

# Debris-Cloud Collision Risk Analysis: Polar-Platform Case Study

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The collision risk to the polar platform ENVISAT-1 following an on-orbit fragmentation event is examined. For the breakup examples considered, it is found that passages of the target (polar platform) through the debris cloud can produce instantaneous collision probabilities as large as five orders of magnitude above the level of the background environment. Probabilistic continuum dynamics is employed to calculate values of the debris density at positions of the target along its orbit after the breakup event. This method does not explicitly assume a cloud shape (e.g., ellipsoid or toroid) or require an overly simplistic breakup model (i.e., isotropic). Also included in the method's spatial-density calculation algorithm is an implicit and accurate technique for determining if/when any encounters between the target object and the debris cloud actually occur.

## Nomenclature

$A$	= collision cross-sectional area, km <sup>2</sup>
$a$	= orbit semimajor axis, km
$e$	= orbit eccentricity
$i$	= orbit inclination, deg
$N$	= number of target orbits after breakup
$P$	= collision probability
$t$	= time, s
$V$	= relative debris velocity, km/s
$\theta$	= true anomaly, deg
$\rho$	= debris density, fragments/km <sup>3</sup>
$\Omega$	= right ascension of the ascending node, deg
$\omega$	= argument of the perigee, deg

## Subscript

0 = value at  $t = 0$

## Introduction

THE collision hazard to an orbiting "target" object resulting from an on-orbit fragmentation event is examined. The study sets out to quantify the risk of a collision with debris experienced by a chosen target object (or objects) as a direct result of the fragmentation event (Fig. 1). This type of study is in contrast to those that strive to model the debris environment as a whole and the subsequent danger that the background population poses to orbiting spacecraft. A shorter-term analysis, one that considers a specific debris cloud and is concerned with time spans of hours to days, as opposed to years, can also be regarded as complementary to such long-term examinations. Interaction with the debris cloud produced by the fragmentation event may result in considerable spikes in the target object's curve of overall (i.e., background plus cloud) instantaneous collision probability versus time. If such order-of-magnitude increases above the background level were predicted to occur often enough, and be of sufficient severity, they could influence mission planning and the shielding strategies employed by a future satellite.

The problem of determining the collision probability for an object passing through a debris cloud has been addressed by a number of authors. The equation generally used in the literature for evaluating  $P$  is

$$P = \rho AVt \quad (1)$$

The collision probability for the target object is monotonically non-decreasing and is obtained by integrating over the time period being considered. This, then, is a cumulative value. Values of the instantaneous collision probability (or more precisely, collision probability per calculation time step) can be obtained from

$$\Delta P = \rho AV \Delta t \quad (2)$$

The instantaneous collision probability can therefore increase or decrease. In this paper, the term "collision probability" refers to instantaneous collision probability, and the  $\Delta$  is omitted.

Undoubtedly the simplest method of cloud propagation and assessment of the collision hazard to spacecraft resulting is the linearized state-transition-matrix method.<sup>1</sup> Although quick and easy to use and implement, such a technique has only limited use and offers no means of detecting whether or not the target object is actually inside the cloud envelope at any given time. Toroidal cloud models<sup>2–4</sup> have also been employed in an attempt to tackle this far from trivial problem, and they have had some success. These methods are still limited by the necessity to make certain simplifying assumptions in their analyses, however, and serve to illustrate the difficulties encountered when using an approach that assumes a cloud shape and looks to compute entry and exit times for objects passing through it. The simplifying assumptions that make such analyses manageable may also be sources of errors in the results that they produce. The most serious of these oversimplifications is the common assumption of average cloud (or subshell) densities. The use of probabilistic continuum dynamics<sup>5,6</sup> has shown that the spatial density of fragments within a cloud can vary by several orders of magnitude. Hence the adoption of mean values can give rise to order-of-magnitude errors in this area alone, which will consequently swamp any inaccuracies that may be present in other parts of the analysis.

The model described in this paper uses a variation on the probabilistic continuum method of exact debris density calculation and applies it to the cloud-target collision hazard assessment problem. The theory behind the model is outlined in the next section. Implemented as a Fortran 77 program, TARGET, the model is then tested using the polar platform ENVISAT-1 as the subject for a case study.

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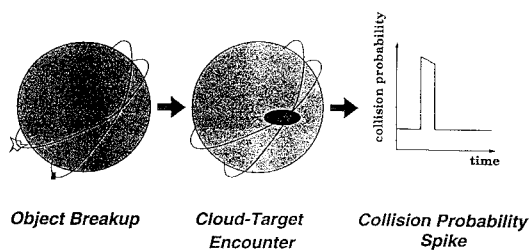


Fig. 1 Problem outline.

