

Formation and Attitude Control for Rotational Tethered Satellite Clusters

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A tethered satellite cluster system, which consists of a cluster of satellites connected by tethers and which can maintain and change formation via active control of tether tension and length to save thruster fuel and improve control accuracy is proposed. The concept can be applied to tethered multisatellites for in-orbit servicing, which can perform various missions, including inspection, casting, capture, recovery, moorage, and deorbiting of an uncontrolled satellite. The rotational motion of such a system that the satellites in formation flying are required to rotate about the center of mass of the system on the same desired plane is considered. The equilibrium conditions that the tether tension imposes on the rotational motion are given, and a coordinated control method for the thrusters, the reaction wheels, and the tether tension/torque is proposed. Numerical simulations and ground experiments show that the control of the tether tension and torque not only saves thruster fuel, but also improves the position and attitude accuracy of formation flying.

Nomenclature

$\mathbf{a}_{jk} = \{\mathbf{i}\}^{T_i} \mathbf{a}_{jk} = \{\mathbf{b}_j\}^{T_b} \mathbf{a}_{jk}$	=	vector from the c.m. of satellite j to the end of arm jk	$\mathbf{q}_{c.m.} = \{\mathbf{i}\}^{T_i} \mathbf{q}_{c.m.}$	=	c.m. of tethered satellite cluster system
$\{\mathbf{b}_j\} = \{\mathbf{b}_{jx} \ \mathbf{b}_{jy} \ \mathbf{b}_{jz}\}^T$	=	body-fixed coordinate system of satellite j	$\mathbf{q}_j = \{\mathbf{i}\}^{T_i} \mathbf{q}_j = \{\mathbf{b}_j\}^{T_b} \mathbf{q}_j$	=	position vector of the c.m. of satellite j
C, K	=	derivative/proportional gains	$\{\mathbf{r}\} = \{\mathbf{r}_x \ \mathbf{r}_y \ \mathbf{r}_z\}^T$	=	rotational coordinate system
$C^j(\theta)$	=	coordinate transformation matrix around the axis j of angle θ	$\{\mathbf{s}\} = \{\mathbf{s}_x \ \mathbf{s}_y \ \mathbf{s}_z\}^T$	=	intermediate coordinate system fixed on the desired rotational plane
d_j	=	distance from the origin of $\{\mathbf{r}\}$ to the c.m. of satellite j in plane	T_{jk}	=	tension of tether jk
$\mathbf{F}_j = \{\mathbf{i}\}^{T_i} \mathbf{F}_j$	=	force vector acting on satellite j as the result of the thruster	α_j, β_j	=	length of the major/minor axis of the ellipse of satellite j
$\mathbf{F}_{jk}, \boldsymbol{\tau}_{jk}$	=	force/torque vectors on satellite j caused by the tension of tether jk	λ_j	=	angle between the two directions of satellites j and l in plane
H_j	=	foot of perpendicular from the c.m. of satellite j to the rotational plane	$\boldsymbol{\tau}_j = \{\mathbf{b}_j\}^{T_b} \boldsymbol{\tau}_j$	=	torque vector acting on satellite j as the result of the reaction wheel
h_j	=	height of the c.m. of satellite out of plane	$\varphi_r, \theta_r, \psi_r$	=	3-2-1 Euler angles between $\{\mathbf{r}\}$ and $\{\mathbf{i}\}$
$\mathbf{I}_j = \{\mathbf{b}_j\}^{T_b} \mathbf{I}_j \{\mathbf{b}_j\}$	=	moment of inertia about a set of axes passing through the c.m. of satellite j	$\dot{\varphi}_r$	=	spin angular velocity of $\{\mathbf{r}\}$ with respect to $\{\mathbf{s}\}$
$\{\mathbf{i}\} = \{\mathbf{i}_x \ \mathbf{i}_y \ \mathbf{i}_z\}^T$	=	Earth-centered inertial coordinate system	$\psi_j, \theta_j, \varphi_j$	=	1-2-3 Euler angles between $\{\mathbf{b}_j\}$ and $\{\mathbf{r}\}$
l_{jk}	=	distance between the c.m. of satellite j and the end of arm jk	$\boldsymbol{\omega}_j = \{\mathbf{b}_j\}^{T_b} \boldsymbol{\omega}_j$	=	angular velocity vector of satellite j
m_j	=	mass of satellite j			
\mathbf{P}	=	angular momentum vector about the c.m. of tethered satellite cluster system			

Introduction

MANY researchers have investigated the concept of satellite cluster systems in which a group of satellites is coordinated to perform various advanced missions. The benefits of such a system are redundancy and distribution. Each satellite in the cluster features smaller size, lighter weight, and lower cost.^{1–3} A typical mission would be, for example, formation flying for interferometer observations in the as part of the of TechSat21⁴ and Darwin⁵ projects. However, coordinated work in orbit requires significant thruster fuel, and this in turn causes a reduction in satellite lifetime. Beard and Hadaegh proposed a finite thrust control method for a formation rotating at a constant rate in free space.⁶ This keeps the relative distances between the spacecraft within specified constraints. This method has the drawback that the reduction of thruster fuel causes decline in the position control accuracy.

As an alternative, many researchers have proposed satellite constellations connected by tethers. Keshmiri et al. formulated the general three-dimensional motion of tethered N-body systems.⁷ Tragesser investigated the stability of the spin motion of a tethered ring formation based on the conical Likins–Pringle relative

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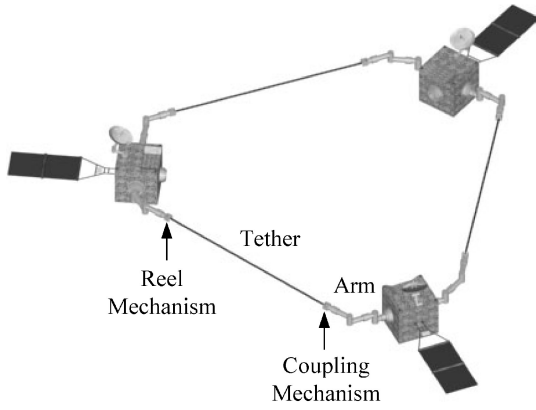


Fig. 1 Tethered satellite cluster system.

