Chinese Chemical Letters Vol. 12, No. 3, pp. 223–226, 2001

Two-Parameter Stochastic Resonance in a Model of Electrodissolution of Fe in H_2SO_4

Zong Xin PI, Hou Wen XIN*

Department of Chemical Physics, University of Science and Technology of China, Hefei 230026

Abstract: Stochastic resonance (SR) is shown in a two-parameter system, a model of electrodissolution of Fe in H_2SO_4. Modulation of two different parameters by a periodic signal in one parameter and noise in the other parameter is found to give rise to SR. The result indicates that the noise can enlarge a weak periodic signal and lead the system to order. The scenario and novel aspects of SR in this system are discussed.

Keywords: Noise, signal, stochastic resonance.

Introduction

Stochastic resonance (SR) is a nonlinear effect which describes the optimal detection of a weak periodic signal by the action of external noise in nonlinear systems. SR was originally proposed to account for periodicity in the Earth's ice ages[1], and it has been observed in a wide range of biological[2-4], physical[5] and chemical[6] systems.

The systems mentioned above which may give rise to SR have a denominator despite many examples of SR in different scientific areas. It is noted that the signal and the noise are assigned to one parameter in the models[2-6], the signal is buried in noise from the identical physical sources. Under some circumstances, the sources of a signal and noise may be different. This kind of SR was called two-parameter SR which was introduced by Amemiya and his colleagues[7].

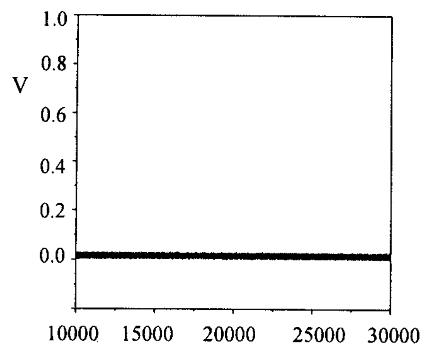
A reaction model, which describes the electrodissolution of Fe in H_2SO_4, is applied in this paper. The two controllable parameters are independent bifurcation parameters in this model. Steady-state close a Hopf bifurcation is modulated by a signal in one parameter and noise in the other parameter. The calculative results show that SR occurs in this system under the condition.

Reaction model and stochastic resonance

Franck and FitzHugh[8] proposed a simple model for the electrodissolution Fe in H_2SO_4 in the region of potential near the active-passive transition. This model was nondimensionalized[9], giving

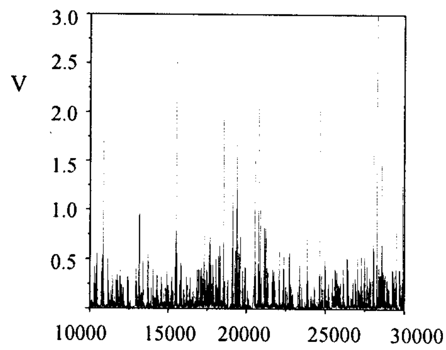
$$\begin{cases} dV/dt=b^*h-V-b^*\theta+V^*G(V,\theta)/c \\ d\theta/dt=V^*G(V,\theta) \end{cases} \qquad (1)$$

Figure 1(a)



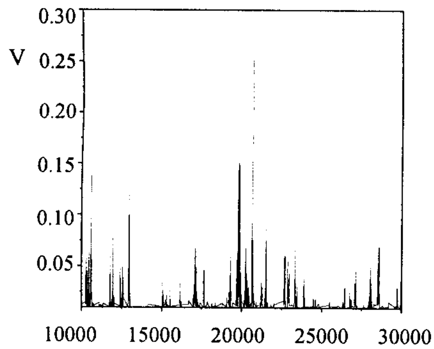
Time series when noise intensity is 0.0

Figure 1(b)



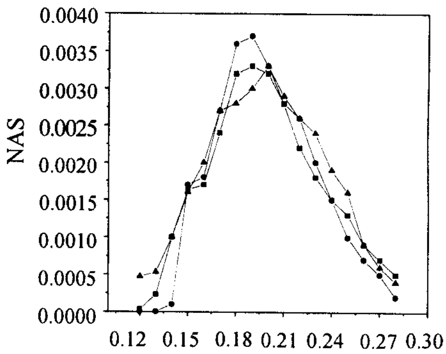
Time series when noise intensity is 0.17

Figure 1(c)



Time series when noises intensity is 0.26

Figure 2



NAS is plotted as a function of the noise intensity under three conditions: the duration of noise is 2[■],3[●],4[▲] respectively. The NAS is divided by the integration time: 30000.

here, b, c, h are the ratios of the rate constants in Franck and FitzHugh's model, and $G(V, \theta) = [\theta + (1-2*\theta)*U(V)]$, where U is the Heaviside step function.

