

# Nonlinear Predictive Attitude Control of Spacecraft Under External Disturbances

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## Introduction

**N**ONLINEAR predictive control law was recently applied to control of aerospace systems such as aircraft, missiles, and spacecraft.<sup>1,2</sup> In particular, the spacecraft large-angle maneuver control law as well as a filtering problem were investigated.<sup>3</sup> The key principle of predictive control is to generate a future reference trajectory and design a control input so that the trajectory of an actual system follows that of the reference system. Predictive control is known to have certain favorable properties such as nonlinear tracking of arbitrary reference trajectory given one step ahead and tracking under actuator saturation.

Crassidis et al. applied the predictive control technique to three-axis spacecraft attitude control.<sup>3</sup> Quaternion attitude parameter and angular velocity of spacecraft body axes were taken as controlled variables. In previous studies, disturbance input was not considered and only control input was taken as a design parameter. A cost function as a weighted combination of output error energy and control input energy was established.<sup>1,3</sup> Predictive control laws under the presence of external disturbances have been studied.<sup>4,5</sup> The  $H_2/H_\infty$  control law with disturbance sources was addressed in Ref. 5. An optimal control law under worst-case disturbance was derived.<sup>5</sup>

In this Note, a predictive control law design is addressed in conjunction with the worst-case disturbance input estimate for spacecraft attitude maneuver. This study is an extension of the previous work<sup>1,3</sup>; it explicitly includes external disturbance in the system dynamics and the cost function. The optimal control input and worst-case disturbance are derived simultaneously. The resultant control law therefore accommodates disturbance for better pointing performance. An adaptive weighting parameter selection strategy is proposed for estimation of the actual disturbance input. For attitude representation, the modified Rodrigues parameter (MRP) is employed instead of quaternion.<sup>3</sup> The MRP is a minimal parameter set, but with 360-deg singularity.<sup>6</sup>

## Problem Statement and Equations of Motion

First, the governing equation of motion and attitude kinematics are described. Euler's governing equation of motion for a perfectly rigid spacecraft model is described as

$$J\dot{\hat{\omega}} = \hat{\omega} \times (J\hat{\omega}) + \mathbf{u} + \mathbf{d} \quad (1)$$

where  $J$  is the system inertia matrix,  $\hat{\omega}$  is angular velocity vector,  $\mathbf{u}$  is control input vector, and  $\mathbf{d}$  is a disturbance input vector.

For attitude representation, the MRP is employed, whereas the quaternion is used in Ref. 3. Recent attitude representation methods have adopted the MRP.<sup>6</sup> The MRP is a minimal set with three

parameters only compared with the quaternion. MRP is defined as

$$\mathbf{p} = \mathbf{q}_{13}/(1 + q_4) = \mathbf{n} \tan(\phi/4) \quad (2)$$

where  $\mathbf{p} = [p_1, p_2, p_3]^T$  is a  $3 \times 1$  vector,  $\mathbf{n}$  is the vector of Euler's principal axis, and  $\phi$  represents Euler's principal angle in the principal axis rotation theorem. The quaternion parameters ( $\mathbf{q}_{13}, q_4$ ) are defined as

$$\mathbf{q} = \begin{bmatrix} \mathbf{q}_{13} \\ q_4 \end{bmatrix} \quad (3)$$

$$\mathbf{q}_{13} = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} \equiv \mathbf{n} \sin \frac{\phi}{2}, \quad q_4 = \cos \frac{\phi}{2} \quad (4)$$

The kinematic equations of motion of MRP are expressed as<sup>6</sup>

$$\dot{\mathbf{p}} = \frac{1}{4} \{ (1 - \mathbf{p}^T \mathbf{p}) \mathbf{I}_{3 \times 3} + 2[\mathbf{p} \times] + 2\mathbf{p}\mathbf{p}^T \} \hat{\omega} = \mathbf{F}(\mathbf{p})\hat{\omega} \quad (5)$$

where

$$[\mathbf{p} \times] = \begin{bmatrix} 0 & -p_3 & p_2 \\ p_3 & 0 & -p_1 \\ -p_2 & p_1 & 0 \end{bmatrix} \quad (6)$$