## Model Description

### Theory

In probabilistic continuum dynamics, debris-cloud motion is treated as a time-dependent mapping from spread-velocity space at the time of breakup to position space at the time of interest. The spatial density of debris at any given position can then be obtained using the Jacobian that relates spread-velocity space to position space.<sup>7</sup> The determination of the point at which to evaluate the Jacobian involves, for Keplerian motion, solution of the classical Gauss-Lambert problem (the problem of orbit determination given two positions and a transfer time). This can be taken to correspond to the calculation of a fragment's initial velocity vector from its position vector at a given, later time. In practice, there can be more than one such velocity vector that satisfies a given Gauss-Lambert problem. The total debris density is obtained by summing the contributions from all solutions.

This approach of calculating exact cloud spatial densities can be taken to form the basis of a simple but effective method of performing a cloud-target collision hazard assessment. For the target-oriented scenario being considered here, the problem is one of determining the orbit or orbits that describe a transfer between the position vector of the breakup object at the time of the breakup ( $t = 0$ ) and the target position vector at some time  $t$  later. From this, the velocity vectors that produce the above orbital transfer can be calculated. For each solution, the velocity vector of the breakup object's center of mass (c.m.) at  $t = 0$  can then be subtracted vectorially from the transfer orbit velocity vector to yield the velocity increment,  $\Delta V$ , that the transfer corresponds to. Hence, from the fragment number and velocity distributions of the breakup model adopted (which can be completely arbitrary), the spatial density of fragments in the relevant partition of spread velocity space can be determined. Then, by calculating the Jacobian that relates to the combination of transfer orbit position and velocity vector at  $t = 0$  and to the orbital transfer time  $t$ , the spatial density of debris at the target position can be evaluated. If the target object is physically outside the bounds of the debris cloud, then the spatial density will be zero. If it lies inside the cloud, then a nonzero value will be returned. Hence, not only is the value of the debris density exactly determined for subsequent calculation of the collision probability at each point, but an implicit and accurate detection method is established for determining if/when any encounters between the target and the debris cloud actually occur.

The above technique can be applied to any scenario and is only limited by its use of ideal orbital theory. Hence its range of validity is dependent solely upon the length of time over which the effects of orbital perturbations can be justifiably neglected. This is, of course, very much scenario dependent, but typically is of the order of a few hours to several days.

### Implementation

The model outlined in the previous section has been encoded as a Fortran 77 program and constitutes the third main module in the S.D.S. (space debris simulation) software suite. Along with programs BREAKUP and EVOLUTION<sup>8</sup> and a number of service graphics routines, program TARGET is part of a fully integrated software package that has been written to operate in X-Windows on the Sun Sparc workstation cluster in the Department of Aeronautics and Astronautics at the University of Southampton.

The model requires the breakup event being considered to be described in terms of the orbit of the breakup-object c.m. before

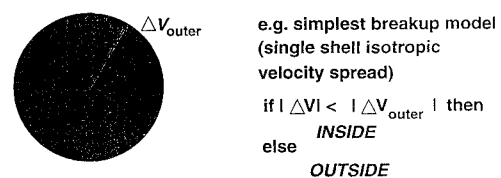


Fig. 2 Encounter detection mechanism.

fragmentation and also the number of fragments generated and their spread velocities in bin form. The cloud data required can be taken directly from the output of program BREAKUP, which can be used to simulate collision-induced and explosive-fragmentation events. A modified version of the Aerospace Corporation's double-cloud kinematic model<sup>2</sup> is installed in the program as one of the options for modeling collision-induced events. If this option is selected, then the centroid velocities of both clouds generated are also passed on to TARGET. Any form of breakup model can, in theory, be used. The current version of BREAKUP uses a relatively simple representation with event-describing fragment distributions (number, velocity, size, mass, ballistic coefficient) being generated by a combination of empirical<sup>9,10</sup> and analytic equations. These distributions are then partitioned (i.e., binned) into constant-density subclouds, or "shells."<sup>11</sup> Each of these shells contains a different number and mass range of fragments and is scaled in BREAKUP by its bounding spread velocities (maximum and minimum). The largest, slowest-moving fragments will be located in the innermost shell, and the smallest particles, which can possess extremely high spread velocities, in the outermost shell. The fragments are then assumed to spread isotropically from the breakup origin. Single- or multiple-shell clouds can be considered by TARGET.

