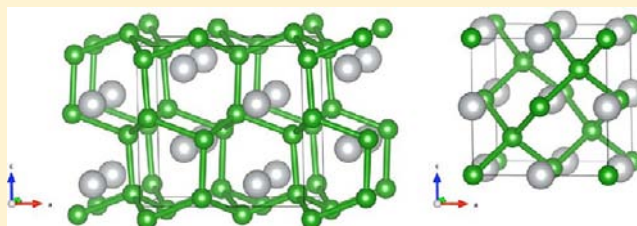


Making Sense of Boron-Rich Binary Be–B Phases

Andreas Hermann,^{*,†} N. W. Ashcroft,[‡] and Roald Hoffmann[†][†]Department of Chemistry and Chemical Biology and [‡]Laboratory of Atomic and Solid State Physics, Cornell University, Ithaca, New York 14853, United States

Supporting Information

ABSTRACT: There is much uncertainty in the literature about the structure of several Be–B phases between 20 and 33 atom % Be. We clarify the structural choices in this region of the phase diagram, proposing structural candidates obtained from a combination of chemical intuition and unbiased solid-state structure searches. In particular, we discuss the structural, dynamical, and electronic properties of the ground states of the BeB₂, BeB₃, and BeB₄ phases, as well as those of the complex (and superconducting) “BeB_{2.75}” phase. For the latter, we find the polyhedral borane cluster electron-counting approach very useful to explain its electronic structure. We can also make sense of the partial and mixed occupancies in the structure by looking at the cavities in a parent structure. A Be₂₉B₈₁ stoichiometry seems most reasonable for the ground state of this phase. The electronic structure points to a region of stability for three additional electrons per unit cell, a 1% difference in total electron count. For BeB₂, which is usually studied computationally in the AlB₂ structure type, we find several other structure types that are more stable, all essentially Zintl phases with 4-connected boron networks. New structure types are also predicted for BeB₃ and BeB₄ as well.



INTRODUCTION

Between BeB₂ and BeB₄ in the Be–B phase diagram, a region that corresponds to binaries with 20–33% of atomic beryllium content, a variety of binary compounds have been suggested in the literature, some simple and some complex. The BeB₂ stoichiometry was the subject of experimental studies several decades ago^{1,2} at room temperature and found to crystallize in *P6/mmm* symmetry with a large unit cell, $a = 9.79$ Å and $c = 9.55$ Å.² Later, the same crystal structure was found but assigned to a *different* stoichiometry, BeB₃.³ In recent years, it has been argued that these older measurements actually detected a Be_{1.09}B₃ phase (or, equivalently, BeB_{2.75}), which crystallizes in this large unit cell, with various partially occupied sites.^{4,5} This is the phase marked as Be_{4.9}B_{13.5} in the experimental Be–B phase diagram.⁶ On the other hand, it has also been postulated that BeB₂ would crystallize, like MgB₂, in the AlB₂ structure,^{7–9} and (theoreticians being particularly prone to wishful thinking) this is especially true in several computational contributions following the discovery of superconductivity in MgB₂.^{10–12} BeB₄, of unknown structure,⁶ completes the experimentally available structures in this region of the Be–B phase diagram.

In this contribution, we aim to bring some order to the complicated experimental situation in the low-temperature region of this section of the Be–B phase diagram. We will concentrate here on the ground states of the stoichiometric BeB₂, BeB₃, BeB₄, and “BeB_{2.75}” phases and briefly discuss some other stoichiometries that are found in other group 2/group 13 compounds. The computational methodology is discussed in the Supporting Information (see also refs 13–20). In another publication we will present a wider range of Be–B

stoichiometries at higher pressures. Here we remain at $P = 1$ atm.

BeB₂

As many have, we looked at the MgB₂ crystal structure (space group 191, *P6/mmm*, one unit per cell, prototype AlB₂) for this phase and in the ground state. We also examined other group 2/group 13 binary structures, of which there are quite a few: the CaAl₂ structure (space group 227, *Fd $\bar{3}m$* , eight units per cell, prototype MgCu₂, the cubic C15 Laves phase²¹); the SrAl₂ structure (space group 74, *Imma*, four units per cell, prototype CeCu₂²²); the CaIn₂ and MgGa₂ structures (space group 194, *P6₃/mmc*, two units per cell^{23,24}); and another MgGa₂ structure (space group 55, *Pbam*, eight units per cell²⁵). Some of these are shown in Figure 1, together with other favorable structure types which we will introduce below.

The MgCu₂ and MgGa₂-*Pbam* structures proved to be quite uncompetitive in enthalpy. The CeCu₂ structure, when optimized, transforms into the AlB₂ structure. The AlB₂ structure for BeB₂ itself is, however, unstable with respect to the CaIn₂ structure, which can be constructed from the AlB₂ structure by doubling the unit cell along z and introducing buckling in the graphitic B sheets so that the B atoms then form a hcp lattice, see Figure 1. Among the known structure types, we find this structure to be the most stable but only at the level of 2 meV/atom, see Figure 2. This corresponds to about 24K in thermal terms.

Received: June 7, 2012

Published: August 8, 2012

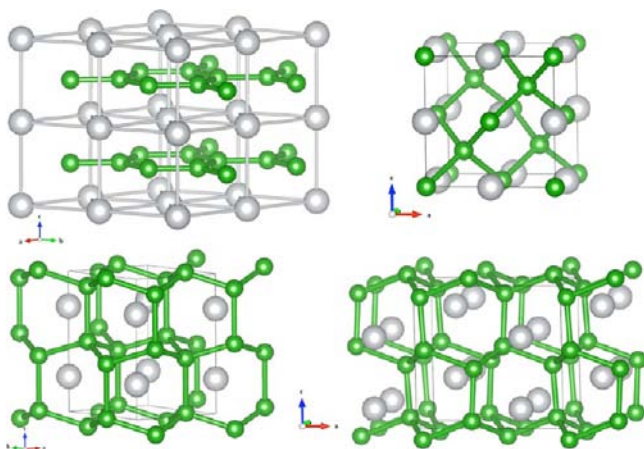


Figure 1. BeB₂ crystal structures. From top left to bottom right: AlB₂, *F43m*, CaIn₂, and *Pnma* structures (see text for details). All are optimized structures at *P* = 1 atm. Note the buckled boron layers in the two structures in the bottom panel. Small green (large gray) spheres denote boron (beryllium) atoms.

