EXPERIMENTAL FACTS PERTAINING TO THE RELATIONSHIP BETWEEN VISCOSITY, MOLECULAR SIZE, AND MOLECULAR SHAPE[1]

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I. INTRODUCTION

Until about ten or twelve years ago, few people thought of interpreting the viscosity increment imparted to a solvent by a solute as being related in any way to the size and shape of the solute particles. Most thinking was dominated by the Einstein equation (12),

$$\frac{\eta}{\eta_0} - 1 = 2.5G \tag{1}$$

where $\eta/\eta_0 - 1$ is the specific viscosity and G is the volume fraction of the solute. According to this equation, the viscosity of a solution or suspension of large spheres is related in no way whatever to the size of the spheres but only to the total volume concentration. It was early recognized that the viscosities of most solutions and suspensions were much too high to be accounted for by the Einstein equation. The usual explanation was that the particles soaked up solvent and occupied more space in solution than in the dry state. It became fashionable to calculate the specific hydrodynamic volume of a colloid from its viscosity, using the Einstein equation, and to regard the ratio of this term to the true specific volume in the dry state as a measure of the degree of solvation of the particles. Table 1, assembled by Kraemer (33) in 1931, shows the ratios of specific hydrodynamic volumes, Φ, calculated from viscosity measurements, to true specific volumes, V, for several colloidal materials. On the basis of the foregoing, it is evident from a consideration of the data in this table that there are two kinds of colloids,—those in which the ratio is near unity and those in which it is considerably greater than unity. The members of the first group were thought to be lyophobic colloids and those of the second, lyophilic colloids. In several instances the specific hydrodynamic volumes of members of the second group may be seen to be several hundred times the true specific volumes. The absurdity of attempting to explain a situation of this sort simply in terms of solvation was beginning to be generally recognized about the time these data were assembled, and Kraemer reflected this trend by suggesting that unusually high viscosities might be caused "by greatly elongated particles or macromolecules that increase resistance to shear through mutual entanglements and interference, but without ordinary flocculation." Here may be seen the germ

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of the notion that there is some connection between the size or shape of a particle and the viscosity of its solution. It is the purpose of the present review to assemble the principal experimental facts upon which this idea rests at the present time. Several excellent, though somewhat less inclusive, considerations of the same general topic are available in the literature (49, 50, 53, 58, 70). The treatment can be divided into two definite sections: First, there will be an examination of those data in the literature from which some sort of a connection between particle size or shape and viscosity can be established on a purely empirical basis. Second, some of the attempts which have been made on theoretical grounds to explain the alleged relationship between viscosity and molecular size and shape will be examined in the light of pertinent experimental findings.

TABLE 1

Ratios of specific hydrodynamic volume to true specific volume for various colloids (33)

COLLOID	Φ/V	COLLOID	Φ/V
Sucrose in water	1.6	Clay in water	9
		Starch in water	20
		Gum arabic in water	50
Egg albumin in water	0.9	Agar in water at 50°C	220
		Isoelectric gelatin at 40°C	6
		Isoelectric gelatin at 20°C	30
Sulfur in water	1.2	Nitrocellulose in ethyl acetate	80
Gamboge in water	1.25	Cellulose acetate in acetone	200
		Polyvinyl acetate in benzene	200–900
Diluted rubber latex	1.00	Rubber in benzene	300–500
		Polystyrene in benzene	4–1000

II. EMPIRICAL APPROACH

A. Linear compounds and polymers

Owing to the pioneering work of Staudinger and his collaborators (73) and to the challenge presented by their findings, an extensive literature is available on the relationship of the viscosity of chain-type compounds and polymers to their molecular weights. In this review, an attempt has been made to sort from the literature those studies in which viscosity and independent molecular-weight measurements of apparently high reliability were carried out on the same preparations of several members of homologous or polymer series of well-defined substances. The materials range in size from the simpler straight-chain paraffin molecules to macromolecular polymers and natural products. The molecular weights of the simpler compounds considered here were determined by the usual procedures of the structural chemist. One or more of three general methods were used for determining the molecular weights of the more complex molecules: (*1*) end-group titrations, (*2*) vapor-pressure, osmotic, and cryoscopic studies, and (*3*) ultracentrifugation-equilibrium measurements. In the case of vapor-pressure and related methods, rather serious difficulties attend the extrapolations to zero concentration, necessary in the case of linear macromolecules. For that reason, in this review only qualitative significance is attached to those studies in which these methods were the only ones available for the independent determination of the molecular weight. In all of the studies considered, viscosities were computed from measurements of the rate of flow through a capillary tube.

In the subsequent considerations, the viscosity of the pure solvent at a given temperature will be represented by the symbol η_0, and that of the solution at the same temperature by the symbol η. Then η/η_0 will be defined as the relative viscosity, η_r, of the solution and $(\eta - \eta_0)/\eta_0$ or $\eta/\eta_0 - 1$ as the specific viscosity, η_{sp}. It is a matter of common experience that the specific viscosity is a function of the volume concentration of the solute. As is illustrated by the data in figure 1, in very dilute systems this function is linear (25, 44). Hence, the mathematical quantity which characterizes any given substance is the expression η_{sp}/G. With most substances, specific viscosity is a linear function of concentration only in rather dilute solutions. It is necessary, therefore, to use the limiting value of the expression η_{sp}/G as G approaches zero. Following Kraemer's suggestion (34), this function will be called "intrinsic viscosity."[2] There are three general

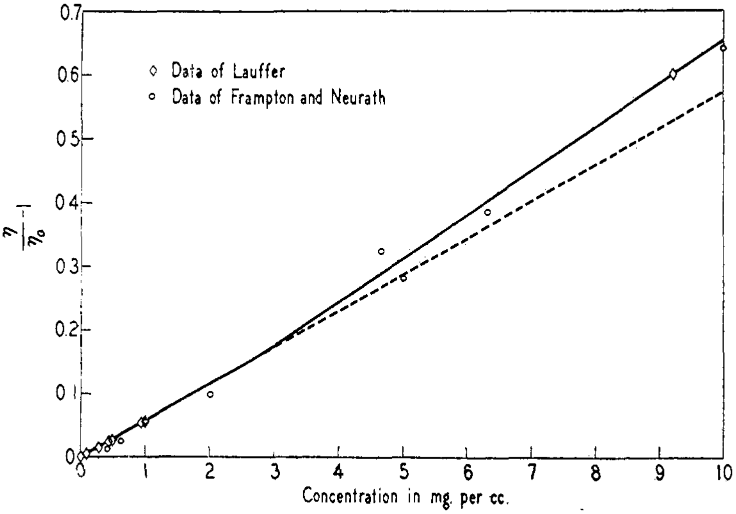

FIG 1. Specific viscosity of tobacco mosaic virus nucleoprotein plotted as a function of concentration in milligrams per cubic centimeter. From Lauffer and Stanley ((47).)

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[2] Kraemer (34) uses the term "intrinsic viscosity" symbolized by $[\eta]$, to define the limiting value of the ratio, η_{sp}/c, for the special case in which c is expressed as grams of solute per 100 cc. of solution. The symbol $[\eta]_v$ is used to represent the analogous function for the case in which concentration is expressed as cubic centimeters of solute per 100 cc. of solution. Since there are many different manners of expressing concentrations in viscosity studies, in this review the term "intrinsic viscosity" will be used to define the limit of η_{sp}/c or η_{sp}/G without restriction to the mode of describing concentration. In each case, the units

methods of evaluating the limit of η_{sp}/G from experimental data. The simplest of the three is to plot specific viscosity as a function of concentration and then to draw a tangent to the experimental curve at the point represented by zero concentration. This procedure was used in the very favorable case of tobacco mosaic virus illustrated in figure 1. The second method is based upon the empirical relationship observed by Arrhenius (2) to apply over a considerable concentration range: $\log \eta_r = kG$, where k is a proportionality factor. On the basis of this relationship, $\log \eta_r$ is plotted against G and $\log \eta_r/G$ is evaluated. When natural logarithms are used, the fraction, $\log \eta_r/G$, is equal to the limit of η_{sp}/G or $[\eta]$, because $\log \eta_r$ is approximately equal to $\eta_r - 1$ or η_{sp}, when η_{sp}

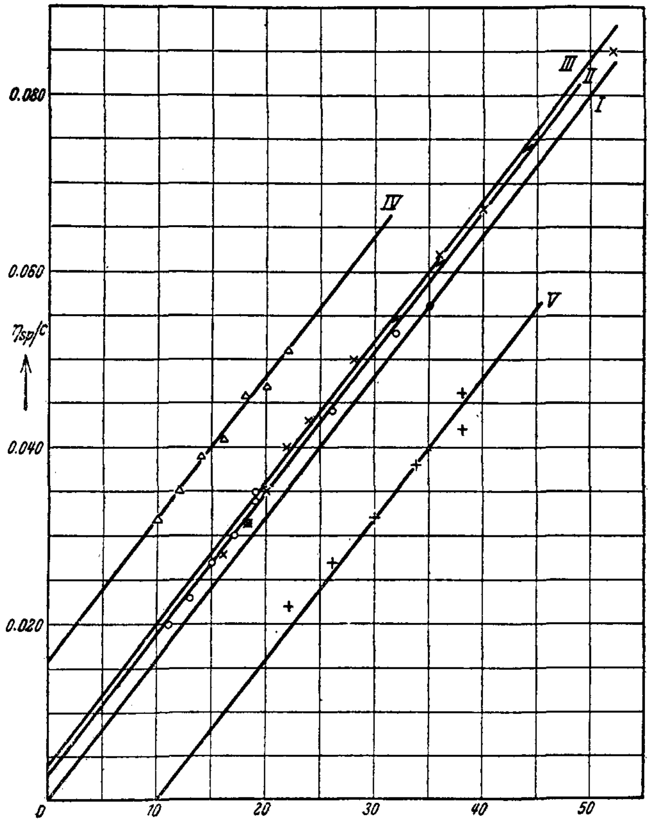

FIG. 2. Base molar intrinsic viscosities of chain molecules (ordinate) plotted as functions of numbers of carbon atoms in chain (abscissa). Curve I, normal paraffins; curve II, normal fatty acid esters; curve III, normal fatty acids; curve IV, normal fatty acids in pyridine; curve V, normal alcohols. (From Staudinger (73).)