As well as a description of the breakup event, the model also requires knowledge of the target orbit, including the target's position at the time of the fragmentation. The target orbit is then divided up into a number of sections, and collision probabilities are calculated for each of the section path lengths. One hundred data points (i.e., orbit sections) are used by the current version of TARGET. This number was selected as providing the optimal compromise between output resolution and speed of program operation. The Gauss-Lambert problem is solved for the midpoint of each orbit section, using the method of universal variables.<sup>12</sup> Problematic situations can arise if the target position vector is either coincident with the breakup event position vector or diametrically opposed. Such situations can generally be avoided, however, through judicious choice of calculation points. These fundamental difficulties are inherent in any attempted solution of the Gauss-Lambert problem and are not specific limitations of the model or program itself.

Once the transfer orbit has been successfully solved, and all the values of  $\Delta V$  relative to the breakup c.m. obtained, calculation of the debris density at the target location is relatively straightforward. For the isotropic (spherical) velocity spreads used, only the magnitude of each  $\Delta V$  is required to determine if the target lies inside the cloud and, if it does, inside which shell (Fig. 2). The spatial density of fragments at the target position is then calculated by evaluating the Jacobian<sup>13</sup> that relates spread-velocity space to position space. Finally, the value of instantaneous collision probability is determined using Eq. (2).

### Validation of Results

The Aerospace Corporation's program DEBRIS is a similar program to TARGET. The program DEBRIS3.1 (Ref. 14) adopts a more general version of the exact debris density algorithm outlined above, using numerical rather than analytical techniques to solve the two-point boundary-value problem (the Gauss-Lambert problem in two-body dynamics) and evaluate the Jacobian. This approach enables perturbation effects to be included in the algorithm. DEBRIS3.1 includes the effects of secular  $J_2$  perturbations in its analysis.

The existence of a similar program to TARGET provides an excellent opportunity for validation of results and comparison of the analytic and numerical approaches adopted. This comparison of software (including fragmentation models) forms the basis of a subsequent paper.<sup>15</sup>

## Polar-Platform Case Study

### Overview

Application of the method can be illustrated by an example. Possible candidates for the target object(s) to be considered include geosynchronous satellites, risk objects such as nuclear power sources, and even satellite constellations. The example considered here, though, is the polar platform (PPF) ENVISAT-1, which is currently due for launch in 1998. ENVISAT-1 has an on-orbit mass of around 8 tons and, with its solar array and ASAR (advanced synthetic aperture radar) antenna deployed, has a maximum cross-sectional area of around 40 m<sup>2</sup>. The proposed orbit (approximately 800 km circular, 98.7-deg inclination) is, in fact, already a particularly hazardous one, corresponding to a peak in the background debris flux.<sup>16</sup> Fragmentation events during the platform's orbital lifetime would further add to this already considerable threat. To examine the nature of this potential additional risk, a comprehensive parametric sensitivity analysis is performed to investigate the effects of various breakup events and also different modeling strategies on the results produced. The parametric investigation can be used, for example, to identify which orbits present the greatest collision hazard to the PPF if a satellite in that orbit were to fragment. The number of objects that currently reside in these critical orbits could then be determined, and the likelihood of a breakup occurring could be assessed.

### Cloud-Target Encounter Detection

The model's encounter detection mechanism can be illustrated, and at the same time validated, by viewing graphically the positions of the cloud and target at the time of a predicted encounter. The operation of the model, and program TARGET, is shown here using a test example. The orbital elements of the breakup object in this scenario are  $a = 8320$  km,  $e = 0.144$ ,  $i = 100.0$  deg,  $\omega = 46.1$  deg,  $\Omega = 350.9$  deg, and  $\theta_0 = -53.9$  deg. The default condition for the analysis carried out in this paper is that PPF values are used for the breakup object unless otherwise stated. For the PPF,  $\omega$ ,  $\Omega$ , and  $\theta_0$  are all set to zero. The debris cloud is represented by a single shell with a bounding spread velocity of 500 m/s and a spatial density (in spread-velocity space) of  $10^6$  fragments/(km/s)<sup>3</sup>. Fragments 1 cm in size and over are considered for this example. This limit is taken to represent the PPF's shielding capability. Figure 3 shows that a brief encounter between the PPF and the cloud occurs during the first orbital revolution of the PPF following the breakup event. The collision probabilities registered during the encounter are over two orders of magnitude above the background level shown. The collision probability due to the background debris population is estimated using a LEO model.<sup>17</sup> The values of  $P$  shown are the sum of the cloud and background contributions. Note that the collision probability falls during the course of the encounter because of the cloud's expansion over that time period.