From the definition in Eq. (2), the MRP is subject to singularity in rotation of more than 360 deg. A strategy of switching to the so-called shadow set of MRPs can be employed to overcome the singularity problem.<sup>6</sup>

## Feedback Control Law Design

In this section, a predictive controller design technique under external disturbance is presented. Because the performance of the controlled system is sensitive to disturbance, the controller design should take the disturbance into account for precision pointing. The disturbance source may not be known exactly in general. In this study, the control law is designed against the worst-case disturbance input.

Let us consider a nonlinear system represented in the following form:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{G}(\mathbf{x})\mathbf{u}(t) + \mathbf{D}(\mathbf{x})\mathbf{d}(t) \quad (7)$$

$$\mathbf{y}(t) = \mathbf{c}(\mathbf{x}) \quad (8)$$

where  $\mathbf{x}(t) \in \mathbb{R}^n$  is a state vector,  $\mathbf{u}(t) \in \mathbb{R}^p$  is a control input vector,  $\mathbf{d}(t) \in \mathbb{R}^r$  is the external disturbance input, and  $\mathbf{y}(t) \in \mathbb{R}^m$  is the system output vector;  $\mathbf{G}(\mathbf{x}) : \mathbb{R}^n \rightarrow \mathbb{R}^{n \times p}$  is a control input distribution matrix,  $\mathbf{D}(\mathbf{x}) : \mathbb{R}^n \rightarrow \mathbb{R}^{n \times q}$  is a disturbance distribution matrix, and  $\mathbf{c}(\mathbf{x}) : \mathbb{R}^n \rightarrow \mathbb{R}^{m \times p}$  is a measurement vector, respectively. The disturbance distribution matrix ( $\mathbf{D}(\mathbf{x})$ ) is assumed to be identified exactly. In general, the disturbance itself is not exactly identified.

As the first step of the predictive control, the output function in Eq. (8) at time  $t + \Delta t$  is represented in terms of other variables at time  $t$  by Taylor series expansion. Using the Lie derivative notations, the following relationship is obtained:

$$\begin{aligned} \mathbf{y}(t + \Delta t) &\simeq \mathbf{y}(t) + \mathbf{w}[\mathbf{x}(t), \Delta t] + \Lambda(\Delta t)\Phi_G(\mathbf{x})\mathbf{u}(t) \\ &\quad + \Lambda(\Delta t)\Phi_D(\mathbf{x})\mathbf{d}(t) \end{aligned} \quad (9)$$

where  $\mathbf{w}[\mathbf{x}(t), \Delta t] \in \mathbb{R}^m$  is also written in terms of the Lie derivative<sup>1,3</sup>:

$$\mathbf{w}_i[\mathbf{x}(t), \Delta t] = \sum_{k=1}^{\gamma_i} \frac{\Delta t^k}{k!} L_f^k(c_i) \quad (10)$$

Note that  $\gamma_i, i = 1, 2, \dots, m$  is the order of derivative of  $c_i[\mathbf{x}(t)]$  for which the control input appears explicitly for the first time.  $L_f^k(c_i)$

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is the  $k$ th-order Lie derivative satisfying<sup>1,3</sup>

$$L_f^k(c_i) \equiv c_i \quad \text{for} \quad k = 0 \quad (11)$$

$$L_f^k(c_i) \equiv \frac{\partial L_f^{k-1}(c_i)}{\partial \mathbf{x}} \mathbf{f} \quad \text{for} \quad k \geq 1 \quad (12)$$

where  $\Lambda(\Delta t) \in R^{m \times m}$  is a diagonal matrix with diagonal components given as<sup>1,3</sup>

$$\lambda_{ii} = \Delta t^{\gamma_i} / \gamma_i!, \quad i = 1, 2, \dots, m \quad (13)$$