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of the concentration term will be stated. In some of the studies discussed, it is doubtful that the limiting values of the ratio have been determined. However, most of the measurements in such cases were made on quite dilute systems, so that the ratios presented may be regarded as reasonable approximations of the intrinsic viscosity.

is very small. The third method of getting the limiting slope depends upon the observation that a power series, $\eta_{sp} = AG + BG^2 + CG^3 + \cdots$, can describe the viscosity–concentration function over a wide concentration range (5). The constants A, B, C, etc., are evaluated from the data by any one of several accepted methods. Since all higher powers of G become insignificant compared to G when G approaches zero, the value of A is the limiting value of η_{sp}/G. Any one of the above three treatments when applied to suitable data should yield a reasonable approximation of the intrinsic viscosity.

In figure 2 are presented some of the data of Staudinger and Ochiai (73, 74, 77) on the viscosity of homologous series of fairly simple, chain-type organic compounds of known molecular weight. Intrinsic viscosity is plotted against

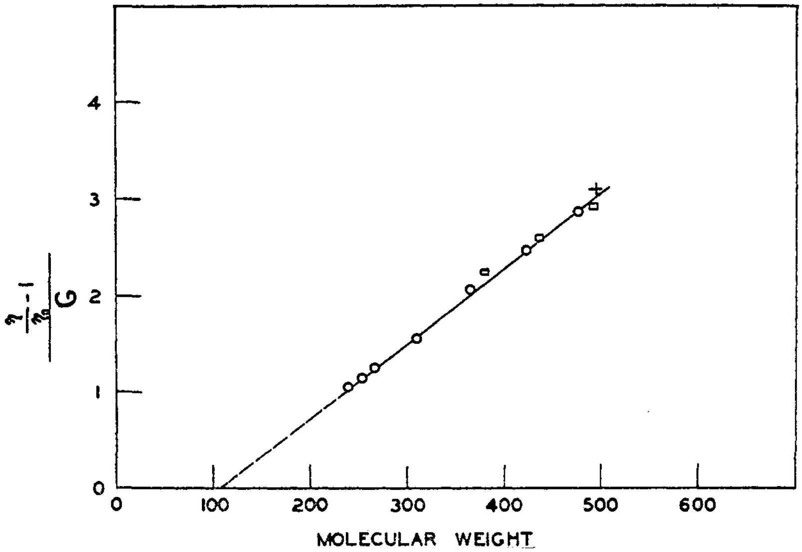

FIG. 3. Volume fraction intrinsic viscosities of normal hydrocarbons plotted against molecular weight. ○, data of Meyer and van der Wyk (54); □, data of Staudinger and Staiger (79); ×, datum of Staudinger and Ochiai (77).

number of carbon atoms in the chain. Staudinger expressed concentrations in terms of the molarity of the basic repeat unit, the CH_2 group in these examples. If these intrinsic viscosities are recalculated on the basis of volume fractions, a new scale of ordinates must be applied to figure 2. Unity on the new scale would coincide with about 0.018 on the old, and 2.5, the coefficient in the Einstein equation for spheres, would fall between 0.04 and 0.045 on the old. This means that the intrinsic viscosities of at least half of the linear molecules represented in this study are less than that predicted for large spheres by the Einstein equation.

Meyer and van der Wyk (54) investigated in a most painstaking manner the viscosities of carbon tetrachloride solutions of normal paraffin hydrocarbons with chain lengths between seventeen and thirty-four carbon atoms. Data were obtained for similar systems by Staudinger and Staiger (79). The intrinsic viscosities calculated on the basis of volume fractions from the data of these two investigations are plotted as a function of molecular weight in figure 3.

The viscosities of essentially monodisperse polyoxyethylene glycol preparations dissolved in carbon tetrachloride and in dioxane were studied by Fordyce and Hibbert (23). These compounds were synthesized by Fordyce, Lovell, and Hibbert (24) according to the reaction:

$$ClCH_2CH_2O[CH_2CH_2O]_xCH_2CH_2Cl + 2KOCH_2CH_2O[CH_2CH_2O]_yCH_2CH_2OH$$
$$\rightarrow HOCH_2CH_2O[CH_2CH_2O]_{(x+2y)}CH_2CH_2OH + 2KCl$$

This should lead to essentially monodisperse compounds of known structure and molecular weight. In figure 4 the data for the intrinsic viscosities of various

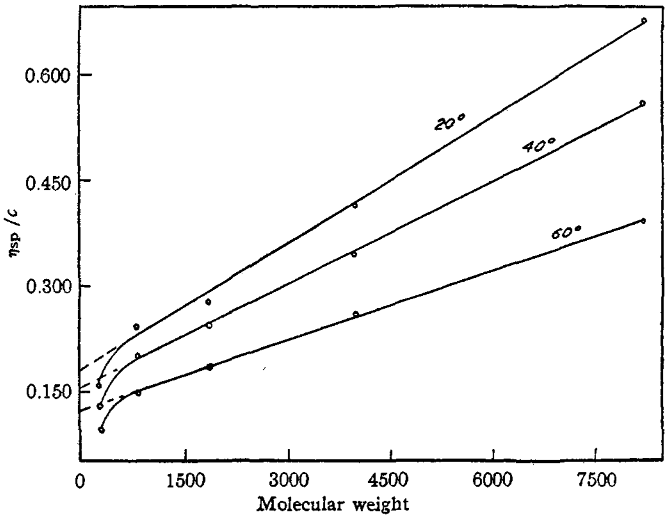

FIG. 4. Base molar intrinsic viscosities at various temperatures of polyoxyethylene glycols in carbon tetrachloride plotted against molecular weight. (From Fordyce and Hibbert (23).)

polyoxyethylene glycols in carbon tetrachloride are plotted as a function of molecular weight. The concentrations are expressed in terms of base molarity, where the $[CH_2CH_2O]$ group represents the repeat unit. If these concentrations had been expressed in terms of the volume fraction, 2.50 on the scale of ordinates would have come close to 0.150 on the present scale. If the data obtained with dioxane as a solvent are plotted in a similar manner, comparable curves are obtained, differing chiefly in having somewhat greater slopes.

Flory and Stickney (22) measured the viscosity of various preparations of decamethylene adipate polymers dissolved in diethyl succinate and in chlorobenzene. These polymers were obtained by allowing equimolar proportions of decamethylene glycol and adipic acid to react to various extents. All of the preparations thus obtained were composed of particles of many sizes. The

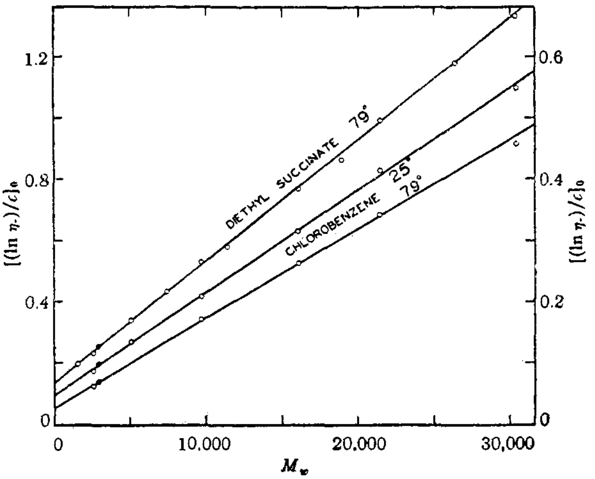

FIG. 5. Base molar intrinsic viscosities of decamethylene adipate polymers in chlorobenzene at 25° and 79°C. (right-hand scale of ordinates) and in diethyl succinate at 79°C. (left-hand scale of ordinates) plotted against weight average molecular weights. (From Flory and Stickney (22).)

weight average[3] molecular weight of each preparation was determined from measurements of the viscosity of the molten polymer.[4] In figure 5, the intrinsic

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[3] In studies on the size and related properties of inhomogeneous materials, it is necessary to distinguish between several different types of averages. The number average molecular weight is simply the total weight of material divided by the total number of moles of particles present. The weight average molecular weight is the limiting value, as the number of fractions approaches infinity, of the average of the molecular weights of the very small practically monodisperse fractions of equal bulk into which the total mass of the material can be divided. The number average molecular weight attaches equal significance to very large and very small molecules. The weight average molecular weight attaches equal significance to equal weights of very small and very large molecules. In a very heterogeneous system, the number average molecular weight can be very much smaller than the weight average molecular weight. Kraemer and Lansing (39) pointed out, and Lovell and Hibbert (48) and Baker, Fuller, and Heiss (4) demonstrated experimentally, that the intrinsic viscosity of a polydisperse material is a function of the weight average molecular weight and not of the number average molecular weight.