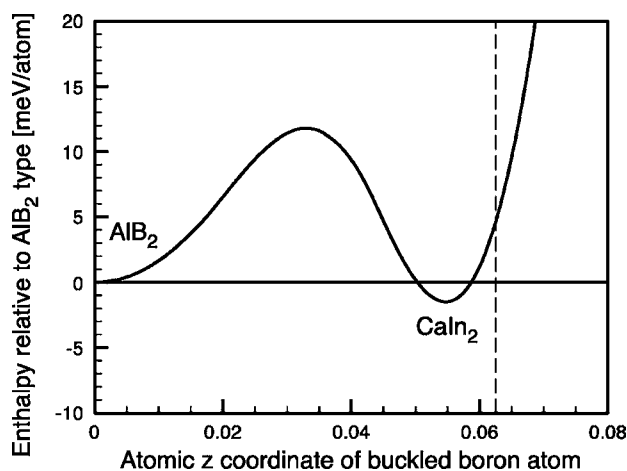


Figure 2. Cohesive energy for BeB₂, relative to the AlB₂ structure type, if buckling of boron layers (see Figure 1) is introduced. Vertical dashed line indicates the buckling needed for ideal tetrahedral coordination in a cell with ideal *c/a* ratio.

However, the CaIn₂ structure is dynamically unstable with respect to an alternate displacement of the Be atoms along *z* and a simultaneous distortion of the hcp sublattice of the B atoms, which lowers the space group symmetry to *Pnma* (see Figure 1) and the enthalpy of formation by another 19 meV/atom. Table 2 in the Supporting Information lists the changes in the atomic positions that correspond to this transition. Note that, among others, PbCl₂ and Co₂Si crystallize in space group *Pnma* with the same Wyckoff site occupations as BeB₂ but with different coordination environments.

The instability of the CaIn₂ structure notwithstanding, it is interesting to study the transition from the AlB₂ to the CaIn₂ structure. We did so by manually introducing buckled layers, i.e., forcing the boron atom to occupy the *4f* Wyckoff site ($\frac{1}{3}, \frac{2}{3}, \frac{1}{2} - z$) instead of the more symmetric ($\frac{1}{3}, \frac{2}{3}, \frac{1}{2}$) site it occupies in MgB₂. Complete cell optimization while keeping *z* constant then allows us to model the enthalpy of formation solely as a function of the buckling, as shown in Figure 2.

One can see that both the AlB₂ and the CaIn₂ structures are local minima along this path and that the CaIn₂ structure is

slightly lower in energy. The optimal buckling for BeB₂ corresponds to *z* = 0.055, slightly less than the ideal value of *z* = 0.0625 for tetrahedral coordination. Together with the *c/a* ratio of 1.663 (ideal 1.633), that means that in the CaIn₂ structure the in-plane B–B distances are significantly shorter (*d* = 1.82 Å) than the out-of-plane B–B distances along the *c* axis (*d* = 1.95 Å). This could be a reflection of the finite size of the beryllium atoms in the interstitial sites, which enforce the elongation of the boron network along the *c* axis. Note that this inequality of bond lengths is partially rectified in the more stable *Pnma* phase, where in-plane B–B distances of *d* = 1.82–1.92 Å are on average much closer to the out-of-plane B–B distances of *d* = 1.87 Å. Note also that the barrier between the AlB₂ and the CaIn₂ structures is only about 12 meV/atom, to be compared with zero-point energies of about 105 meV/atom in the *Pnma* phase.

From an electronegativity perspective, the stability of the CaIn₂ and *Pnma* structures can be explained by beryllium donating its two valence electrons to a (B[−]) network that then becomes isoelectronic to carbon. We have then a classical Zintl system, typified by NaTl.^{26,27} In contrast to MgB₂, and many other metal diborides which form graphitic layers^{28,29} and thus three-connected boron networks, we find that the most stable structures for BeB₂ are inherently three dimensional and with four-connected boron atoms. We would argue that the smaller size of beryllium allows boron to form a three-dimensional network (with the slight anisotropy discussed above), which is more stable. A similar effect is found in alkaline metal digallides, where a transition from the CaIn₂ structure (with a three-dimensional gallium sublattice) to the AlB₂ structure (with planar gallium networks) occurs between Ca and its larger analogues Sr and Ba.³⁰

We then wondered if this Zintl perspective might also be useful in designing other BeB₂ structures in their ground states and beyond. If the (B[−])–sublattice is isoelectronic to carbon, could a diamond-like boron network be stabilized in BeB₂, with beryllium occupying tetrahedral holes? This indeed proves to be the case: such a structure, of *F43m* symmetry and depicted in Figure 1, is actually more stable at *P* = 1 atm than the other structure types discussed above and is the only BeB₂ structure we find which is more stable than the elements, see Table 1,

Table 1. Enthalpies of Formation for Different BeB₂ Crystal Structures

| structure type | AlB ₂ | “CaIn ₂ ” | MgGa ₂ | MgCu ₂ | “AlLiSi” |
|------------------------|------------------|----------------------|-------------------|-------------------|-------------|
| space group | <i>P6/mmm</i> | <i>Pnma</i> | <i>Pbam</i> | <i>Fd3m</i> | <i>F43m</i> |
| ΔH_f [eV/atom] | +0.120 | +0.099 | +0.179 | +0.687 | −0.016 |

although not by much (note that dynamical contributions, such as zero-point energies, are not included here). We find this structure dynamically stable. The phase is a binary version of the AlLiSi structure, a half-Heusler compound, with Be on the Li sites and B on both Al and Si sites.³¹ The B–B distance is *d* = 1.87 Å, longer than intrapolyhedral bond lengths in pure boron which are about 1.7–1.8 Å.

A structure search with *Z* = 8 formula units at *P* = 1 atm confirmed the proposed *F43m* structure as the global minimum structure for BeB₂. In line with the electronegativity reasoning above, this structure is a semiconductor, in contrast to all other structure types presented for the BeB₂ phase (see Figure 3 for electronic densities of states (DOS) for the most stable structures).

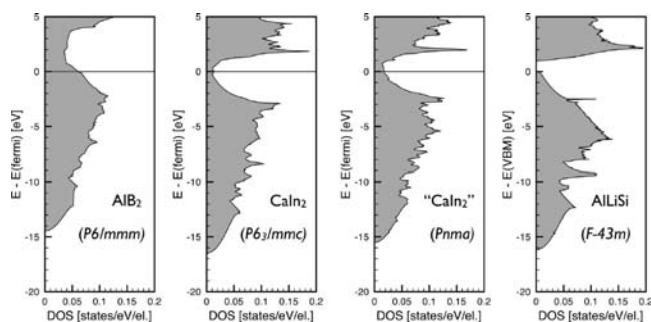


Figure 3. Electronic DOS per electron for BeB_2 in various structure types (from left those corresponding to AlB_2 , CaIn_2 , “ CaIn_2 ”- $Pnma$, and AlLiSi , with space groups indicated), all at $P = 1$ atm. Energies are given with respect to the respective Fermi energy or valence band maximum (VBM, the highest occupied crystal orbital).