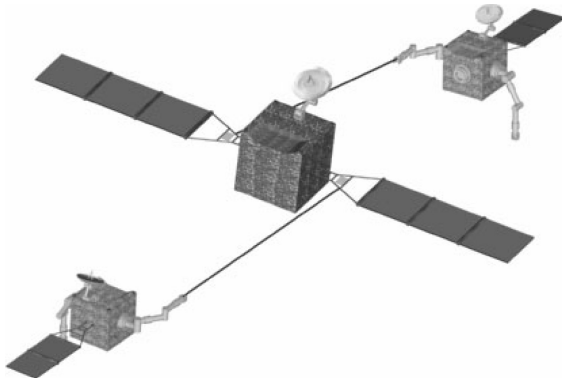


Fig. 2 Tethered multisatellites for in-orbit servicing.

equilibrium.⁸ DeCou investigated the dynamics for a triangular formation of satellites.⁹ Maccone considered a tethered system to get magnified radio pictures of the galactic center.¹⁰ Yasumitsu and Natori showed the dynamical behavior of a tethered system microsatellite constellation.¹¹ However, in these studies each satellite is defined as a point mass, and the torque caused by the tether offset is not discussed.

This paper proposes a new tethered satellite cluster system as sketched in Fig. 1. The system consists of tether-connected robotic satellites, which maintain and change formation using active control of the tension and length of each tether. Each satellite has arms and can change the tether offsets by controlling the endpoints of the arms. Thus tether tension can be used for attitude control^{12–15} as well as for position control. Such a system can be used in formation flying by using the tether tension not only for saving thruster fuel but also improving the control accuracy of the position and attitude of each satellite. The concept can be applied to tethered multisatellites for in-orbit servicing,^{16–19} performing various missions such as autonomic inspection, casting, capture, recovery, moorage, and deorbiting of an uncontrolled satellite, as shown in Fig. 2.

We consider the rotational motion with respect to the c.m. of the system in a formation pattern. Circular motion is treated as the most fundamental case. The equilibrium conditions for maintaining the circular motion using the tether tension are shown, and a control method based on the equilibrium conditions for minimizing thruster fuel is investigated. Next, the equilibrium conditions for a similar transformation of the system are derived, and the control methods for saving fuel are shown. Numerical simulations demonstrate that these control methods are effective in decreasing the relative position/attitude errors of the satellites as well as in reducing the satellite fuel consumption. In addition, we develop a ground experiment system,^{20–22} including a reel mechanism^{12,23–25} to simulate the tethered satellite cluster system. We outline the system components and verify the dynamics and control methods with two-dimensional experiments.

Tethered Satellite Cluster System

The main features of the proposed tethered satellite cluster system are as follows:

- 1) The system is reconfigurable by tether connection/disconnection between the satellites.
- 2) The tether tension and length are controlled by a reel mechanism, and the tether connection/disconnection is effected with a coupling mechanism.
- 3) The reel or coupling mechanisms are installed on the endpoints of the arms of each satellite.
- 4) Each satellite maintains formation flying by a coordinated control of position and attitude using the reel, the arm, the thruster, and the reaction wheel.
- 5) The control system results in reduced fuel consumption and improved relative position/attitude accuracy through control of the tether tension.

We consider the following two fundamental motions: translational motion, which is moving the satellites to an arbitrary position for the transportation of supplies, such as for a space elevator, or for rendezvous docking, etc.; and rotational motion, which is rotating the system about the c.m. while maintaining or changing the formation. This is useful for interferometry measurement, or for stabilization of a solar sail^{26,27} or a space solar power system.²⁸

Combining the two motions, the system can perform various advanced services, such as capturing an uncontrolled satellite, mooring to space structures, and operating a tethered camera for satellite inspection.

Modeling and Formulation

Figure 3 shows an analytical model of the system. We consider the motions of the n satellites connected by tethers in an Earth-centered inertial coordinate system $\{i\}$. Each satellite, tether, and arm is modeled as follows where $j, k = 1, 2, \dots, n$, and $j \neq k$.

For satellite j , the body-fixed coordinate system is $\{b_j\}$; the mass is m_j ; the position of the c.m. is \mathbf{q}_j ; the moment of inertia about a set of axes passing through its c.m. is \mathbf{I}_j ; and the angular velocity is $\boldsymbol{\omega}_j$.

For tether jk (connects satellites j and k), the tension is $T_{jk} (= T_{kj}) \geq 0$; and the mass, twist moment, bending moment, and tensile strain are ignored.

For arm jk , a three-degree-of-freedom (DOF) arm mounted on satellite j , and controlling the position of the arm end $\mathbf{a}_{jk} (\neq \mathbf{a}_{kj})$ and the orientation of the reel or the coupling mechanism connected to tether jk .

In this paper, each satellite is in the neighborhood of the c.m. of the system. By the Newton–Euler equations, the motion of each satellite can be formulated as follows:

$$m_j \ddot{\mathbf{q}}_j = \frac{\mu m_j}{|\mathbf{q}_j|^3} \mathbf{q}_j + \mathbf{F}_j + \sum_k \mathbf{F}_{jk} \quad (1)$$

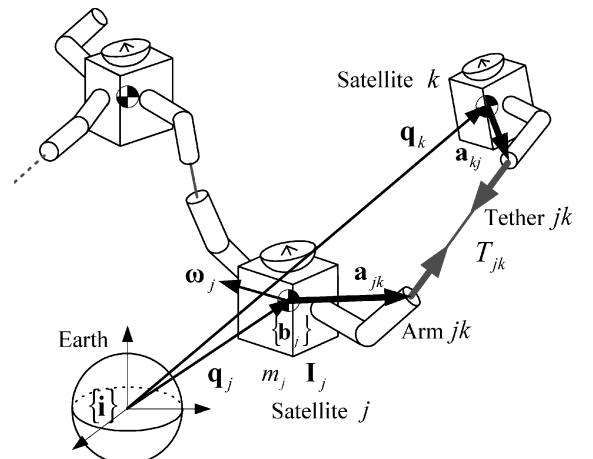


Fig. 3 Analytical model.