[4] Flory (21) has shown that there is a linear relationship between the logarithm of the melt viscosity of polyester preparations and the square root of the weight average molecular weight. In establishing this relationship, melt viscosities and number average molecular weights obtained by end-group titrations were measured on various polyester preparations. Weight average molecular weights were calculated from the number average molecular weights on the basis of a theoretical distribution function previously derived (19), and then these values were used in connection with the viscosity data to establish the relationship in question. In view of this, the weight average molecular weight determinations of Flory and Stickney depend ultimately upon end-group titrations and upon the validity of Flory's distribution theory. The introduction of this theoretical consideration destroys somewhat the empiricism of the approach to the relationship between the intrinsic viscosity and the weight average molecular weight of the polyesters. However, Flory's distribution function was derived by straightforward statistical considerations on the basis of the

viscosities of decamethylene adipate polymers are plotted as a function of the weight average molecular weight. Intrinsic viscosities were computed on the basis of concentration expressed in terms of base molarity, the repeat unit being considered to be something a little bigger than a methylene group. The right-hand scale of ordinates applies to the polyester dissolved in chlorobenzene and the left-hand scale to the polymer dissolved in diethyl succinate. Had concentrations been calculated as volume fractions, the value of the ordinate 2.5 would coincide approximately with 0.08 on the present scale.

The polyesters just discussed cover about the same molecular weight range as the ω-hydroxydecanoic acid polymers prepared by Carothers and van Natta (8) and studied in the viscometer by Kraemer and van Natta (41). Number average molecular weights were determined by means of an end-group titration. Since these polymers were fractionated by crystallization, even now that a distribution theory (19) is available, it is not possible to convert number average molecular weights into weight average molecular weights. Sedimentation-equilibrium measurements made by Kraemer and Lansing (37) on the largest polymer of the series did, however, yield a value for the average molecular weight—presumably the weight average—not too different from the value obtained by titration. All in all, this study represents a very important contribution, because it first clearly demonstrated that the relationship between the molecular weight and the intrinsic viscosity of polymeric materials is one of approximate linearity rather than direct proportionality.

More recently Baker, Fuller, and Heiss (3) studied the viscosity of polymer preparations obtained by allowing pure ω-hydroxyundecanoic acid to polymerize at 200°C. Number average molecular weights were determined by means of titration with alcoholic sodium hydroxide. From Flory's distribution theory (19) it can be determined that the weight average molecular weight is approximately equal to 1.94 times the number average molecular weight for all of the polymer preparations considered in the above work. On this basis, weight average molecular weights were computed from the data presented in the publication. Viscosities were measured in reagent-grade chloroform. In figure 6, intrinsic viscosity is plotted as a function of weight average molecular weight. The concentration term involved in the evaluation of the intrinsic viscosity was expressed in volume fractions. A more or less comparable study has been carried out by Staudinger and Nuss (76). From a gross point of view, the data obtained by these workers are in agreement with those presented in figure 6. However, Staudinger and Nuss regard their results as a confirmation

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assumption that the reactivity of the end groups does not change as the polymers increase in size. Kinetic studies demonstrated the substantial validity of this assumption (20). Hence, the theory can be considered to be established experimentally, at least in part. The distribution theory provides that the weight average–number average molecular weight ratio is approximately a constant for all preparations of the polymer considered by Flory and Stickney. Hence, if the number average molecular weights of the polyester, which can be obtained entirely experimentally, were plotted against intrinsic viscosity, linear relationships similar to those of figure 5, but differing in slope, would be obtained.

of the Staudinger rule of direct proportionality between intrinsic viscosity and molecular weight. In order to evaluate the conflicting interpretations, it is necessary to observe that the data of the latter workers do not seem to possess quite the same degree of internal consistency as those of Baker, Fuller, and Heiss.

There has long been a discussion as to whether or not a simple relationship of viscosity to molecular size obtains for the true macromolecules,—particles with molecular weights up to half a million. Staudinger (75) has assembled some data on polystyrene. Styrene was allowed to polymerize at each of several temperatures, and then each batch was fractionated. The number average molecular weight of each fraction was determined by osmotic-pressure measurements and the intrinsic viscosity of each fraction dissolved in toluene was measured. The results show approximately linear relationships between the intrinsic viscosities and the number average molecular weights of the fractions of each batch, but both the slopes and the intercepts of the lines vary from batch to batch. Signer and Gross (64, 65, 66) measured the molecular weights of some polystyrene fractions by means of sedimentation equilibrium. Sufficiently dilute solutions were studied so that the extrapolation hazards involved in osmotic measurements could be avoided. Viscosities of the same materials dissolved in chloroform were obtained. These results demonstrated an approximate proportionality between intrinsic viscosity and molecular weight.

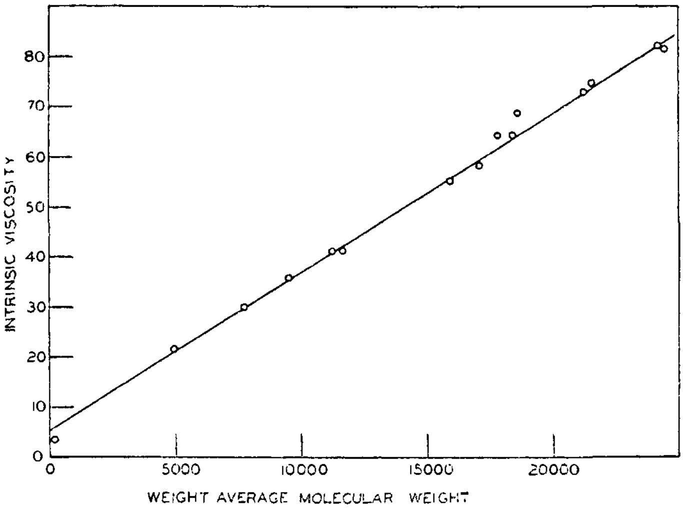

FIG. 6. Volume fraction intrinsic viscosity of ω-hydroxyundecanoic acid self-polymers in chloroform plotted against weight average molecular weight. (Data of Baker, Fuller, and Heiss (3).)

Staudinger (75) has also assembled data on the intrinsic viscosities and the osmotically determined number average molecular weights of various nitrocellulose preparations dissolved in acetone, of various degraded methyl celluloses dissolved in water, and of various polyethylene oxides dissolved in water. Later, Staudinger and Reinecke (78) presented data on the viscosities of various fractions of ethyl cellulose, ethyl acetyl cellulose, methyl cellulose, and methyl acetyl cellulose. Here too, molecular weights were estimated osmotically. All of these studies indicated approximate proportionality between the number average molecular weights and the intrinsic viscosities for each type of material.

Kraemer and his associates made extended studies of the viscosities and molecular weights of a great many degradation products of natural polymers. They measured molecular weights by the sedimentation-equilibrium method, carrying out the calculations (43) in such a manner as to obtain weight average molecular weights. In figure 7, curve II, the weight average molecular weights of various polydisperse cellulose and cellulose derivative systems (34, 38, 39, 40) are plotted against intrinsic viscosities calculated on the basis of volume fractions. Signer and collaborators (65, 67, 68) carried out an analogous study on methyl cellulose fractions dissolved in water. Their data, calculated in the same manner as in the preceding case, are presented in figure 7, curve I. In a comparable manner, Kraemer and associates (40) studied rubber fractions in ether and polychloroprene in n-butyl chloride. The results of these studies, computed as described above, are presented in curves III and IV of figure 7.

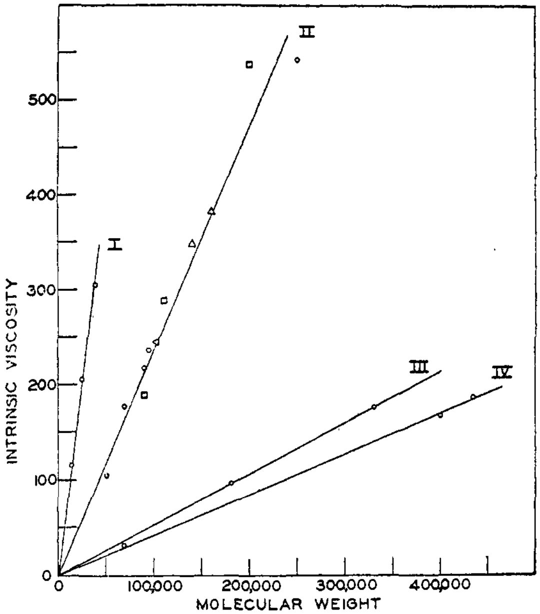

FIG. 7. Volume fraction intrinsic viscosities of various natural and synthetic polymers plotted against weight average molecular weights. Curve IV, rubber in ether (Kraemer and Nichols (40)). Curve III, polychloroprene in n-butyl chloride (Kraemer and Nichols (40)). Curve II: □, cellulose in cuprammonium; ○, cellulose acetate in acetone; △, nitrocellulose in acetone (Kraemer (34)). Curve I, methyl cellulose in water (Signer (65, 67, 68)).

Kemp and Peters (32) have recently considered the intrinsic viscosities of numerous rubber degradation fractions dissolved in benzene and in hexane in relation to the average molecular weights determined cryoscopically. Extrapolated values of the molecular weight showed an approximately linear relationship to intrinsic viscosity.

When all of the data that have just been presented on linear molecules and linear polymers are considered, four important facts are outstanding: (*1*) It is clear that for all of the cases examined, including the very simplest chain molecules, the largest macromolecular structures, and all intermediates, the intrinsic viscosities can be expressed at least approximately as linear functions of molecular weights or chain lengths, according to equations of the form,

$$[\eta] = \beta + KM \tag{2}$$

where M represents the molecular weight. (*2*) The data show that the constants β and K can vary considerably from system to system. The variability in β is evident from a consideration of the data of Staudinger and Ochiai (77) on simple chain molecules, presented in figure 2. From these same data, however, one might be led to expect that the value of K is a general constant applicable for a given solvent to all molecules which have the carbon atom as a recurring unit. In conformity with this idea, Staudinger (73, 74) represented the viscosity–molecular weight relationship of the normal hydrocarbons by a line drawn parallel to the other lines of figure 2, through the origin and through a single datum. However, the graph of the relationship between intrinsic viscosity and the molecular weight of the normal hydrocarbons representing the results of the more extended and more exacting study of Meyer and van der Wyk (54) may be seen in figure 3 not to extrapolate to the origin. Since the data of Staudinger and Ochiai (77) and of Staudinger and Staiger (79) fit this graph fairly well, this means that the slope of the hydrocarbon graph is actually not quite the same as that of the other systems represented in figure 2. Hence, K must be regarded as a specific rather than a general constant. (*3*) These data show that, within a given homologous or polymeric series, the intrinsic viscosity–molecular weight relationship varies from solvent to solvent and from temperature to temperature. The studies of Flory and Stickney (22), of Fordyce and Hibbert (23), and of Kraemer and van Natta (41) bring out the facts that both the intercepts and the slopes of the graphs may be varied by changing solvents and by changing the temperature. (*4*) As was pointed out previously, several of the various studies here considered show that it is possible for chain molecules to have lower intrinsic viscosities than that predicted by Einstein for large spheres. The significance of this point will be discussed later in connection with a consideration of the theoretical aspects of the question.