In addition, both  $\Phi_G(\mathbf{x}(t)) \in R^{m \times p}$  and  $\Phi_D(\mathbf{x}(t)) \in R^{m \times q}$  can be defined in terms of matrices with the row elements<sup>3</sup> as

$$(\Phi_G)_i = \{L_{G_1}[L_f^{\gamma_i-1}(c_i)], \dots, L_{G_p}[L_f^{\gamma_i-1}(c_i)]\} \quad i = 1, 2, \dots, m \quad (14)$$

$$(\Phi_D)_i = \{L_{D_1}[L_f^{\gamma_i-1}(c_i)], \dots, L_{D_q}[L_f^{\gamma_i-1}(c_i)]\} \quad i = 1, 2, \dots, m \quad (15)$$

for which the Lie derivatives satisfy

$$L_{G_j}[L_f^{\gamma_j-1}(c_i)] = \frac{\partial L_f^{\gamma_j-1}(c_i)}{\partial \mathbf{x}} G_j, \quad j = 1, 2, \dots, p \quad (16)$$

$$L_{D_j}[L_f^{\gamma_j-1}(c_i)] = \frac{\partial L_f^{\gamma_j-1}(c_i)}{\partial \mathbf{x}} D_j, \quad j = 1, 2, \dots, q \quad (17)$$

For the control law design, a cost function is defined first as a weighted combination of squares of tracking error, control input, and external disturbance. The cost function is a Lyapunov function given in the form

$$J[\mathbf{u}(t), \mathbf{d}(t)] = [\tilde{\mathbf{y}}(t + \Delta t) - \mathbf{y}(t + \Delta t)]^T Q [\tilde{\mathbf{y}}(t + \Delta t) - \mathbf{y}(t + \Delta t)] + \mathbf{u}^T(t) R \mathbf{u}(t) - \mathbf{d}^T(t) H \mathbf{d}(t) \quad (18)$$

where  $\tilde{\mathbf{y}}(t + \Delta t)$  is the desired output at time  $t + \Delta t$ ;  $Q \in R^{m \times m}$ ,  $R \in R^{p \times p}$ , and  $H \in R^{q \times q}$  represent positive definite weighting matrices on each term. The cost function is different from that of previous studies.<sup>1,3</sup> In the previous cases, only the control input was considered in the cost function. For a filtering problem, modeling error was solely employed in the cost function. Note that a negative sign is assigned to the weighted square of the disturbance so that the larger the disturbance is, the more control input is required. The cost function, therefore, corresponds to a typical minimax problem in the differential game theory. The control input and disturbance conflict with each other for the given cost function. Namely, the control input should compensate the output error as well as the disturbance.

Now using the given cost function, partial derivatives are taken with respect to  $\mathbf{d}$  and  $\mathbf{u}$ . This operation is targeted to find the optimal control and worst-case disturbance simultaneously in the sense of a minimax optimization problem. Simple algebra starting from  $\partial J / \partial \mathbf{u} = 0$ ,  $\partial J / \partial \mathbf{d} = 0$  yields an optimal predictive control command and the worst-case disturbance, respectively:

$$\begin{bmatrix} \mathbf{u}^*(t) \\ \mathbf{d}^*(t) \end{bmatrix} = \begin{bmatrix} A_{GG} & B_{GD} \\ B_{DG} & A_{DD} \end{bmatrix}^{-1} \begin{bmatrix} C_G \\ C_D \end{bmatrix} \quad (19)$$

where  $\mathbf{d}^*(t)$  represents the worst-case disturbance estimated, and each parameter is given as

$$A_{ij} = [\Lambda(\Delta t) \Phi_i(\mathbf{x})]^T Q [\Lambda(\Delta t) \Phi_j(\mathbf{x})] + M$$

$$B_{ij} = [\Lambda(\Delta t) \Phi_i(\mathbf{x})]^T Q [\Lambda(\Delta t) \Phi_j(\mathbf{x})]$$

