

## An Engineering Survey of Radiating Shock Layers

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*"In science, by a fiction as remarkable as any to be found in law, what has once been published, even though it be in the Russian language, is spoken of as known, and it is too often forgotten that the rediscovery in the library may be a more difficult and uncertain process than the first discovery in the laboratory."*

*Lord Rayleigh—1884*

### Introduction

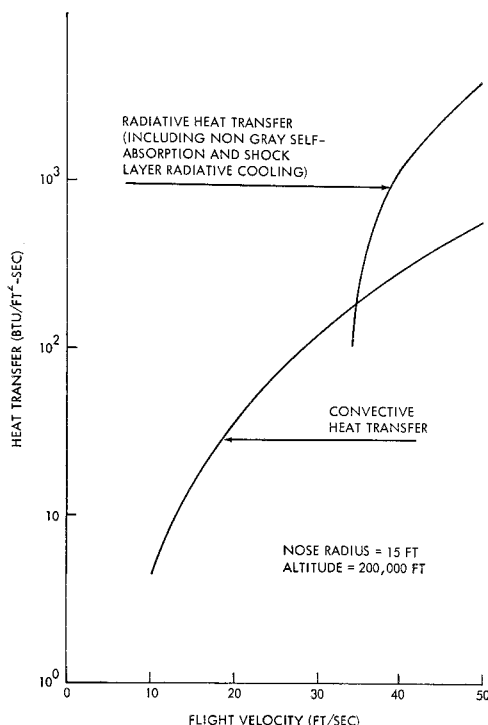
THE intense radiative heat transfer from a high temperature, partially ionized shock layer to the surface of a large, blunt superorbital re-entry vehicle is an important engineering consideration in the design of a thermal protection system; this is in contrast to the relatively cooler and thinner shock layers about IRBM, ICBM, and orbital re-entry vehicles where radiative energy transport within the shock heated gas is negligibly small. In fact, at superorbital re-entry velocities, the radiative heat transfer to the stagnation region can readily exceed the convective heating rates. To illustrate this point clearly, Fig. 1 compares radiative and convective stagnation point heat-transfer rates to a re-entry body with a nose radius of 15 ft at an altitude of 200,000 ft as a function of flight velocity. The convective heat-transfer rates are obtained from the approximate formula of Lees<sup>1</sup>; consequently, they do not include the slight moderation due to coupling between convective heating and shock-layer radiative energy transport, nor do they include the effect of mass addition or ablation from the surface. The continuum radiative heat-transfer rates are obtained from an approximate formula<sup>2</sup> that includes the important effects of radiative cooling and nongray self-absorption within the shock heated gas. Atomic line radiation and absorption by ablation products are not included. These effects of shock-layer radiative cooling, nongray self-absorption, atomic line radiation, absorption by ablation products, and radiation-convection coupling, among others, will be discussed at length in subsequent sections. However, at this point, the main purpose of Fig. 1 is to illustrate clearly the importance of shock-layer radiative heat transfer at high velocities in comparison to the

convective heat transfer. In fact it is precisely this type of comparison that leads to the suggestion that future interplanetary vehicles may have a conical rather than a blunt re-entry configuration in order to reduce the re-entry radiative heat transfer.<sup>3-5</sup>

Spurred by the superorbital re-entry heat-transfer problem, intensive efforts have been made in recent years to obtain detailed and accurate calculations of energy transfer from a high-temperature, conducting and radiating shock layer, aided by shock-tube experimental measurements of the radiative properties of high-temperature gases. These endeavors require the simultaneous application of principles in quantum mechanics, radiative transfer, physical chemistry, and gas dynamics. The purpose of the present paper is to provide a concise and useful state-of-the-art engineering survey of these recent radiative gas dynamic calculations and experiments, as well as to delineate the important future problems in the field. However, in addition to its purpose as a survey, the present paper presents new results for the effects of radiative gas dynamic coupling on surface skin friction, an area in which little information has been reported in the literature.

### Scope of the Survey

The focal point of the present survey is stagnation-point heat transfer inasmuch as it is the maximum heating rate for conventional smooth, blunt re-entry vehicles at zero degrees angle of attack, and it is close to the maximum heating rate for moderate angles of attack. As a result, the present paper will emphasize those radiating flowfield analyses and experiments which have direct bearing on the engineering calculation of stagnation-point heat transfer. Knowledge of the



**Fig. 1 Comparison of radiative and convective stagnation-point heat transfer.**

physical radiative properties of high-temperature air is a necessary prerequisite for such heat-transfer calculations, and extensive quantum mechanical analyses have been made in this regard. Brief references to these analyses will be made throughout the present survey as they pertain to radiating shock-layer computations. However, it is not the present purpose to make a detailed survey of the microscopic physics of gas radiation; indeed, a clear and concise survey and discussion of this discipline is contained in the recent paper by Wilson and Grief.<sup>9</sup>

The existing radiating shock-layer analyses can usually be categorized by means of the radiation transport model employed, i.e., whether the gas is treated as transparent, gray, or nongray. In a transparent gas, fluid elements are assumed to emit locally but not absorb radiative energy. On the other hand, in both a gray and a nongray gas, fluid elements locally emit radiative energy as well as absorb some of the incident radiative intensity that originates from other fluid elements in the shock layer. However, the effect of this self-absorption differs markedly between a gray gas, where by definition the gas absorption coefficient is independent of wavelength, and a nongray gas, where the wavelength variation of the gas absorption coefficient is included, either approximately or in detail. Of the preceding categories, the nongray model is by far the most realistic and important for atmospheric re-entry; in fact, the wavelength variation of absorption coefficient should be taken into account for engineering analyses, as will be shown in subsequent sections. In addition, each of the three categories can be subdivided according to whether or not the gas dynamic flowfield is assumed to be coupled with the radiative energy transport. If no coupling is assumed, the computation of local flowfield variables neglects local emission or absorption of radiation, and the radiative energy flux is subsequently calculated by integration over the resulting (nonradiating) flowfield density and temperature profiles. On the other hand, the more realistic case of radiative gas dynamic coupling (or radiation gas dynamic interaction) takes into account the fact that the flowfield variables depend upon radiative emission and absorption (the flowfield becomes nonadiabatic), and in turn the radiative emission and absorption depend on the flowfield variables. A large part of

the present survey will be organized according to the aforementioned categories and subdivisions, which are illustrated in a block diagram in Fig. 2. This organization also allows a reasonable chronological survey of radiating shock-layer analyses.