It appears, then, on the basis of a wealth of reasonably good measurements of the intrinsic viscosities of simple chain-type molecules and of chain-type polymers with well-characterized molecular weights, that a linear relationship between intrinsic viscosity and molecular weight exists for each homologous or polymeric series.[5] This is valid for systems ranging in size from the simplest small molecules to the macromolecular polymers. The constants characterizing these relationships vary not only from series to series, but from solvent to solvent and from temperature to temperature within a given series. From the practical point of view, these facts mean that the very simple viscosity technique can be adapted to the measurement of the molecular weight of chain-type compounds or polymers, but only after the method has been standardized by an independent and reliable procedure for each series of molecules or particles in a given solvent at a given temperature. An illustration of such a use of viscosity data was recently afforded by Mead and Fuoss (50a). From a theoretical point of view, a relationship of such a degree of generality as that here obtained suggests strongly that there is actually some fundamental connection between the size of a chain-type molecule and its solution viscosity. However, that hypothetical relationship must be one in which several other factors play a significant part.

B. Proteins

Many protein solutions are known to have intrinsic viscosities far greater than that predicted for spheres by the Einstein equation. In view of the relationship between viscosity and molecular weight established for linear polymers, it is natural to inquire whether such a relationship exists for proteins also. If intrinsic viscosities are plotted against molecular weights for the many proteins which have been investigated, no correlation of any significance is observed. However, if the intrinsic viscosities of these same proteins are plotted against the molecular shape as determined independently, a very real correlation may be obtained. This is illustrated in figure 8, where the volume fraction intrinsic viscosities of several proteins and related materials are plotted against the axial ratios of the rod-shaped ellipsoids of revolution assumed to represent the protein particles.[6] It may be observed that all of these intrinsic viscosities exceed the Einstein value for large spheres.

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[5] Staudinger draws a slightly different conclusion from many of the same data that have been considered here. He insists that the intrinsic viscosity is directly proportional to the molecular weight of linear macromolecules,—in other words, the constant β of equation 2 is 0. For those systems in which a graph of intrinsic viscosity *versus* molecular weight fails to extrapolate to the origin, Staudinger believes that the lower members of the series are associated or aggregated and that the viscosity studies can be used to measure the extent of association. K. H. Meyer has recently reviewed much of the literature pertaining to this issue and has concluded that the Staudinger law of direct proportionality does not adequately describe the relationship between viscosity and molecular weight (53). Very recently Flory (21a) reported that the intrinsic viscosity of polybutene is accurately proportional to the 0.606th power of the osmotically determined molecular weight within the range from 5000 to 500,000. Not only is this result at variance with the views of Staudinger, but also it constitutes an exception to the rather general approximately linear dependence of intrinsic viscosity upon the molecular weight of chain-type polymers.

[6] Kraemer (35, 36) has carried out viscosity, sedimentation, and diffusion studies on gelatin in various non-gelating solvents. If his value for intrinsic viscosity were plotted against the axial ratio calculated from the sedimentation and diffusion data, the point would

The data here presented were obtained or assembled by Polson (62), by Neurath and associates (55, 56), by Carter and Hall (9, 10), and by workers at the Rockefeller Institute (11, 45). The materials represented are egg albumin, serum albumin, hemoglobin, amandin, gliadin, serum globulin, thyroglobulin, *Octopus*, *Homarus*, and *Helix* hemocyanins, pepsin, lactoglobulin, thymus nucleohistone, tobacco mosaic virus, and tobacco mosaic virus nucleic acid. The axial ratios here shown were calculated, for the most part, from diffusion, sedimentation, and specific-volume data.[7] The molecular weight of the protein was first computed by use of the equation

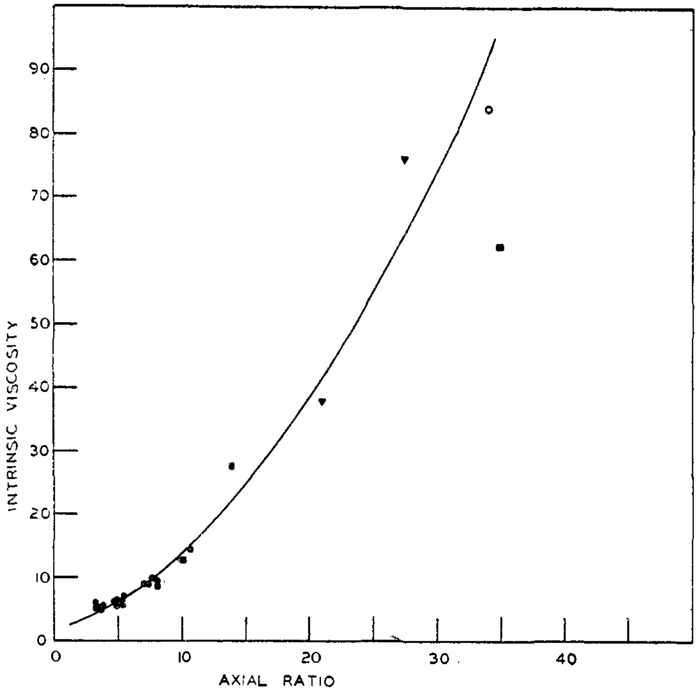

FIG. 8. Volume fraction intrinsic viscosities of various proteins and protein derivatives plotted against axial ratios obtained from independent data for the case of rod-like shape. ●, data of Polson (62) and of Neurath (55, 56); ○, thymus nucleohistone (9, 10); ▼, tobacco mosaic virus (45); ■, tobacco mosaic virus nucleic acid (11).

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fall far to the right of those shown in figure 8. This may be due to the difficulties which, as Kraemer emphasized, are experienced in this case in extrapolating diffusion data to zero concentration.

[7] The axial ratios of the two tobacco mosaic virus preparations (▼ in figure 8) were determined by measuring the particle lengths from electron micrographs, by computing the weight average length (forty or more particles), and by then dividing this by the particle thickness established by x-ray studies. The absolute error in these values is probably less than 10 per cent. Viscosities were measured in a capillary viscometer. The preparation having the lower intrinsic viscosity gave a single boundary in the ultracentrifuge and a unimodal distribution of particle lengths in the electron micrograph (45). Its viscosity is of the same order of magnitude as that reported previously by Stanley (71). The prepara-

$$M = \frac{RTS}{D(1 - V\rho)} \tag{3}$$

R is the gas constant, T the absolute temperature, S the sedimentation constant, D the diffusion constant, V the partial specific volume of the protein, ρ the density of the solution, and M the molecular weight (81). From the molecular weight and partial specific volume, the radius of a sphere having the same weight as each particle was next calculated. Then, by use of the Einstein–Sutherland (13, 80) equation,

$$D = \frac{RT}{6\pi\eta_0 rN} \tag{4}$$

where r is the radius and N is Avogadro's number, the value, D_0, that the diffusion constant would have if the particles were spheres of specific volume V was computed. The real measured diffusion constant was next divided by this artificial or ideal diffusion constant to get the quotient D/D_0. When this quotient is unity, it is justifiable to conclude that the protein molecules under question are really spheres with specific volume V. In the cases of all of the proteins involved in these considerations, the ratios were less than unity. There are three possible interpretations of such a situation: (*1*) The protein molecules might be rods instead of spheres. A relationship has been derived (26, 28, 60) between D/D_0 and b/a for elongated ellipsoids of revolution which can be expressed as follows when $b/a \gg 1$:

$$\frac{D}{D_0} = \frac{(a/b)^{2/3}}{\sqrt{1 - (a/b)^2}} \ln\left(\frac{1 + \sqrt{1 - (a/b)^2}}{a/b}\right) \tag{5}$$

where a is the minor and b the major axis. Hence, when D/D_0 is known, a/b can be evaluated. (*2*) The protein particles might be plate-like ellipsoids of revolution. A relationship comparable to equation 5 has been derived expressing D/D_0 as a function of the axial ratio of plate-like ellipsoids. (*3*) The various particles might have partial specific volumes in solution greater than those used in computing the equivalent spheres, that is, the particles in solution may actually be hydrated. In the computation of the b/a values presented in figure 8, it was assumed that the protein particles are rod-like ellipsoids of revolution and equation 5 was used for the evaluations.

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tion having the higher intrinsic viscosity showed a double boundary in the ultracentrifuge and a bimodal distribution of particle lengths in the electron micrograph (45), both results indicating the first stage of end-to-end aggregation of a fraction of the particles. Its viscosity is about the same as that reported for different preparations by the author (44) and by Frampton and Neurath (25). As was pointed out by the author (44) and demonstrated by Robinson (63), the viscosity of tobacco mosaic virus measured in a capillary viscometer should be somewhat too low, owing to the partial orientation of the particles in the flowing stream. This effect was particularly pronounced in the case of the very highly aggregated virus material studied by Robinson. It should certainly be of a much lower magnitude in the cases of the unaggregated material and the very slightly aggregated material employed in the present instance.