$$C_i = [\Lambda(\Delta t) \Phi_i(\mathbf{x})]^T Q [\tilde{\mathbf{y}}(t + \Delta t) - \mathbf{y}(t) - \mathbf{z}(\mathbf{x}, \Delta t)] \quad (20)$$

where the subscripts are arranged in such a way that if  $i \equiv G$ ,  $j \equiv D$ , then  $M \equiv R$ , and if  $i \equiv D$ ,  $j \equiv G$ , then  $M \equiv -H$ . Equation (19) indicates that the final control law depends on the estimated disturbance

coupled with system and weighting parameters. The control law is essentially close to that of Ref. 5, where the worst-case disturbance was handled by the  $H_2/H_\infty$  approach. The estimated disturbance  $\mathbf{d}^*$  is not the actual one, but it is implicitly related to the actual disturbance  $\mathbf{d}$  through the output  $\mathbf{y}$ . The estimated disturbance  $\mathbf{d}^*$  is not directly used for the control command synthesis. It is just an estimation of the actual disturbance  $\mathbf{d}$ , which is generally unknown and actually activated during the simulation. But the control command may compensate  $\mathbf{d}^*$  for better pointing accuracy for which the estimated disturbance is matched with the actual one via weighting parameters adaptation, which is explained later.

### Scalar System Example

To have better understanding of the relationship between the control input and disturbance, we examine the proposed method for a scalar system. Consider a nonlinear scalar system expressed as

$$\dot{x} = -x|x| + Gu(t) + Dd(t) \quad (21)$$

$$y = x \quad (22)$$

where  $G$  and  $D$  are assumed constant for convenience. From the formulation in Eq. (9), it can be written as follows:

$$y(t + \Delta t) = y(t) - \Delta t|x|x + \Delta tGu + \Delta tDd \quad (23)$$

Furthermore, from Eq. (21) the predictive control command and the worst-case disturbance input are derived as

$$u^*(t) = K_1(1/\Delta t G)[\tilde{y}(t + \Delta t) - y(t) + \Delta t|x|x] \quad (24)$$

$$d^*(t) = -K_2(1/\Delta t D)[\tilde{y}(t + \Delta t) - y(t) + \Delta t|x|x] \quad (25)$$

where

$$K_1 = \frac{\Delta t^2 Q G^2 H}{H R + \Delta t^2 Q (G^2 H - D^2 R)} \quad (26)$$

$$K_2 = \frac{\Delta t^2 Q D^2 R}{H R + \Delta t^2 Q (G^2 H - D^2 R)}$$

The worst-case disturbance is affected by the actual disturbance through the output  $y(t)$  and the gain parameter  $K_2$ . The control command is also affected by  $d^*$  and the gain parameters. The control gains  $K_1$  and  $K_2$  can be selected by adjusting the design parameters. The terms inside the bracket can be approximated as

$$\tilde{y}(t + \Delta t) - y(t) + \Delta t|x|x \simeq e(t) + \Delta t \dot{e}(t) \quad (27)$$

where  $e(t) = \tilde{y}(t) - y(t)$ . Thus, the final control law is a negative feedback form, and the disturbance acts against the control command for positive values of  $K_1$  and  $K_2$ . Examination of the system stability is performed by substituting Eq. (23) into Eq. (24). A sufficiently small  $R$  for large control input and small  $\Delta t$  are assumed. Consequently, we arrive at

$$\dot{e}(t) = -e(t)/\Delta t - Dd(t) \quad (28)$$

The tracking performance is thus subject to the disturbance. If the external disturbance is not present, the output error tends to asymptotically converge to zero with  $R = 0$ .<sup>1</sup>

Because there are three parameters ( $Q$ ,  $R$ ,  $H$ ) to be determined, the parameter selection strategy is not unique. If  $K_1$  and  $K_2$  are set to unity, it follows that

$$\Delta t^2 Q = R/G^2 = H/D^2 \quad (29)$$

Because  $G$  and  $D$  are given system matrices,  $H$  and  $R$  are determined by specifying  $\Delta t$  and  $Q$ . Substitution of Eq. (28) with Eq. (29) into Eq. (25) provides a useful relationship:

$$\lim_{t \rightarrow \infty} d^*(t) = d(t)$$

That is, the worst-case disturbance approaches the actual disturbance with the adaptation law in Eq. (29).