Finally, it is not the intention of the present paper to describe completely the mathematical formulation of radiative gas dynamics; authoritative formulations are easily found in the existing literature such as Refs. 10–15. However, at this point it will be very useful to present a few important quantitative expressions that will have bearing on subsequent sections. Consider a radiating shock layer about a blunt body with a shock detachment distance equal to  $\delta$ , as shown in Fig. 3. Making the assumptions of 1) local thermodynamic and chemical equilibrium, 2) one-dimensional radiative energy transport, 3) a cold, nonemitting black surface, and 4) neglecting radiation absorption and emission upstream of the bow shock wave, the radiative heat flux (energy per second per unit area) to the vehicle stagnation point is given by

$$q_R = 2\pi \int_0^\infty \int_0^{\tau_{ps}} B_\nu(t) \mathcal{E}_n(t) dt d\nu \quad (1)$$

where  $B_\nu$  is the spectral blackbody function,  $B_\nu = 2h\nu^3/[c^2(e^{h\nu/kT} - 1)]$ ,  $\mathcal{E}_n(t)$  is the exponential integral of order  $n$ ,

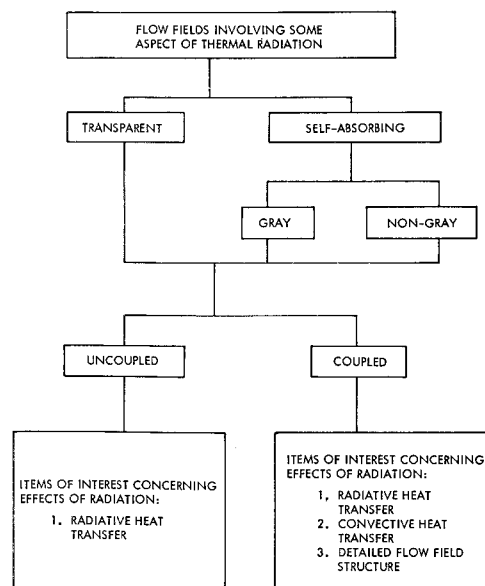
$$\mathcal{E}_n(t) = \int_0^1 \omega^{n-2} e^{-t/\omega} d\omega$$

$\tau_{ps}$  is the spectral optical thickness of the shock layer,

$$\tau_{ps} = \int_0^\delta \kappa_\nu(x) dx$$

$\nu$  is the radiation frequency,  $t$  represents the optical length to a point in the shock layer, and  $\kappa_\nu$  is the spectral volumetric absorption coefficient, which is a property of the gas and depends on local temperature and density.  $q_R$ , as given by Eq. (1), can easily be obtained by numerical integration after the temperature and density profiles are known across the shock layer. In order to find these flowfield profiles, the shock-layer continuity, momentum, and energy equations must be solved. The practical influence of shock-layer radiation enters this system of equations by means of the divergence of the radiative heat flux vector, which appears in the energy equation. For the preceding assumptions, this added term is given by

$$\nabla \cdot \mathbf{q}_R = 4\pi \int_0^\infty \kappa_\nu B_\nu d\nu - 2\pi \int_0^\infty \kappa_\nu \int_0^{\tau_{ps}} B_\nu(t) \mathcal{E}_1 \times (\tau_{ps} - t) dt d\nu \quad (2)$$



**Fig. 2 Diagram of various assumptions for the analysis of radiating shock layers.**

On a physical basis, the first term on the right-hand side of Eq. (2) is the local radiative energy emitted per unit volume and time, and the second term is the radiative energy absorbed locally. Consequently, the general energy equation for a viscous, compressible, reacting, and radiating gas can be written as

$$\rho Dh/Dt = \nabla \cdot (k \nabla T) - \nabla \cdot \sum_i \rho_i \mathbf{U}_i h_i - \nabla \cdot \mathbf{q}_R + \Phi + Dp/Dt \quad (3)$$

where  $h$  is the static enthalpy, and the terms on the right-hand side represent the contributions of thermal conduction, mass diffusion, radiation, viscous dissipation, and flow work, respectively. For practical re-entry body shock layers, the only appearance of radiation in the shock-layer equations is through  $\nabla \cdot \mathbf{q}_R$ ; however, this is sufficient to couple the two disciplines of radiative transfer and gas dynamics.

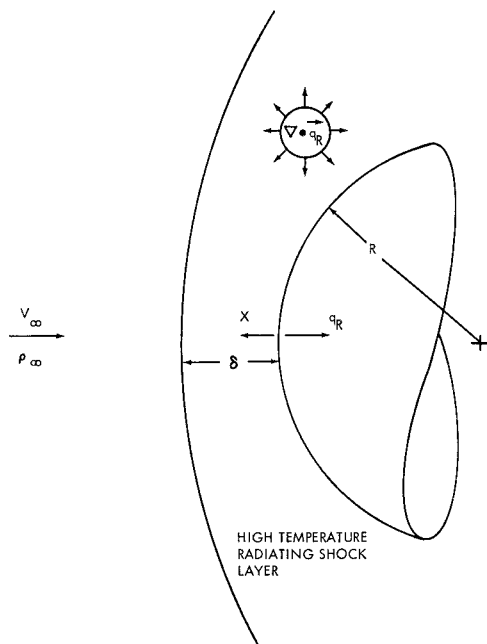
## Transparent Shock Layers

Prior to 1962, the majority of radiating shock-layer analyses made use of a transparent gas model for the radiative transport. This was in part due to the great simplification obtained in the calculation of  $q_R$  and  $\nabla \cdot \mathbf{q}_R$ . Specifically, for a transparent gas, Eq. (2) reduces to the local emission term only;

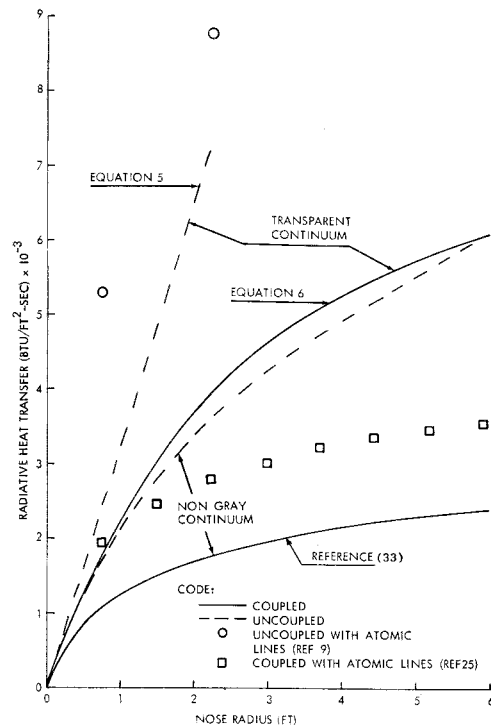
$$\nabla \cdot \mathbf{q}_R = 4\pi \int_0^\infty \kappa_\nu B_\nu d\nu = 4\kappa_p \sigma T^4 \quad (4)$$

where  $\kappa_p$  is the Planck mean absorption coefficient and  $\sigma$  is the Stefan-Boltzmann constant. For a given gas mixture in local thermodynamic and chemical equilibrium,  $\kappa_p$ , and consequently  $\nabla \cdot \mathbf{q}_R$ , is a function of the local gas density and temperature. Thus, Eq. (3) reduces to a differential equation for a transparent gas as opposed to the more general integro-differential equation when self-absorption is included. The transparent gas assumption leads to realistic results when the shock-layer optical thickness

$$\tau_\nu = \int_0^\delta \kappa_\nu(x) dx$$



**Fig. 3 Sketch of a radiating shock layer about a blunt body.**



**Fig. 4 Results for stagnation-point radiative heat transfer reflecting various stages of the state-of-the-art;  $V_\infty = 50,000$  fps, altitude = 200,000 ft.**

is very small ( $\tau_v \lesssim 0.01$ ) for all wavelengths. In practice, such conditions are reasonably obtained for a re-entry vehicle traveling at a velocity low enough that radiation effects just begin to be noticeable (on the order of 30,000 fps). Also, the shock layers about blunt models in shock tubes and ballistic ranges are reasonably transparent simply because of the geometrically small detachment distance.\*