A structure with a three-dimensional, four-connected boron network could have interesting mechanical properties. An estimate of the hardness³² of the $F\bar{4}3m$ structure is $H = 29.5$ GPa (with bulk modulus $B_0 = 219$ GPa), which is much less than the hardness of diamond (95 GPa³³) but compares well to transition metal diborides such as TiB_2 , ZrB_2 , or ReB_2 with reported hardness values of 20–40 GPa.^{34–37} For the $Pnma$ structure we find $H = 29.3$ GPa (bulk modulus 170 GPa, and we only count the B–B bonds as covalent bonds in these structures), and for the AlB_2 -type structure we obtain $H = 12.4$ GPa (bulk modulus 172 GPa, and considering also Be–B bonds along the c axis for the hardness estimate).

■ BeB_3

A more boron-rich phase than BeB_2 is BeB_3 . This is the stoichiometry found in MgIn_3 (space group 221, $Pm\bar{3}m$, one formula unit per cell), which crystallizes in the AuCu_3 structure type. Using this structure for BeB_3 results in a very high enthalpy of formation ($\Delta H_f = +0.86$ eV/atom with respect to the elemental crystals).

Evolutionary structure searches at $P = 1$ atm and 160 GPa, using $Z = 4$ formula units per cell led to a variety of low-enthalpy phases, with the best ones stable at $P \geq 160$ GPa. Note that by using 12 boron atoms in the unit cell, structures with B_{12} or B_6 clusters were accessible to the evolutionary structure search algorithms. However, the best metastable structures at atmospheric pressure, of $C2/m$, $P2_1/m$, and Cm symmetry, are basically planar, as shown in Figure 4, and do not indicate that cluster formation is favorable for this stoichiometry.

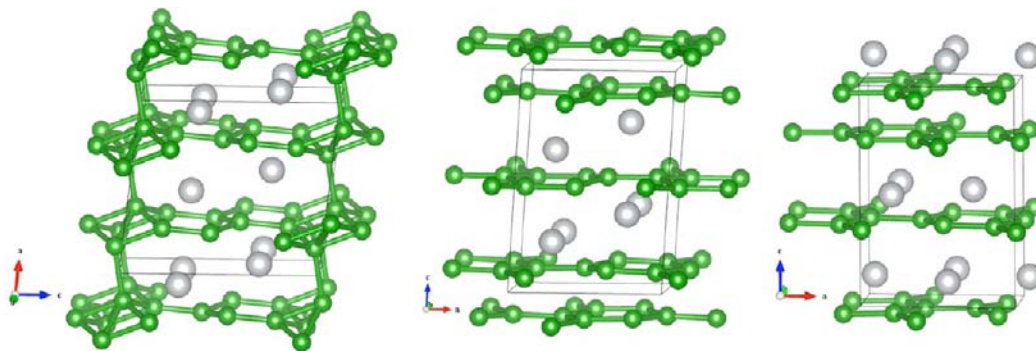


Figure 4. Predicted BeB_3 crystal structures: (left) $C2/m$ phase; (middle) $P2_1/m$ phase; (right) Cm phase. All shown to the same scale and optimized at $P = 1$ atm.

etry (one reviewer commented that this is surprising for a rather boron-rich system). The enthalpies of formation for these structures are listed in Table 2: the $C2/m$ structure is stable with respect to the elements but not by much and, as we shall see below, is unstable with respect to other binary stoichiometries.

Table 2. Relative Enthalpies of Formation for the Most Stable BeB_3 Structures

| space group | $C2/m$ | $P2_1/m$ | Cm |
|------------------------|--------|----------|--------|
| ΔH_f [eV/atom] | −0.018 | −0.002 | +0.043 |

The $P2_1/m$ and Cm structures exhibit features of the AlB_2 structure: graphite-like boron sheets, separated by triangular beryllium nets. However, because of the higher atomic boron content in BeB_3 , an additional graphitic boron sheet is present in the unit cell (or, equivalently, one beryllium net is missing). The structures differ slightly in that the $P2_1/m$ structure has buckled boron and beryllium layers, compared to flat sheets in the Cm structure. B–B distances within the graphitic layers range from 1.67 to 1.77 Å in both structures; the nearest B–B distances between adjacent layers range from 1.78 to 1.88 Å and are thus very much comparable to the intralayer distances.

The $C2/m$ structure is also similar to the AlB_2 structure but solves the stoichiometry mismatch in a different way: instead of extended graphene-like sheets, one-dimensional graphitic ribbons of boron are formed, with direct B–B contacts between the ribbons (see Figure 5, where these contacts are

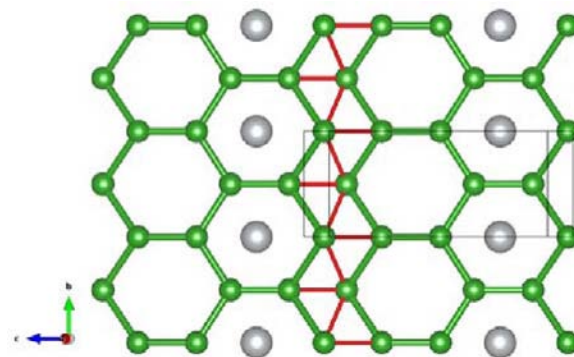


Figure 5. $C2/m$ structure of BeB_3 at $P = 1$ atm, as seen along the $[100]$ direction. Recall that the boron ribbons are buckled; their connections are shown in red.

drawn in red). The B–B distances within the ribbons range from 1.63 to 1.82 Å, while the distances between the ribbons themselves are 1.72 Å. The closest B–B distance between layers is 1.89 Å.

The calculated electronic structures confirm the viewpoint of these structures as low-dimensional constructs. Their electronic DOS, shown in Figure 6, share the square onset at low energies

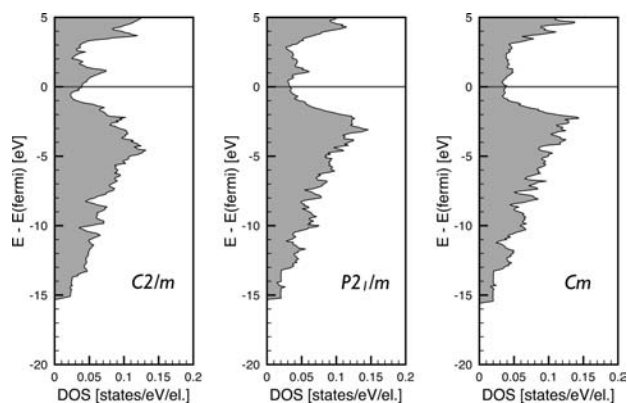


Figure 6. Electronic DOS of three candidate structures for BeB_3 at $P = 1$ atm.

that is associated with a two-dimensional electron system. These lowest valence states are associated with boron's $2s$ level and hence suggest a basic two dimensionality of the boron sublattices in these structures. Notice also that all structures are metallic, but the density of states at the Fermi energy is significantly depleted.

