | 300 | 0.33×10^{-5} |
| Solid-fuel booster | 10×10^{-3} | 180 | 0.56×10^{-4} |
| Fast-burn booster | 20×10^{-3} | 50 | 0.4×10^{-3} |

$$B_{\text{tot}} = N_{\text{sat}} \cdot B_L \quad (27)$$

where B_L can be obtained from Eq. (26) and N_{sat} from Eq. (4) or, equivalently, from Fig. 2. The total system energy is given by

$$E_{\text{tot}} = N_{\text{sat}} \cdot B_L \cdot (\pi \alpha^2 / 4) \cdot T \quad (28)$$

In order to make predictions with this model, one chooses an F/T value from Table 3 and uses Eq. (26) or, equivalently, Fig. 2 to obtain the relationship between laser radiant intensity and N_{sat} . For 1400 SS-18's and $R_{\max} = 6600$ km (a constellation size of 15 lasers from Fig. 2), Eq. (26) gives

$$B_L \approx 2.97 \times 10^{14} \text{ MW/sr} \quad (29)$$

This individual laser radiant intensity approximately corresponds to a diffraction limited 25 MW high-frequency chemical laser with a wavelength of $2.7 \mu\text{m}$ and a 10 m mirror. The total system radiant intensity from Eq. (27) is

$$B_{\text{tot}} \approx 4.58 \times 10^{15} \text{ MW/sr} \quad (30)$$

and since the assumed beamwidth, $\alpha = 0.33 \mu\text{rad}$, the total system energy from Eq. (28) is

$$E_{\text{tot}} \approx 1.17 \times 10^5 \text{ MJ} \quad (31)$$

However, suppose the threat is more formidable, say 1400 fast-burn boosters; then, if the laser radiant intensity is given by Eq. (29), four of the lasers are needed—which corresponds to $R_{\max} = 1000$ km. The total system radiant intensity is now

$$B_{\text{tot}} \approx 121.5 \times 10^{15} \text{ MW/sr} \quad (32)$$

while the total system energy is

$$E_{\text{tot}} \approx 5.18 \times 10^5 \text{ MJ} \quad (33)$$

The total energies given by Eqs. (31) and (33) can be translated into Shuttle loads, e.g., the transport of the fuel. Using an *optimistic value* of 1 kg of chemicals for every megajoule radiated¹ and the fact that a Shuttle load capacity is 15,000 kg§ the point threat of 1400 SS-18's corresponds to 8 Shuttle loads, while the point threat of 1400 fast-burn boosters corresponds to 35 Shuttle loads. Note that the weight of the fuel depends on the square of the laser beam width. In order to put these requirements into perspective, Fig. 5 plots our predictions for the individual laser power and the number of Shuttle loads required to transport the fuel vs the weapon range (i.e., number of satellites) for the ICBM threats postulated above, always assuming a $0.33 \mu\text{rad}$ laser beamwidth.

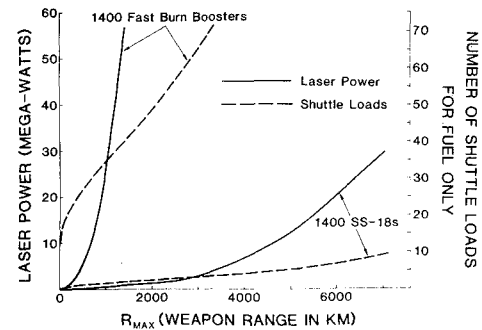


Fig. 5 Plot of the required laser power P (solid curves) for point threats corresponding to 1400 SS-18's and 1400 fast-burn boosters vs weapon range R_{\max} . Also plotted are the required Shuttle loads (dotted curves) for these threats vs weapon range R_{\max} . A laser beam width of $0.33 \mu\text{rad}$ is assumed.