### Uncoupled Shock Layers

In addition to the simplifying assumption of a transparent gas, many early investigators such as Kivel,<sup>16</sup> Yoshikawa,<sup>17</sup> and Wick,<sup>17,18</sup> among others, obtained radiative heat-transfer results that neglected the influence of radiative energy transport on the shock-layer variables, i.e., neglected the radiative gas dynamic coupling. In the stagnation region, this is tantamount to assuming a constant property transparent gas, which reduces Eq. (1) to the very simple form

$$q_R = E_s \delta / 2 \quad (5)$$

In Eq. (5),  $E_* = 4\kappa_p \sigma T_s^4$ , which physically represents the radiative energy emitted by the gas per second per unit volume evaluated at the equilibrium conditions behind a normal shock wave. For re-entry conditions associated with Earth parabolic velocities or higher, Eq. (5) considerably overestimates the radiative heat transfer, as will be discussed at length in subsequent sections. This fact is markedly illustrated in Fig. 4, which shows  $q_R$  obtained from Eq. (5) as a function of nose radius for the trajectory point:  $V_\infty = 50,000$  fps, altitude = 200,000 ft. Additional results which are more realistic are also shown in Fig. 4 and will be discussed later. In fact, Fig. 4 is a pivotal illustration in the present paper, and will be used to compare results that reflect various advances in the state-of-the-art. All curves in Fig. 4 (except the points labeled as including atomic line radiation) are calculated for continuum radiation; of these, all the curves

\* In a recent private communication, M. Silbulkin of Brown Univ. suggests that, for any continuum flow, atomic line centers may be optically thick.

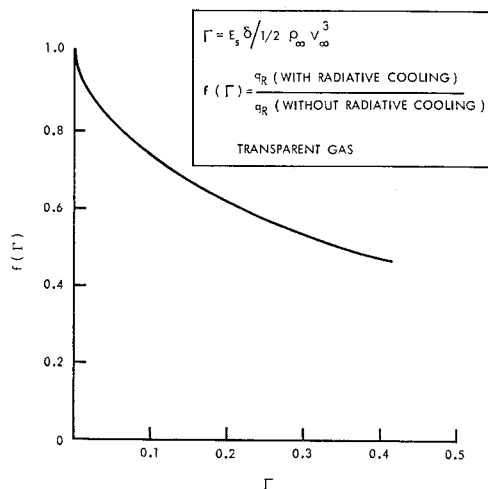


Fig. 5 Reduction in stagnation-point radiative heat transfer due to radiative cooling of a transparent shock layer.

except the lowest were obtained using the absorption coefficient data of Hahne<sup>19</sup> below 1100 Å and one-half the radiance values of Nardone et al.<sup>20</sup> above 1100 Å. These radiation properties appear to be reasonable estimates for continuum radiation, as described later.

#### Coupled Shock Layers

The effect of radiative gas dynamic coupling on the transparent shock layer markedly reduces  $q_R$  in comparison to the uncoupled case, as shown by Wick,<sup>18</sup> Wilson and Hoshizaki,<sup>21,22</sup> and Nerem,<sup>23</sup> among others. References 18 and 21 were inviscid analyses, whereas Refs. 22 and 23 treated viscous shock layers. Wick and Nerem considered the stagnation region only, whereas Wilson and Hoshizaki included a more extended blunt body flowfield using an approximate integral and finite difference method. On a physical basis, these investigations showed that the shock-layer gas can be greatly cooled by radiative energy loss, and consequently  $q_R$  can be considerably reduced. For this case,

$$q_R = (E_s \delta / 2) f(\Gamma) \quad (6)$$

where  $\Gamma$  is the radiation loss parameter<sup>21</sup> defined by  $\Gamma = E_s \delta / (1/2 \rho_\infty V_\infty^3)$ , and  $f(\Gamma) \leq 1$ . Physically,  $\Gamma$  represents the one-dimensional radiative energy flux out of a constant property, transparent shock layer divided by the enthalpy convected through the bow shock from the freestream. The numerical variation of  $f(\Gamma)$  has been found by Wilson and Hoshizaki,<sup>21</sup> and is independent of the radiative properties of the gas as indicated in a subsequent numerical solution<sup>24</sup> and as later shown analytically by Page et al.<sup>25</sup> Figure 5 illustrates  $f(\Gamma)$ . Results obtained from Eq. (6) are shown in Fig. 4, and markedly illustrate the effect of shock-layer radiative cooling on radiative heat transfer from a transparent shock layer.

An interesting and important anomaly is associated with a shock layer that is both transparent and inviscid. Because a radiating fluid element traversing the bow shock and moving along the stagnation streamline requires an infinite amount of time to reach the stagnation point, the element radiates away all of its energy, and consequently an artificial zero enthalpy condition exists along the body streamline. This anomaly does not greatly influence the radiative heat transfer, but it does preclude the calculation of convective heat transfer by the usual boundary-layer techniques, which require the inviscid results for conditions at the boundary-layer outer edge. However, this anomalous situation does not occur when the energy mechanism of thermal conduction is included in the shock-layer calculations;<sup>22,23,26</sup> in this case, a finite wall enthalpy is one of the prescribed boundary condi-

tions. In particular, the previously mentioned analyses of Hoshizaki and Wilson,<sup>22</sup> and Nerem<sup>23</sup> include viscous effects from the body to the bow shock; in this way, by treating the shock layer as entirely viscous (as opposed to the more conventional model of matching an inviscid shock layer with a viscous boundary layer near the surface) the coupling between radiative transport and thermal conduction within the shock layer is suitably taken into account. An alternative to these numerical solutions has been offered by Burggraf,<sup>26</sup> who has solved the viscous, transparent, radiating shock layer by matched asymptotic expansions. In addition to thermal conduction, the mechanism of self-absorption within the radiation cooled layer next to the surface also removes the anomaly and results in a finite enthalpy at the wall.<sup>27</sup> Consequently, these results prompted more realistic engineering shock-layer calculations that included both viscous and self-absorption mechanisms, which will now be discussed.

#### Gray Shock Layers

Around 1962, several analyses<sup>27-30</sup> made the next logical step, which was to include gray gas self-absorption in addition to local radiative emission. Among these was the particularly interesting work of Howe and Viegas,<sup>28</sup> which dealt not only with self-absorption, but with a viscous stagnation region shock layer as well. Howe and Viegas showed that the radiating stagnation region shock layer lends itself to a self-similar solution, thus dealing with ordinary rather than partial differential equations. These ordinary differential equations are subsequently solved numerically by a complex iterative process.

