# **Modeling the Entrainer Effects on Solubility of Solutes in Supercritical Carbon Dioxide**

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**Applicability of a previously published equation for calculating the solubility of solutes in supercritical carbon dioxide was extended to calculate the solubility in entrained supercritical carbon dioxide employing 42 experimental data sets collected from the literature. The accuracy of the proposed model was evaluated by calculating both the average absolute relative deviation and the individual absolute relative deviation. The proposed model showed superiority to a previously published similar model, from both correlation and prediction points of view.**

**Key words** solubility; supercritical carbon dioxide; entrainer; calculation

Supercritical fluids (SCF) have been employed in the pharmaceutical industry for production of fine drug particles with narrow size distribution, separation of active ingredients and preparation of microemulsions and sustained drug delivery systems.<sup>1)</sup> Supercritical carbon dioxide (SC-CO2) is a very popular fluid for pharmaceutical applications, as it is nontoxic, non-flammable and of low cost. Furthermore, because of its low critical temperature and pressure, SC-CO2 is ideal for thermal-liable compounds. In addition to the pharmaceutical applications, supercritical fluids are very important solvents in many other industrial processes, including decaffeination of coffee, extraction of hops, spices and seed oils, as well as decontamination of environmental wastes. However, the solubility of most pharmaceutical/chemical compounds is very low in SC-CO2 and an entrainer is usually added to enhance the solubility.

Solubility data are required for designing SCF processes but experimental measurement of solubility is time-consuming and costly. Only a limited number of reports has been published on the solubility of pharmaceutical/chemical compounds in entrained supercritical fluids. Mathematical modelling of solubility data in  $SC-CO2 +$ entrainer could provide useful information for pharmaceutical/chemical engineers to speed up the process. Thermodynamic models employing equations of state, lattice gas equations or expanded liquid models have been used for modelling such data. These methods require complicated computations and a number of physico-chemical properties which are often unavailable. The aim of this study was to present a simple and easy to use empirical model to calculate the solubility in  $SC-CO2 + en$ trainer with respect to the concentration of the entrainer, pressure, temperature and density of pure SC-CO2. The accuracy of the proposed model is evaluated employing published experimental data sets and compared with that of a model from the literature. Reported experimental data sets containing solubility data of pharmaceutical/chemical compounds at different temperatures, pressures and entrainer concentrations were included in this study. Data sets containing the solubility at only one temperature, pressure or entrainer concentration, as well as data sets with data points numbering less than 30 (five points for each independent variable) were excluded from this study.

#### **Computational Methods**

An empirical equation has been reported<sup>2)</sup> to correlate the mole fraction solubility  $(y_2)$  of a solute in pure SC-CO2 with respect to the density of pure SC-CO2  $(\rho)$ , pressure  $(P)$  and temperature  $(T)$ :

$$
\ln y_2 = K_0 + K_1 \rho + K_2 P^2 + K_3 P T + \frac{K_4 T}{P} + K_5 \ln \rho \tag{1}
$$

where  $K_0$ — $K_5$  are the model constants computed using a least squares analysis.2) The accuracy of Eq. 1 has been evaluated employing 23 data sets and a mean correlation error  $(\pm S.D.)$  of 12.6  $(\pm 7.4)$ % was obtained. The model is applicable for calculating the solubility in the absence of an entrainer, and the introduction of one more term representing the concentration (in mole, weight and/or volume fraction) of the entrainer (*X*) enables it to compute the solubility in SC-CO2+entrainer. The proposed model is:

$$
\ln y_2 = M_0 + M_1 X + M_2 \rho + M_3 P^2 + M_4 P T + \frac{M_5 T}{P} + M_6 \ln \rho
$$
 (2)

where  $M_0$ — $M_6$  are the model constants and the numerical value of  $\rho$  is calculated by Eq.  $3.^3$ )

$$
\ln \rho = -27.091 + 0.609\sqrt{T} + \frac{2966.170}{T} - \frac{3.445P}{T} + 0.401\sqrt{P}
$$
 (3)

The  $M_0$ — $M_6$  terms should be computed using a least squares method by employing experimental data of entrained SC-CO2; one cannot use the numerical values of  $K_0$ — $K_5$  terms from pure SC-CO2 solubility data. The next limitation of Eq. 2 is that it could be used to predict the solubility data with an interpolation technique.

Gonzalez and co-workers<sup>4)</sup> extended the mass action law-based Chrastil model to correlate the solubility data in entrained SC-CO2. Their model could be rewritten as:

$$
\ln S = a + \frac{b}{T} + c \ln X + d \ln \rho \tag{4}
$$

where *S* is the solubility of a solute (g/l), and  $a$ ,  $b$ ,  $c$  and  $d$  are the model constants. The authors noticed that *a* is equal to  $[q - d\ln(M_c) - d\ln(c)]$  $d \ln(M_e) + \ln(M_s + dM_c + cM_e)$ ] = constant, in which  $M_s$ ,  $M_c$  and  $M_e$  are the molecular weights of the solute, carbon dioxide and entrainer, respectively, *q* is a constant,  $b = \Delta H_{total}/R$  in which  $\Delta H_{total}$  is the total reaction enthalpy and *R* the molar gas constant, *c* the association number of entrainer and *d* the association number of carbon dioxide. Although the Chrastil model was originally proposed to correlate solubility (in grams per liter) with density and temperature, the model was employed to correlate the mole fraction solubilities as well.<sup>5,6)</sup> In addition, it has been shown that for Eq. 4 replacing  $g/l$ solubility with mole fraction solubility provides more accurate calculations.<sup>3)</sup> Therefore, the logarithm of mole fraction solubility ( $\ln y_2$ ) was used as a dependent variable of the Chrastil model in this work. The main limitation of Eq. 4 is that it is not applicable to pure SC-CO2 since ln(0) is not a defined value.