The results of the linear stability analysis of equations (1) are given elaborately[9]. According to the linear stability diagram[9], the boundary lines divide the region into excitable steady state (SS) and oscillatory state (OSC).

One parameter (h) and the other parameter (c) that are modulated by a periodic or a stochastic component respectively, and expressed as follows:

$$c = c_0(1+D\,\xi(t)) \qquad (2)$$
$$h = h_0(1+\beta \sin(2\pi\,\omega t)) \qquad (3)$$

where c_0, h_0 are the constant components, their values are 0.5 and 0.525, respectively. D is the noise intensity, $\xi(t)$ is the white noise (the durations of the noise may be 0.5, 1 , 2, 3 or 5), β is the amplitudes of the sinusoidal signal, ω is the frequency of the signal, the values of β and ω are 0.03, 0.04, respectively.

Equations (1)-(3) are calculated numerically using the fourth-order Runge-Kutta method with the time step being taken as 0.01. The time series of the data (30000) are analyzed by examining an interspike distribution at the period of the signal to detect SR. An interspike histogram is made[7], but the time interval is 22.5 and 27.5 in this paper. The number of adjacent spikes obtained from six calculations using different random number was averaged at each value of the noise intensity.

The system shows no response when a periodic signal (β = 0.03, ω = 0.04) is imposed on the parameter h without the noise in the parameter c (**Figure 1(a)**). With the help of noise, irregular spikes of oscillations begin to show. Increasing the noise intensity, more spikes appear, and the interspike intervals approach the period of the sinusoidal signal in the parameter h (**Figure 1(b)**). When the noise amplitude is further increased , the number of adjacent spikes whose time intervals are between 22.5 and 27.5 decreases, the result is shown in **Figure 1(c)**. A maximum of the number of adjacent spikes is the feature of SR[7]. SR is shown in this electrochemical system. **Figure 2** shows the number of adjacent spikes as a function of the noise intensity.

We find that the durations of the noise can influence the appearance of stochastic resonance. According to **Figure 2**, when the duration of the noise is 1, 2 or 3, stochastic resonance is shown, but when the duration is 0.5 or 5, the system shows disordered interspike distributions (not shown).

Stochastic resonance occurs near Hopf bifurcation point when the periodic signal and noise are added to the two different parameters, respectively. The Hopf bifurcation point is used as a constant threshold for SR in this paper. The results of this paper can be explained by the threshold scenario[10]. A periodic signal in one parameter h changes the threshold of the parameter c. The parameter c can give rise to stochastic resonance when the system passes the periodically perturbed threshold with the help of the noise.

Conclusion

Computational study of a model of the electrodissolution of Fe in H_2SO_4 shows that two-parameter stochastic resonance occurs when a periodic signal and noise are added to different parameters. The noise can play a constructive role to detect a weak signal in this electrochemical system. Noise can induce oscillations, and the interspike distribution at the frequency of the weak periodic signal passes through a maximum at an optimal noise intensity, in other words, noise at optimal intensity can lead the system to order.

Two-parameter system has features that one parameter system does not have. Because the two parameters act different roles in this electrochemical system, and noise in one parameter can enhance a signal in the other parameter. We may assume that selective enhancement in a signal in one of the two parameters might take place.

Acknowledgment

This research was supported by the National Natural Science Foundation of China (No.29892165) and the National Laboratory of Theoretical and Computational Chemistry of China.

References

1. R. Benzi, A. Sutera, A. Vulpiani, *J. Phys. A.*, **1981**, *14*, L453.
2. J. K. Douglass, L. Wilkens, E. Pantazelou, F. Moss, *Nature*, **1993,** *365*, 337.
3. J. E. Levin, J. P. Miller, *Nature*, **1996**, *380*, 165.
4. F. Moss, J. K. Douglass, L. Wilkens, D. Pierson, *Ann. N. Y. Acad. Sci*., **1993**, *26*, 706.
5. S. Fauve, F. Heslot, *Phys. Lett.*, **1983**, *97A*, 5.
6. L. F. Yang, Z. H. Hou, H. W. Xin, J. Chem. Phys., **1998**, 109, 5, 2002.
7. T. Amemiya, T. Ohmori, T. Yamamoto, T. Yamaguchi, J. Phys. Chem. A, **1998**, 102, 4537.
8. U. F. Franck, R. FitzHugh, *Z. Electrochem.*, **1961**, *65*, 156.
9. A. J. Pearlstein, A. J. Jacqueline, *J. Electrochem. Soc.*, **1989**, *136, 5*, 1290.
10. P. Jung, *Phys. Rev. E.*, **1994,** *50*, 2513.

Received 22 May, 2000
Revised 11 December, 2000