However, concerning the radiative energy transport, Hoshizaki<sup>31</sup> has shown that the assumption of gray gas self-absorption using the Planck mean absorption coefficient gives essentially the same results for  $q_R$  as the transparent gas assumption when practical re-entry conditions are considered. The same holds true for  $q_c$  as shown in Ref. 24. This is due to the relatively low shock-layer optical thickness associated with the Planck mean absorption coefficient. Therefore, for the time being, we shall dismiss further discussion of the gray gas case in light of later developments described below.

#### Nongray Shock Layers

By 1965, the striking reduction in radiative heat transfer due to radiative cooling of the shock layer had been well established. However, at about this time, two new investigations<sup>32,33</sup> clearly demonstrated that further substantial reductions in shock-layer radiative heat transfer were obtained by means of continuum, nongray self-absorption within the shock heated gas. In particular, Olstad,<sup>32</sup> using a crude, approximate step model to represent the wavelength variation of absorption coefficient, first indicated that nongray self-absorption could have a strong effect on shock-layer structure and heat transfer. This indication was put on a firm quantitative basis by Hoshizaki and Wilson<sup>33</sup> in their important paper presented in 1966, which is a sequel to their earlier work.<sup>21,22</sup> Hoshizaki and Wilson employed detailed spectral variations of the gas continuum absorption coefficient obtained from exhaustive quantum mechanical calculations,<sup>34</sup> and showed that shock-layer radiative energy transport is strongly attenuated by nongray self-absorption. More precisely, the continuum nongray nature of high-temperature air results in strong vacuum ultraviolet self-absorption, whereas the shock layer is relatively transparent to the long wavelength. This behavior is implied by the spectral variation of the continuum absorption coefficient for high-temperature air reported in Ref. 33 and shown in Fig. 6. The solid curves in Fig. 6 reflect the detailed quantum mechanical calculations of Armstrong et al.<sup>34,35</sup> as well as some approximate results of Biberman<sup>36</sup>; the dotted line labeled step model A will be dis-

cussed later. The large increase in absorption coefficient at short wavelengths is particularly evident. Unfortunately, the widely different self-absorption characteristics of different parts of the spectrum physically cannot be accounted for by means of a gray gas assumption employing either the Planck or Rosseland mean absorption coefficients. Consequently, for engineering shock-layer calculations, the assumption of a gray gas using one of the common mean absorption coefficients became passe; the wavelength variation of the absorption coefficient must be taken into account. In fact, the combined influence of shock-layer radiative cooling and non-gray self-absorption can reduce the radiative heat transfer by as much as an order of magnitude in comparison to the early predictions made on the basis of a constant property, transparent shock layer. This type of comparison is particularly evident in Fig. 4; note the striking reduction shown by the lower curve obtained from Ref. 33, which accounts for radiative cooling and nongray self-absorption, compared to the upper curve obtained from Eq. (5). Also, for the sake of comparison, the dotted curve labeled "nongray continuum" is shown to illustrate the single effect of nongray self-absorption by itself. [This curve is obtained from a closed form integration of Eq. (1), assuming a constant property shock layer and a two-step absorption coefficient model based on the results of Refs. 19 and 20, as described below.] In light of these results, the conclusion that future superorbital re-entry vehicles need to be conical, which was based on early predictions, has been questioned,<sup>33</sup> and it has been suggested that a more conventional blunt body shape may be sufficient for superorbital missions envisioned for the future.

Subsequent to the work of Olstad<sup>32</sup> and Wilson and Hoshizaki,<sup>33</sup> further experimental and analytical investigations of stagnation-point radiative heat transfer<sup>37-41</sup> also delineated strong nongray self-absorption within the shock heated gas. A similar nongray influence has been noted on end-wall radiative heat transfer from the hot radiating gas behind a strong reflected shock wave in air.<sup>42,43</sup> In fact, Refs. 42 and 43 constitute a coordinated theoretical and experimental study of radiating reflected shock waves. Shock-tube end-wall radiative heat transfer behind a strong reflected shock wave has practical significance for at least two reasons: 1) it can be used to measure the radiative properties of a high-temperature air plasma,<sup>43</sup> and 2) there is a loose analogy between shock-tube end-wall and blunt body stagnation-point radiative and convective heat transfer.<sup>42</sup>

Recognizing that the wavelength variation of the absorption coefficient must be taken into account, the question quite naturally arises as to how much spectral detail is required for engineering calculations of shock-layer radiative heat transfer. Referring again to Fig. 6, the continuum absorption coefficient takes almost a step increase in the vacuum ultraviolet region below 1100 Å; this variation suggests that a two-step model absorption coefficient might adequately approximate the spectral distribution. Such a crude step model was first employed in shock-layer calculations by Olstad,<sup>32</sup> and served the purpose of at least qualitatively delineating the nongray effect in comparison to a gray gas. Subsequently, Anderson<sup>44-46</sup> has shown that a simple two-step model (step model A in Fig. 6) constructed from existing quantum mechanical calculations,<sup>19,20</sup> can lead to accurate quantitative estimates for shock-layer nongray continuum radiative heat transfer. (Note that Refs. 44-46 use an improved model for the continuum absorption coefficient in comparison to the earlier models employed in Refs. 24 and 47.) As shown by Eqs. (1) and (2), calculations of nongray radiative energy transport require integration over wavelength (or frequency); thus, step models allow a significant simplification in comparison to the use of detailed spectrally varying absorption coefficients. In terms of numerical solutions on a high-speed computer, this simplification allows a considerable reduction in execution time. In addition, the use of a step model ab-

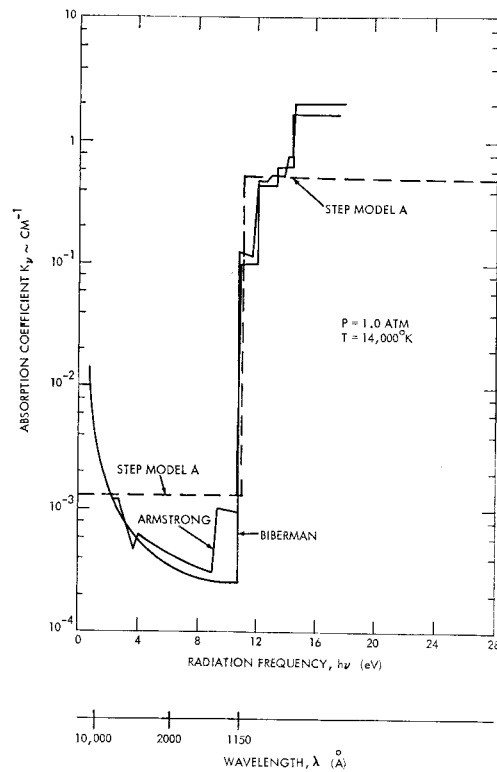


Fig. 6 Comparison of step model with detailed continuum absorption coefficients for air.

sorption coefficient leads to an approximate, closed form method for rapid hand calculation of stagnation-point radiative heat transfer that includes the effects of shock-layer nongray self-absorption and radiative cooling, thus precluding lengthy computer calculations when only approximate results (within about 15%) are required.<sup>2</sup>

Of course, a further simplification of the radiative properties would be the use of a frequency independent absorption coefficient, i.e., a gray gas. As discussed earlier, gray gas analyses using either a Planck or Rosseland mean absorption coefficient are not sufficiently accurate for practical re-entry conditions. On the other hand, Sampson,<sup>48,49</sup> Traugott,<sup>50</sup> Patch,<sup>51,52</sup> and Finkleman and Chien,<sup>53</sup> among others, have proposed other types of mean (frequency independent) absorption coefficients that are aimed at accounting for the non-gray behavior of high-temperature air. However, the accuracy of these mean absorption coefficients for practical re-entry shock-layer conditions has not yet been ascertained.