Under pressure, the graphitic boron sheets are unstable with respect to formation of three-dimensional boron networks, and hence the most stable high-pressure structures are similar to the CaIn_2 structure type, which we discussed for BeB_2 : hexagonal close-packed boron lattices with beryllium atoms in the interstitial positions. At low pressures, however, the low-temperature phases are dominated by the complicated “ $\text{BeB}_{2.75}$ ” structure, which is discussed further below.

■ BeB_4

This stoichiometry is known experimentally at room temperature, even though no details of its crystal structure are presently available.⁶ The same stoichiometry is, however, found in a variety of group 2/group 13 binaries: MgB_4 , in Pnm

symmetry, with four formula units per unit cell;³⁸ CaB_4 , in $P4/m\bar{3}m$ symmetry, with four formula units per cell;³⁹ the BeAl_4 structure of $I4/m\bar{3}m$ symmetry with two formula units per cell, which is also found in the CaGa_4 , SrGa_4 , BaGa_4 , and BaIn_4 systems;⁴⁰ and a distorted variant of the last structure with $C2/m$ symmetry and two formula units per cell, as found in CaGa_4 .⁴¹ For BeB_4 , the last structure optimizes to an orthorhombic structure of $Fmm2$ symmetry with one formula unit per cell.

In our calculations, we find no structure of BeB_4 to be stable with respect to the elements at atmospheric pressure (note that an electron-counting argument can be made against the stability of a pure CaB_4 -type structure⁴²). Yet a phase of that composition, of as yet unknown structure, has been reported. If, as our calculations indicate, BeB_4 at $P = 1$ atm is metastable, it is interesting to see which structure theory might predict for it.

At $P = 1$ atm, we find that among the known structure types the MgB_4 structure has the lowest enthalpy of formation for BeB_4 . However, this structure is not dynamically stable, and optimizing it within the lower space group symmetry $P2_12_12_1$ leads to a structure that is 70 meV/atom lower in enthalpy, which is also dynamically stable (see Table 5 in the Supporting Information on how these structures are related). The $P2_12_12_1$ structure features a 3D boron network with beryllium atoms in interstitial sites, see Figure 7. The main structural motifs of the boron sublattice are singly capped edge-sharing pentagons, with bonds involving the “capping” B atoms providing the three-dimensional structure. The B–B separations in this structure range from 1.68 to 1.85 Å; on the other hand, Be–B distances are 1.91 Å or longer (comparable or even shorter B–Be separations have been found in molecules with H bridges between Be and B: 1.78⁴³ or 1.92 Å;⁴⁴ longer separations of 2.05 Å occur in molecules with direct Be–B bonds;⁴⁵ and separations of 2.02–2.07 Å have been found in solid BeB_2C_2 ⁴⁶). A numerical scan for cavities in the B sublattice reveals that the Be atoms are close to the centers of the four largest “holes” of the B network, which illustrates their interstitial character.

We find that the $P2_12_12_1$ structure is a metal but only just: the band overlap at the Fermi energy is very small (see the Supporting Information for the electronic band structure) as is the absolute electronic DOS, see Figure 7. In fact, we cannot rule out that the vanishing band gap is an artifact of the semilocal GGA approximation to the exchange-correlation

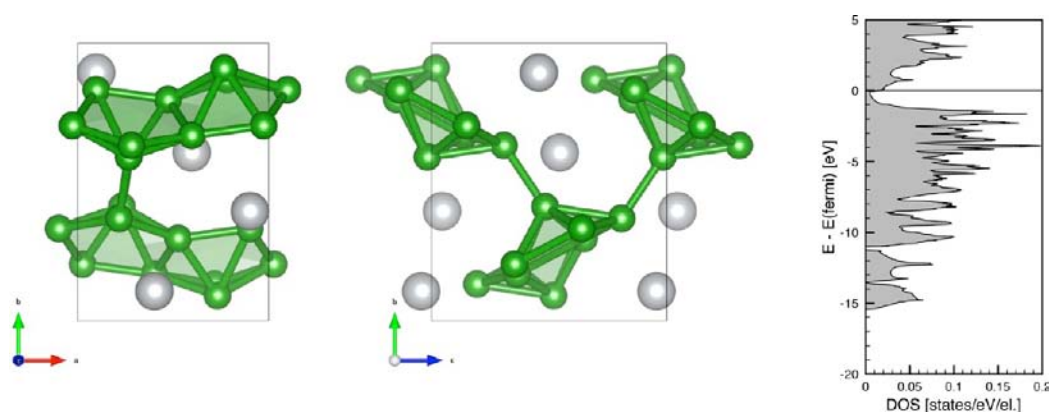


Figure 7. (Left) Two views of the $P2_12_12_1$ ground state structure of BeB_4 . Capped B pentagons are shaded. (Right) Electronic DOS, featuring a severe depletion of states at the Fermi energy.

Table 3. Relative Enthalpies of Formation ΔH_f Per Atom for Various BeB_4 Phases Relative to the Elemental Crystals

| structure type | "MgB ₄ " | CaB ₄ | BaAl ₄ | "CaGa ₄ " | structure search | structure search | structure search |
|------------------------|---|------------------|-------------------|----------------------|------------------|------------------|-------------------------|
| space group | <i>P2₁2₁2₁</i> | <i>P4/mbm</i> | <i>I4/mmm</i> | <i>Fmm2</i> | <i>P-1</i> | <i>Cm</i> | <i>P2₁/m</i> |
| ΔH_f [eV/atom] | +0.019 | +0.467 | +0.331 | +0.169 | +0.096 | +0.140 | +0.139 |