A global fitness test for the multiple regression model was evaluated using the *F* value, found by splitting the total variations of the response variable into one part due to the regression and one part due to the residual, or error. The numerical value of *F* is the ratio of the mean squares due to regression to that of the residuals, and the higher the *F* value is, the more significant the correlation. The significance level (s.l.) of the *F* value (with degrees of freedom (df) related to the number of independent variables (*V*) and number of data points in each set (*N*), *i.e.* df= $N-V-1$ ) is provided by statistical software such as SPSS (Statistical Package for Social Sciences). A significant *F* value means that not all the model constants (*i.e.*,  $M_0$ — $M_6$  in this work) are zero although some of them might be, depending on the data set.

In order to provide a reliable accuracy criterion to compare the models possessing different numbers of curve-fitting parameters, the average absolute relative deviations (AARD) were used, and were calculated by:

$$
AARD = \frac{100}{(N-Z)} \sum \frac{|y_2^{\text{calulated}} - y_2^{\text{observed}}|}{y_2^{\text{observed}}}
$$
 (5)

where *N* is the number of data points in each set and *Z* the number of curvefitting parameters for each model (*Z* is equal to 7 and 4 for Eqs. 2 and 4, respectively). To test the prediction capability of the models after training using a minimum number of experimental data points, the individual absolute relative deviations (IARD) of predicted solubilities from observed values were calculated by:

$$
IARD = 100 \left( \frac{|y_2^{\text{calulated}} - y_2^{\text{observed}}|}{y_2^{\text{observed}}} \right)
$$
 (6)

Since the aim of the present study was to compare the proposed equation with a previously published model, and because the number of curve-fitting parameters was not equal in the two models, AARD values were preferred over the IARD. It is obvious that the mean IARD values were less than the corresponding AARDs.

## **Results and Discussion**

Table 1 shows details of the systems studied, the references, the number of data points in each set and the AARDs for correlation studies. The minimum AARD (3.8) is observed for perylene in  $SC-CO2 +$ methanol and the maximum AARD (24.2) for naproxen in  $SC-CO2+1$ -propanol whereas the mean  $(\pm S.D.)$  of AARD is  $10.9 \pm 5.4\%$ . The corresponding minimum and maximum values of AARD for Eq. 4 are 4.0% for perylene in SC-CO2+methanol and  $44.8\%$  for

Table 1. Details of Systems Studied, Number of Correlated Data Points in Each Set (*N*) and Average Absolute Relative Deviation (AARD) for Eqs. 2 and 4 Using Correlative Study