### Adaptive Weighting Matrices

For the selection of appropriate weighting parameters, a procedure similar to the scalar example can be applied to the general system. The weighting matrices ( $Q$ ,  $R$ ) then satisfy the following adaptation law:

$$\Lambda Q \Lambda = \Phi_G^{-T} R \Phi_G^{-1} = \Phi_D^{-T} H \Phi_D^{-1} \quad (30)$$

or

$$\begin{aligned} R(\Delta t, \mathbf{x}) &= \Phi_G^T(\mathbf{x}) \Lambda(\Delta t) Q \Lambda(\Delta t) \Phi_G(\mathbf{x}) \\ H(\Delta t, \mathbf{x}) &= \Phi_D^T(\mathbf{x}) \Lambda(\Delta t) Q \Lambda(\Delta t) \Phi_D(\mathbf{x}) \end{aligned} \quad (31)$$

The control law and estimated disturbance are then rewritten as

$$\begin{aligned} \mathbf{u}^*(t) &= R(\Delta t, \mathbf{x})^{-1} \Phi_G^T(\mathbf{x}) \Lambda(\Delta t) Q [\tilde{\mathbf{y}}(t + \Delta t) - \mathbf{y}(t) - \mathbf{w}(\mathbf{x}, \Delta t)] \\ \mathbf{d}^*(t) &= -H(\Delta t, \mathbf{x})^{-1} \Phi_D^T(\mathbf{x}) \Lambda(\Delta t) Q [\tilde{\mathbf{y}}(t + \Delta t) - \mathbf{y}(t) - \mathbf{w}(\mathbf{x}, \Delta t)] \end{aligned} \quad (32)$$

or

$$\begin{aligned} \mathbf{u}^*(t) &= [\Lambda(\Delta t) \Phi_G(\mathbf{x})]^{-1} [\tilde{\mathbf{y}}(t + \Delta t) - \mathbf{y}(t) - \mathbf{w}(\mathbf{x}, \Delta t)] \\ \mathbf{d}^*(t) &= -[\Lambda(\Delta t) \Phi_D(\mathbf{x})]^{-1} [\tilde{\mathbf{y}}(t + \Delta t) - \mathbf{y}(t) - \mathbf{w}(\mathbf{x}, \Delta t)] \end{aligned} \quad (33)$$

From Eq. (32) one can see that the control and disturbance weighting matrices are functions of control update time and output error weighting matrices. Moreover, because  $\Phi_G$  and  $\Phi_D$  are functions of state variables, the weighting parameters need to be updated adaptively at every time step. This leads to asymptotic convergence of the estimated worst-case disturbance to the actual one.

### Application to Spacecraft Attitude Control

To apply the control algorithm to the spacecraft attitude control problem, the state vector is introduced as  $\mathbf{x} = [\mathbf{p} \ \hat{\omega}]^T$  and the governing equations of motion are

$$\begin{bmatrix} \dot{\mathbf{p}} \\ \dot{\hat{\omega}} \end{bmatrix} = \begin{bmatrix} F(\mathbf{p})\hat{\omega} \\ J^{-1}[J\hat{\omega} \times \hat{\omega}] \end{bmatrix} + \begin{bmatrix} 0 \\ J^{-1} \end{bmatrix} \mathbf{u}(t) + \begin{bmatrix} 0 \\ D \end{bmatrix} \mathbf{d}(t) \quad (34)$$