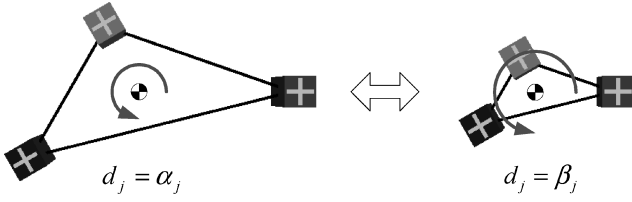
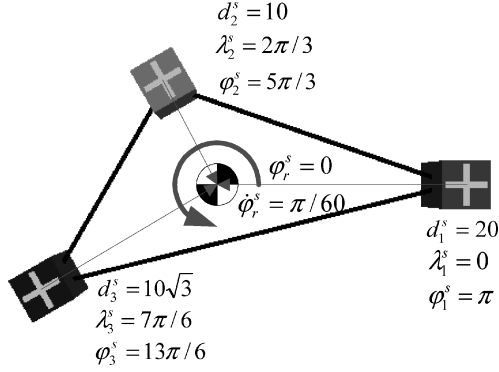
Fig. 6 Similar transformation sequence in the $r_x r_y$ plane.

Fig. 7 Simulation conditions.

transformation. In the case of $\ddot{d}_j/d_j = 0$, Eq. (24) becomes equal to Eq. (12). Replacing the $\dot{\varphi}_r^2$ terms with $(\dot{\varphi}_r^2 - \ddot{d}_j/d_j)$ in Eqs. (15), (16), (18), and (19), we arrive at the equilibrium tension T_{jk}^d for the similar transformations. The equilibrium formations for the similar transformations are the same as those for circular motions. Because $T_{jk} \geq 0$, the relationship $\ddot{d}_j/d_j \leq \dot{\varphi}_r^2$ needs to be satisfied.

Control Method for Similar Transformations

The system can take advantage of the equilibrium tension for similar transformations in the control scheme. The coordinated control method for circular motions is also efficient for decreasing fuel usage for similar transformations. The thruster/reaction-wheel control, Eqs. (20) and (21), transfers the system to a desired motion, and the reel/arm control, Eqs. (22) and (23), maintains it.

Numerical Simulations

Consider the rotational motion of three satellites ($n = 3$), all connected to each other, as shown in Fig. 7. The following numerical simulations verify the control methods just proposed.

Simulation Conditions

The following parameters describe the system in meter-kilogram-second (MKS) units.

Satellite shape:

$$m_j = 50, \quad {}^b I_j = \text{diag}\{2, 2, 2\}, \quad l_{jk} = 1 \quad (25)$$

Orbital motion:

$$|{}^i q_{c.m.}| = 570,000 \quad (\text{circular orbit of which period is } 5764 \text{ s}) \quad (26)$$

Rotational axis and rotational plane:

$$\psi_r^d = \pi/6, \quad \theta_r^d = \pi/6 \quad (27)$$

Initial conditions:

$$\varphi_r^s = 0, \quad \dot{\varphi}_r^s = \pi/60 \quad (28)$$

$$d_1^s = 20, \quad d_2^s = 10, \quad d_3^s = 10\sqrt{3} \quad (29)$$

$$\lambda_2^s = 2\pi/3, \quad \lambda_3^s = 7\pi/6, \quad h_j^s = 0 \quad (30)$$

Table 1 Average of position/attitude errors: circular motion simulation

Tension	$ \Delta q_2 $, mm	$\sqrt{ \Delta \psi_2 ^2 + \Delta \theta_2 ^2 + \Delta \varphi_2 ^2}$, deg
$T_{jk} = T_{jk}^d$	0.000	0.000
$T_{jk} = 0$	0.273	0.000
$T_{jk} = 1.5T_{jk}^d$	0.137	0.000

$$\psi_j^s = 0, \quad \theta_j^s = 0, \quad \varphi_j^s = \lambda_j^s + \pi \quad (31)$$

$$\dot{\mathbf{q}}_j^s = \{\mathbf{r}\}^T [-\dot{\varphi}_r^s d_j^s \sin \lambda_j^s \quad \dot{\varphi}_r^s d_j^s \cos \lambda_j^s \quad 0]^T \quad (32)$$

$$\boldsymbol{\omega}_j^s = \{\mathbf{b}\}^T [0 \quad 0 \quad \dot{\varphi}_r^s]^T \quad (33)$$

where the superscript s indicates the initial value. Equation (28) shows the spin angle and angular velocity of the system. Equations (29) and (30) show the triangle formation. Equation (31) shows the attitude of each satellite. These initial conditions are sketched in Fig. 7. Equations (32) and (33) show the velocity and angular velocity vectors of each satellite, respectively.

Circular Motion Simulation

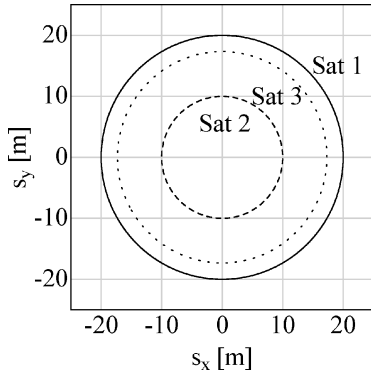
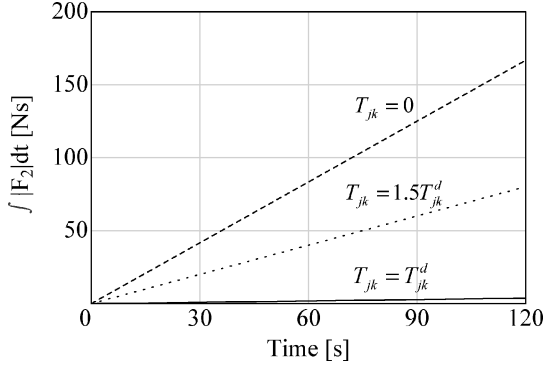
When the initial formation is maintained, each satellite of the system makes a circle. Because $\dot{\varphi}_r^s = \pi/60$, the cycle of the circular motion is 120 s, which is a typical condition for interferometer observations. This motion is set as the desired one. Each satellite can perform the desired motion by the prescribed thruster/reaction-wheel control in Eqs. (20) and (21), adding equilibrium tension T_{jk}^d calculated by Eq. (19).