| $\mathrm{SN}^{a)}$ | Solute                                  | Entrainer     | Reference | $\cal N$ | Eq. $2$            | Eq. $4$             |
|--------------------|---|---------------|-----------|----------|--------------------|---------------------|
| $\mathbf{1}$       | $2,2',3,4,4',5,5'$ -Heptachlorobiphenyl | <b>Butane</b> | 11        | 96       | 10.1               | 41.0                |
| $\overline{2}$     | 2,2',3,4,4',5,5'-Heptachlorobiphenyl    | Methanol      | 11        | 96       | 10.8               | 37.0                |
| 3                  | 2,2',4,4',5,5'-Hexachlorobiphenyl       | <b>Butane</b> | 11        | 96       | 8.6                | 33.6                |
| $\overline{4}$     | 2,2',4,4',5,5'-Hexachlorobiphenyl       | Methanol      | 11        | 96       | 8.1                | 30.6                |
| 5                  | 2,2',4,5,5'-Pentachlorobiphenyl         | <b>Butane</b> | 11        | 96       | 9.9                | 39.7                |
| 6                  | 2,2',4,5,5'-Pentachlorobiphenyl         | Methanol      | 11        | 96       | 9.6                | 35.5                |
| $\overline{7}$     | 2,4',5-Trichlorobiphenyl                | <b>Butane</b> | 11        | 96       | 6.8                | 27.5                |
| 8                  | 2,4',5-Trichlorobiphenyl                | Methanol      | 11        | 96       | 6.8                | 24.8                |
| 9                  | 2-2'-Dichlorobiphenyl                   | <b>Butane</b> | 11        | 96       | 9.3                | 35.2                |
| 10                 | 2-2'-Dichlorobiphenyl                   | Methanol      | 11        | 96       | 9.2                | 32.9                |
| 11                 | 2-Monochlorobiphenyl                    | <b>Butane</b> | 11        | 96       | 6.7                | 25.5                |
| 12                 | 2-Monochlorobiphenyl                    | Methanol      | 11        | 96       | 6.6                | 24.1                |
| 13                 | 3,3',4,4'-Tetrachlorobiphenyl           | <b>Butane</b> | 11        | 96       | 10.9               | 44.8                |
| 14                 | 3,3',4,4'-Tetrachlorobiphenyl           | Methanol      | 11        | 96       | 10.6               | 41.8                |
| 15                 | 4,4'-Dichlorobiphenyl                   | <b>Butane</b> | 11        | 96       | 9.0                | 31.4                |
| 16                 | 4,4'-Dichlorobiphenyl                   | Methanol      | 11        | 96       | 8.6                | 27.7                |
| 17                 | 4-Monochlorobiphenyl                    | <b>Butane</b> | 11        | 96       | 8.5                | 31.9                |
| 18                 | 4-Monochlorobiphenyl                    | Methanol      | 11        | 96       | 8.4                | 31.5                |
| 19                 | Anthracene                              | <b>Butane</b> | 12, 13    | 164      | 8.7                | 13.7                |
| 20                 | Anthracene                              | Ethane        | 12, 13    | 172      | 7.7                | 12.4                |
| 21                 | Anthracene                              | Methanol      | 12, 13    | 99       | 8.7                | 10.0                |
| 22                 | Anthracene                              | Propane       | 12, 13    | 176      | 9.0                | 15.2                |
| 23                 | Behenic acid                            | Ethanol       | 14        | 53       | 19.4               | 18.4                |
| 24                 | Behenic acid                            | $n$ -Octane   | 15        | 52       | 16.4               | 26.4                |
| 25                 | Behenic acid                            | Pentane       | 15        | 49       | 22.6               | 37.2                |
| 26                 | beta-Carotene                           | Ethanol       | 16        | 31       | 22.7               | 7.1                 |
| 27                 | beta-Carotene                           | Vegetable oil | 16        | 37       | 11.5               | 8.5                 |
| 28                 | Naproxen                                | 1-Propanol    | 17        | 33       | 24.2               | 12.1                |
| 29                 | Naproxen                                | 2-Propanol    | 17        | 40       | 21.0               | 16.5                |
| 30                 | Naproxen                                | Acetone       | 17        | 51       | 17.4               | 17.7                |
| 31                 | Naproxen                                | Ethanol       | 17        | 42       | 18.8               | 16.0                |
| 32                 | Naproxen                                | Ethyl acetate | 17        | 36       | 15.4               | 17.6                |
| 33                 | Naproxen                                | Methanol      | 17        | 44       | 18.7               | 21.3                |
| 34                 | Perylene                                | Methanol      | 12, 13    | 42       | 3.8                | 4.0                 |
| 35                 | Phenanthrene                            | Methanol      | 12, 13    | 108      | 6.7                | 10.6                |
| 36                 | Phenanthrene                            | Methanol      | 12, 13    | 92       | 5.4                | 15.9                |
| 37                 | Phenanthrene                            | Propane       | 12, 13    | 120      | 5.8                | 7.6                 |
| 38                 | Phenanthrene                            | Ethane        | 12, 13    | 128      | 4.6                | 11.0                |
| 39                 | Pyrene                                  | <b>Butane</b> | 12, 13    | 184      | 8.5                | 4.8                 |
| 40                 | Pyrene                                  | Ethane        | 12, 13    | 184      | 5.6                | 12.6                |
| 41                 | Pyrene                                  | Methanol      | 12, 13    | 112      | 6.8                | 14.6                |
| 42                 | Pyrene                                  | Propane       | 12, 13    | 184      | 8.6                | 10.7                |
|                    |   |               |           |          | $10.9 \pm 5.4^{b}$ | $22.3 \pm 11.6^{b}$ |

*a*) SN is the system number. *b*) The mean AARD difference of Eqs. 2 and 4 is statistically significant (paired *t*-test, s.l. < 0.0005).

3,  $3'$ , 4, 4' tetrachlorobiphenyl in SC-CO2+butane, respectively, and the mean  $(\pm S.D.)$  is  $22.3 \pm 11.6\%$ . Both the mean and standard deviation of Eq. 4 are twice those of Eq. 2. The low error value for Eq. 2 could be considered as acceptable when it is compared with the relative standard deviation, which is around 15%, for repeated experiments under the same experimental conditions. $^{7)}$ 

Table 2 shows the correlation coefficients (c.c.), standard error (s.e.), *F* and significance levels (s.l.) for Eqs. 2 and 4. The correlation coefficients of Eq. 2 vary between 0.974 to 0.999, showing a good correlation between the independent variable (*i.e.*,  $\ln y_2$ ) and dependent variables (*i.e.*  $\rho$ , *X*,  $P^2$ ,  $P \cdot T$ ,  $T/P$  and  $\ln(\rho)$ ) as also indicated by high *F* values, low standard errors and significance levels of  $< 0.0005$ . Equation 4 in only two cases produces c.c. values greater than the corresponding c.c. values for Eq. 2, indicating that the proposed model is able to provide better correlations than the previously published Eq. 4.