To summarize at this point, during the period 1965-1967, the important effects of shock-layer nongray self-absorption and radiative cooling on radiative heat-transfer calculations were well established. This amounted to a "quantum jump" in realism compared to the earlier transparent and gray gas results, as graphically shown in Fig. 4. However, an additional quantum jump in realism has recently occurred, namely, the recognition that atomic line radiation can contribute approximately as much as continuum radiation to the power emitted from a high-temperature air plasma. In fact, the importance of atomic line radiation has made the earlier continuum calculations quantitatively obsolete, as will be discussed below.

### Atomic Line Radiation

The aforementioned numerical analyses of transparent, gray, and nongray shock layers employed continuum and molecular band radiative properties for high-temperature air represented by Refs. 19, 20, 34, 35, and 54-57. The primary microscopic mechanisms for continuum radiation are the de-

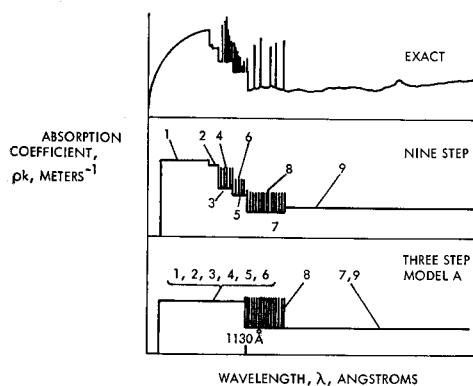


Fig. 7 Comparison of step models with detailed continuum and atomic line absorption coefficients for air, obtained from Ref. 68.

ionization of  $N^+$  and  $O^+$  ions (free-bound radiation) and the acceleration of free electrons in the vicinity of ions and neutral atoms (free-free radiation). In the case of air, molecular band radiation is negligible for the high-temperature, dissociated and partially ionized shock layer about a superorbital re-entry vehicle. Of course,  $O_2$ ,  $N_2$ , and  $NO$  will be present in the cooler boundary layer at the vehicle surface, but their presence has little effect upon stagnation-point continuum radiative heat transfer.<sup>33</sup> (Molecular band radiation from foreign species and ablation products can be important, and this matter will be discussed in the next section.) However, there is yet another radiative mechanism that has not been included in the preceding calculations, namely, atomic line radiation from the hundreds of different bound electronic transitions that take place within the atomic species. The importance of atomic line radiation for shock-layer radiative heat transfer was markedly shown by the calculations of Biberman et al.<sup>58,59</sup> in the USSR; indeed, these calculations showed that atomic line radiation can contribute as much as continuum radiation to the shock-layer radiative energy flux. Consequently, Biberman's results have prompted some recent experimental and analytical work on atomic line radiation from high-temperature air plasmas. This contemporary work has strong ramifications on the state-of-the-art for radiating shock layers. In particular, Nerem et al.,<sup>38,43</sup> Gruszczynski and Warren,<sup>37</sup> and Wood et al.<sup>39</sup> have made experimental shock-tube measurements of the integrated (over wavelength) radiative heat transfer from blunt body shock layers as well as the high-temperature plasma behind a strong reflected shock wave. In all these measurements, the strong contribution of atomic lines could be delineated, amounting in some cases to almost a factor of two increase from the continuum radiative flux. Therefore, Biberman's earlier analytical predictions have been somewhat substantiated by experiment.

Recognizing the important influence of atomic line radiation, several quantum mechanical calculations and tabulations of high-temperature air radiative properties including atomic lines have recently been made available for detailed shock layer analysis.<sup>35,60-64</sup> (Ref. 61 gives results for a nitrogen plasma, only.) Of particular note is the convenient compilation of the spectral absorption coefficients of carbon, nitrogen, and oxygen atoms and ions by Wilson and Nicolet.<sup>62</sup> Also, Ref. 63 is a recent, authoritative, and extensive source for high-temperature air properties in general. In turn, these radiative properties including atomic lines have recently been employed in several engineering shock-layer calculations.<sup>9,25,40,65-69</sup> (Note that Ref. 66 is a modified and improved calculation in comparison to the earlier results presented in Ref. 40.) In particular, Wilson and Grief<sup>9</sup> have made radiative transport calculations from an atomic plasma assuming a plane-parallel layer with either a constant or a linear temperature distribution across the layer. Some of their constant

property results, which represent the uncoupled self-absorbing case including atomic lines, are shown in Fig. 4. Also of particular note is the recent analyses of Coleman et al.,<sup>65</sup> which is an extremely thorough and interesting investigation of heat shielding requirements as influenced by uncertainties in the shock-layer and ablation properties. Among many types of uncertainties, Ref. 65 treats blunt body shock-layer radiative heat transfer including radiative cooling, nongray self-absorption, and detailed atomic line radiation. This analysis indicates the important point that, even though the spectral absorption coefficient can be greatly increased because of atomic lines, the radiative heat transfer itself is increased to a much lesser extent because of the mitigation by radiative cooling and nongray self-absorption. This point is further substantiated by the results of Page et al.,<sup>25</sup> which are shown in Fig. 4 and which illustrate the coupled, self-absorbing case including atomic lines. Similar results have been reported by Rigdon et al.<sup>66</sup> and Wilson.<sup>67</sup>

As an interim summary, Fig. 4 allows a graphic comparison of stagnation-point radiative heat-transfer calculations as influenced by recent advances in the state-of-the-art. In light of Fig. 4, the following observations are of particular importance: 1) in comparison to the early, transparent, uncoupled (constant property) shock-layer calculations, the influences of radiative coupling (radiative cooling) and nongray self-absorption have strong and somewhat comparable effects, and their combined influence can result in an order of magnitude reduction in continuum radiative heat-transfer predictions; 2) for a constant property shock layer, the contribution of atomic line transport approximately doubles the radiative heat transfer in comparison to the nongray continuum case. On the other hand, the influence of radiative cooling substantially mitigates this increase. In fact, it appears that stagnation-point radiative heating including radiative cooling, nongray self-absorption, and atomic lines is only approximately 50% above the purely continuum results (obtained from Ref. 33) shown in Fig. 4. However, additional work on accurate, radiation coupled shock-layer analyses including atomic lines is warranted in order to complement the aforementioned results.