The output equation is prescribed as<sup>3</sup>

$$\mathbf{y} = \mathbf{x} \quad (35)$$

Applying the predictive control theory yields  $\gamma_i = 2$  for  $i = 1, 2, 3$  and  $\gamma_i = 1$  for  $i = 4, 5, 6$  and, in addition,

$$\mathbf{w} = \begin{bmatrix} w_p \\ w_\omega \end{bmatrix} \quad (36)$$

where

$$\begin{aligned} w_p &= \left\{ \Delta t F(\mathbf{p}) + (\Delta t^2/2) [M F(\mathbf{p}) + F(\mathbf{p}) J^{-1} [J \hat{\omega} \times \hat{\omega}]] \right\} \hat{\omega} \\ w_\omega &= \Delta t J^{-1} [J \hat{\omega} \times \hat{\omega}] \end{aligned} \quad (37)$$

The matrices defined in Eqs. (13–15) are given by

$$\Lambda(\Delta t) = \begin{bmatrix} \frac{\Delta t^2}{2} I_{3 \times 3} & 0 \\ 0 & \Delta t I_{3 \times 3} \end{bmatrix} \quad (38)$$

$$\Phi_G(\mathbf{x}(t)) = \begin{bmatrix} M & F(\mathbf{p}) \\ 0 & I_{3 \times 3} \end{bmatrix} \begin{bmatrix} 0 \\ J^{-1} \end{bmatrix} = \begin{bmatrix} F(\mathbf{p}) J^{-1} \\ J^{-1} \end{bmatrix}$$

$$\Phi_D(\mathbf{x}(t)) = \begin{bmatrix} M & F(\mathbf{p}) \\ 0 & I_{3 \times 3} \end{bmatrix} \begin{bmatrix} 0 \\ D \end{bmatrix} = \begin{bmatrix} F(\mathbf{p}) D \\ D \end{bmatrix} \quad (39)$$

where the parameter  $M$  is represented in terms of the MRP as

$$\begin{aligned} M &= \frac{1}{2} \begin{bmatrix} \mathbf{p}^T \hat{\omega} & [-p_2 \ p_1 \ 1] \hat{\omega} & [-p_3 \ -1 \ p_1] \hat{\omega} \\ [-p_2 \ p_1 \ -1] \hat{\omega} & \mathbf{p}^T \hat{\omega} & [1 \ -p_3 \ p_2] \hat{\omega} \\ [p_3 \ 1 \ -p_1] \hat{\omega} & [-1 \ p_3 \ -p_2] \hat{\omega} & \mathbf{p}^T \hat{\omega} \end{bmatrix} \\ &= \begin{bmatrix} M_1 \\ M_2 \\ M_3 \end{bmatrix} \end{aligned} \quad (40)$$

Because MRP is used instead of the quaternion<sup>3</sup> in this study the controller parameters are different. Finally, the control law synthesis and the worst-case disturbance for the three-axis attitude maneuver with the MRP as attitude parameters can be accomplished from Eq. (33) by combining Eqs. (36–40).

For validation of the control law, a sample simulation run is conducted. The mass moment of inertia of the spacecraft, the control input, and the disturbance distribution matrices are given by  $J = \text{diag}(20, 10, 15)$  ( $\text{kg} \cdot \text{m}^2$ ),  $G = J^{-1}$ ,  $D = 0.2 I_{3 \times 3}$ . Weighting parameters for each term in the cost function are assumed as

$$H = 0.5 I_{3 \times 3}, \quad R = 0.5 I_{3 \times 3}, \quad Q = \begin{bmatrix} q_p I_{3 \times 3} & 0_{3 \times 3} \\ 0_{3 \times 3} & q_\omega I_{3 \times 3} \end{bmatrix}$$

where  $q_p = 10$  and  $q_\omega = 5$ . The control input is made bounded up to  $\pm 0.1$  Nm. The total simulation time is set to 900 s, and the control update time is 1.0 s. Initial errors are imposed between the reference and actual systems.