Figure 8a shows the trajectory of each satellite in the $s_x s_y$ plane. The solid, dashed, and dotted lines represent satellites 1, 2, and 3, respectively, and the system maintains the desired circular motion. The radius of the circular motion of each satellite is d_j^s . Figures 8b and 8c show the accumulated thruster impulse and the accumulated reaction-wheel angular impulse acting on satellite 2. The solid, dashed, and dotted lines represent the three cases of $T_{jk} = T_{jk}^d$ or 0 or $1.5T_{jk}^d$. Table 1 shows the average of the relative position/attitude errors of satellite 2 for each case. The accumulated thruster impulse is proportional to the fuel consumption. In the case of $T_{jk} = T_{jk}^d$, the sum of the tether tensions acting on the satellite is equal to the centrifugal force of the desired circular motion. Thus the thruster force is needed only to correct errors caused by the orbital motion, and the accumulated impulse is small. On the other hand, in the case of $T_{jk} = 0$ or $1.5T_{jk}^d$, the thruster force for maintaining the circular motion is equal to the centrifugal force $m_2 d_2^d \dot{\varphi}_r^{d2}$ or $-0.5m_2 d_2^d \dot{\varphi}_r^{d2}$, respectively. The same is true of the other satellites. Therefore, in the case of $T_{jk} = \alpha T_{jk}^d$, where α is a constant, the accumulated thruster impulse acting on satellite j is represented as follows:

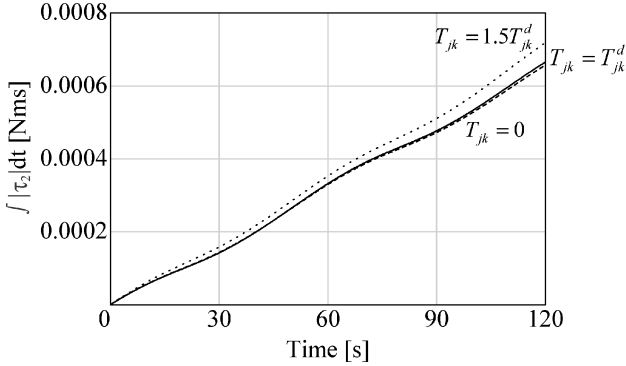
$$\int |F_j| dt \approx |1 - \alpha| m_j d_j^d \dot{\varphi}_r^{d2} t \quad (34)$$

The accumulated reaction-wheel angular impulse is also related to the thruster fuel consumption because the thruster is substituted for reaction-wheel attitude control. Figure 8c indicates that it is nearly equal to zero in all three cases. The reason is that no torque is required for maintaining the circular motion and that the reaction wheel is used only to correct attitude errors.

In Table 1, the average of the position error is the smallest for the case of $T_{jk} = T_{jk}^d$, and it becomes larger in the cases of $T_{jk} = 1.5T_{jk}^d$ and 0 in that order. The average of the attitude error is equal to zero in the three cases. Similar results are obtained for accumulated thruster impulse and reaction-wheel angular impulse. Therefore the proposed feedforward tension is useful in improving the position control accuracy as well as in reducing the fuel consumption of the thruster.

a) Trajectory in the $s_x s_y$ plane

b) Accumulated thruster impulse



c) Accumulated reaction-wheel angular impulse

Fig. 8 Circular motion simulation.

Similar Transformation Simulation

The following ellipse motion is considered as a desired similar transformation:

$$d_j^d = \frac{\alpha_j \beta_j}{\sqrt{\alpha_j^2 \cos^2 \varphi_r^d + \beta_j^2 \sin^2 \varphi_r^d}} \quad (35)$$

where α_j and β_j are the lengths of the major and minor axes of the ellipse in the $s_x s_y$ plane, respectively. They are set as follows:

$$\alpha_j = d_j^s, \quad \beta_j = 0.5d_j^s \quad (36)$$

The equilibrium tension T_{jk}^d and the desired angular velocity $\dot{\varphi}_r^d$ are calculated by the following equations:

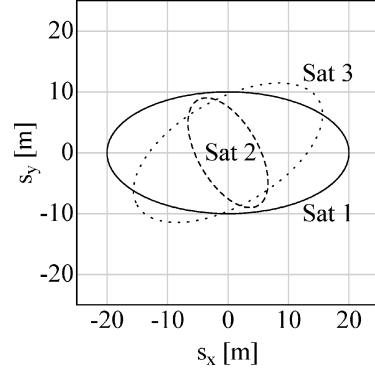
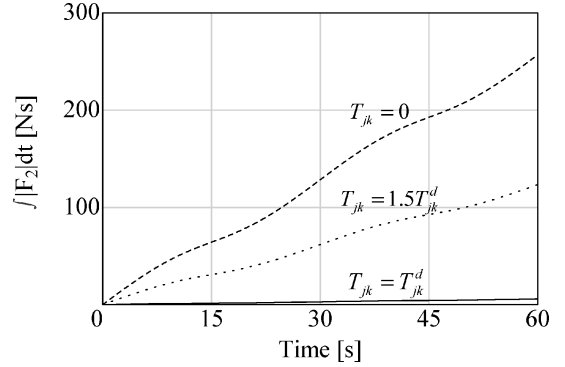
$$T_{jk}^d = m_j m_k \left| \mathbf{q}_k^d - \mathbf{q}_j^d \right| \frac{(\dot{\varphi}_r^{d^2} - \ddot{d}_j^d / d_j^d)}{\sum_{l=1}^n m_l} \quad (37)$$

$$m_j d_j^{d^2} \dot{\varphi}_r^d = m_j d_j^{s^2} \dot{\varphi}_r^s \quad (38)$$

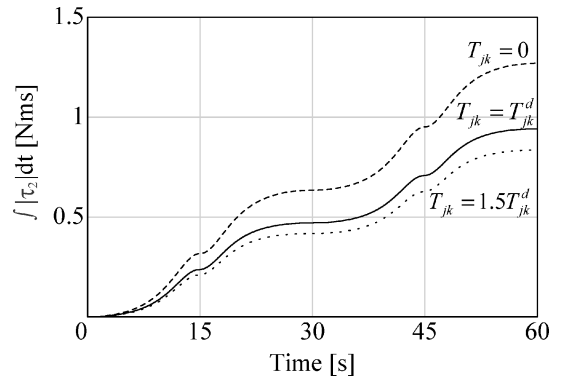
Because $\dot{\varphi}_r^s = \pi/60$, the cycle of the elliptical motion is 60 s.