However, in estimating the required Shuttle loads, the reader should bear in mind that the weight of the lasers, optics, platforms, etc., must be added to the weight of the fuel. This weight cannot be estimated without details of the actual technology.⁴ Also note that a crucial assumption in our model is the neglect of the target switching time³ compared to the average dwell time needed to kill a booster. In Fig. 6, we present predictions for the inverse of the average kill rate vs the weapon range of the same ICBM threats considered above. Roughly speaking, in order for the switching time to be negligible, it must be less by a factor of 3–10 compared to the inverse of the average kill rate.

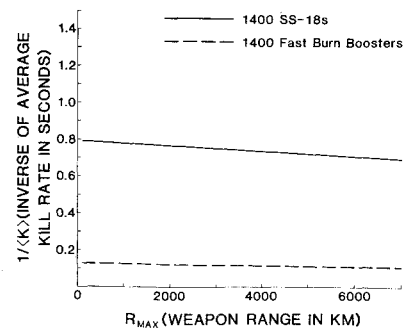


Fig. 6 Plot of the inverse of the average kill rate, $1/\langle K \rangle$, where $\langle K \rangle$ is given by Eq. (21), of a constellation of lasers adequate to defend against point threats corresponding to 1400 SS-18's (solid curve) and 1400 fast-burn boosters (dotted curve) vs weapon range R_{\max} .

§This is the Shuttle payload to polar orbit. When launched in an easterly direction, Earth rotation gives a higher payload of $\sim 25,000$ kg.

B. Distributed Threat Model

The model for a geographically distributed threat is depicted in Fig. 4 where the extent in latitude of the Soviet Union is described by the angle χ (about 30 deg). This model reduces to the point-threat model in the limit $\chi \rightarrow 0$. Obviously, the cap is much larger than the point-threat situation, which means the number of satellites that can engage the Soviet ICBM's is greater. This is summarized in Table 2.

According to Fig. 4, the shortest distance between a satellite and a target directly below it is constant in region B, but varies in regions A and C. Regions A and C, which are the same, contain satellites that can engage targets on the edge of the Soviet Union, i.e., on the rim of the cap subtended by χ . Of course, such satellites can engage targets within the region bounded by the rim at the expense of dwelling on a target for a longer time. However, in the following discussion, we will assume that engagements involving the satellites in regions A and C will be restricted to the rim; therefore, the effectiveness of these satellites will be overestimated.

For region B, the weapon range is now given by

$$R'_{\max} = \sqrt{D^2 + r^2 - 2Dr \cos(\gamma/2)} \quad (34)$$

where r is given by Eq. (19) and the angle γ is indicated in Fig. 4. R'_{\max} will be determined by the requirement that the satellites in region B completely cover the area of the Earth subtended by the angle χ . To this end, the area of the Earth subtended by the angle γ is given by

$$A = 4\pi D^2 \sin^2 \gamma / 4 \quad (35)$$

Hence, R'_{\max} is determined by the condition

$$A_{\text{su}} = N_{\text{dt}}^B \cdot A \quad (36)$$

where N_{dt}^B is the number of satellites on the cap subtended by χ given by

$$N_{\text{dt}}^B = N_{\text{sat}} / \eta_{\text{dt}}(\theta = 0) \quad (37)$$

with $\eta_{\text{dt}}(\theta = 0)$ given by Eq. (14) and where A_{su} is the area of the Soviet Union subtended by χ .

A laser beam at a distance R' away from a target has a dwell time given by

$$\tau' = R'^2 / \beta \quad (38)$$

To find the total engagement time for a laser, it is necessary to sum the individual dwell times required to kill each target. This is given by

$$\tau_{\text{tot}}^B = \frac{1}{\beta} \int_0^{2\pi} d\Phi' \int_0^{\gamma/2} \sin\theta' d\theta' \sigma_t D^2 R'^2 \quad (39)$$

where σ_t is the target density over the Soviet Union and

$$R'^2 = D^2 + r^2 - 2Dr \cos\theta' \quad (40)$$

Since, roughly, 1400 ICBM's are uniformly distributed over half the Soviet Union, σ_t is given by