The justification of the assumption that hydration is not the most important factor contributing to the inequality of D and D_0 is not difficult. For most of the data represented in figure 8, it would be necessary to assume that the particles swell to from three to forty times their dry volumes in order to explain the observed D/D_0 values solely in terms of hydration. Such an assumption seems absurd. Furthermore, when the supposed hydrodynamic volumes obtained from D/D_0 ratios are plotted against the measured viscosities, a line with a slope about fifty per cent greater than the 2.50 required by the Einstein equation (equation 1) for spheres is obtained. Since, as will be demonstrated later, the Einstein relationship is quantitatively correct for large spheres, this is strong evidence against the belief that hydration plays the dominant rôle in determining the D/D_0 ratios of proteins. However, it may account for a small portion of the effect.

It is not certain that the choice of the rod shape over the plate shape as the model for all protein particles is entirely justifiable, for it is not possible to decide with assurance, on the basis of an empirical consideration of sedimentation, diffusion, and viscosity data, whether protein particles are rod-shaped or plate-shaped. Some may have one shape and some the other. There is, however, excellent independent evidence that at least a few of the proteins represented in figure 8 are rod-like rather than plate-like. Oncley (57) and Ferry and Oncley (18) have shown that the dielectric dispersion curves of serum albumin and serum pseudoglobulin depart markedly from the Debye prediction for spheres. They were able to demonstrate, moreover, that the experimental curves can be anticipated theoretically if the two proteins are regarded as being rod-shaped ellipsoids with dipole angles of 45°. It was not found possible to predict these curves on the assumption of plate-like shapes. In the case of tobacco mosaic virus protein, a great deal of evidence has been amassed to show that the component particles are rod-shaped (46, 47). Recently obtained electron micrographs demonstrate the correctness of this conclusion, as is illustrated in figure 9 (72). Finally, as will be shown later, if the theoretical viscosity equations of Simha are regarded as being essentially valid, it is possible to conclude that viscosity, sedimentation, and diffusion data favor the interpretation that most of the proteins thus far investigated are rods rather than plates. Nevertheless, it is necessary to consider the possibility that some proteins are plate-like in shape. Whatever their shapes may be, it seems possible to conclude cautiously and tentatively that there is a quantitative correlation between the axial ratios of protein particles and their intrinsic viscosities.

C. Rod-like models

Experiments on the intrinsic viscosity of rod-like models have been reported by Eirich, Margaretha, and Bunzl (15). Natural silk fibers with a diameter of 40 μ were cut into several distinct lengths in order to produce relatively homogeneous batches of rod-like particles. The smallest had an axial ratio of 5:1, and the largest a ratio of 23:1. Portions of some of these preparations were agitated so as to split the fibers. Various homogeneous batches with axial ratios between 32:1 and 140:1 were obtained in this manner. Then artificial silk strands with diameters of 9 μ were cut into various lengths to give particles with axial ratios of 25:1, 50:1, 75:1, and 100:1. Naturally, all of these dimensions were checked by methods involving the use of the microscope. The various rod models were suspended in a mixed solvent of tetrachloroethane and olive oil, and their viscosities were studied in Couette, Ostwald, and pressure viscometers. In figure 10 are shown the data obtained in a Couette viscometer. Here specific viscosity is plotted against concentration for the various axial ratio values. It is evident that increasing axial ratios of such rod-like models parallel increasing viscosities. Here, then, there is a very definitely established correlation between viscosity and axial ratio. There is no correlation between intrinsic viscosity and particle weight, particle length, or particle diameter, however.

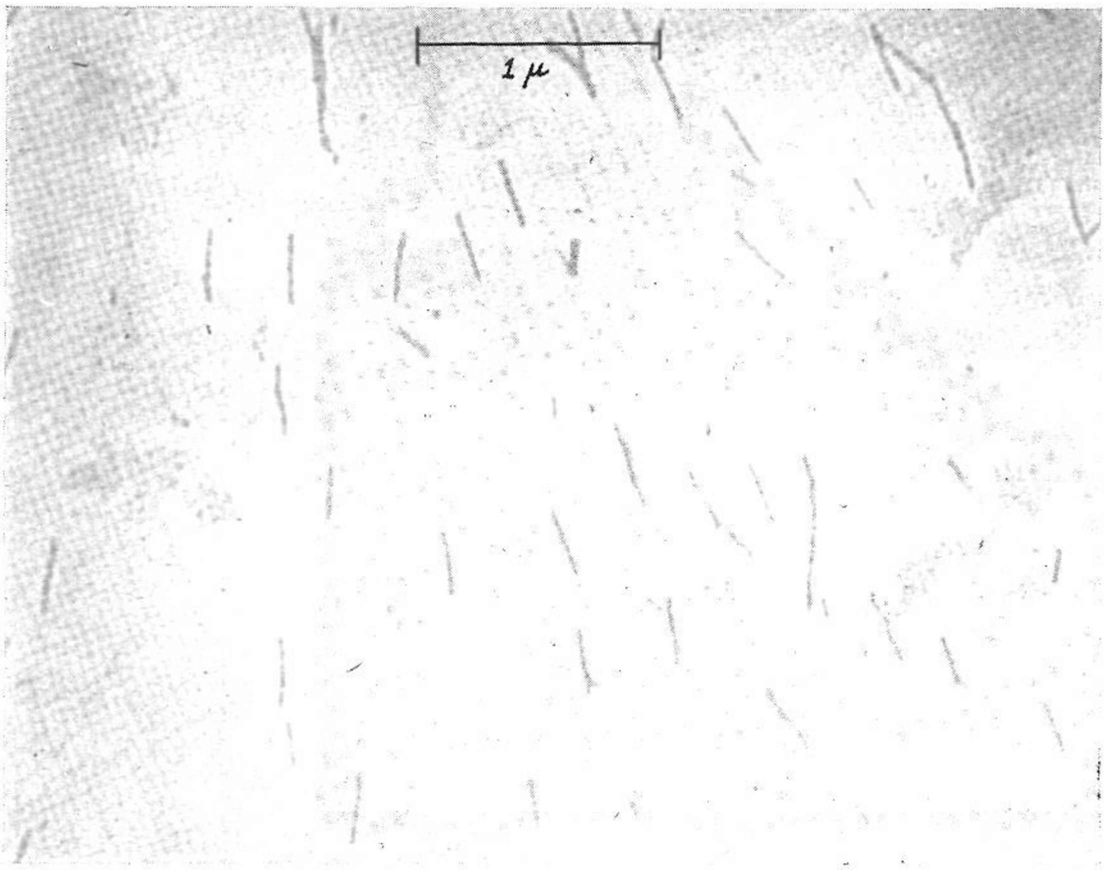

FIG. 9. Electron micrograph of tobacco mosaic virus particles × 24,000. (From Stanley and Anderson (72).)

When these data and those on proteins are considered together, it seems that a good case can be established for the existence of a connection between the intrinsic viscosity and the axial ratio of rigid particles. In reality, this conclusion is in complete harmony with the previously drawn conclusion that, within homologous series of linear polymers or linear molecules, intrinsic viscosity depends upon molecular weight. In an homologous series, all of the particles are probably of the same thickness. Hence, molecular weights within a series are proportional to axial ratios. Therefore, all of the data considered thus far point to the conclusion that the intrinsic viscosity of a suspension or solution of anisometric particles is a function of the ratio of length to thickness of the particles.

III. AN EXPERIMENTAL EXAMINATION OF THE HYDRODYNAMIC APPROACH

Many attempts have been made in recent years to account theoretically for the relationship just observed between the intrinsic viscosity and the shape of particles in suspension or solution. Most of these approaches have been based upon hydrodynamic considerations. Stated crudely, the hydrodynamic viewpoint is that, in a pure liquid undergoing plane laminar flow, infinitesimal layers of the liquid glide over one another, each layer moving with a velocity slightly less than that of its neighbor on one side and slightly greater than that of its neighbor on the other side. In this process energy is dissipated, accounting for the viscosity of the liquid. If a rigid solid object, large compared to the infini-

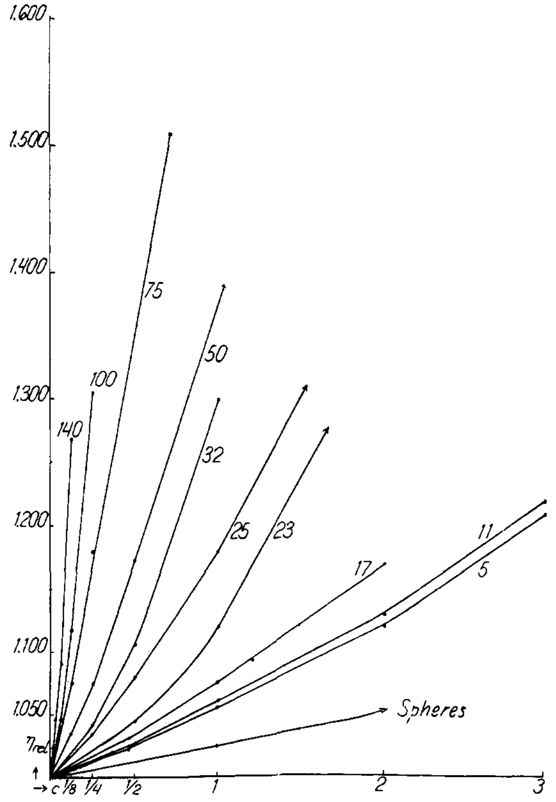

Fig. 10. Relative viscosities of various suspensions of rod-like models plotted as a function of volume per cent concentration. Numbers indicate the ratios of length to thickness of the particular particles for which each curve was obtained. (From Eirich, Margaretha, and Bunzl (15).)

tesimal layers of liquid, i.e., large compared to the dimensions of the fluid particles, is placed in such a flowing system, some layers of the liquid will move faster than the particle and some slower. Hence, liquid will have to flow around the obstruction. This disturbance in the motion of the fluid results in an added dissipation of energy by the system, that is, in increased viscosity.