Figure 4 shows the positions of the PPF (represented by +) and the cloud immediately before, during, and just after the encounter. The

time step (i.e., the number of target orbits described after breakup) that each plot corresponds to is shown in the top right-hand corner of the plot window, and the large circle in each case depicts the Earth. A geocentric inertial coordinate system is used with the  $X$  axis taken to point towards the vernal equinox, the  $Z$  axis north, and the  $Y$  axis to complete a right-handed triad. The positions of the PPF and the debris fragments are evolved from the time of the breakup using Keplerian propagation. The collision-probability spike in Fig. 3 can clearly be seen to correspond to the passage of the PPF through the cloud viewed graphically in Figure 4. Note that in practice, for the target to be deemed "inside" the cloud, all three planar views must show the target within the projected cloud shape. The corresponding  $X$ - $Y$  and  $X$ - $Z$  views can be shown to support Fig. 4b in this case. In contrast, the evidence of just one planar view is sufficient for the object to be deemed "outside" the cloud.

### Object Orbits and Initial Positions

The orbits of the target (PPF) and the breakup object before fragmentation, and their locations at the time of the breakup event, influence when, or indeed if, the PPF passes through the debris cloud. The timing of a cloud-target encounter governs the magnitude and duration of the collision-probability spike produced. An encounter close to one of the cloud's pinch locations (the high concentrations of debris that appear to occur at, and close to, the cloud's half and full multiple revolution points) is more dangerous by orders of magnitude than a passage through a more expanded section of the cloud.<sup>14</sup>

The six Keplerian orbital elements ( $a, e, i, \omega, \Omega, \theta_0$ ) result in twelve parameters to vary when considering both the target and breakup-object orbits. There are too many variables to consider the whole variation matrix, so there must be some reduction in the number of parameters to be examined. Firstly, the effects of varying the target orbit parameters will be essentially the same as those of varying the breakup orbit parameters, since it is in essence the relative motion between the two objects that is of interest. As a target object has been chosen to be the focus of this case study, the target orbit parameters are held constant throughout, and the effects of varying the orbital elements of the breakup object are examined. Secondly, it should also be noted that the six Keplerian orbital elements can be grouped into three pairs. These element pairs are linked in the way that they describe an orbit and also by virtue of the influence they have when varied from their PPF values (Table 1). Varying the two elements within each of these three element pairs enables any cross-element dependences and/or general trends to be observed for any element combination, since, by definition, there is no interdependence between elements that are in different pairs. Hence, by varying only the breakup orbit parameters and adopting the element-pair method outlined above, the problem can be transformed to a manageable size.

### Variation of $a$ and $e$

Variation of the semimajor axis  $a$  and/or the eccentricity  $e$  of the breakup orbit effects the timing and duration of any cloud-target interceptions. As  $a$  and  $e$  are varied from their PPF values, less time is spent inside the cloud. If  $e$  is set to zero (the value used for the PPF) and  $a$  varied, symmetrical behavior on either side of the PPF semimajor-axis value is observed. As  $\Delta a$  increases, the relative velocities associated with potential collisions rise, but this increase in risk is offset by fewer encounters occurring close to the cloud's pinch locations. Varying  $e$  has a similar effect. As  $\Delta e$  increases, the cloud-target encounters become much shorter in duration (thinner spikes on the  $P$ -vs- $N$  curves), but away from the cloud's pinch locations

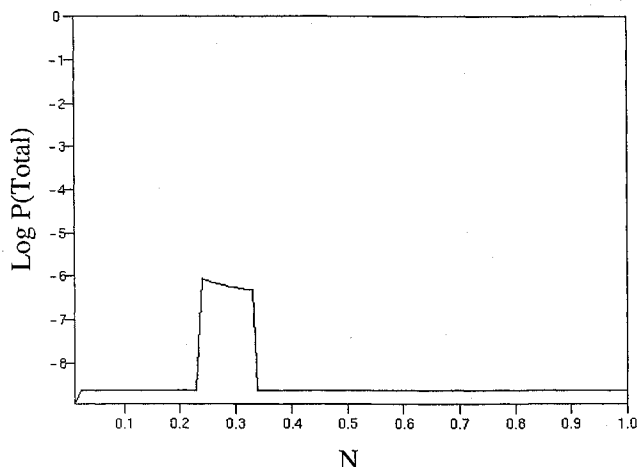


Fig. 3 Collision-probability spike caused by cloud encounter.