$$\sigma_t = 0.9 \times 10^{-4} \text{ ICBM/km}^2 \quad (41)$$

An integration of Eq. (39) gives

$$\tau_{\text{tot}}^B = N_t \cdot \langle R_B^2 \rangle / \beta \quad (42)$$

where N_t is the number of targets within range R'_{\max} of a laser, which is given by

$$N_t = 4\pi\sigma_t D^2 \sin^2(\gamma/4) \quad (43)$$

and the average square range of a laser in region B, $\langle R_B^2 \rangle$, is given by

$$\langle R_B^2 \rangle = \frac{(D^2 + r^2) \sin^2(\gamma/4) - Dr \sin^2(\gamma/2)/2}{\sin^2(\gamma/4)} \quad (44)$$

From Eq. (37), the total kill rate in region B is given by

$$K_t^B = N_{\text{dt}}^B \cdot \langle K^B \rangle \quad (45)$$

where $\langle K^B \rangle$ is the average kill rate per laser given by

$$\langle K^B \rangle = \beta / \langle R_B^2 \rangle \quad (46)$$

The average number of ICBM's killed by a laser in region B is given by

$$\langle N_B' \rangle = \langle K^B \rangle \cdot T \quad (47)$$

For regions A and C, the kill rate is determined by the minimum distance to a target located on the circular rim. If that distance is R , then the kill rate is

$$K = \beta / R^2 \quad (48)$$

where

$$R^2 = D^2 + r^2 - 2Dr \cos\theta' \quad (49)$$

with the angle θ' restricted such that

$$(\chi/2) \leq \theta' \leq (\chi/2) + \theta \quad (50)$$

The total kill rate is obtained by summing over the individual kill rates for each satellite on the portion of the cap associated with regions A and C, which is given by

$$K_t^{AC} = \beta \int_0^{2\pi} d\phi \int_{\chi/2}^{\chi/2+\theta} \sin\theta' d\theta' \frac{\rho r^2}{D^2 + r^2 - 2Dr \cos\theta'} \quad (51)$$

where ρ is the satellite density described in the Appendix and angle θ is given by Eq. (8). For a uniform satellite density, Eq. (51) becomes

$$K_t^{AC} = N_{\text{dt}}^{AC} \cdot \langle K^{AC} \rangle \quad (52)$$

where N_{dt}^{AC} is the number of satellites on the portion of the cap associated with regions A and C, which is given by

$$N_{\text{dt}}^{AC} = N_{\text{sat}} \cdot [1.5\delta / (1 + 0.5\delta)] \quad (53)$$

where

$$\delta = \frac{\cos(\chi/2) - \cos(\chi/2 + \theta)}{2} \quad (54)$$

and the average kill rate per laser $\langle K^{AC} \rangle$ is given by

$$\langle K^{AC} \rangle = \beta \langle 1/R_{AC}^2 \rangle \quad (55)$$

where

$$\frac{1}{\langle R_{AC}^2 \rangle} = \ell n \left(\frac{D^2 + r^2 - 2Dr \cos(\chi/2 + \theta)}{D^2 + r^2 - 2Dr \cos(\chi/2)} \right) / (4Dr\delta) \quad (56)$$

The average number of ICBM's killed by a laser on this portion of the cap is given by

$$N'_{AC} = \langle K^{AC} \rangle \cdot T \quad (57)$$

The total number of ICBM's killed by satellites on the entire cap is given by

$$N_{ICBM} = K_t \cdot T \quad (58)$$

where

$$K_t = K_t^B + K_t^{AC} \quad (59)$$

with K_t^B given by Eq. (45) and K_t^{AC} by Eq. (52). The average number of ICBM's killed by a laser on the cap is defined by

$$\langle N' \rangle = \frac{N_{ICBM}}{N_{dt}^B + N_{dt}^{AC}} \quad (60)$$

where N_{dt}^B is given by Eq. (37) and N_{dt}^{AC} by Eq. (53). $\langle N' \rangle$ is plotted in Fig. 7 along with the corresponding curve for the point-threat situation.