A. The viscosity of spherical particles

The simplest application of the hydrodynamic approach to this general question was made by Einstein (12) in deriving equation 1, which describes the viscosity of a suspension of rigid spheres, large with respect to the dimensions of the particles of the fluid and small with respect to the dimensions of the viscometer. If an explanation of the relatively complex viscosity behavior of rod-like or thread-like materials is to be sought on the basis of hydrodynamic theory, it is of the utmost importance that the validity of this simplest of applications of such theory be ascertained. Eirich, Bunzl, and Margaretha (14) have considered the matter experimentally. They studied the viscosity of glass spheres with a radius of 80 microns suspended in a mercuric nitrate–nitric acid solution, that of mushroom spores with a radius of 4 microns dispersed in an olive oil–tetrachloroethane mixture, and that of yeast with an average radius of 2.5 microns dispersed in water, using Couette, capillary, and falling-ball vis-

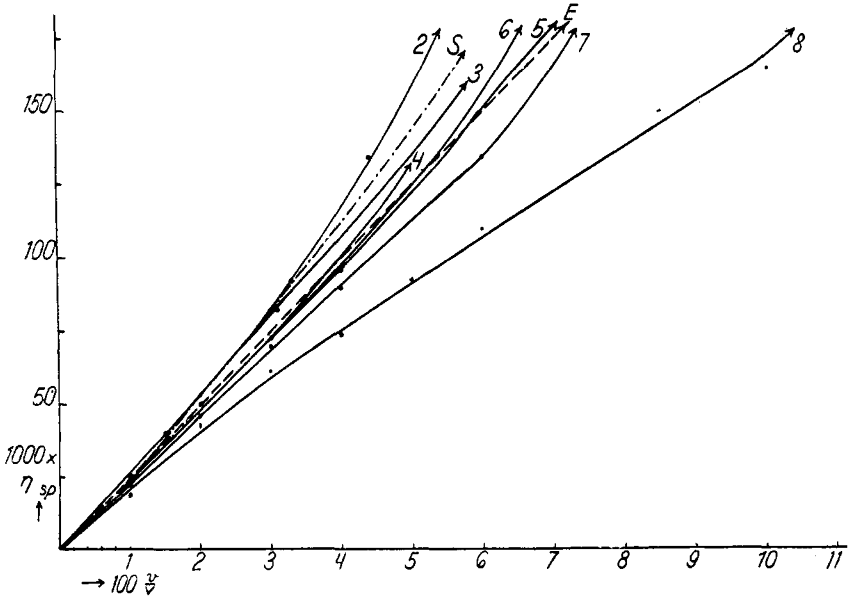

FIG. 11. Specific viscosities plotted against volume concentrations for glass spheres in mercuric nitrate-nitric acid (4, 5, 7, 8), mushroom spores in olive oil-tetrachloroethane (2, 3), and yeast in water (6). Curve E is graph of Einstein equation (equation 1), and curve S is graph of Einstein equation with second-order term. (From Eirich, Bunzl, and Margaretha (14).)

cometers. The spherical natures of all of these particles were ascertained by microscopic examination. In figure 11 specific viscosities are plotted against volume concentrations for the three suspensions. It is evident that the limiting slopes of all of these viscosity–concentration curves approach that demanded by the Einstein equation, with a maximum deviation of less than 10 per cent. These results demonstrate quite conclusively the applicability of the Einstein equation to the situations for which it was derived. In view of this, it seems justifiable to proceed to the more complex problem, namely, the examination of the application of hydrodynamic theory to the relationship between the viscosity and the shape of non-spherical particles.

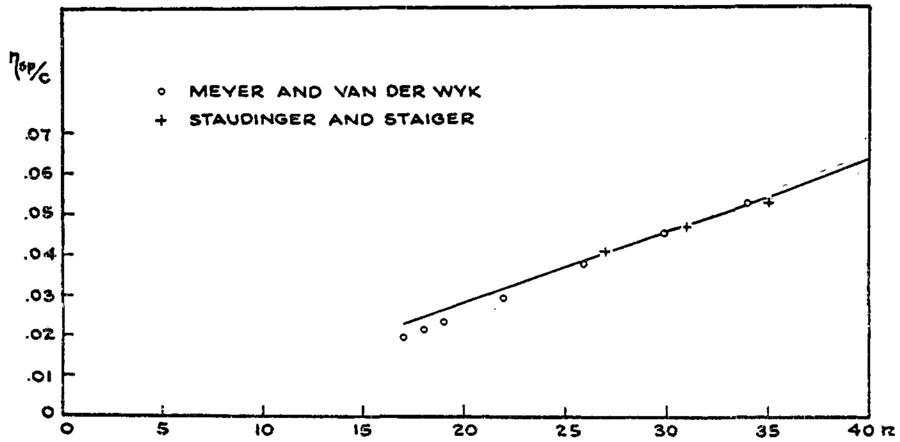

FIG. 12. Base molar intrinsic viscosity of normal hydrocarbons plotted against number of carbon atoms in chain. Smooth curve is plot of Huggins' theoretical equation for randomly kinked chain. Data were taken from Staudinger and Staiger (79) and from Meyer and van der Wyk (54). (From Huggins (30).)

B. The viscosity of randomly kinked chains

Huggins (29, 30) has extended Kuhn's (42) hydrodynamic treatment of the case of a chain of spheres to consider the case of a randomly kinked chain of spheres. Such a system ought to represent a fair model of most of the natural and synthetic linear polymers and compounds considered earlier in this review. An expression which can be reduced to the form of equation 2, that is, to a form which expresses intrinsic viscosity as a linear function of the molecular weight or of the number of repeat units, was obtained. Figure 12, taken from one of Huggins' publications (30), shows how his equation may be made to fit the data of Meyer and van der Wyk and of Staudinger and Staiger discussed earlier. Agreement of this sort can be regarded as being most encouraging. Because it was assumed that the interaction between the individual spheres of the kinked chains and the surrounding medium was that of very small spheres, Huggins' treatment is able to account for intrinsic viscosities less than 2.5. Alfrey, Bartovics, and Mark (1) have been able to account for the effect of temperature and solvent type on the intrinsic viscosity of polymer solutions on the basis of theoretical considerations similar to those of Huggins.

C. *The viscosity of rigid ellipsoids*

It is yet necessary to discuss only the various attempts that have been made to relate the intrinsic viscosity of rigid ellipsoids to axial ratios. These treatments are continuations and modifications of Jeffrey's (31) original extension of the Einstein development for large spheres. In considering the intrinsic viscosity of anisometric particles, three distinct cases must be recognized: (*1*) the case in which the particles are under the influence of complete Brownian movement; (*2*) the case in which there is no Brownian movement; (*3*) the intermediate case. A great many equations have been derived relating axial ratios to the intrinsic viscosity of both rod-like and plate-like ellipsoids under the influence of complete Brownian motion (6, 7, 17, 27, 42, 59, 61, 69). These are summarized graphically in figure 13. The various equations predict that the intrinsic viscosities of rod-like ellipsoids will be greater than that of spheres by amounts which are roughly functions of the squares of the axial ratios. The Simha (69) equation predicts a much higher viscosity than the others. This is due to the fact that the effect of Brownian motion is considered more rigorously in this derivation than in the others. Compared to that of rods, the viscosity of plate-like ellipsoids is relatively insensitive to changes in the axial ratio.

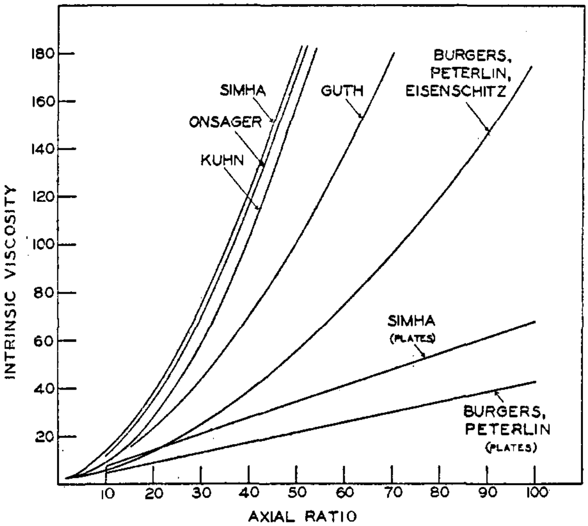

FIG. 13. Graphs of various theoretical equations expressing relationships between the volume fraction intrinsic viscosities of colloidal solutions with complete Brownian motion and the axial ratios of rod-like (undesignated graphs) and plate-like ellipsoids of revolution considered as models of the colloidal particles.

The smooth curve fitting the data in figure 9 is Simha's theoretical curve for rod-like particles. The agreement between the theoretical relationship of particle shape to intrinsic viscosity and the data obtained with proteins is indeed very satisfactory. Using some of these same data, Mehl, Oncley, and Simha (52) computed the axial ratios from diffusion and sedimentation measurements, first, on the assumption that the particles are rod-shaped and, second, on the assumption that they are plate-shaped. Then, using Simha's equation for rods and for plates, they calculated the axial ratios from viscosity data for both cases. It can be observed in table 2, which summarizes their results, that the agreement between the values calculated from diffusion and sedimentation on the one hand and from viscosity on the other hand is very excellent when it is assumed that the particles are rods and just fair when it is assumed that they are plates. As was mentioned previously, this may be taken as a rather feeble indication that most of the proteins are indeed rods.