Table 2 Average of position/attitude errors: similar transformation simulation

Tension	$ \Delta q_2 $, mm	$\sqrt{ \Delta \psi_2 ^2 + \Delta \theta_2 ^2 + \Delta \varphi_2 ^2}$, deg
$T_{jk} = T_{jk}^d$	0.000	0.049
$T_{jk} = 0$	0.843	0.066
$T_{jk} = 1.5T_{jk}^d$	0.420	0.043

a) Trajectory in the $s_x s_y$ plane

b) Accumulated thruster impulse



c) Accumulated reaction-wheel angular impulse

Fig. 9 Similar transformation simulation.

Figure 6 shows the transformation sequence of the motion in the $r_x r_y$ plane. Figure 9a shows the trajectory of each satellite in the $s_x s_y$ plane. The solid, dashed, and dotted lines represent the data for satellites 1, 2, and 3, respectively. Figure 9a indicates that the formation of the system changes in size similarly in the $r_x r_y$ plane and that each satellite performs the desired elliptical motion in the $s_x s_y$ plane. Figures 9b and 9c show the accumulated thruster impulse and the accumulated reaction-wheel angular impulse working on satellite 2. The solid, dashed, and dotted lines represent the three cases of $T_{jk} = T_{jk}^d$, 0, and $1.5T_{jk}^d$. Table 2 shows the average of the relative position/attitude errors of satellite 2 for each case. In Fig. 9b, the sum of the tether tensions in the case of $T_{jk} = T_{jk}^d$ is

equal to the centrifugal force of the desired elliptical motion, and the accumulated thruster impulse is small. The larger the difference between T_{jk} and T_{jk}^d , the larger the accumulated thruster impulse becomes. In general, the accumulation of the thruster impulse for the case of $T_{jk} = \alpha T_{jk}^d$, where α is a constant, is shown as follows:

$$\int |F_j| dt \approx |1 - \alpha| \int m_j d_j^d \left(\dot{\varphi}_r^{d^2} - \frac{\ddot{d}_j^d}{d_j^d} \right) dt \quad (39)$$

The graph inclination in Fig. 9b changes cyclically, which corresponds to the centrifugal force of the elliptical motion.

The larger the tension becomes, the smaller the accumulated reaction wheel angular impulse becomes in Fig. 9c. In a similar transformation, the desired angular velocity $\dot{\varphi}_r^d$ changes cyclically. Therefore a torque $I_j^d \ddot{\varphi}_r^d$ is necessary for the attitude control of each satellite. In the case of $T_{jk} = 0$, the accumulated reaction-wheel angular impulse is nearly equal to

$$\int I_j^d \ddot{\varphi}_r^d dt$$

On the other hand, in the case of $T_{jk} \neq 0$, each tether provides the satellites with torque to correct all attitude errors. Because the torque is proportional to the tension, the accumulated reaction-wheel angular momentum becomes smaller with an increase of the tension.

Figures 9b and 9c and Table 2 indicate that the relative position and attitude errors become smaller with a decrease of the thruster impulse and the reaction-wheel angular impulse. Substituting the tether tension/torque for the thruster and the reaction wheel, we see that each satellite can save thruster fuel and improve the position and attitude control accuracy. The accumulated thruster impulse becomes the smallest in the case of $T_{jk} = T_{jk}^d$. However, the accumulated reaction-wheel angular impulse becomes the smallest in the case of $T_{jk} = 1.5T_{jk}^d$ (maximum tension case).

Sketch of a Two-Dimensional Ground Experiment System

Figure 10 shows the two-dimensional ground experiment system^{20,21} described in this paper. The system consists of satellite simulators and a ground station as shown in Fig. 10b. Next is a description of the components.