Table 1 Orbital-element pairs

Element pair	Role in orbit description	Influence on cloud-target encounters
$a, e$	Determine orbit size and shape	Timing and duration
$i, \Omega$	Determine orbit orientation	Primarily timing
$\omega, \theta$	Determine position within orbit	Timing and duration

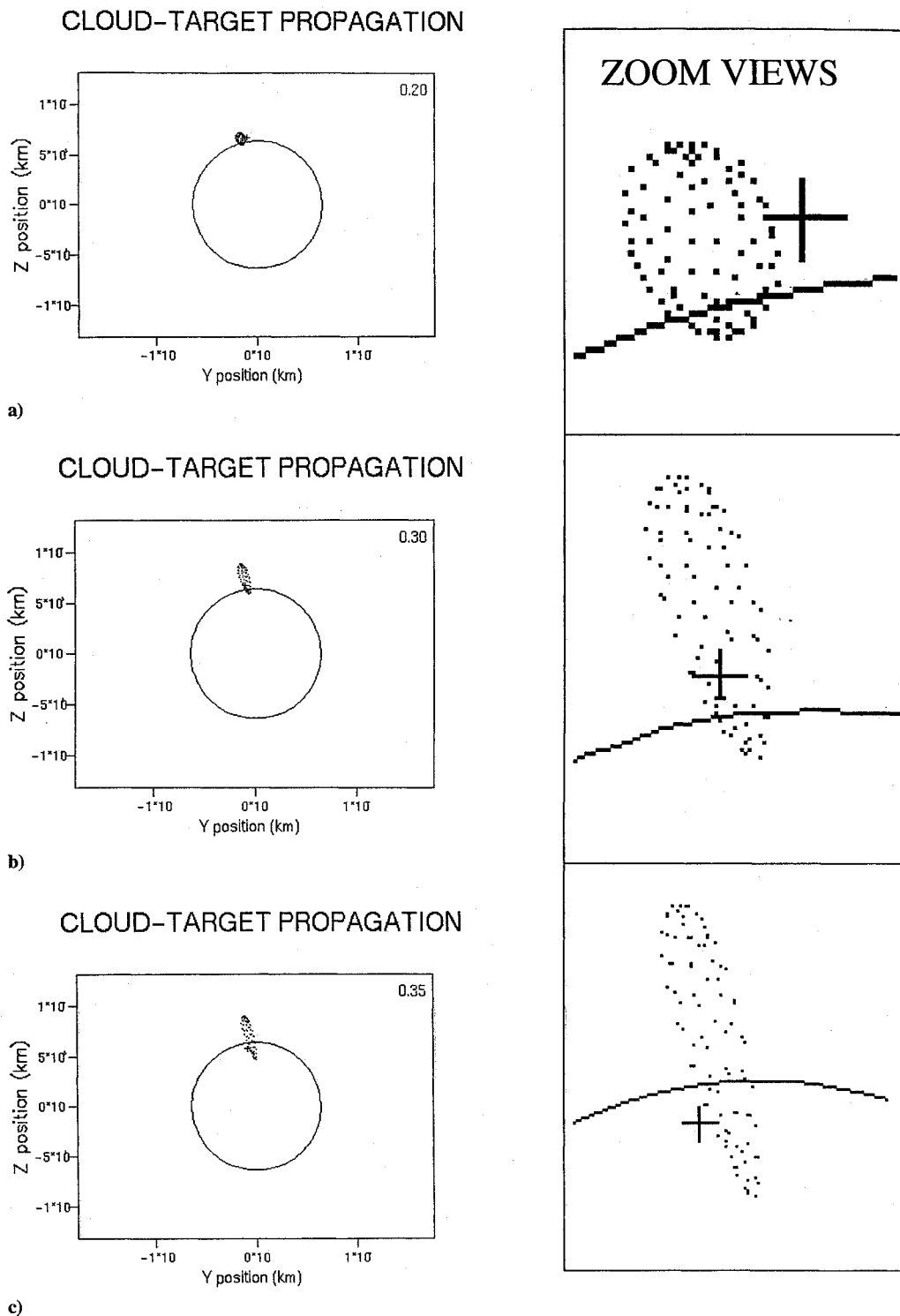
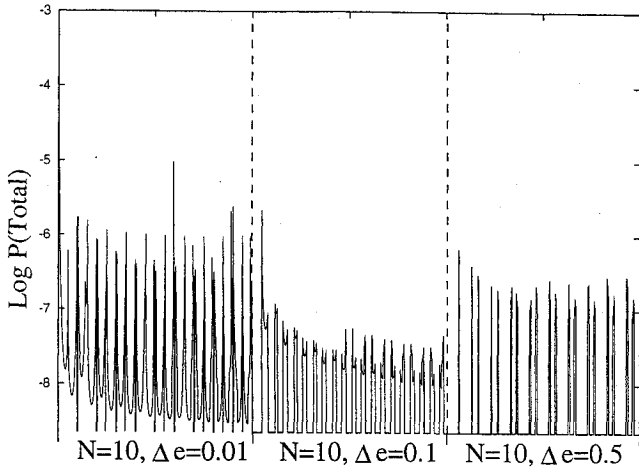
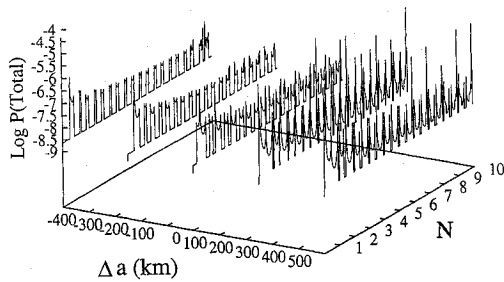
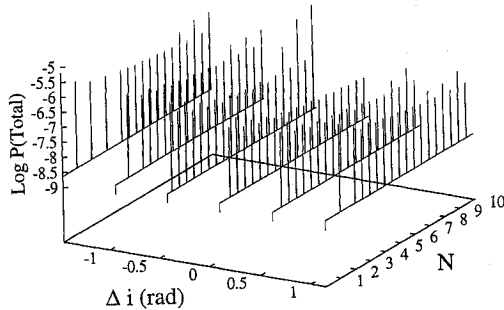
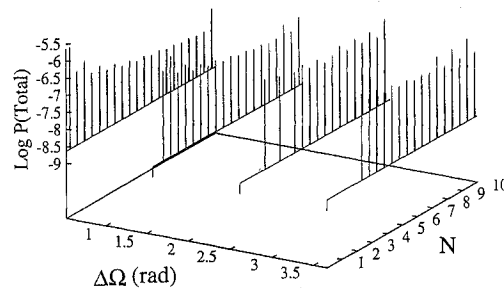