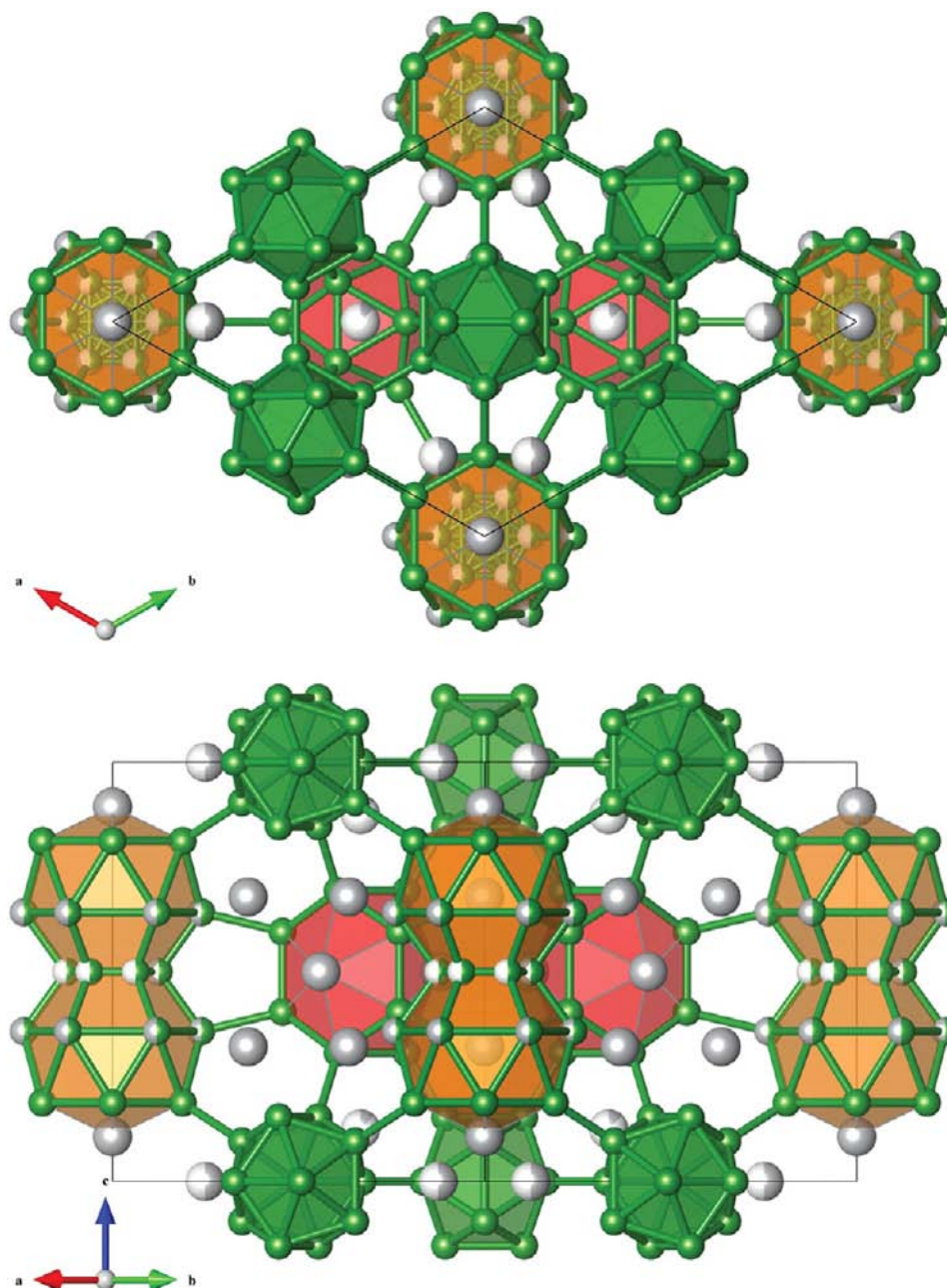


Figure 8. Experimental crystal structure of "BeB_{2.75}" at $T = 120 \text{ K}$,⁴ seen along the c axis (top) and within the a – b plane (bottom). Green (gray) spheres denote boron (beryllium) atoms, and partially filled spheres indicate partially occupied lattice sites. Green polyhedra indicate B₁₂ icosahedra, red polyhedra the Be₃B₁₂ subunits, and orange polyhedra the Be₈B₂₁ subunit with most of the partial occupancies (see text).

energy functional; it is unusual for a boron-rich metal boride to be metallic.

The MgB₄-based *P2₁2₁2₁* structure becomes rapidly unstable with respect to a variety of other structures when pressure is increased from $P = 1 \text{ atm}$, as we will discuss elsewhere. Several of these more stable structures were obtained from a structure search at $P = 80 \text{ GPa}$ (where relative volume compressions are about $V_0/V = 1.3$), with two formula units per unit cell. The first of these, of *P-1* symmetry, becomes stable with respect to

the *P2₁2₁2₁* structure even under very moderate pressures and so may be a candidate for a metastable structure at $P = 1 \text{ atm}$. This structure is dynamically stable as well; its structural properties, along with other candidate structures listed in Table 3, are given in the Supporting Information.

■ "BeB_{2.75}" AND NEARBY STOICHIOMETRIES

The unusual composition "BeB_{2.75}" was assigned from single-crystal X-ray studies of a phase that crystallizes in a large

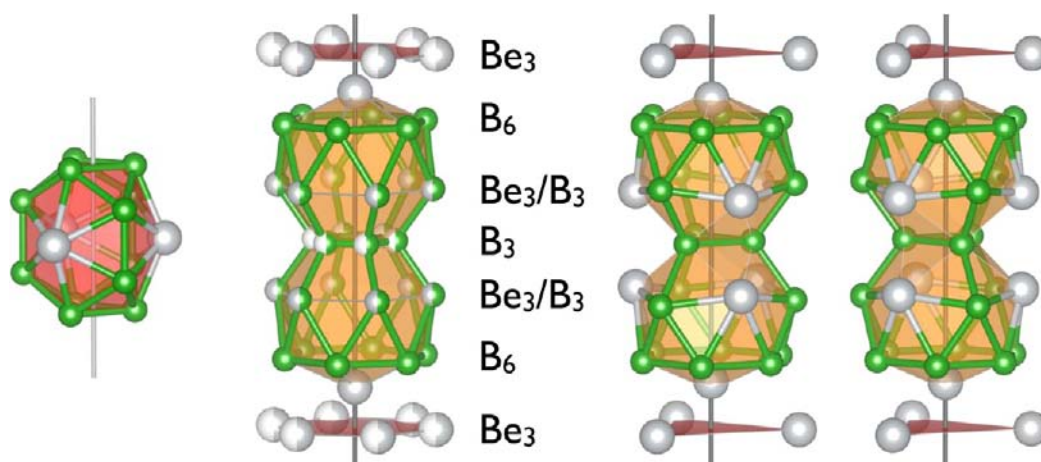


Figure 9. (From left) Experimental Be_3B_{12} polyhedron in “ $\text{BeB}_{2.75}$ ”, Be_8B_{21} polyhedron of “ $\text{BeB}_{2.75}$ ”, which contains most of the partially occupied lattice sites (note mixed occupancies on some sites), and two most stable theoretical occupations of the partial sites within the primitive unit cell.

hexagonal unit cell⁴ (the experimentally assigned stoichiometry is close to but not exactly $\text{BeB}_{2.75}$). This phase potentially explains existing discrepancies in the literature about Be–B phases ranging from BeB_2 to BeB_3 , as we discussed above. In experiment, the “ $\text{BeB}_{2.75}$ ” phase was found to be a superconductor.⁵ The unit cell is large (about 110 atoms) and has various polyhedra with mixed elements in it. Its main features are highlighted in Figure 8: in the unit cell one finds no less than three B_{12} icosahedra, two Be_3B_{12} 15-vertex polyhedra, and one Be_8B_{21} 29-vertex polyhedron which in turn consists of two face-fused Be_4B_{12} polyhedra. Note that this structural interpretation is slightly different from the experimental description of the structure⁴ but will prove useful in the deduction of its electronic properties below. These polyhedra include all boron and about one-half of the beryllium atoms. The remaining beryllium atoms (in the experimental structural refinement) are in a fully occupied $12o$ site, a half-occupied $6l$ site, and a $4h$ site with occupancy 0.125. All of these are located in the interstitial regions between the polyhedra.