It is interesting to note that, on an engineering basis, the effects previously summarized also have a strong influence on the variation of  $q_R$  with flight velocity. Early predictions<sup>55</sup> based on a transparent, constant property shock layer indicated that  $q_R \propto V_\infty^{10}$ ; however, the combined effects of shock-layer radiative cooling, nongray self-absorption, and atomic lines considerably reduce the dependence of  $q_R$  on  $V_\infty$ , leading to  $q_R \propto V_\infty^3$  indicated by Ref. 25 or  $q_R \propto V_\infty^5$  as indicated by Ref. 65.

As a final note regarding atomic lines, there has been some anticipation that the detailed spectral variation of the gas absorption coefficient may be adequately represented by step models in the same spirit of the engineering simplification shown by Refs. 44-46 for continuum radiation. Indeed, this appears to be the case as demonstrated by Page et al.<sup>25</sup> In this investigation, a two-step model absorption coefficient was assumed for the continuum contribution; superimposed on this model was an additional step in the visible and infrared and an equivalent, strongly self-absorbed line with a Lorentz profile in the ultraviolet in order to account for atomic lines. Using this simple, combined continuum and atomic line absorption coefficient to calculate radiative energy transport from a high-temperature shock layer, Page et al.<sup>25</sup> have shown reasonable agreement with more detailed spectral calculations and experiment. Concurrently, in an interesting stagnation region analysis using a time-dependent finite-difference procedure, Callis<sup>68</sup> has employed a nine-step absorption coefficient model originally developed by Olstad.<sup>70</sup> In this analysis, Callis makes the point that a simpler three-step absorption coefficient simulates the effects of combined continuum and atomic line radiation about as well as a nine-

step model. The step models used by Callis are compared with the detailed spectral absorption coefficients in Fig. 7, which was obtained from Ref. 68. As emphasized in the previous section, such simple step models provide striking and useful engineering simplifications in the calculation of radiative energy transport in comparison to the use of detailed spectral variations. The apparent successful simulation of the detailed spectral absorption coefficient by means of step models may stem in part from the general lack of sensitivity of  $q_R$  to reasonably large uncertainties in the absorption coefficient. In fact, within the space of six months, three investigations<sup>25,44,65</sup> have demonstrated the important point that uncertainties up to a factor of two in the magnitudes of the  $v$ - $u$ - $v$  continuum and atomic line absorption coefficients can be tolerated for engineering predictions of  $q_R$ .

### Ablation Effects

The preceding discussions do not constitute the whole story of engineering radiating shock-layer calculations. Of several matters that remain for discussion, the effect of surface ablation on the radiative heat flux is of most importance. The injection and diffusion of highly absorbing (and emitting) molecules, atoms, and ions from an ablating surface into the radiating shock layer can change the radiative energy transport towards the vehicle. In practical cases, the ablation products are usually concentrated in the relatively cool thermal boundary layer near the surface; consequently, they may absorb more radiative energy than they emit, and the result can be an attenuation of the radiative heat transfer towards the surface. Of course, at the same time, radiative energy absorption within the thermal boundary layer acts to increase the convective heating. On one hand, several quantitative investigations<sup>28,44,71</sup> of this effect, using simplified radiative properties, have indicated less than a 10% reduction in the total (radiative plus convective) heat transfer to the surface due to the presence of a highly absorbing foreign gas in the thermal boundary layer. On the other hand, a very detailed analysis has been made by Hoshizaki and Lasher<sup>41</sup> that realistically includes 20 air and ablation product chemical species combined with detailed continuum spectral absorption coefficients for each species. This analysis shows that carbon atoms and ions diffuse far into the shock layer and act as strong radiation absorbers. For a nylon phenolic sphere with a nose radius of 1 ft flying at 41,000 fps at 180,000 ft, the net reduction in total heat transfer due to ablation was calculated to be more than a factor of two. The recent work of Coleman et al.<sup>65</sup> has extended these calculations to include atomic line radiation. A substantial reduction, amounting to more than a factor of two, was observed in the radiative heat transfer towards the surface, due to absorption by the air-ablation products. This reduction is graphically shown as a function of ablation product mass injection rate in Fig. 8, which was obtained from Coleman et al.<sup>65</sup> The recent work of Chin<sup>69</sup> complements these results.

From these analyses, it may be concluded<sup>72</sup> that realistic quantitative calculations of total heat transfer from a high-temperature, radiating shock layer to the surface of superorbital re-entry vehicles should definitely account for the effect of mass addition and ablation as well as the detailed spectral variation of the absorption coefficients of each important air-ablation chemical species.

### Additional Considerations

#### Freestream Absorption

For a moment, let us re-orient our thinking and consider the shock-layer radiative energy flux that is transported upstream through the strong bow shock wave. For the high shock-layer temperatures associated with superorbital re-entry velocities (on the order of 15,000°K at 50,000 fps), a

considerable portion of the upstream radiative energy flux is in the vacuum ultraviolet wavelength region, and is consequently readily absorbed by the air ahead of the bow shock wave. By means of this upstream (or precursor) absorption, the freestream is forewarned of the presence of the re-entry vehicle, and as a result the streamlines and Mach number ahead of the shock layer may be effectively different from ambient conditions.<sup>3,73</sup> The practical consequence of upstream absorption on heat transfer to the vehicle is that a portion of the shock-layer energy that would ordinarily be lost to the surrounding atmosphere can now be returned to the shock layer and subsequently to the vehicle surface. However, using one-dimensional radiative transfer calculations, investigators at the NASA Ames Research Center<sup>72,74</sup> have estimated that upstream absorption increases the surface heat transfer by only 10% for velocities on the order of 60,000 fps. Similar results have been found by Coleman et al.,<sup>65</sup> and Lasher and Wilson.<sup>93</sup> Three-dimensional effects will tend to lessen the effect of this phenomenon. Consequently, upstream absorption may not have a practical effect on engineering shock-layer heat-transfer calculations for velocities less than 60,000 fps.

#### Nonequilibrium Effects

All of the preceding discussions have dealt with shock layers that are in local thermodynamic and chemical equilibrium. This will occur in practice when the shock-layer densities are high (favored by high velocities and low altitudes) and consequently the gas particle collision frequencies are sufficiently high to maintain equilibrium within the internal energy modes and chemical species. On the other hand, when the gas densities are low (favored by low velocities and high altitudes), the collision frequencies are not high enough to establish local equilibrium; as a result, nonequilibrium conditions prevail within the shock layer. These nonequilibrium conditions result in higher temperatures within portions of the shock layer, which in turn result in local increases in radiative intensity. Thus, nonequilibrium radiation has been of interest and has prompted investigations such as Refs. 75-77. However, the flight conditions that generate noticeable radiative heat-transfer rates are the same as those that generate equilibrium shock-layer conditions, namely, high velocities and low altitudes.