Fig. 4 Cloud and target positions a) immediately before, b) during, and c) just after the encounter.

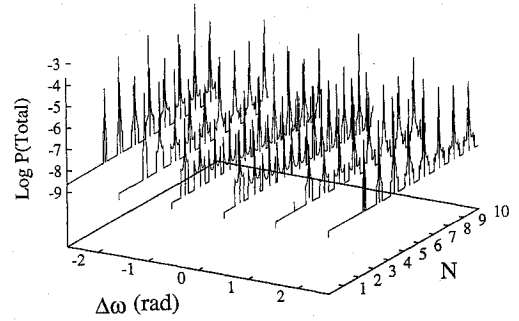
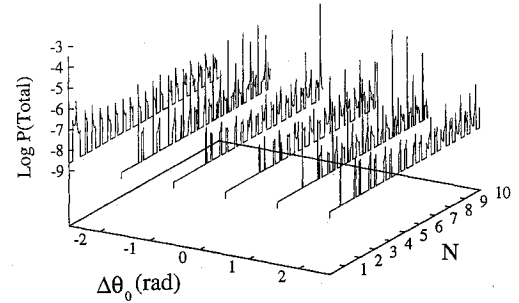
they generally have a higher level of risk associated with them (Fig. 5). Coupling between the two elements occurs when they are varied together, i.e., the effects of varying  $a$  are explored for a number of different values of  $e$  (e.g., Fig. 6). The addition of small eccentricities to the breakup orbit results in the loss of the symmetrical behavior about the PPF value of  $a$ . The most time spent in the cloud then occurs for a semimajor axis greater than that of the PPF. Combinations of  $a$  and  $e$  that result in the breakup orbit having its apogee close to the PPF's circular-orbit altitude can produce cloud-target encounters that are significantly more dangerous than would generally be the case for those values of  $a$  or  $e$  alone (longer durations, higher collision probabilities). A similar but less pronounced effect also occurs for combinations whose perigees are close to the PPF altitude.