The experimental crystal structure has partial occupancies of various lattice sites, as shown in Figure 8. However, if the lattice site with the smallest occupancy is ignored ($\text{Be}13$ in the original paper,⁴ in site $4h$ with occupancy 0.125), the system can be modeled using the original unit cell, with stoichiometry $\text{Be}_{29}\text{B}_{81}$ (approximately $\text{BeB}_{2.79}$, compared to $\text{BeB}_{2.75}$ in the experimental structure). In that case, all partially occupied lattice sites have 6-fold rotational symmetry, see Figure 9.

There are several ways to distribute the atoms over these partially occupied lattice sites. If some restrictions on minimum Be–Be and B–B distances are adhered to, six different unit cells (four of $P\bar{6}m2$ and two of $P3m1$ symmetry) can be constructed. The enthalpies of formation for these different unit cells range from -40 to -125 meV/atom. For the two most stable structures, the occupations of the partial atom sites are shown in Figure 9. The enthalpy of formation agrees quite well with a value of -96 meV/atom obtained in a recent computational study which probably used the stoichiometry $\text{Be}_{30}\text{B}_{81}$ to describe this structure.⁴⁷ The structural parameters of the optimized structure also agree well with experimental findings: theoretical lattice constants $a = 9.778$ Å and $c = 9.524$ Å can be compared to experimental values $a = 9.774$ Å and $c = 9.547$ Å. The atomic coordinates are given in the Supporting Information.

The rather large enthalpy of formation means a significant stabilization of this phase over all other phases studied here; at atmospheric and slightly elevated pressures the $\text{BeB}_{2.79}$ ($\text{Be}_{29}\text{B}_{81}$) phase is the only stable ground state point in the stoichiometry range of the Be–B phase diagram we study here.

In experiment, “ $\text{BeB}_{2.75}$ ” was found to be a superconductor, with $T_c = 0.72$ K⁵ (compared with pure Be whose $T_c = 0.026$ K⁴⁸). Our calculations confirm that $\text{Be}_{29}\text{B}_{81}$ is a metal (see Figure 10), and the relatively low DOS at the Fermi level also

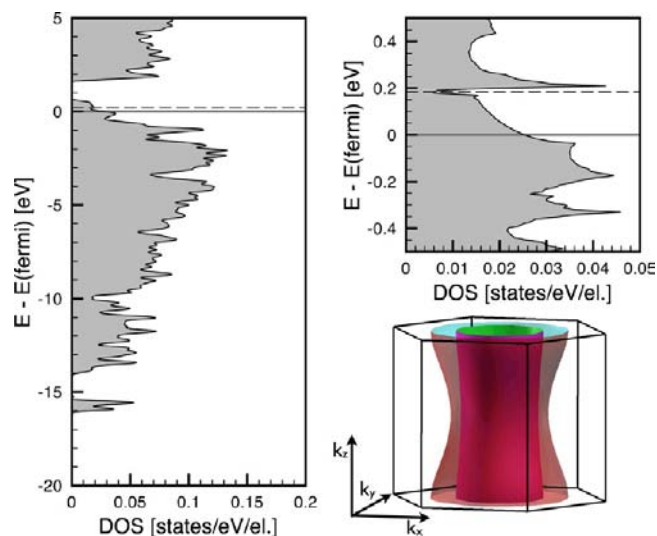


Figure 10. Electronic DOS (left and upper right) and Fermi surface for the $\text{Be}_{29}\text{B}_{81}$ model of “ $\text{BeB}_{2.75}$ ” (lower right). Solid (dashed) horizontal line in the DOS plots indicates the position of the Fermi level for 301 (302) valence electrons per unit cell, respectively (see text).

suggests a low transition temperature T_c (there are simply not many electrons available to form paired states); however, since we did not investigate the electron–phonon coupling strength for this system, we cannot compare the experimental T_c directly to our calculations.

We note that the experimental structure features an additional 0.5 beryllium atoms per unit cell, which we neglected here. If we assume that the effect of this additional atom on the structure and electronic structure is negligible (this could be

reasonable since it is located experimentally in the interstitial space between three B_{12} icosahedra and the Be_3B_{12} polyhedron, its effect reduces to adding an additional valence electron to the unit cell. The resulting shift in the Fermi energy is shown in the DOS plots in Figure 10. However, as is also shown in the enlarged DOS plot around the Fermi energy in Figure 10, the DOS has a drastic drop just at a valence count of 302 per unit cell. This is not an artifact of the finite k -point set choice but corresponds to complete filling of electronic bands, as the band structure in Figure 11 shows.

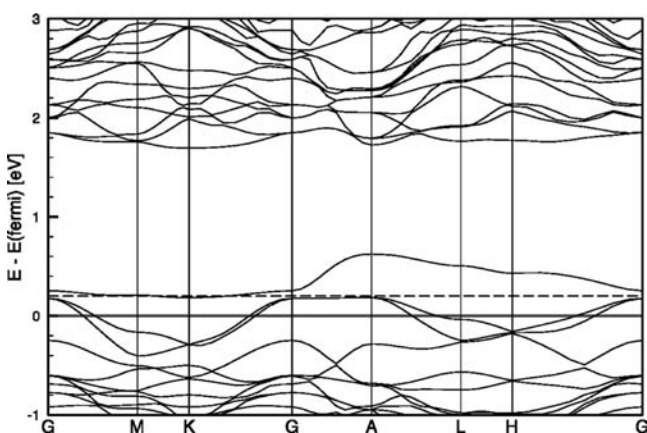


Figure 11. Electronic band structure of $Be_{29}B_{81}$. Fermi energy for 301 electrons is indicated by the solid line. Dashed line indicates the Fermi energy for 302 valence electrons per unit cell.