The RSD values for repeated experiments using the same operating conditions in the same laboratory was reported to be relatively high, *i.e.*, 10—25% (see refs. 7—9). However, the discrepancies between solubility data for a given solute obtained under the same conditions from different laboratories were even higher (*e.g.*, differences by a factor of 10, see ref. 10 and references herein). These differences may be caused by the presence of impurities, differences in pressure or temperature calibrations, or technical variations during solubility measurements. In other words, there is no reliable solution to the use of previously collected data from another laboratory to design proper SCF technology. On the other hand, solubility measurements are costly and also time-consuming and therefore, it is not recommended that solubility be measured under all operational conditions of interest. As an alternative solution, a minimum number of experimental solubility data at pressures, temperatures and entrainer concentrations of interest could be collected. Then the data

Table 2. The Correlation Coefficients (c.c.), Standard Error (s.e.), F and Significance Levels (s.l.) for Eqs. 2 and 4 for Correlative Studies

| $SN^{(a)}$     | Eq. $2$ |        |                           |          | Eq. 4 |        |                           |          |
|----------------|---------|--------|---------------------------|----------|-------|--------|---------------------------|----------|
|                | c.c.    | s.e.   | $\boldsymbol{\mathrm{F}}$ | s.1.     | c.c.  | s.e.   | $\boldsymbol{\mathrm{F}}$ | s.1.     |
| $\mathbf{1}$   | 0.991   | 0.2634 | 42.564                    | < 0.0005 | 0.963 | 0.4965 | 18.899                    | < 0.0005 |
| $\overline{c}$ | 0.991   | 0.2581 | 41.344                    | < 0.0005 | 0.964 | 0.4783 | 17.981                    | < 0.0005 |
| 3              | 0.992   | 0.2177 | 31.741                    | < 0.0005 | 0.951 | 0.4881 | 13.634                    | < 0.001  |
| 4              | 0.990   | 0.2330 | 30.743                    | < 0.0005 | 0.957 | 0.4486 | 13.046                    | < 0.001  |
| 5              | 0.991   | 0.2663 | 44.798                    | < 0.0005 | 0.965 | 0.4858 | 19.112                    | < 0.0005 |
| 6              | 0.991   | 0.2711 | 45.355                    | < 0.0005 | 0.965 | 0.4943 | 19.827                    | < 0.0005 |
| $\overline{7}$ | 0.989   | 0.1808 | 15.815                    | < 0.0005 | 0.950 | 0.3416 | 6.482                     | < 0.001  |
| 8              | 0.989   | 0.1777 | 15.682                    | < 0.0005 | 0.948 | 0.3481 | 6.402                     | < 0.001  |
| 9              | 0.991   | 0.2342 | 35.033                    | < 0.0005 | 0.961 | 0.4571 | 15.086                    | < 0.0005 |
| $10\,$         | 0.991   | 0.2346 | 35.014                    | < 0.0005 | 0.959 | 0.4675 | 15.126                    | < 0.001  |
| 11             | 0.990   | 0.1787 | 17.619                    | < 0.0005 | 0.957 | 0.3300 | 7.108                     | < 0.001  |
| 12             | 0.992   | 0.1665 | 18.037                    | < 0.0005 | 0.955 | 0.3462 | 7.502                     | < 0.001  |
| 13             | 0.991   | 0.3042 | 55.875                    | < 0.0005 | 0.964 | 0.5651 | 24.852                    | < 0.0005 |
| 14             | 0.991   | 0.2982 | 56.433                    | < 0.0005 | 0.963 | 0.5776 | 25.502                    | < 0.0005 |
| 15             | 0.989   | 0.1757 | 15.329                    | < 0.0005 | 0.937 | 0.3670 | 5.864                     | < 0.002  |
| 16             | 0.991   | 0.1632 | 15.585                    | < 0.0005 | 0.942 | 0.3621 | 6.230                     | < 0.001  |
| 17             | 0.991   | 0.2301 | 32.574                    | < 0.0005 | 0.966 | 0.3983 | 13.328                    | < 0.0005 |
| 18             | 0.990   | 0.2434 | 33.241                    | < 0.0005 | 0.961 | 0.4373 | 14.025                    | < 0.0005 |
| 19             | 0.991   | 0.1502 | 13.995                    | < 0.0005 | 0.966 | 0.2926 | 9.622                     | < 0.0005 |
| 20             | 0.998   | 0.0806 | 16.737                    | < 0.0005 | 0.960 | 0.3595 | 12.138                    | < 0.0005 |
| 21             | 0.980   | 0.2064 | 8.421                     | < 0.0005 | 0.953 | 0.2359 | 2.777                     | < 0.005  |
| 22             | 0.998   | 0.0797 | 15.324                    | < 0.0005 | 0.964 | 0.3201 | 10.836                    | < 0.0005 |
| 23             | 0.986   | 0.2274 | 38.368                    | < 0.0005 | 0.983 | 0.1931 | 17.847                    | < 0.0005 |
| 24             | 0.982   | 0.2300 | 27.059                    | < 0.0005 | 0.950 | 0.3680 | 17.588                    | < 0.0005 |
| 25             | 0.986   | 0.2405 | 37.170                    | < 0.0005 | 0.917 | 0.4963 | 18.280                    | < 0.0005 |
| 26             | 0.974   | 0.2561 | 18.523                    | < 0.0005 | 0.984 | 0.0877 | 2.117                     | < 0.0005 |
| 27             | 0.995   | 0.0939 | 11.106                    | < 0.0005 | 0.989 | 0.0635 | 1.597                     | < 0.0005 |
| 28             | 0.991   | 0.2517 | 23.196                    | < 0.0005 | 0.993 | 0.1645 | 6.121                     | < 0.001  |
| 29             | 0.989   | 0.2580 | 24.773                    | < 0.0005 | 0.981 | 0.2669 | 7.425                     | < 0.003  |
| 30             | 0.987   | 0.1821 | 13.375                    | < 0.0005 | 0.959 | 0.2644 | 5.645                     | < 0.0005 |
| 31             | 0.991   | 0.2257 | 21.413                    | < 0.0005 | 0.983 | 0.2480 | 7.015                     | < 0.002  |
| 32             | 0.985   | 0.2126 | 10.110                    | < 0.0005 | 0.980 | 0.2595 | 4.954                     | < 0.008  |
| 33             | 0.989   | 0.2426 | 27.354                    | < 0.0005 | 0.967 | 0.3464 | 10.438                    | < 0.001  |
| 34             | 0.997   | 0.0555 | 2.247                     | < 0.0005 | 0.987 | 0.0858 | 1.417                     | < 0.0005 |
| 35             | 0.996   | 0.1254 | 21.116                    | < 0.0005 | 0.993 | 0.1543 | 13.440                    | < 0.0005 |
| 36             | 0.996   | 0.1240 | 15.111                    | < 0.0005 | 0.981 | 0.2454 | 7.702                     | < 0.001  |
| 37             | 0.996   | 0.1088 | 16.305                    | < 0.0005 | 0.990 | 0.1460 | 8.627                     | < 0.0005 |
| 38             | 0.999   | 0.0570 | 19.719                    | < 0.0005 | 0.978 | 0.2617 | 12.192                    | < 0.0005 |
| 39             | 0.976   | 0.2737 | 16.414                    | < 0.0005 | 0.995 | 0.0989 | 7.802                     | < 0.0005 |
| 40             | 0.999   | 0.0574 | 19.131                    | < 0.0005 | 0.971 | 0.2933 | 10.071                    | < 0.0005 |
| 41             | 0.997   | 0.1303 | 17.666                    | < 0.0005 | 0.978 | 0.3128 | 8.684                     | < 0.003  |
| 42             | 0.985   | 0.2151 | 17.964                    | < 0.0005 | 0.988 | 0.1600 | 9.618                     | < 0.0005 |