#### *Variation of $i$ and $\Omega$*

If the breakup being examined occurs in an orbit plane that is highly inclined with respect to the PPF orbit plane (by virtue of the difference between orbital inclinations  $\Delta i$  and/or ascending-node positions  $\Delta \Omega$ ), then any encounters will be brief and will generally occur, either at or close to the orbits' intersection points. Hence  $\Delta i$  and  $\Delta \Omega$  govern when/if any cloud-target encounters occur, but have only a minor influence in comparison on their duration. Encounters will generally occur either once or twice per target orbit revolution, depending upon the relative orientation of the target and breakup orbit planes and the time elapsed since the fragmentation event (Fig. 7). The higher the values of  $\Delta i$  and/or  $\Delta \Omega$ , the longer the time until the first encounter (Fig. 8). The magnitudes of the

Fig. 5 Target-breakup-object  $\Delta e$  variation.Fig. 6 Target-breakup-object  $\Delta a$  variation,  $\Delta e = 0.05$ .Fig. 7 Target-breakup-object  $\Delta i$  variation,  $\Delta \Omega = \pi/4$  rad.Fig. 8 Target-breakup-object  $\Delta \Omega$  variation.

collision-probability spikes will again be governed by two main factors: the relative debris flux velocities and the closeness of the encounters to the cloud's pinch locations. Here, the relative velocities associated with passages of the target through the cloud will generally be large, potentially as large as the sum of the target's and fragments' orbital velocities for a head-on encounter. Again, though, as with the eccentricity variation described in the previous section, the vast majority of encounters will occur away from the cloud's pinch locations.

Fig. 9 Target-breakup-object  $\Delta \omega$  variation.Fig. 10 Target-breakup-object  $\Delta \theta_0$  variation,  $\omega = 150$  deg,  $\Delta e = 0.1$ .

#### Variation of $\omega$ and $\theta_0$

For a given breakup-target-orbit combination, the position of the fragmentation event within the breakup c.m.'s orbit can significantly effect the level of risk posed to the target. The relative initial positions of the PPF and breakup object will determine how much time the PPF spends inside the cloud and, in particular, the number of encounters that are in close proximity to the cloud's pinch locations. Figure 9 shows that the most time is spent in the cloud for values of  $\omega + \theta_0$  close to the PPF value, but large collision-probability spikes are registered for all starting configurations. The addition of eccentricity complicates the problem. Figure 10 shows that the curve representing equal  $\omega + \theta_0$  for the PPF and the breakup object again results in the quickest first encounter, but other combinations can in fact produce the largest instantaneous levels of risk.

#### Debris Cloud Structure

In addition to the orbits of the PPF and breakup object, and their relative locations at the time of the breakup, the size and internal structure of the debris cloud influence when cloud-target encounters occur and their severity. The cloud's bounding spread velocity determines when the target passes through the cloud, and the inner-shell values of  $\Delta V$  (if a multishell representation is used) dictate what regions (and hence what size of fragments) the PPF will encounter. The magnitudes of the collision probabilities associated with cloud encounters depend upon the cloud or shell spatial densities. For a given event,  $\rho$  can vary by several orders of magnitude, depending upon the timing of encounters with respect to the cloud's pinch locations. This then is the critical parameter in determining the level of collision risk to the target. The event type (i.e., collision-induced or explosive) is also highly significant, as it determines which regions of the cloud are the most dangerous for the PPF to pass through. Again, order-of-magnitude differences in  $P$  are found, and not only for different events, but also for different simulation approaches.

#### Fragment Spread

Although the  $P$ -vs-time curves shown in Figs. 5–10 are very spiky in nature, each individual encounter section is generally smooth because of the simple single-shell breakup model used. The jaggedness observed on some curves for the later orbital revolutions is caused by encounters with fragments that have described either more or less revolutions than the PPF, since the time of the breakup, i.e., multiple solutions to the Gauss-Lambert problem. The risk due to these additional encounters is added to that from the fragments that have described the same number of orbits as the PPF, resulting in a departure

from the smooth curve in a positive sense. As the cloud's bounding  $\Delta V$  and the number of PPF orbits both increase, the number of potential additional encounters rises. Hence, the more energetic the breakup and the longer the time over which the risk to the target is assessed, the more jagged the collision-probability curve is likely to be. In terms of the operation of program TARGET, the possibility of encounters with fragments that have described more or less orbits than the PPF since the time of the breakup means that the transfer orbit has to be solved for a number of different cases at each time step. If the PPF has described  $N$  complete orbits since the event, for example, it may be necessary to consider encounters with fragments that have described  $N - 1$ ,  $N$  or  $N + 1$  orbits, say. The range of orbits that need to be solved for at each point is determined from the maximum and minimum fragment orbital periods. This increased workload for large cloud  $\Delta V$  means that run times are affected. An analysis of ten orbit periods with the current version of TARGET takes between 30 s and 3 min on a Sun Sparc 2 workstation.