Interestingly, the DOS and band structure show a substantial band gap about 0.6 eV above the Fermi level, which corresponds to a valence electron count of 304 per unit cell (the $Be_{29}B_{81}$ unit cell has 301 valence electrons). A gap at just this electron count can be understood in terms of the stabilizing electron count for the polyhedra which form the backbone of this structure.^{49–51} According to Wade's rules, a closed ("closo") cluster with n vertices is stabilized with $2n + 1$

electron pairs. That means a B_{12} icosahedron (with 36 valence electrons) would need a charge of 14^- to reach an electron count of 50 (25 electron pairs) and thus be stable. However, if some electrons can be obtained by electron sharing through external bonds, the required charge can be reduced significantly: $B_{12}H_{12}$ requires only a charge of 2^- , as the shared electrons in the B–H bonds contribute to the cluster electron count.⁵² For the " $BeB_{2.75}$ " structure, we can examine the electron counts and required charges for the stability of the various polyhedra, which have to be compared to the available electronic charge (provided by interstitial beryllium atoms).

There are three B_{12} polyhedra in the unit cell; in all of them all boron vertices form two-center two-electron (2c-2e) bonds to other boron atoms: four each to other B_{12} units ($d = 1.61 \text{ \AA}$), the Be_3B_{12} polyhedron ($d = 1.71 \text{ \AA}$), and the Be_8B_{21} polyhedron ($d = 1.79 \text{ \AA}$). Hence, all of them require charge 2^- , similar to $B_{12}H_{12}$ as explained above. The two Be_3B_{12} polyhedra form 12 2c-2e bonds, involving all their boron atoms: six to B_{12} icosahedra (pure B–B bonds, $d = 1.71 \text{ \AA}$) and six to the Be_8B_{21} polyhedron (B–Be/B bonds, $d = 1.90 \text{ \AA}$). With a valence count of 42, plus 12 electrons from external bonds, the required charge to reach stability (31 electron pairs) is 8^- . Lastly, the 29-vertex Be_8B_{21} polyhedron consists of two face-sharing *closo*- Be_4B_{12} polyhedra, which each form six bonds to B_{12} icosahedra, and six (from the shared Be/B sites) to the Be_3B_{12} polyhedra. The total valence count of Be_8B_{21} is 79, plus 24 electrons from the external bonds, so the charge required for stability (57 electron pairs) is 11^- . In total, the charge required per unit cell is thus 33^- , which corresponds to a total valence count per unit cell of 304 electrons. *This is exactly where our DFT calculations find a substantial gap.*

The electron count also explains why the experimental structure is metallic: 16.5 beryllium atoms per unit cell would provide a charge of -33 , but only 15.5 are found in experiment. Why the system does not take up an additional beryllium atom per cell is not immediately clear, but it could be related to the number of available cavities in the interstitial space, see below.

Filling Cavities in the Polyhedral Network. How can other stoichiometries close to $BeB_{2.75}$ be probed? Starting from

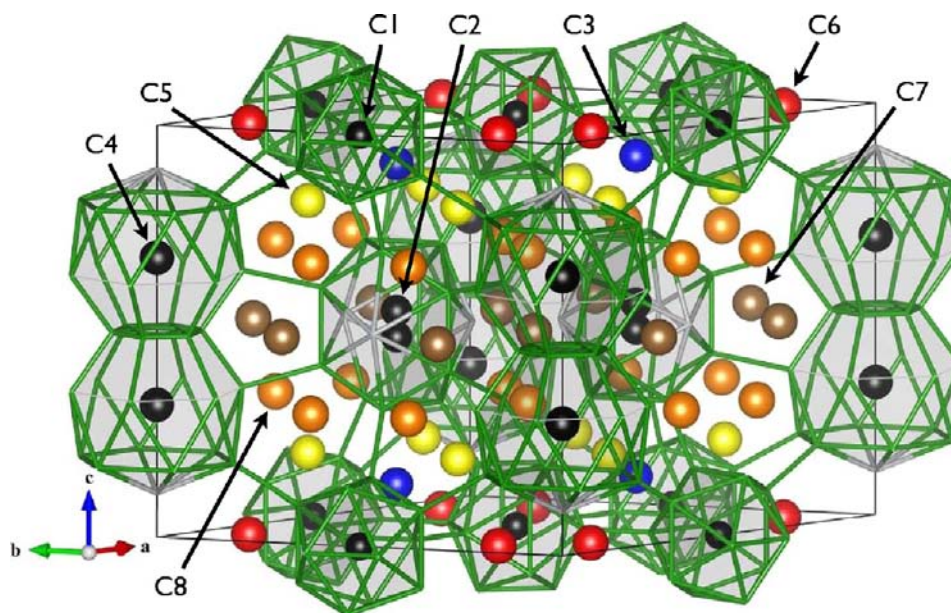


Figure 12. Polyhedral network of " $BeB_{2.75}$ " with all cavity sites indicated and labeled according to Table 4.

Table 4. Positions and Sizes of Cavities in the Polyhedral Network of “BeB_{2.75}” (experimental structure)

| site | Wyckoff position | atomic coordinates | | | size [Å] | comments |
|------|------------------|--------------------|----------|----------|-------------|--|
| | | <i>x</i> | <i>y</i> | <i>z</i> | | |
| C1 | 3 <i>f</i> | 0.5 | 0 | 0 | 1.64 | inside B ₁₂ |
| C2 | 4 <i>h</i> | 0.3333 | 0.6667 | 0.5316 | 1.82 | inside Be ₃ B ₁₂ |
| C3 | 4 <i>h</i> | 0.3333 | 0.6667 | 0.1105 | 1.97 | Be13 in exp |
| C4 | 2 <i>e</i> | 0 | 0 | 0.3211 | 2.00 | inside Be ₈ B ₂₁ |
| C5 | 12 <i>o</i> | 0.4205 | 0.2102 | 0.1947 | 2.03 | |
| C6 | 6 <i>l</i> | 0.2564 | 0.1282 | 0 | 2.08 | Be12 in exp |
| C7 | 6 <i>k</i> | 0.3077 | 0 | 0.5 | 2.11 (1.89) | 1.89 Å to C8 |
| C8 | 12 <i>n</i> | 0.3795 | 0 | 0.3158 | 2.13 | Be10 in exp |

the polyhedral network of the “BeB_{2.75}” structure (with only the lattice sites of the B₁₂, Be₃B₁₂, and Be₈B₂₁ polyhedra occupied), we proceed to analyze the cavities in this network and subsequently fill them with varying amounts of beryllium atoms, Figure 12.

As can be seen in Table 4, the largest cavity site C8 (measured by the radius of the largest sphere that can be inserted to just touch the polyhedral network) is indeed the site that is fully occupied by beryllium atoms in experiment. The second-largest cavity site C7 is in conflict with a simultaneous occupation of C8, but the next smaller site, C6, can be occupied and is found to be so in experiment. Cavity site C5, though not in close proximity to any other atom or cavity, is curiously empty (but its full occupation would mean a large change in the overall stoichiometry). Among the smaller sites C1–C4, we find all center points of the polyhedral units and the experimentally assigned Be13 site with occupancy 0.125.