TABLE 2

The shapes of protein molecules (52)

PROTEIN	D_0/D	η_{sp}/G	b/a, ELONGATED		a/b, FLATTENED	
			Diffusion	Viscosity	Diffusion	Viscosity
Egg albumin	1.17	5.7	3.8	5.0	4.0	6.7
Serum albumin	1.25	6.5	5.0	5.6	5.4	7.7
Hemoglobin	1.16	5.3	3.7	4.6	3.9	6.0
Amandin	1.28	7.0	5.4	6.0	6.0	8.5
Octopus hemocyanin	1.38	9.0	7.2	7.3	8.2	11.4
Gliadin	1.60	14.6	10.9	10.5	13.6	21
Homarus hemocyanin	1.27	6.4	5.2	5.5	5.8	7.5
Helix pomatia hemocyanin	1.24	6.4	4.8	5.5	5.2	7.5
Serum globulin	1.41	9.0	7.6	7.3	8.9	11.4
Thyroglobulin	1.43	9.9	7.8	7.9	9.2	12.7
Lactoglobulin	1.26	6.0	5.2	5.1	5.7	6.9
Pepsin	1.08	5.2	2.5	4.5	2.6	5.8
Helix hemocyanin (pH 8.6)	1.89	18.0	16.6	12.0	23.9	26

Because of the large size of the particles involved, the data of Eirich and collaborators (15) on visible rod-like models exemplify the condition in which there is no Brownian motion. In figure 14 the intrinsic viscosities of Eirich's various rod-like model suspensions are plotted against the axial ratios of the fractions. The two lower curves are the results of the two theoretical treatments of this case (16, 31). Although there is qualitative agreement, as yet no satisfactory quantitative agreement between theoretical and observed relationships of intrinsic viscosity to axial ratios has been attained for the case of rod-like particles not under the influence of thermal movement.

Solutions of tobacco mosaic virus, especially those of highly aggregated preparations, are representative of systems with intermediate Brownian motion. The virus particles in solution do not have dominant or complete Brownian motion, because the solutions exhibit double refraction of flow,—a condition due to the partial or complete orientation of the rod-like particles by the flowing stream (46). However, Brownian movement is not completely overshadowed by the orienting tendency of the flowing field in the case of virus particles, because the intensity of double refraction of flow is a function of the flow-velocity gradient, and because the average orientation of the particles as measured by the angle of isocline, a physical consideration related to the double refraction of flow, increases with increasing velocity gradients (51, 63). For intermediate systems such as this, it is possible to make three definite predictions from hydrodynamic theory in its present state of development: (*1*) The viscosity of even infinitely dilute systems of this sort should be anomalous,—that is, the viscosity coefficient measured in very low velocity gradients should be greater than that measured in higher gradients. This follows simply from the hydrodynamic consideration that rods randomly oriented contribute more to the viscosity of a system than rods oriented parallel to the direction of flow.

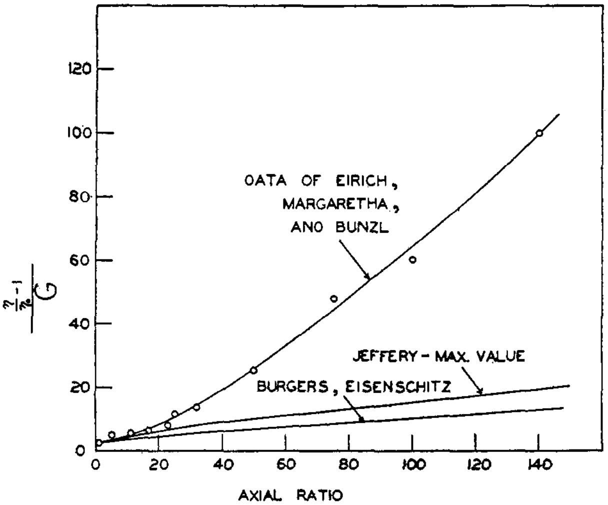

FIG. 14. Volume fraction intrinsic viscosities of suspensions of rod-like models plotted against axial ratios. Upper curve, data of Eirich, Margaretha, and Bunzl (15); lower two curves, graphs of theoretical equations derived for particles with no Brownian motion (16, 31).

(*2*) The decrease in the viscosity coefficient as the velocity gradient is increased should be paralleled by an increase in the intensity of double refraction of flow and by an increase in the average orientation of particles as determined by measurements of the angle of isocline. (*3*) For intermediate velocity gradients, a *positive* temperature coefficient of relative or of intrinsic viscosity should be obtained, but at very high and at very low gradients the temperature coefficient should approach zero. This follows simply from the fact that in intermediate gradients where the rods are partially oriented, increasing the temperature increases the randomness of the particles, thereby increasing their contribution to viscosity. In very high gradients, where Brownian motion is completely dominated, or in very low gradients where it is completely dominant, no such effect can ensue.

Robinson (63) has studied exceedingly dilute tobacco mosaic virus solutions in a Couette-type viscometer equipped with an optical system for measuring double refraction of flow and the angle of isocline. Figure 15 is a diagram showing the relative viscosity of a 0.02 per cent tobacco mosaic virus solution plotted

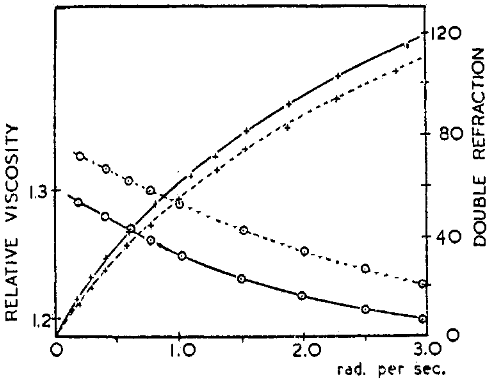

FIG. 15. Relative viscosity (○) and intensity of double refraction of flow (+) of 0.02 per cent tobacco mosaic virus solution plotted against angular velocity of rotating cylinder of viscometer. ———, at 14.4°C.; - - - -, at 19.6°C. (From Robinson (63).)

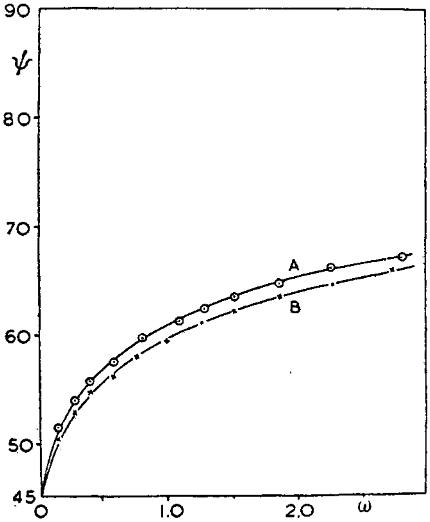

FIG. 16. Angle of isocline (ψ) of 0.02 per cent tobacco mosaic virus solution plotted against angular velocity (ω) of rotating cylinder. Curve A at 14.4°C.; curve B at 19.6°C. (From Robinson (63).)

against the angular velocity of the rotating cylinder. Exactly in accordance with the first prediction, the relative viscosity decreases as the velocity gradient increases. Intensity of double refraction of flow is also plotted against velocity of flow in this figure, and in figure 16 the angle of isocline is plotted against velocity of flow. Exactly in accordance with the second prediction, the decrease of relative viscosity with increase of velocity gradient is paralleled by an increase in the intensity of double refraction and by an increase in the angle of isocline. It may also be observed in figure 15 that the relative viscosity of tobacco mosaic virus is higher at 19.6°C. than at 14.4°C. This is exactly in accordance with the third prediction. From an examination of figure 17, it is evident that the positive temperature coefficient of the relative viscosity has a maximum value for moderate velocity gradients, also exactly as predicted from pure hydrodynamic considerations. None of the currently discussed theories of viscosity based on interaction concepts would predict a temperature relationship such as this. Furthermore, the relative viscosities of substances, such as relatively concentrated gelatin solutions, for which one can reasonably ascribe the viscosity to structure, have negative temperature coefficients. In view of this, it would seem that these experiments constitute perhaps the best confirmation available at present of the validity of the hydrodynamic approach to the description of the viscosity behavior of suspensions of rigid ellipsoids.

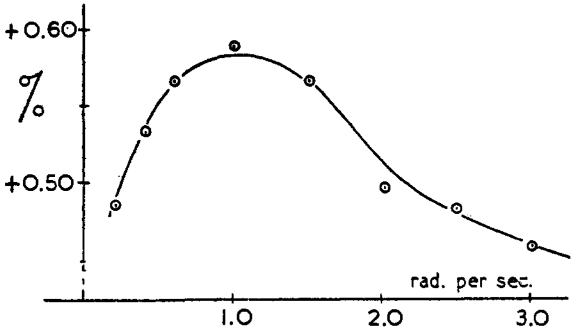

FIG. 17. Temperature coefficient of relative viscosity (ordinate) of 0.02 per cent tobacco mosaic virus solution plotted against angular velocity. (From Robinson (63).)

D. General conclusions

The experimental data which have been examined in connection with the hydrodynamic approach have brought out several facts: (*1*) The equation derived by the hydrodynamic method for the viscosity of spheres in suspension was found to be entirely in accord with the results of experiments designed to test it critically. (*2*) The hydrodynamic method can be used to obtain an equation predicting a linear relationship between the intrinsic viscosity and the molecular weight of chain molecules and polymers. Without making any unreasonable assumptions, this equation can be fitted to actual data with reasonable success. (*3*) The results of the hydrodynamic treatment for the case of rigid ellipsoids undergoing complete Brownian movement can be used to rationalize quantitatively the experimental data obtained on proteins by viscosity methods, on the one hand, and diffusion and sedimentation methods, on the other hand. (*4*) Results of hydrodynamic consideration are in qualitative, though not quantitative, accord with viscosity data on rod-like models too large to have Brownian motion. (*5*) Finally, hydrodynamic theory leads in a seemingly unique way to an explanation of the positive temperature coefficient of relative viscosity observed in solutions of rods with intermediate Brownian motion. Whether or not these encouraging findings justify the acceptance as strictly valid of the equations thus far obtained by the hydrodynamic approach, they certainly justify further consideration of this type of theory in the search for an explanation of the connection between the intrinsic viscosity and the shape of suspended or dissolved particles.