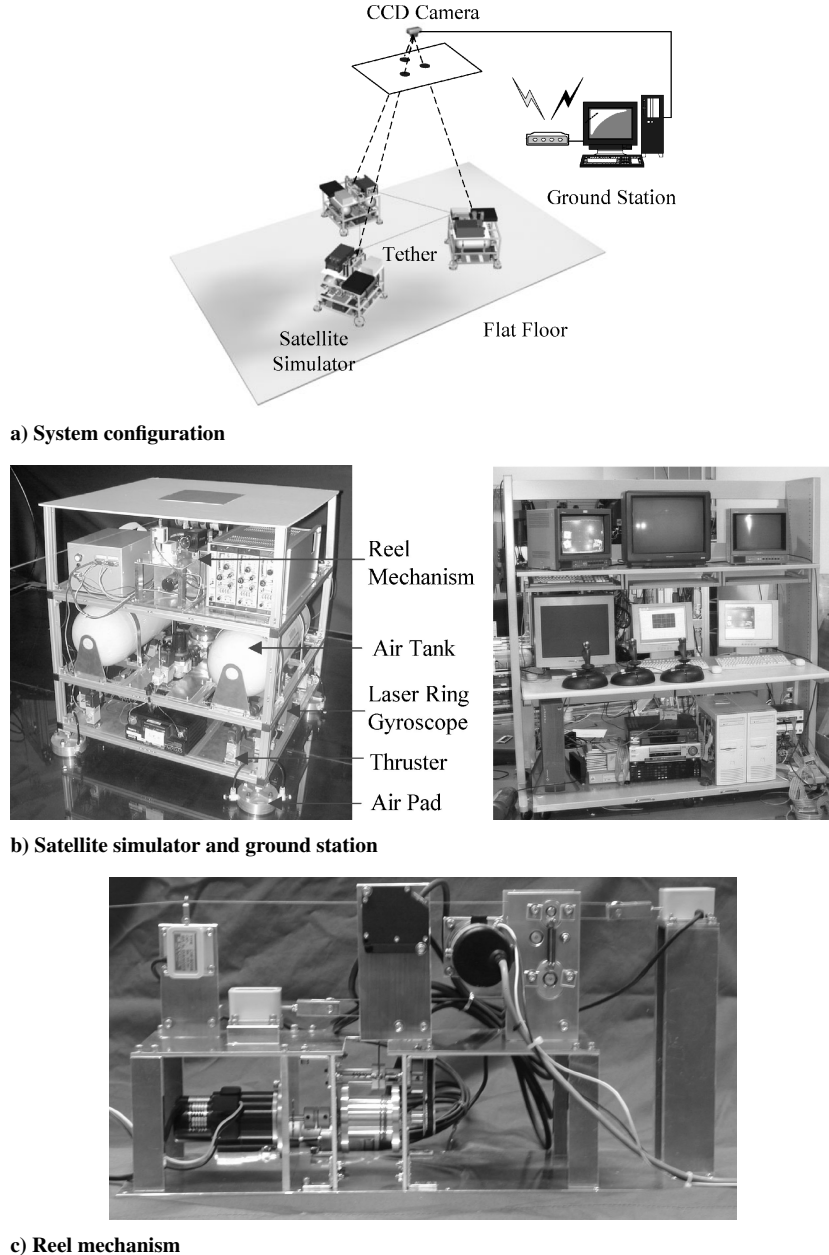


Fig. 10 Two-dimensional ground experiment system.

Satellite Simulator

Each satellite is a solid rectangular parallelepiped ($0.6 \times 0.6 \times 0.7$ m, $m_j = 58$ kg, $I_j = 3.73$ kgm²), floating on a flat floor (3×5 m) with four air pads. The driving force/torque is generated with eight thrusters. In these experiments, the satellites are not equipped with reaction wheels. Thus the thrusters control both the position and attitude of each satellite. The force and torque of one thruster is 2.22 N and 0.444 Nm. (Specific impulse is 24.1 s.) Fuel for the thrusters and air pads is supplied by two air tanks. A laser ring gyroscope provides the attitude data of the satellite and the onboard PC communicates with the ground station by wireless LAN (TCP/IP protocol). In the experiments, the reel mechanisms^{12,23} as shown in Fig. 10c are mounted on the satellite. The size is $140 \times 400 \times 200$ mm, and the weight is 3.95 kg. The main function of the reel mechanism is to deploy and retract the tether. The reel mechanism also serves the following functions: 1) to avoid a jam in the case the tension is nearly zero, 2) to measure tension and direction of the tether, 3) to wind the tether on the reel uniformly, and 4) to measure the deployed length of the tether.

Ground Station

A charge-coupled-device camera captures a picture of the whole motion area, and a PC calculates the position of the satellites by pattern matching. The PC also controls the position/attitude of each satellite through the wireless network.

Ground Experiments

We performed two experiments with the tethered satellite cluster system; a circular motion experiment using three satellite simulators ($n = 3$) and a similar transformation experiment using two satellites ($n = 2$).

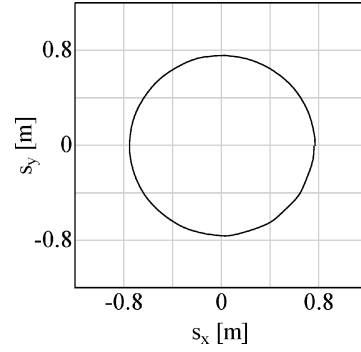
Experimental Conditions

Figure 11 shows the experimental conditions. The system rotates about the c.m. of the system, and the desired formations are a regular triangle for the circular motion and a line segment for the similar transformation. The tether connection point for each satellite is fixed ($l_{jk} = 0.26$ m). The initial conditions are set as $\varphi_i^s = 0$, $\dot{\varphi}_i^s = 2\pi/30$ rad/s, $d_j^s = 0.8$ m, $\lambda_j^s = 2\pi(j-1)/n$, and $\varphi_j^s = \lambda_j^s + \pi$. In this ground experiment system, the motion of each satellite is limited to two dimensions, and $\psi_j^s = 0$, $\theta_j^s = 0$, and $h_j^s = 0$ are satisfied.

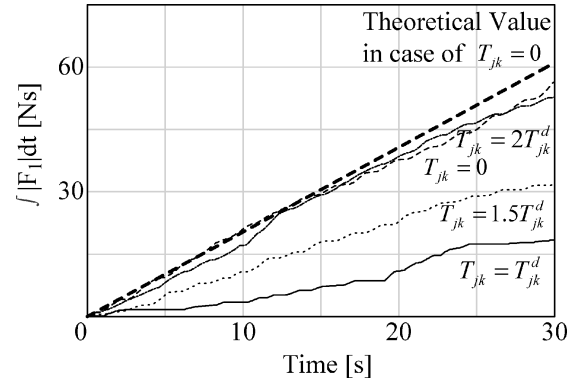
Circular Motion Experiment

The desired motion of each satellite is defined as the circular motion that is caused by maintaining initial motions. Each satellite conducts coordinated control of the reel and thruster to achieve the desired motion. The cycle period of the circular motion is 30 s because $\dot{\varphi}_i^s = 2\pi/30$ rad/s. The equilibrium tension T_{jk}^d of each tether is 1.18 N by Eq. (19).