For example, consider Fig. 9, which is a velocity-altitude map showing the region where significant chemical nonequilibrium conditions prevail within the shock layer for a blunt body with a nose radius of 5 ft. This region is defined by a collection of data points obtained from existing nonequilibrium blunt body results<sup>78-81</sup> scaled to a 5-ft radius by means of binary scaling. Figure 9 shows that, at superorbital

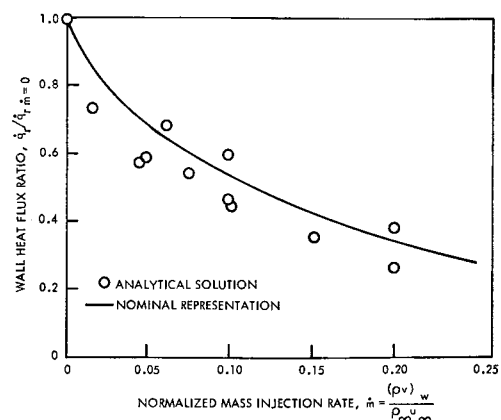
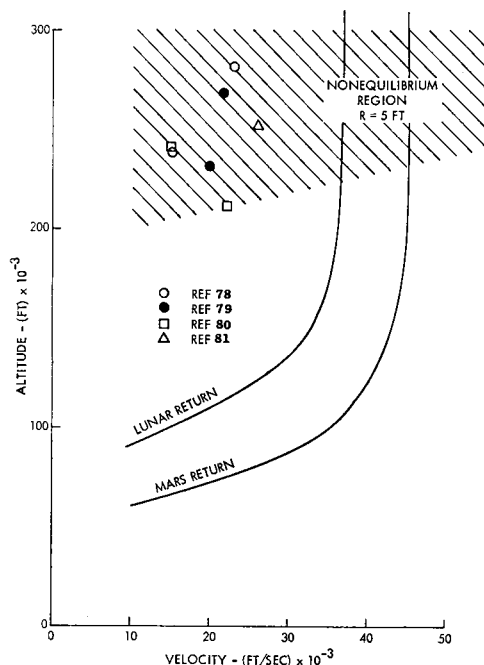


Fig. 8 Reduction in stagnation-point radiative heat transfer due to ablation layer absorption, obtained from Ref. 65.





**Fig. 9 Velocity-altitude map showing area where chemical nonequilibrium effects are important for the stagnation region shock layer;  $R = 5$  ft.**

re-entry velocities, chemical equilibrium conditions will prevail in the stagnation region shock layer about a blunt body for altitudes at least as high as 200,000 ft. In addition, the high radiative heat-transfer rates normally occur at altitudes around or below 200,000 ft.<sup>65</sup> (Several re-entry trajectories, obtained from Ref. 28, are also shown in Fig. 9.) Consequently, nonequilibrium shock-layer effects are not important for engineering predictions of Earth atmospheric re-entry radiative heat transfer.

It should be noted that nonequilibrium effects may be important for entry into other planetary atmospheres, notably the tenuous Martian atmosphere. This situation has prompted investigations of both equilibrium and nonequilibrium radiation from various CO<sub>2</sub> and N<sub>2</sub> mixtures simulating foreign planetary atmospheres.<sup>62</sup>

### Convective Heat Transfer and Skin Friction (Reynolds Analogy)

The processes of local radiative emission and absorption within the shock layer cause the flowfield to be generally nonadiabatic. In turn, this nonadiabatic nature influences surface convective heating and skin friction. For example, shock-layer radiative cooling reduces the enthalpy potential across the thermal boundary layer; consequently, convective heat transfer to the surface is also reduced. The effect of this radiation-convection coupling on stagnation-point heat transfer has been examined by several investigators. At first, several analyses<sup>22,23,28</sup> employing either a transparent or gray gas model indicated that convective heat transfer was strongly reduced by shock-layer radiative cooling, amounting to as much as a factor of two. However, more recent analyses<sup>33,44</sup> which include nongray self-absorption show that, due to local absorption, a considerable amount of energy is trapped within the shock layer that would otherwise escape from a transparent gas. As a result, nongray self-absorption mitigates the radiation-convection coupling, and convective heating is reduced only by 0–20% because of nonadiabatic shock-layer effects. This reduction in convective stagnation-point heat transfer for both a transparent and nongray gas is illustrated in Fig. 10 for several different trajectory points. The results in Fig. 10 were obtained from the viscous, radiating

stagnation region analysis described in Refs. 24 and 44–46. The radiative properties used for graph I are those employed in Ref. 24; graph II, Ref. 43; and graphs III and IV, Refs. 44–46. Also, these graphs do not include the effects of mass injection or ablation. The expression  $q_c/(q_c)_{AD}$  is the ratio of stagnation-point convective heat transfer with and without radiative coupling. Of course, for a given trajectory point, the influence of radiation grows as  $R$  (and consequently the shock-layer thickness) increases.

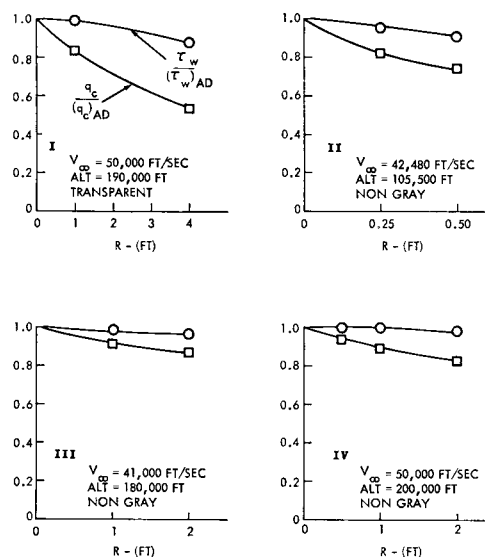
Very little information has been reported concerning the effect of shock-layer radiation on surface skin friction. Such information in the form of the ratio of shear stress with and without radiation,  $\tau_w/(\tau_w)_{AD}$ , is also shown in Fig. 10. (Note that, even though  $\tau_w = 0$  at a stagnation point, the skin-friction coefficient and the ratio  $\tau_w/(\tau_w)_{AD}$  have nonzero values.) From these results, it can be seen that shock-layer radiative cooling has an effect on local shear stress, but for practical flight conditions, it appears to be negligible. Similar results have been reported by Sibulkin and Dispaux<sup>53</sup> for a radiating flat plate boundary layer.

It is of some interest to examine the effect of radiative gas dynamic coupling on Reynolds analogy at a stagnation point. This effect is shown in Fig. 11, which gives the variation of  $(C_H/C_f)/(C_H/C_f)_{AD}$  with an effective nongray radiation loss parameter  $\Gamma_{eff}$ , defined in Ref. 2, for a two-step, continuum, nongray absorption coefficient model. An increase in  $\Gamma_{eff}$  is associated with an increase in shock-layer radiative cooling. Again, the results shown in Fig. 11 are obtained from the analysis of Refs. 24 and 44–46. Data from several different trajectory points and nose radii are shown in Fig. 11, and on an empirical basis, appear to be reasonably correlated by  $\Gamma_{eff}$ .

Evaluating the information shown in Figs. 10 and 11 in light of a practical engineering point of view, it appears that nongray radiative coupling has little influence on both convective heat transfer and shear stress for typical superorbital re-entry conditions, assuming no mass transfer or ablation effects.