#### Event Type

Typically, on-orbit explosions and collision-induced breakups will produce quite different debris clouds. This is primarily due to the difference in fragmentation energies associated with the two types of event. Collision-induced events generate enormous quantities of very small fast-moving fragments and hence produce very large debris clouds. In contrast, explosions produce relatively few small fragments but are far more effective at producing large (i.e., trackable) fragments than collision-induced events of "equivalent" breakup energy.<sup>18</sup> So, although the chance of a collision with any fragment is greater for a collision-induced event, the risk of a collision with a large (and hence potentially damaging) fragment is greater for an explosive breakup. Note that the different internal cloud structures associated with different events are usually best represented by a multishell cloud model.

#### Collision-Probability Calculation

The method employed to calculate the collision probabilities that result from a cloud-target encounter is, of course, itself significant in determining the values that are returned. Program TARGET uses the formula shown in Eq. (2). This is derived from the kinetic theory of gases and Poisson statistics and is widely accepted in the literature. The usage and interpretation of the constituent parameters ( $\rho$ ,  $A$ ,  $V$ ,  $t$ ) are, however, a subject for discussion. The calculation of  $\rho$  is fundamental to the model being presented in this paper, and so the method of its evaluation will not be treated as a variable here. Its value does depend upon the breakup model being used, though, as well as on the input parameters of the breakup model and the cloud representation adopted, i.e., the number of cloud shells and the spread type (isotropic or nonisotropic). Large uncertainties still exist in even the most up-to-date models, and these uncertainties will be carried through to the values of  $\rho$  calculated. Better breakup models are required to match the increases in accuracy being made in debris-cloud evolution and risk analysis methods. Care must be taken, however, when using a more complex breakup model, to insure that the reverse problem is not encountered. The resolution of shell spread velocities chosen to represent the debris cloud must not exceed the level of accuracy to which the Gauss-Lambert problem can be solved, for example.

The extension of the current nondirectional model to incorporate debris flux directionality and the evaluation of collision probabilities for different spacecraft surfaces is an area for future work. Allowing  $A$  to depend upon the orientation of the target with respect to the debris flux would enable a more precise and potentially more realistic analysis to be carried out. The constant value of  $A$  adopted for the PPF in this study represents the maximum spacecraft cross-sectional area the debris can encounter. Hence the collision probability values obtained can be taken to represent worst case scenarios. The actual values experienced by the individual spacecraft surfaces would in practice be somewhat lower, proportionally weighted by their areas projected normal to the direction of the debris flux.

Finally, some studies have used a path-length approach, i.e., they have combined the relative-velocity and time-step terms into a single distance variable. In this particular study, the path length in

question would most obviously be taken to correspond to the distance between calculation points on the target orbit. The distance traveled over each section could be calculated either using linear interpolation between the target positions or more precisely along the actual orbital track. Either way, this method implicitly assumes a relative velocity between the target and the debris that is equal (or approximately equal) to the target's orbital velocity. This is obviously incorrect in general terms and can be erroneous by an order of magnitude or more. When the target and breakup orbit planes have only a small mutual inclination, the relative velocities associated with potential collisions are generally much smaller than the objects' orbital velocities, possibly less than 1 km/s. For cases where the orbital planes are highly inclined, relative velocities greater than orbital velocities are incurred. It is therefore vital to use the actual relative velocity calculated for each encounter examined.

#### Conclusions

The model described in this paper provides a versatile approach to the problem of assessing the additional collision risk to orbiting spacecraft caused by a debris cloud. It avoids making many of the simplifying assumptions characteristic of earlier studies, thus enabling more accurate and realistic analyses to be performed. Written to implement the new model, program TARGET currently operates in a fairly simple mode in terms of the breakup model utilized and its nondirectional collision-probability calculation method. The model does, however, lend itself well to the incorporation of improved cloud representations and a fully directional collision risk analysis. Its main drawback is the use of ideal orbital theory. The solution of the Gauss-Lambert problem that is fundamental to the model may preclude any future inclusion of perturbation effects into the model's current analytical framework. This would essentially limit the model to short-term usage only. The use of generalized numerical methods in place of the analytic two-body algorithms may be the most practical means of considering the effects of secular  $J_2$ , atmospheric drag, etc.

The case study performed on the proposed polar platform ENVISAT-1 showed that for certain event scenarios the collision risks associated with passages of the PPF through a debris cloud could be as much as five orders of magnitude higher than the background level. The timing and duration of cloud-target encounters are governed by a number of different parameters. These include the orbits of the two objects and their relative positions at the time of the breakup, and the size and growth rate of the debris cloud itself. The severity of the encounter (i.e., the increase in collision probability) depends on the spatial density of fragments experienced (which in turn depends on the event type, i.e., collision-induced or explosion) and, of course, on the method employed to actually calculate the collision probability.

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