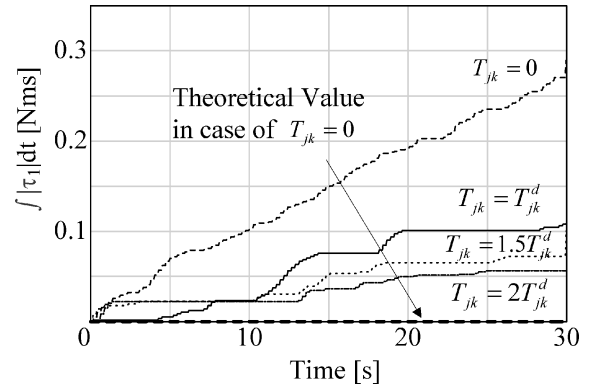
Figure 12a shows the trajectory of satellite 1 in the $s_x s_y$ plane. The trajectories of satellites 2 and 3 are almost the same. The results



a) Trajectory of satellite 1 in the $s_x s_y$ plane



b) Accumulated thruster impulse



c) Accumulated thruster angular impulse

Fig. 12 Circular motion experiment.

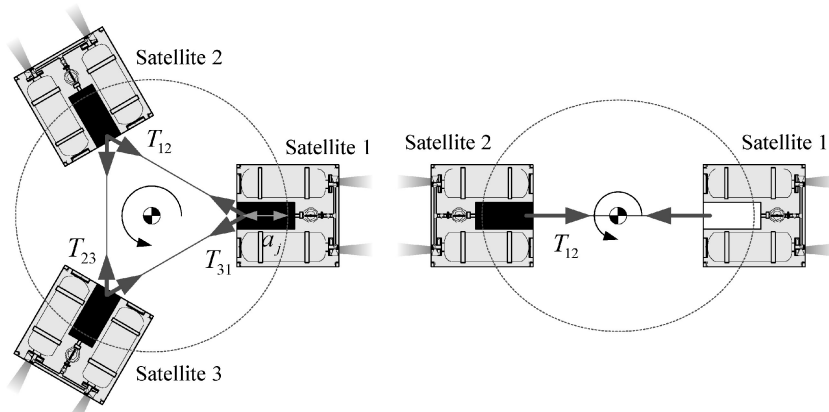


Fig. 11 Experimental conditions (circular and ellipse cases).

Table 3 Average of position/attitude errors: circular motion experiment

Tension	$ \Delta q_1 $, mm	$ \Delta \varphi_1 $, deg
$T_{jk} = T_{jk}^d$	24.7	4.36
$T_{jk} = 0$	31.3	7.00
$T_{jk} = 1.5T_{jk}^d$	34.1	3.61
$T_{jk} = 2T_{jk}^d$	45.7	4.83

indicate that the system maintains the desired circular motion. Figures 12b and 12c show the accumulated thruster impulse and the accumulated thruster angular impulse acting on the satellite 1. The solid, dashed, dotted, and dot-dashed lines represent, respectively, the four cases of $T_{jk} = T_{jk}^d$ or 0 or $1.5T_{jk}^d$ or $2T_{jk}^d$. The bold broken line represents the following theoretical values in the case of $T_{jk} = 0$ (no tether):

$$\int |F_1| dt = m_1 d_1^d \dot{\varphi}_r^{d^2} t, \quad \int |\tau_1| dt = \int |I_1^d \ddot{\varphi}_r^d| dt = 0 \quad (40)$$

In the case of $T_{jk} = T_{jk}^d$, the accumulated thruster impulse is the smallest, however, not equal to zero, because of the friction and inclination of the flat floor, tension control error, and pattern matching error. The accumulated impulse in the case of $T_{jk} = 0$ is nearly equal to the theoretical value as shown in Eq. (40). It is also nearly equal to the case of $T_{jk} = 2T_{jk}^d$. Note that thrust directions in the cases of $T_{jk} = 0$ and $2T_{jk}^d$ are opposite to each other. The accumulated impulse in the case of $T_{jk} = 1.5T_{jk}^d$ becomes the average value of the impulses in the case of $T_{jk} = T_{jk}^d$ and $2T_{jk}^d$. The fuel mass difference between the two cases $T_{jk} = T_{jk}^d$ and 0 is 0.0726 kg (specific impulse is 24.1 s) per circular cycle. These results indicate that the tether tension saves thruster fuel in the case of $T_{jk} = T_{jk}^d$.

None of the accumulated thruster angular impulses in all cases of T_{jk} is equal to zero because of the friction and inclination of the flat floor and position control error. The larger the tension becomes, the smaller the accumulated angular impulse becomes. This is because the tether torque to correct the attitude error is proportional to the tension.

Table 3 shows the average of the position/attitude errors of satellite 1 for each case. The results for other satellites are nearly equal to those of satellite 1. The average of the position error is the smallest in case of $T_{jk} = T_{jk}^d$. The average of the attitude error is the largest in case of $T_{jk} = 0$ because the tension torque is the smallest (zero) in this case.

Similar Transformation Experiment

As a similar transformation, each satellite performs the elliptical motion defined in Eq. (35). The long and short radii are set as $\alpha_j = d_j^s$, $\beta_j = 0.85d_j^s$, respectively. The cycle period of the elliptical motion is 25.5 s.

Figure 13a shows the trajectory of satellite 1. Figures 13b and 13c show the accumulated thruster impulse and the accumulated thruster angular impulse acting on satellite 1. The solid, dashed, and dotted lines represent the three cases of $T_{jk} = T_{jk}^d$ or 0 or $1.5T_{jk}^d$. The bold broken line represents the following theoretical values in the case of $T_{jk} = 0$ (no tether):

$$\int |F_1| dt = \int m_1 d_1^d \left(\dot{\varphi}_r^{d^2} - \frac{\ddot{d}_1^d}{d_1^d} \right) dt$$