An entirely analogous equation to that of Eq. (26) for the distributed threat of ICBM's is given by

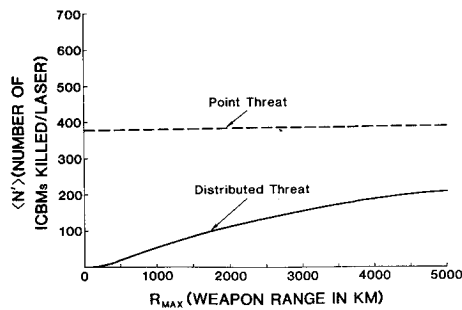


Fig. 7 Plot of the average number of ICBM's killed per laser $\langle N' \rangle$ for a point threat (dotted curve) of 1400 ICBM's and a distributed threat (solid curve) of 1400 ICBM's vs weapon range R_{max} .

$$\frac{B_L}{(F/T)} = \langle N' \rangle \cdot \frac{N_{dt}^B + N_{dt}^{AC}}{N_{dt}^B / \langle R_B^2 \rangle + N_{dt}^{AC} / (1/R_{AC}^2)} \quad (61)$$

This, of course, reduces to the point-threat situation in the limit when $\chi \rightarrow 0$. Equation (61) is plotted in Fig. 8 as a function of R_{max} along with N_{sat} from Eq. (4). The total system radiant intensity is simply given by Eq. (27) with B_L given by Eq. (61). The total system energy is again given by Eq. (28).

Again, in order to make accurate predictions with this model, one uses Table 3 for the F/T value and then uses Eq. (61) or, equivalently, Fig. 8 to find the relationship between laser radiant intensity B_L and N_{sat} . For 1400 SS-18's and $R_{max} = 7000$ km, i.e., 14 satellites, Eq. (61) gives

$$B_L \approx 2.96 \times 10^{14} \text{ MW/sr} \quad (62)$$

which, again, approximately corresponds to a diffraction-limited 25 MW high-frequency chemical laser with a wavelength of $2.7 \mu\text{m}$ and a 10 m mirror. The total system radiant intensity from Eq. (32) is

$$B_{tot} = 4.24 \times 10^{15} \text{ MW/sr} \quad (63)$$

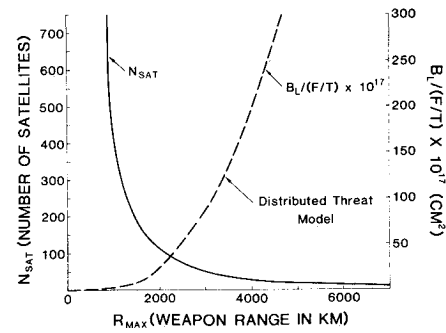


Fig. 8 Plot of the required number of satellites N_{sat} (solid curve) for coverage over the Soviet Union and the oceans vs weapon range R_{max} , in kilometers. Also plotted is the required individual laser radiant intensity B_L (dotted curve) for a distributed threat of 1400 ICBM's divided by the ratio of the target fluence F and the boost time minus the warning time T vs weapon range R_{max} .

and since $\alpha = 0.33 \mu\text{rad}$, the total system energy from Eq. (28) is

$$E_{tot} = 108,380 \text{ MJ} \quad (64)$$

However, suppose the threat is more formidable—say, 1400 fast-burn boosters—then the laser radiant intensity given by Eq. (62) approximately corresponds to a R_{max} of 1440 km, i.e., 197 satellites. The total system radiant intensity is now

$$B_{tot} \approx 6 \times 10^{16} \text{ MW/sr} \quad (65)$$

while the total system energy is

$$E_{tot} \approx 255,554 \text{ MJ} \quad (66)$$

As previously discussed, the total energies given by Eqs. (64) and (66) can be translated into Shuttle loads just to transport the fuel. The distributed threat of 1400 SS-18's corresponds to approximately 7 Shuttle loads, while 1400 fast-burn boosters corresponds to approximately 14 Shuttle loads. Again, the reader should bear in mind that the weight of the lasers, platforms, etc., must be added to this.