First, the reference trajectory for MRPs is generated corresponding to a large-angle attitude maneuver. The actual disturbance input acting on the spacecraft body is displayed in Fig. 1. Figure 2

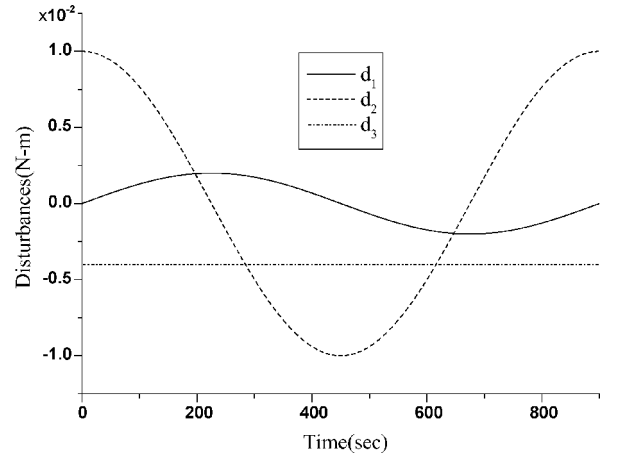


Fig. 1 Actual external disturbances.

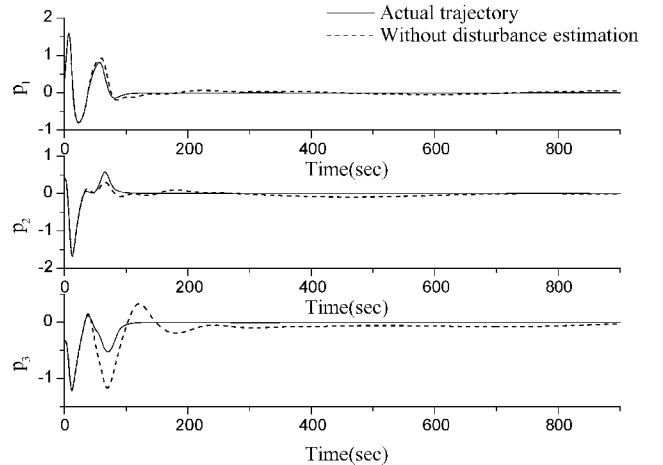
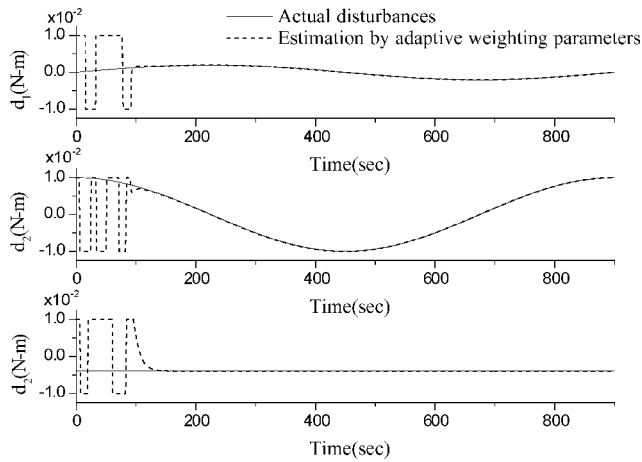


Fig. 2 Attitude tracking error responses for MRPs.



**Fig. 3 Actual and estimated disturbance histories.**

shows attitude (MRP) tracking error between reference and actual trajectories. Satisfactory tracking performance for the reference trajectories after about 100 s is achieved. The tracking error with the disturbance-estimation technique was as low as an order of  $10^{-3}$  in MRPs. The actual disturbance and estimated disturbance by the adaptive weighting parameter strategy [Eq. (30)] are presented in Fig. 3. Without the adaptation scheme, there exists a scale factor error between the actual and the worst-case estimated disturbances. The simulation results in Fig. 2 also show that the time response with estimation of the disturbance leads to better performance compared to the case without estimation.

### Conclusions

An extension of the predictive control approach has been made by explicitly including external disturbance with attitude representation

by MRP. The control law design reflects the worst-case disturbance input in the sense of the minimax game problem. Desired tracking performance is achieved by the proposed control law. The worst-case disturbance information is used to estimate the actual disturbance by using an adaptive weighting parameter selection strategy. The new predictive control approach was found to be applicable to the spacecraft attitude maneuver control under external disturbance and achieved the desired performance.

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