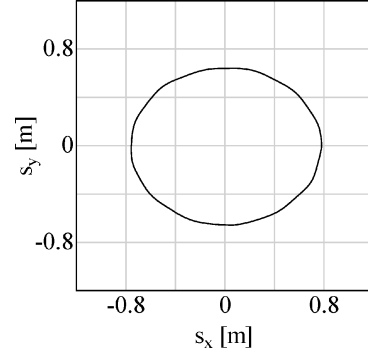
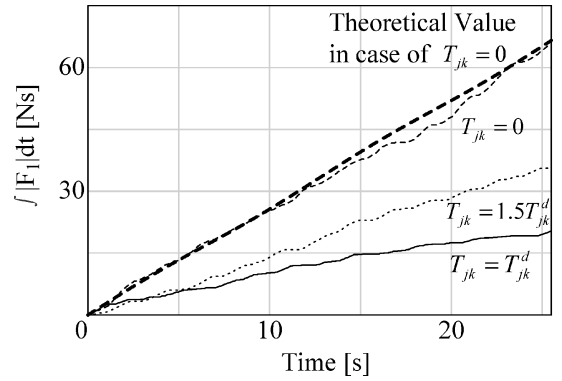
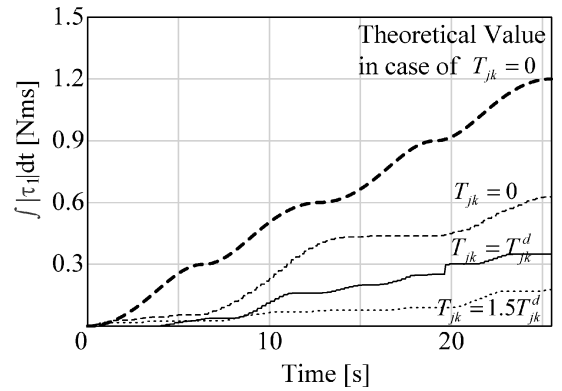
$$\int |\tau_1| dt = \int |I_1^d \ddot{\varphi}_r^d| dt \quad (41)$$

Table 4 shows the average of the position/attitude errors of satellite 1 for each case.

In the case of $T_{jk} = T_{jk}^d$, the accumulated thruster impulse is the smallest, however, not equal to zero, because of the same errors as mentioned in the preceding circular motion experiment. The accumulated impulse in the case of $T_{jk} = 0$ is nearly equal to the theoretical value.

Table 4 Average of position/attitude errors: similar transformation experiment

Tension	$ \Delta q_1 $, mm	$ \Delta \varphi_1 $, deg
$T_{jk} = T_{jk}^d$	21.2	3.31
$T_{jk} = 0$	33.0	9.20
$T_{jk} = 1.5T_{jk}^d$	37.3	3.36

**a) Trajectory of satellite 1 in the $s_x s_y$ plane****b) Accumulated thruster impulse****c) Accumulated thruster angular impulse****Fig. 13** Similar transformation experiment.

The accumulated thruster angular impulse is the largest in the case of $T_{jk} = 0$. The larger the tension, the smaller the accumulated angular impulse.

The average of the position error is the smallest in the case of $T_{jk} = T_{jk}^d$, and the average of the attitude error is the largest in the case of $T_{jk} = 0$ in Table 4. These are the same conclusions as with the circular motion experiment.

Conclusions

This paper has discussed the dynamics of tethered satellite clusters in orbit. The control for a same-plane formation rotating with respect to the c.m. of the system was developed and evaluated. The following conclusions were made:

1) The paper presented the equilibrium conditions for maintaining system formation and performing a circular motion by tether tension.

2) The paper presented a coordinated control method for circular motion that minimizes thruster fuel by using the reel, arm, thruster, and reaction wheel.

3) The paper presented the equilibrium conditions for changing the system formation in size similarly.

4) The paper presented a coordinated control method for similar transformations.

5) Numerical simulations and ground experiments were conducted and showed that these control methods can save thruster fuel and improve control accuracy of the position/attitude of each satellite, using the tether tension/torque in addition to the thruster force/reaction-wheel torque.

Appendix: Equation Form of Circular Motion

Newton–Euler Equations (1) and (2) are described by elements of vectors as follows:

$$m_j \ddot{q}_j = \frac{\mu m_j}{|q_j|^3} q_j + {}^i F_j + \sum_k \frac{{}^i q_k + {}^i a_{kj} - {}^i q_j - {}^i a_{jk}}{|{}^i q_k + {}^i a_{kj} - {}^i q_j - {}^i a_{jk}|} T_{jk} \quad (A1)$$

$$\begin{aligned} {}^b I_j {}^b \ddot{\omega}_j + {}^b \tilde{\omega}_j {}^b I_j {}^b \dot{\omega}_j &= \frac{3\mu}{|{}^b q_j|^3} {}^b \tilde{q}_j {}^b I_j {}^b \dot{q}_j + {}^b \tau_j \\ &+ \sum_k \frac{{}^b \tilde{a}_{jk} ({}^b q_k + {}^b a_{kj} - {}^b q_j - {}^b a_{jk})}{|{}^b q_k + {}^b a_{kj} - {}^b q_j - {}^b a_{jk}|} T_{jk} \end{aligned} \quad (A2)$$

The last terms of Eqs. (A1) and (A2) are relating to tension and torque of tether, respectively. They are unique to tethered satellite cluster system. Each satellite can control the motion by modifying parameters T_{jk} and ${}^b a_{jk}$ as well as ${}^i F_j$ and ${}^b \tau_j$. If Eqs. (22) and (23) are satisfied, these terms become $({}^i q_k - {}^i q_j / |{}^i q_k - {}^i q_j|) T_{jk}^d$ and 0, respectively. In case of the circular motion of all-connection type, Eqs. (A1) and (A2) are rewritten as follows, considering Eqs. (5) and (19):

$$m_j \ddot{q}_j = \frac{\mu m_j}{|q_j|^3} q_j + {}^i F_j + m_j |q_k - {}^i q_{c.m.}| \dot{\phi}_r^2 \quad (A3)$$

$${}^b I_j {}^b \ddot{\omega}_j + {}^b \tilde{\omega}_j {}^b I_j {}^b \dot{\omega}_j = \frac{3\mu}{|{}^b q_j|^3} {}^b \tilde{q}_j {}^b I_j {}^b \dot{q}_j + {}^b \tau_j + 0 \quad (A4)$$

The motion equations of the other connection types are described as the same. Thus the circular motion can be maintained, if ${}^i F_j = -(\mu m_j / |q_j|^3) q_j$ and ${}^b \tau_j = -(3\mu / |{}^b q_j|^3) {}^b \tilde{q}_j {}^b I_j {}^b \dot{q}_j$ are satisfied by the feedback controls of Eqs. (20) and (21). In this paper, they are small relatively because the orbital rate is supposed to be much larger than the spin rate of the formation.

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