In order to put these requirements into perspective, Fig. 9 shows the predictions for the individual laser power and the number of Shuttle loads required to transport the fuel vs the weapon range (i.e., number of satellites) for the ICBM threats postulated above, always assuming a laser beamwidth of $0.33 \mu\text{rad}$.

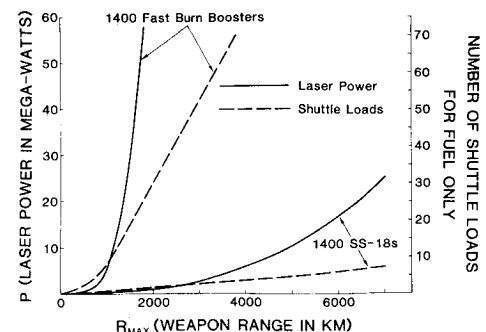


Fig. 9 Plot of the required laser power P (solid curves) for distributed threats corresponding to 1400 SS-18's and 1400 fast-burn boosters vs weapon range R_{max} , in kilometers. Also plotted are the required Shuttle loads (dotted curves) for these threats vs weapon range R_{max} . A laser beam width of $0.33 \mu\text{rad}$ is assumed.

Again, in order for the switching time to be negligible, it must be less by a factor of 3–10 compared to the inverse of the average kill rate, which is plotted as a function of the weapon range in Fig. 10.

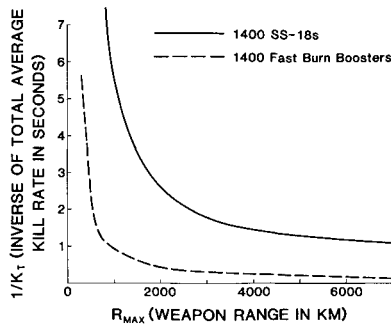


Fig. 10 Plot of the inverse of the average kill rate $1/K_i$, where K_i is given by Eq. (59) of a constellation of lasers for distributed threats corresponding to 1400 SS-18's (solid curve) and 1400 fast-burn boosters (dotted curve) vs weapon range R_{\max} .

Comparing Figs. 2 and 8, it is clear that the required individual laser radiant intensity for a point threat approaches that required for a distributed threat (same number of ICBM's) for satellite altitudes greater than 500 km (in our model this means $R_{\max} \approx 2500$ km or less than 70 satellites). In terms of the distributed-threat model of Fig. 4, this says that for satellite altitudes, $h \approx 500$ km, the contribution to the total ICBM kill rate from satellites on the A and C portions of the cap is much greater than that from portion B. This is due to the number of satellites at high altitudes on the A and C portions, which is an order of magnitude greater than those on the B portion. Therefore, as a "rule of thumb," a threat distributed over half the Soviet Union begins to look like a point threat for altitudes of $h \approx 500$ km or $R_{\max} \approx 2500$ km.

VII. Threat Scaling of N_{sat}

This section discusses how the number of satellites in the constellation must scale if the system is to be able to engage point and distributed ICBM threats of varying size.