*a*) SN is the system number and the details are the same as in Table 1.

could be used to train the proposed model and after successful training, the solubility data at any pressure, temperature and entrainer concentration could be predicted using the interpolation technique within the range studied.

In order to test the prediction capability of the proposed model, a minimum number of experimental data points was used to train the models and the solubility at other data points was predicted using trained models. The training data points included the experimental solubility data at the highest, the intermediate and the lowest temperatures and pressures of interest for each entrainer concentration. Both models were trained using the same training data points and these points were selected from previously published data sets; when the model is trained, it is capable of predicting solubility at other temperatures, pressures and entrainer concentrations of interest using the interpolation technique. Although the trained model could be used under the same experimental conditions as in the training process, there is no guarantee that the model can be employed for other conditions. However, the main advantage of the proposed model is its capability to reduce the number of experiments required in designing the supercritical process. This is a valuable tool in practice, since many variations were reported for the solubility of the solutes from various laboratories.<sup>10)</sup> Tables 3 and 4 listed the numerical values of the model constants, the number of predicted data points in each set and AARD values, respectively, for Eqs. 2 and 4. The mean values  $(\pm S.D.)$  of AARD for Eqs. 2 and 4 are  $14.23 \pm 6.13$  and  $20.85 \pm 7.24$ , respectively, and the difference between the means was statistically significant (s.l. $\leq 0.0005$ ). A summary of the IARD values for Eqs. 2 and 4 sorted in three error subgroups is shown in Fig. 1. The frequency of IARD being  $\leq 10\%$  is the highest and that of  $IARD>30\%$  is the lowest for Eq. 2. The probability of solubility prediction with an error of less than 30% is 0.95, indicating a good agreement between the model predicted solubilities and the experimental results. The corre-

Table 3. The Model Constants of Eq. 2 Calculated Based on a Minimum Number of Training Data Points, Number of Predicted Data Points in Each Set (*N*) and Average Absolute Relative Deviation (AARD)