### Experimental Measurements of Air Radiative Properties

To this point, the present survey has emphasized various theoretical calculations of the flow properties and heat transfer from a radiating shock layer. In turn, these radiative gas dynamic calculations require values for the gas absorption coefficient, which are usually obtained from quantum mechanical calculations. Until recently, large uncertainties existed in the accuracy of these theoretical absorption coefficients, due mainly to the simplified atomic models that were



**Fig. 10 Radiative gas dynamic coupling effects on stagnation-point convective heat transfer and skin friction.**



assumed and to the neglect of several contributing radiative mechanisms such as atomic lines and negative ion continuum. (This is in partial analogy to the large uncertainties that have existed in calculations of thermal conductivity, viscosity coefficients and diffusion coefficients for high-temperature air due to simplified models assumed for the intermolecular potential.) Experimental measurements of the radiative properties of high-temperature air have played an important role in singling out the theoretical absorption coefficients that seem to be most accurate, and because of their importance to engineering shock-layer calculations, these experiments will be summarized briefly.

Most of these experiments have been carried out in shock tubes because of the high gas temperatures required to simulate superorbital re-entry conditions, and have usually involved the measurement of integrated (over wavelength) radiative emission from the shock layers about small blunt models inserted in the flow behind the incident shock wave or from the hot gas behind a reflected shock wave. Because of the shortwave cutoff associated with sapphire and quartz windows, experiments using these windows in blunt-nosed models<sup>84,85</sup> and on shock-tube end walls<sup>43</sup> have measured only the portion of integrated radiative emission above 1700 Å. However, because of the importance of vacuum ultraviolet continuum and atomic line radiation, Gruszczinski and Warren,<sup>37</sup> and Wood et al.,<sup>39</sup> have recently measured integrated emission over all wavelengths of interest by using various windowless techniques. Also using a windowless model, Nerem<sup>38</sup> has measured the vacuum ultraviolet emission for wavelengths less than 1100 Å.

In light of results obtained from these experiments, the state-of-the-art for radiative properties of high-temperature air appears to be as follows. Theoretical comparisons of the integrated radiative heat flux from a constant property air plasma calculated for temperatures and densities corresponding to superorbital re-entry conditions and using separately the quantum mechanical calculations of Allen,<sup>60</sup> Armstrong et al.,<sup>35</sup> Thomas and Menard,<sup>85</sup> Biberman et al.,<sup>59</sup> and Hunt and Sibulkin (for N<sub>2</sub> only)<sup>61</sup> agree within a factor of two. The recent experiments of Wood et al.,<sup>39</sup> and Gruszczinski and Warren,<sup>37</sup> agree within a factor of four and bracket the theoretical results. Consequently, as stated by Wood et al.,<sup>39</sup> the state-of-the-art agreement between theory and experiment for integrated radiative emission appears to be within a factor of two.

It must be emphasized that the preceding comments apply to integrated radiative emission from essentially constant property shock layers. Consequently, the detailed quantum mechanical calculations of spectral absorption coefficients have been only indirectly confirmed by experiment. However, the factor of two agreement between theory and experiment for radiative emission integrated over all wavelengths is moderately comfortable compared to the status that existed a few years ago; this prompts an intuitive feeling that the theory is also somewhat accurate on a spectral basis. On the other hand, because the spectral absorption coefficients (and not the integrated emission properties from a constant property gas) are required for calculations of non-gray, nonadiabatic shock layers, further experiments should emphasize detailed spectral measurements. Such data would complement the recent spectral measurements by Morris et al.<sup>86</sup> of radiation from N<sub>2</sub> and O<sub>2</sub>, and the spectral data for air measured by Thomas and Menard.<sup>85</sup>

## Concluding Discussion

What important problems with regard to radiating shock layers remain to be solved? From the previous sections of this survey, and particularly from the curves shown in Fig. 4, it can be seen that the state-of-the-art for calculating superorbital re-entry stagnation-point heat transfer has taken

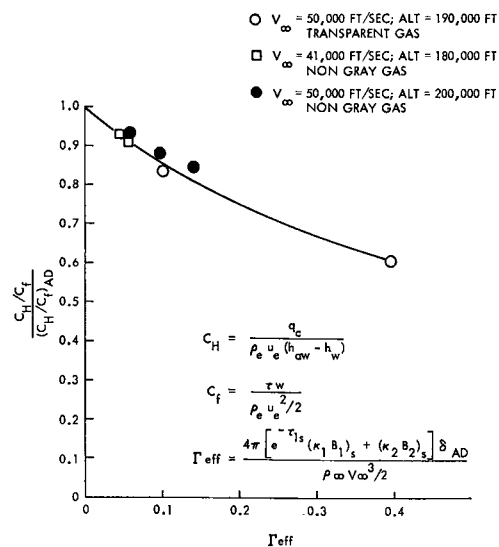


Fig. 11 Radiative gas dynamic coupling effects on Reynolds analogy at a stagnation point.

several quantum jumps in the past few years. At present, taking into account uncertainties in the gas absorption coefficient as well as the estimated effect of radiation absorption by ablation products,  $q_R$  is known within a factor of two. This translates into a factor of two uncertainty in heat shield weight at velocities on the order of 55,000 fps. Taking into account only the uncertainties in absorption coefficient,  $q_R$  can probably be determined within 50%. This is a considerable improvement over the status of shock-layer radiative heat transfer that existed as recently as three years ago. However, at least the following additional work still remains to be accomplished:

1) Because of the extreme importance of radiative heat transfer in the design of a thermal protection system for superorbital re-entry vehicles, additional radiative coupled blunt body shock-layer analyses should be made that include atomic lines and absorption by ablation products in order to complement the few existing, recent calculations of these phenomena.

2) In comparison with the conventional assumption of one-dimensional radiative energy transport, the influence of three-dimensional radiative transport on regions of the shock layer away from the stagnation region should be accurately ascertained. This effect may have bearing on the accurate computation of radiative and convective heat-transfer distributions along the surface of a blunt body. (An interesting analysis of such distributions has recently been made by Burns and Oliver.<sup>87</sup>)

3) Although the aforementioned efforts stress computational detail and accuracy, parallel endeavors should be made to find engineering simplifications to the complex analysis of radiating shock layers without undue sacrifice in accuracy. The contemporary investigations of step-model absorption coefficients in lieu of detailed spectral variations are an example of such endeavors. Also, the useful correlations recently made by Olstad<sup>88</sup> are in this vein. In fact, it is not entirely unreasonable to hope that someday relatively simple but accurate hand estimates of shock-layer radiative heat transfer, which do not rely upon detailed computer solutions, will be possible.

4) In order to confirm the spectral details of the theoretically obtained absorption coefficients, additional experiments emphasizing spectral measurements of air radiative properties should be made.

Finally, this survey has emphasized the engineering calculation of stagnation-point heat transfer; consequently, many important contributions to other aspects of radiative gas dynamics have not been reviewed. However, other aspects are

discussed in earlier surveys by Nerem,<sup>89,90</sup> and Goulard,<sup>91</sup> and in some recent books.<sup>13,15,92</sup>

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