Consider a point threat of 1400 solid-fuel missiles and taking $R_{\max} = 2240$ km, i.e., 87 satellites, Eq. (26) gives an individual laser radiant intensity of

$$B_L \approx 2.97 \times 10^{14} \text{ MW/sr} \quad (67)$$

which again corresponds to a diffraction-limited 25 MW high-frequency chemical laser with a wavelength of $2.7 \mu\text{m}$ and a 10 m mirror. However, suppose an additional 1400 solid-fuel missiles are deployed in the form of a point threat and we require that the laser radiant intensity remain fixed. According to Eq. (26), R_{\max} now must be 1700 km and, therefore, the number of additional lasers needed is 59, according to Fig. 2. So, doubling the threat results in an increase in the number of lasers that is less than double the initial value. In this model, this sublinear dependence of the number of required lasers on the threat size is entirely due to the fact that the altitude of the satellites dropped as their number increased, which helped shorten the range to the target (and hence the dwell time). An examination of Eq. (26) shows that if we had held the satellite altitude constant (i.e., R_{\max} constant) we would have had to double the number of lasers per satellite to deal with a doubling of the number of ICBM's. In some cases, keeping the altitude constant may be more realistic, since atmospheric drag will shorten a satellite's lifetime if it drops below about 300 km altitude.

Now consider a distributed threat of 1400 solid-fuel missiles. For $R_{\max} = 2520$ km, Eq. (62) gives approximately the same radiant intensity as Eq. (67). The required constellation size is now 70 such lasers. If the threat is doubled, then R_{\max} must be reduced to 2000 km in order to hold B_L constant. The number of additional lasers needed is approximately 33. In this case, the sublinear behavior is even more pronounced.

Quantitatively this corresponds to an approximate square root scaling law—namely, if the distributed threat is doubled and the weapon range is decreased appropriately, then the required number of lasers with fixed radiant intensity must increase by only a factor of approximately $\sqrt{2}$. This sublinear behavior is due to an increase in the average kill rates of satellites on the cap in Fig. 4 that results from the shorter dwell times. Hence, by lowering the satellite altitude and increasing the number of satellites (remember the number of satellites is a function of the altitude in this model), the dwell time is shortened because the targets within the range of any given satellite are closer to that satellite. Note also that the number of satellites over the Soviet Union increases in this case, but not for the point-threat situation. (See Tables 1 and 2.)

In summary, the number of satellites required can be linear or sublinear, with the linear dependence being associated with a concentrated point threat and a constant satellite altitude. These dependences are important because a sublinear behavior suggests that the marginal cost exchange ratio could become more favorable to the defense as the number of ICBM's increases.

We can also conclude that, for the case of distributed threats, it is best to have one laser per satellite and to spread these satellites out rather than to mount several lasers on a single satellite, as long as the target switching times are short enough to take advantage of the reduced target ranges. From the point of view of minimizing total system radiant intensity, operating at the lowest possible altitude (as was done in this model) is also to be preferred, but, once again, to take advantage of the shorter ranges that result, rapid target switching times will be needed³ (instantaneous target switching is assumed in the model presented). However, considerations of satellite survivability may also dictate higher altitudes.

Appendix

In this Appendix, we will estimate the enhancement of the density of satellites on each of the caps of Figs. 3 and 4 relative to the density of satellites on the complement of the respective orbital spheres. We will argue qualitatively that this ratio depends in a fairly specific way upon the geometry chosen for the constellation configuration as described in Sec. II.

Throughout this paper, we have made the approximation that the satellite densities on each of the respective caps of Figs. 3 and 4 are constant. We have also assumed that the satellite densities are constant on the complement of the orbital spheres, but with different values than their respective caps. A rationale for this assumption can be derived by noting that the choice of ICBM launch time is an unknown, i.e., purely a choice made by the Soviets. At different times, satellites will be located at different points on the respective caps of Figs. 3 and 4 as well as in the complementary regions of the respective orbital spheres. This can result in satellites being closer or further away from the Soviet ICBM launch sites. Our approach to analyzing this problem involves a model for which there are a large number of equally likely launch times. This results in a corresponding average density of satellite configurations on the caps and the complementary regions of the orbital sphere, which are essentially uniform. Therefore, modeling the satellite density as constant will result in a statistical analysis and hence to average coverage and capability requirements for the satellite constellation. This should not be confused with *minimum coverage requirements*,

which would involve a specific launch time and hence a specific pointwise satellite configuration. In a realistic situation, the latter is the more relevant quantity, but our purpose in this paper is to develop an approach that will lead to easily reproducible estimates of satellite constellation capabilities that are in the right "ballpark."