| $SN^{(a)}$     | $M_0$      | $M_1$   | $M_2$      | $M_{\rm{3}}$     | $M_{4}$         | $M_{\rm s}$ | $M_{6}$   | $N^{b)}$ | $N^{c)}$ | <b>AARD</b>      |
|----------------|------------|---------|------------|------------------|-----------------|-------------|-----------|----------|----------|------------------|
| $\mathbf{1}$   | 1.8480     | 6.4350  | $-14.1690$ | $-2.4880E - 05$  | $6.254E - 05$   | 0.0169      | 10.3570   | 18       | 78       | 10.9             |
| $\overline{c}$ | 1.4900     | 4.6260  | $-14.1310$ | $-2.6550E - 05$  | $6.667E - 05$   | 0.0667      | 10.2570   | 18       | 78       | 11.6             |
| $\mathfrak z$  | 3.8110     | 4.9070  | $-14.4680$ | $-7.7430E - 06$  | $3.259E - 05$   | 0.1840      | 11.0470   | 18       | 78       | $10.0\,$         |
| $\overline{4}$ | 1.2710     | 3.7720  | $-13.0920$ | $-2.1380E - 05$  | $5.541E - 05$   | 0.4040      | 10.2960   | 18       | 78       | 10.7             |
| 5              | 4.3510     | 5.6720  | $-14.3510$ | $-1.2310E - 05$  | $3.959E - 05$   | 0.2320      | 11.7180   | 18       | 78       | 12.2             |
| 6              | 3.4080     | 4.0380  | $-14.1410$ | $-1.8590E - 05$  | $5.145E - 05$   | 0.4180      | 11.8040   | 18       | 78       | 12.9             |
| $\overline{7}$ | 0.5790     | 4.7740  | $-9.5970$  | $-1.8000E - 05$  | $4.365E - 05$   | 0.3000      | 7.4590    | 18       | 78       | 9.2              |
| 8              | 1.3370     | 3.8250  | $-10.0610$ | $-1.4120E - 05$  | $3.782E - 05$   | 0.1890      | 7.5220    | 18       | 78       | 8.6              |
| 9              | 4.9340     | 5.1820  | $-13.2680$ | $-1.8460E - 05$  | $4.916E - 05$   | 0.0694      | 9.9780    | 18       | 78       | 12.0             |
| 10             | 5.7400     | 4.0850  | $-13.6750$ | $-1.4240E - 05$  | $4.217E - 05$   | $-0.0611$   | 10.0200   | 18       | 78       | 11.4             |
| 11             | 1.2460     | 4.3360  | $-9.6760$  | $-1.5810E - 05$  | $3.902E - 05$   | 0.4570      | 8.2090    | 18       | 78       | 9.9              |
| 12             | 2.1810     | 3.3070  | $-10.2210$ | $-1.1220E - 05$  | $3.153E - 05$   | 0.3690      | 8.4800    | 18       | 78       | 9.3              |
| 13             | 0.7770     | 5.8740  | $-15.8890$ | $-3.4350E - 05$  | $8.102E - 05$   | 0.1230      | 11.7720   | 18       | 78       | 13.4             |
| 14             | 0.2990     | 3.8970  | $-15.9950$ | $-3.9640E - 05$  | $9.076E - 05$   | 0.2460      | 11.9390   | 18       | 78       | 14.2             |
| 15             | 1.3100     | 5.1770  | $-11.1920$ | $-1.7880E - 05$  | $4.494E - 05$   | $-0.1820$   | 6.9250    | 18       | 78       | 10.1             |
| 16             | 1.3450     | 3.8430  | $-11.2350$ | $-1.8300E - 05$  | $4.579E - 05$   | $-0.2160$   | 6.8650    | 18       | 78       | 9.9              |
| 17             | 3.2600     | 5.5150  | $-12.2880$ | $-2.7970E - 05$  | $6.539E - 05$   | 0.5390      | 10.1310   | 18       | 78       | 12.6             |
| 18             | 3.1150     | 4.0660  | $-12.4320$ | $-3.1150E - 05$  | $7.089E - 05$   | 0.5870      | 10.2640   | 18       | 78       | 13.2             |
| 19             | $-0.3200$  | 6.2380  | $-9.6910$  | $-5.6560E - 06$  | $1.853E - 05$   | $-0.6930$   | 6.0550    | 18       | 146      | 12.7             |
| 20             | 9.5790     | 2.8380  | $-15.2550$ | $2.523E - 05$    | $-4.4770E - 05$ | $-1.8990$   | 8.7990    | 18       | 154      | 17.0             |
| 21             | 6.5000     | 10.4080 | $-11.1480$ | $2.749E - 05$    | $-5.2430E - 05$ | $-2.3080$   | 5.4420    | 15       | 84       | 12.9             |