REFERENCES

(1) ALFREY, T., BARTOVICS, A., AND MARK, H.: J. Am. Chem. Soc. **64**, 1557 (1942).
(2) ARRHENIUS, S.: Z. physik. Chem. **1**, 285 (1887).
(3) BAKER, W. O., FULLER, C. S., AND HEISS, J. H., JR.: J. Am. Chem. Soc. **63**, 2142 (1941).
(4) BAKER, W. O., FULLER, C. S., AND HEISS, J. H., JR.: J. Am. Chem. Soc. **63**, 3316 (1941).
(5) BUNGENBERG DE JONG, H. G., KRUYT, H. R., AND LENS, W.: Kolloid-Beihefte **36**, 429 (1932).
(6) BURGERS, J. M.: Second Report on Viscosity and Plasticity, Chap. III. Amsterdam (1938).
(7) BURGERS, J. M.: Proc. Acad. Sci. Amsterdam **43**, 307 (1940).
(8) CAROTHERS, W. H., AND VAN NATTA, F. J.: J. Am. Chem. Soc. **55**, 4714 (1933).
(9) CARTER, R. O.: J. Am. Chem. Soc. **63**, 1960 (1941).
(10) CARTER, R. O., AND HALL, J. L.: J. Am. Chem. Soc. **62**, 1194 (1940).
(11) COHEN, S. S., AND STANLEY, W. M.: J. Biol. Chem. **144**, 589 (1942).
(12) EINSTEIN, A.: Ann. Physik **19**, 289 (1906); **34**, 591 (1911).
(13) EINSTEIN, A.: Ann. Physik **17**, 549 (1905); Z. Elektrochem. **14**, 235 (1908).
(14) EIRICH, F., BUNZL, M., AND MARGARETHA, H.: Kolloid-Z. **74**, 276 (1936).
(15) EIRICH, F., MARGARETHA, H., AND BUNZL, M.: Kolloid-Z. **75**, 20 (1936).
(16) EISENSCHITZ, R.: Z. physik. Chem. **A158**, 78 (1931).
(17) EISENSCHITZ, R.: Z. physik. Chem. **A163**, 133 (1933).
(18) FERRY, J. D., AND ONCLEY, J. L.: J. Am. Chem. Soc. **60**, 1123 (1938).
(19) FLORY, P. J.: J. Am. Chem. Soc. **58**, 1877 (1936).
(20) FLORY, P. J.: J. Am. Chem. Soc. **61**, 3334 (1939).
(21) FLORY, P. J.: J. Am. Chem. Soc. **62**, 1057 (1940).
(21a) FLORY, P. J.: Abstracts, Buffalo Meeting of the American Chemical Society, 11Q (September, 1942).
(22) FLORY, P. J., AND STICKNEY, P. B.: J. Am. Chem. Soc. **62**, 3032 (1940).
(23) FORDYCE, R., AND HIBBERT, H.: J. Am. Chem. Soc. **61**, 1910, 1912 (1939).
(24) FORDYCE, R., LOVELL, E. L., AND HIBBERT, H.: J. Am. Chem. Soc. **61**, 1905 (1939).
(25) FRAMPTON, V. L., AND NEURATH, H.: Science **87**, 436 (1938).
(26) GANS, R.: Ann. phys. [4] **86**, 628 (1928).
(27) GUTH, E.: Kolloid-Z. **74**, 147 (1936).
(28) HERZOG, R. O., ILLIG, R., AND KUDAR, H.: Z. physik. Chem. **A167**, 329 (1933).
(29) HUGGINS, M. L.: J. Phys. Chem. **42**, 911 (1938); **43**, 439 (1939).
(30) HUGGINS, M. L.: J. Applied Phys. **10**, 700 (1939).
(31) JEFFERY, G. B.: Proc. Roy. Soc. (London) **A102**, 163 (1923).
(32) KEMP, A. R., AND PETERS, H.: Ind. Eng. Chem. **33**, 1263 (1941).
(33) KRAEMER, E. O.: Chapter XX of Taylor's *Treatise on Physical Chemistry*, 2nd edition. D. Van Nostrand Company, New York (1931).
(34) KRAEMER, E. O.: Ind. Eng. Chem. **30**, 1200 (1938).
(35) KRAEMER, E. O.: J. Phys. Chem. **45**, 661 (1941).
(36) KRAEMER, E. O.: J. Phys. Chem. **46**, 177 (1942).

(37) Kraemer, E. O., and Lansing, W. D.: J. Am. Chem. Soc. **55**, 4319 (1933).
(38) Kraemer, E. O., and Lansing, W. D.: Nature **133**, 870 (1934).
(39) Kraemer, E. O., and Lansing, W. D.: J. Phys. Chem. **39**, 153 (1935).
(40) Kraemer, E. O., and Nichols, J. B.: Part IV, 1, of Svedberg and Pedersen's *The Ultracentrifuge.* University Press, Oxford (1940).
(41) Kraemer, E. O., and Van Natta, F. J.: J. Phys. Chem. **36**, 3174 (1932).
(42) Kuhn, W.: Kolloid-Z. **62**, 269 (1933).
(43) Lansing, W. D., and Kraemer, E. O.: J. Am. Chem. Soc. **57**, 1369 (1935).
(44) Lauffer, M. A.: Science **87**, 469 (1938); J. Biol. Chem. **126**, 443 (1938).
(45) Lauffer, M. A.: Unpublished results.
(46) Lauffer, M. A., and Stanley, W. M.: J. Biol. Chem. **123**, 507 (1938).
(47) Lauffer, M. A., and Stanley, W. M.: Chem. Rev. **24**, 303 (1939); Kolloid-Z. **91**, 62 (1940).
(48) Lovell, E. L., and Hibbert, H.: J. Am. Chem. Soc. **62**, 2140 (1940).
(49) Mark, H.: *Physical Chemistry of High Polymeric Systems.* Interscience Publishers, Inc., New York (1940).
(50) Mark, H., and Simha, R.: Nature **145**, 571 (1940).
(50a) Mead, D. J., with Fuoss, R. M.: J. Am. Chem. Soc. **64**, 277 (1942).
(51) Mehl, J. W.: Cold Spring Harbor Symposia Quant. Biol. **6**, 218 (1938).
(52) Mehl, J. W., Oncley, J. L., and Simha, R.: Science **92**, 132 (1940).
(53) Meyer, K. H.: Kolloid-Z. **95**, 70 (1941).
(54) Meyer, K. H., and van der Wyk, A.: Helv. Chim. Acta **18**, 1067 (1935).
(55) Neurath, H.: J. Am. Chem. Soc. **61**, 1841 (1939).
(56) Neurath, H., Cooper, G. R., and Erickson, J. O.: J. Biol. Chem. **138**, 411 (1941).
(57) Oncley, J. L.: J. Phys. Chem. **44**, 1103 (1940).
(58) Oncley, J. L.: Ann. N. Y. Acad. Sci. **41**, 121 (1941).
(59) Onsager, L.: Phys. Rev. **40**, 1028 (1932).
(60) Perrin, F.: J. phys. radium **5**, 497 (1934); **7**, 1 (1936).
(61) Peterlin, A.: Kolloid-Z. **86**, 230 (1939).
(62) Polson, A.: Kolloid-Z. **88**, 51 (1939).
(63) Robinson, J. R.: Proc. Roy. Soc. (London) **A170**, 519 (1939).
(64) Signer, R.: Trans. Faraday Soc. **32**, 296 (1936).
(65) Signer, R.: Part IV, A, 2 of Svedberg and Pedersen's *The Ultracentrifuge.* University Press, Oxford (1940).
(66) Signer, R., and Gross, H.: Helv. Chim. Acta **17**, 58, 335, 726 (1934).
(67) Signer, R., and Liechti, J.: Helv. Chim. Acta **21**, 530 (1938).
(68) Signer, R., and von Tavel, P.: Helv. Chim. Acta **21**, 535 (1938).
(69) Simha, R.: J. Phys. Chem. **44**, 25 (1940).
(70) Simha, R.: J. Applied Phys. **13**, 147 (1942).
(71) Stanley, W. M.: J. Phys. Chem. **43**, 55 (1938).
(72) Stanley, W. M., and Anderson, T. F.: J. Biol. Chem. **139**, 325 (1941).
(73) Staudinger, H.: *Die hochmolekularen organischen Verbindungen.* J. Springer, Berlin (1932).
(74) Staudinger, H.: Trans. Faraday Soc. **29**, 18 (1933).
(75) Staudinger, H.: Trans. Faraday Soc. **32**, 97 (1936).
(76) Staudinger, H., and Nuss, O.: J. prakt. Chem. **157**, 283 (1941).
(77) Staudinger, H., and Ochiai, E.: Z. physik. Chem. **A158**, 35 (1931).
(78) Staudinger, H., and Reinecke, F.: Ann. **535**, 47 (1938).
(79) Staudinger, H., and Staiger, F.: Ber. **68**, 707 (1935).
(80) Sutherland, W.: Phil. Mag. **9**, 781 (1905).
(81) Svedberg, T.: Chem. Rev. **14**, 1 (1934); **20**, 81 (1937).