In order to estimate the enhancement ratio described above, it is necessary to first estimate the average number of orbit segments on each of the respective caps in Figs. 3 and 4. Based upon the geometry of the satellite constellation described in Sec. II, it is quite easy to convince oneself that there will be, on average, two orbit segments on the point-threat cap and four such segments on the distributed-threat cap. For example, in the case of the point threat, there will be one segment whose ground track will traverse the Soviet Union along the northeast or southwest direction and another along the northwest or southeast direction. These segments on the caps will, on average, be large (roughly greater than half the maximum possible arc length on the caps), since small incremental encroachments of the orbital segments onto the caps result in larger and larger segments on the caps because of the significant curvature of the caps. The reason for the larger number of orbital segments in the case of the distributed threat has to do with the larger cap size compared to the point-threat situation. Based upon the horizon range requirement, it is also easy to see that there will be on the average three or four satellites on the point-threat cap and seven or eight on the distributed-threat cap.

The satellite density according to this model is given by

$$\rho(\theta') = \rho^< S(\delta - \theta') + \rho^> S(\theta' - \delta) \quad (A1)$$

where the angle δ is given by

$$\delta = \theta \quad (A2)$$

for the point threat and

$$\delta = \theta + 15 \text{ deg} \quad (A3)$$

for the distributed threat and S is the usual step function. The density $\rho^<$ corresponds to the density on the cap, while $\rho^>$ corresponds to the density on the complement of the orbital sphere. The enhancement factor is defined by

$$\epsilon = \frac{\rho^<}{\rho^>} = \frac{N^<}{N^>} \cdot \frac{A^>}{A^<} \quad (A4)$$

where $N^<$ and $N^>$ are the number of satellites on the cap and complement, respectively, and $A^<$ and $A^>$ the corresponding areas. These quantities satisfy the relation

$$N^< + N^> = N_{\text{sat}} \quad (A5)$$

and

$$A^< + A^> = A_T \quad (A6)$$

where N_{sat} and A_T are, respectively, the total number of satellites defined by Eq. (4) and total area of the orbital sphere. Substituting Eqs. (4), (A5), and (A6) into Eq. (A4), we obtain

$$\epsilon = \left[\frac{N^< (\theta/\pi)^2}{1 - N^< (\theta/\pi)^2} \right] \cdot \cot^2 \left(\frac{\delta}{2} \right) \quad (A7)$$

where δ is defined by Eqs. (A2) and (A3). For the point-threat situation, the enhancement factor ϵ is a rather smooth function of the weapon range R_{max} . The average value of the enhancement factor for weapon ranges between 500 and 5000 km is $\langle \epsilon \rangle = 1.73$ when $N^< = 4$ for the point threat. For the case of the distributed threat, the dependence on the R_{max} is a little more sensitive, especially at the shorter ranges that correspond to low satellite altitudes according to our model. However, these lower altitudes should not be taken seriously, because satellite survivability requires that the satellite altitudes be high enough to nullify any effects of the Earth's atmosphere. Even so, the average value for weapon ranges between 500 and 5000 km is $\langle \epsilon \rangle = 1.29$ when $N^< = 8$. Therefore, as a crude estimate, it is reasonable to take $\langle \epsilon \rangle = 1.5$ for both the distributed and point threats. For this value of the enhancement factor, both Tables 1 and 2 result in consistent values for the number of satellites over the Soviet Union compared to those discussed above. Note that, for the distributed threat, there is some inconsistency in the values for the number of satellites over the Soviet Union for weapon ranges roughly below 2500 km; however, the corresponding satellite altitudes are much too low in this range and therefore should be ignored.

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