| 22             | 3.2210     | 4.1220  | $-12.3090$ | $-1.5130E - 08$  | $6.062E - 06$   | $-0.9160$   | 7.9280    | 18       | 158      | 15.1             |
| 23             | $-5.9000$  | 59.3580 | $-11.9290$ | $\boldsymbol{0}$ | $9.845E - 05$   | 0.6250      | 6.4550    | 27       | 26       | 18.7             |
| 24             | 3.2070     | 40.0520 | $-20.7600$ | $\boldsymbol{0}$ | $8.890E - 05$   | $-0.1870$   | 9.2740    | 25       | 27       | 15.4             |
| 25             | 17.5510    | 30.5320 | $-42.9180$ | $\theta$         | $1.472E - 04$   | 0.3980      | 19.4470   | 24       | 25       | 23.2             |
| 26             | $-28.2020$ | 99.6510 | 2.0330     | $-1.6180E - 04$  | $2.730E - 04$   | 2.0310      | 2.3800    | 22       | 9        | 18.3             |
| 27             | 8.3220     | 56.8630 | $-15.2530$ | $-5.8440E - 06$  | $-1.6030E - 05$ | $-3.9080$   | 8.1230    | 20       | 17       | 18.1             |
| 28             | $-7.5770$  | 70.1520 | $-1.7830$  |                  | $8.567E - 06$   | $-1.2010$   | 0.4340    | 13       | 20       | 25.6             |
| 29             | $-25.0990$ | 71.5650 | $-2.0970$  | $-2.7940E - 04$  | $3.756E - 04$   | 3.2080      | 6.4960    | 15       | 25       | 33.7             |
| 30             | $-8.2460$  | 45.5030 | $-9.3020$  | $-7.9330E - 05$  | $1.341E - 04$   | 0.9530      | 7.6670    | 18       | 33       | 22.2             |
| 31             | $-17.0070$ | 66.2120 | $-5.6200$  | $-1.9410E - 04$  | $2.685E - 04$   | 2.0860      | 7.1080    | 15       | 27       | 28.2             |
| 32             | $-4.3430$  | 46.2820 | $-8.8850$  | $\theta$         | $3.892E - 05$   | $-0.0506$   | 6.1880    | 13       | 23       | 24.4             |
| 33             | $-2.3980$  | 63.2030 | $-16.2150$ | $9.110E - 05$    | $-9.2610E - 05$ | $-1.6490$   | 8.5140    | 17       | 27       | 21.0             |
| 34             | $-10.5910$ | 4.9580  | $-1.8040$  | $-9.5790E - 06$  | $1.580E - 05$   | $-1.7910$   | $-0.6950$ | 12       | 30       | 3.6              |
| 35             | $-7.1050$  | 6.5580  | $-4.4260$  | $-4.1570E - 05$  | $9.109E - 05$   | 0.5830      | 4.1260    | 18       | 90       | 12.4             |
| 36             | 1.0620     | 9.2230  | $-8.1150$  | $-7.1120E - 06$  | $2.172E - 05$   | $-0.5240$   | 5.5950    | 15       | 77       | 6.2              |
| 37             | 6.9210     | 5.1840  | $-10.6620$ | $1.571E - 05$    | $-2.4850E - 05$ | $-1.6610$   | 5.6130    | 18       | 102      | 13.4             |
| 38             | 2.7290     | 2.4820  | $-9.4370$  | $-4.8200E - 06$  | $1.693E - 05$   | $-0.6830$   | 6.1990    | 18       | 110      | 6.5              |
| 39             | $-3.9640$  | 8.6040  | $-5.7290$  | $-1.1170E - 05$  | $3.097E - 05$   | $-0.3910$   | 3.9340    | 18       | 166      | 11.8             |
| 40             | 6.1650     | 2.7470  | $-12.4970$ | $1.415E - 05$    | $-2.0100E - 05$ | $-1.1990$   | 8.2350    | 17       | 167      | 10.4             |
| 41             | 8.9830     | 8.0240  | $-14.0840$ | $2.632E - 05$    | $-4.2660E - 05$ | $-1.4390$   | 9.1030    | 14       | 98       | 12.8             |
| 42             | $-5.2270$  | 6.6460  | $-5.8690$  | $-2.1160E - 05$  | $5.116E - 05$   | 0.0333      | 4.6520    | 19       | 165      | 11.7             |
|                |            |         |            |                  |                 |             |           |          |          | $14.23 \pm 6.13$ |

*a*) SN is the system number and the details are the same as in Table 1. *b*) *N* is the number of training data points. *c*) *N* is the number of predicted data points.

Table 4. The Model Constants of Eq. 4 Calculated Based on a Minimum Number of Training Data Points, Number of Predicted Data Points in Each Set (*N*) and Average Absolute Relative Deviation (AARD)

| $SN^{(a)}$               | $\boldsymbol{a}$ | $\boldsymbol{b}$ | $\boldsymbol{c}$ | $\overline{d}$ | $N^{b)}$ | <b>AARD</b>      |
|--------------------------|------------------|------------------|------------------|----------------|----------|------------------|
| $\,1\,$                  | 10.7720          | $-5817.1690$     | $\boldsymbol{0}$ | 4.2570         | 78       | 29.5             |
| $\overline{c}$           | 11.5960          | $-6127.1850$     | $\boldsymbol{0}$ | 4.1550         | 78       | 28.3             |
| 3                        | 2.9800           | $-3217.2610$     | $\boldsymbol{0}$ | 3.5820         | $78\,$   | 28.2             |
| $\overline{\mathcal{L}}$ | 5.2500           | $-3976.6640$     | $\boldsymbol{0}$ | 3.5250         | $78\,$   | 26.1             |
| 5                        | 5.7680           | $-3799.2430$     | $\boldsymbol{0}$ | 4.2410         | 78       | 29.0             |
| 6                        | 6.3050           | $-3986.8120$     | $\boldsymbol{0}$ | 4.3230         | $78\,$   | 29.1             |
| $\boldsymbol{7}$         | 3.4910           | $-2851.3040$     | $\boldsymbol{0}$ | 2.4860         | 78       | 20.7             |
| 8                        | 4.3870           | $-3157.2210$     | $\boldsymbol{0}$ | 2.4760         | 78       | 20.2             |
| 9                        | 10.0930          | $-4586.4010$     | $\boldsymbol{0}$ | 3.7970         | 78       | 27.6             |
| 10                       | 10.6970          | $-4797.9360$     | $\boldsymbol{0}$ | 3.8050         | 78       | 27.5             |
| 11                       | 2.3690           | $-2356.5050$     | $\boldsymbol{0}$ | 2.5880         | 78       | 20.2             |
| 12                       | 2.0560           | $-2262.4680$     | $\boldsymbol{0}$ | 2.6530         | 78       | 21.1             |
| 13                       | 11.8850          | $-6711.4810$     | $\boldsymbol{0}$ | 4.8820         | 78       | 33.9             |
| 14                       | 12.9340          | $-7072.3560$     | $\boldsymbol{0}$ | 4.9470         | 78       | 34.1             |
| 15                       | 8.1960           | $-4694.3370$     | $\boldsymbol{0}$ | 2.3580         | 78       | 23.9             |
| 16                       | 9.2540           | $-5047.7720$     | $\boldsymbol{0}$ | 2.4250         | 78       | 23.8             |
| 17                       | 7.8830           | $-3757.0910$     | $\boldsymbol{0}$ | 3.5580         | 78       | 25.6             |
| 18                       | 8.8330           | $-4073.1690$     | $\overline{0}$   | 3.6540         | 78       | 26.7             |
| 19                       | 7.3500           | $-4574.7090$     | 0.5530           | 2.7620         | 92       | 18.3             |
| 20                       | 6.5970           | $-4583.5970$     | 0.3090           | 3.1640         | 100      | 22.7             |
| $21\,$                   | 6.4320           | $-4023.2380$     | 0.7950           | 2.0080         | 30       | 10.5             |
| 22                       | 6.3500           | $-4466.4420$     | 0.3130           | 3.1110         | 104      | 24.5             |
| 23                       | 20.1530          | $-7158.6190$     | 1.4590           | 2.0310         | 20       | 21.2             |
| 24                       | 32.4910          | $-11692.0000$    | 1.0470           | 2.2140         | 22       | 21.9             |
| 25                       | 38.0930          | $-13391.7800$    | 0.9820           | 4.0640         | 19       | 27.0             |
| 26                       | 8.3770           | $-5644.4230$     | 0.4080           | 2.8350         | 5        | 8.4              |
| 27                       | $-1.4580$        | $-2063.3450$     | 0.9200           | $-2.0810$      | 11       | 12.5             |
| 28                       | 0.5030           | $\mathbf{0}$     | 1.9610           | 3.1980         | 9        | 14.1             |
| 29                       | 17.9630          | $-5691.5540$     | 2.0270           | 3.5960         | 14       | 21.0             |
| 30                       | 11.8860          | $-4732.0850$     | 1.3980           | 3.6350         | 22       | 23.4             |
| 31                       | 16.4050          | $-5372.2730$     | 1.8420           | 3.9180         | 16       | 21.1             |
| 32                       | $-1.6670$        | $\mathbf{0}$     | 1.5220           | 3.9470         | 12       | 21.4             |
| 33                       | $-0.9040$        | $-3634.0530$     | 1.7740           | 3.8290         | 16       | 18.7             |
| 34                       | 7.2840           | $-6241.6840$     | 0.2340           | 2.8310         | 23       | 4.3              |
| 35                       | 12.5320          | $-5336.5570$     | 0.3900           | 2.9760         | 40       | 13.0             |
| 36                       | 10.0830          | $-4401.6830$     | 0.4460           | 3.1960         | 27       | 14.5             |
| 37                       | 11.3420          | $-5086.1680$     | 0.3050           | 2.7460         | 52       | 9.9              |
| 38                       | 8.9900           | $-4546.6120$     | 0.0784           | 3.1130         | 60       | 16.2             |
| 39                       | 8.2490           | $-4540.8240$     | 0.4870           | 2.4660         | 108      | 6.6              |
| 40                       | 6.4770           | $-4140.5770$     | 0.3310           | 3.1910         | 109      | 16.9             |
| 41                       | 2.9220           | $-2613.7890$     | 0.5550           | 3.4480         | 40       | 21.1             |
| 42                       | 7.8220           | $-4484.3630$     | 0.4120           | 2.7450         | 107      | 10.9             |
|                          |                  |                  |                  |                |          | $20.85 \pm 7.24$ |

*a*) SN is the system number and the details are the same as in Table 1. *b*) *N* is the number of predicted data points.



Fig. 1. The Individual Absolute Relative Deviation (IARD) Values for Predictive Eqs. 2 and 4 Trained Using a Minimum Number of Data Points

sponding probability for Eq. 4 is 0.57, and in 23% of the cases, the solubility could not be predicted.

### **Conclusions**

The proposed model shows a simple and readily available least squares method to calculate the solubility in SC- $CO2$ +entrainer systems with reasonable accuracy. It is more accurate than a similar model from the literature and is also able to predict solubility data under operational conditions with acceptable prediction error. It is suggested that the model can be employed in the pharmaceutical/chemical industry to speed up the process of SCF technology development.

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