We examined various alternatives to fill these cavity sites that differ from the experimental structure refinement. For these we ignored the cavities within the polyhedral units (C1, C2, and C4, these being unlikely to be occupied) and also C7, which is too close to C8. The enthalpies of formation for our alternative structures (relative to the elemental crystals) are shown in Figure 13. Out of range for Figure 13 are calculations for the polyhedral network of “BeB_{2.75}” only (Be₁₄B₈₁, very unstable

with $\Delta H_f = +0.136$ eV/atom) and for a complete occupation of cavities C5 and C8, plus 50% of the C6 sites (Be₄₁B₈₁, also very unstable with $\Delta H_f = +0.123$ eV/atom).

The tieline drawn in Figure 13 is the convex hull of this small section of the phase diagram with respect to elemental boron and beryllium crystals and reveals that the cavity occupation found in experiment is indeed most stable. We can also see that (i) occupation of cavity C8 is always favored over occupation of C5, (ii) a more boron-rich phase BeB_{3.23} (Be₂₆B₈₄) comes closest to stability (it is discussed below), and (iii) the change in enthalpy for various occupations of the Be₈B₂₁ polyhedron is of the same magnitude as typical enthalpy changes associated with changes in stoichiometry. For the last point, it would then seem reasonable that the mixed occupations in the Be₈B₂₁ subunit are not completely arbitrary but rather consist of the two most stable local configurations only (shown in Figure 9), which alternate (possibly disordered) throughout the crystal.

Returning to the stable electron-count argument, there is no option to add another beryllium atom to the unit cell without breaking the experimental space group symmetry, although changing the occupancy of the Be13 site to 0.375 would achieve this but not make the structure more computationally tractable. Other ways to achieve the valence electron count of 304 (and thus probably reach a stable semiconducting phase) could be, for example, (a) adding one aluminum atom per unit cell, (b) replacing a beryllium atom by nitrogen, or (c) replacing three beryllium atoms by boron atoms.

The last option corresponds to a more boron-rich composition, approximately BeB_{3.23}. We constructed this structure by occupying six of the mixed occupancy Be/B sites with boron atoms only. The resulting unit cell has Be₂₆B₈₄ stoichiometry (\sim BeB_{3.23}) and *P3m1* symmetry. When optimized, we find it to be semiconducting (the band gap in our DFT calculations is 1.35 eV) and unstable with respect to decomposition into Be₂₉B₈₁ and pure boron by only 4 meV/atom, see Figure 13. It is not inconceivable that more competitive structures of this stoichiometry could be constructed, and it is then not immediately clear how to reconcile this with the experimental finding that “BeB_{2.75}” is superconducting.

Known A₄B₁₁ Structures. The nearest “simple” stoichiometry close to the experimental “BeB_{2.75}” phase is Be₄B₁₁. Some A₄B₁₁ structures are known in the literature: two phases of Mo₄O₁₁^{53,54} and transition metal binaries including Ir₄Sc₁₁⁵⁵, Se₄Ti₁₁⁵⁶ and Mn₄Al₁₁.⁵⁷ However, none of these structure types is calculated to be stable with respect to the elements in the Be–B phase diagram.

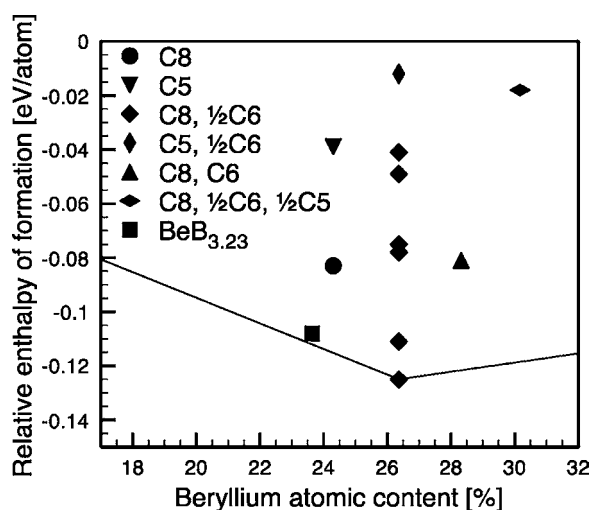


Figure 13. Relative enthalpies for various cavity occupation of the “BeB_{2.75}” structure. Legend indicates which cavities are occupied (some only at 50%), see Table 4, Supporting Information, for their definition. Six data points for the “C8, 1/2C6” cavity occupation correspond to the different choices to occupy the partial sites in the Be₈B₂₁ polyhedron (see Figure 9 for the two most stable ones).

■ TRYING YET OTHER STOICHIOMETRIES

Several other binary phases might exist between group 2/group 13 elements within the stoichiometry range considered in this work. We go through these below.

Be₃B₁₁. This phase is found in the Sr–In phase diagram⁵⁸ and crystallizes at room temperature in the orthorhombic space group 71, *Immm*, with two formula units per unit cell. We calculated a very large positive enthalpy of formation, +0.55 eV/atom, for this phase, and it is hence very unlikely to be stabilized in the Be–B system.

Be₃B₈. This phase, present in the Ca–Ga system,⁴¹ crystallizes at room temperature in the *Immm* space group (structure type Eu₃Ga₈), with two formula units per cell. However, as with Be₃B₁₁, we calculate a positive enthalpy of formation, +0.14 eV/atom. This stoichiometry (BeB_{2.67}) is also very close to the very stable “BeB_{2.75}” phase discussed in detail above; it is unlikely to be synthesized, as the driving force toward formation of “BeB_{2.75}” is very large. Note that the prototype of this structure, Eu₃Ga₈, seems to feature disordered substitution of Eu atoms by Ga trimers, and its actual stoichiometry was determined to be Eu_{3–x}Ga_{8+3x} with $x = 0.12$.⁵⁹ The same effect was also found in Sr₃Ga₈ (now determined to be Sr_{3–x}Ga_{8+3x} with $x = 0.15$ ⁶⁰). Hence, a more complex substitutional structure might prove more favored in the Be–B system as well, but this is beyond the scope of this study.

Be₂B₅. This structure, found in the Mg–Ga phase diagram,⁶¹ crystallizes at room temperature in the tetragonal space group *I4/mmm*. It is interesting because it would feature layers of boron atoms (in puckered six-membered rings), some of which are 5-fold coordinated. However, again, a positive enthalpy of formation of +0.20 eV/atom from our calculations makes its synthesis unlikely.

■ SUMMARY

We presented results from computational studies on a variety of structures in the boron-rich side of the Be–B binary phase diagram. We explore and clarify the experimental uncertainty around the existence and stability of phases between BeB₂ and BeB₃, making a case for the stoichiometry actually being Be₂₉B₈₁ or BeB_{2.79}. However, the computed electronic properties of this structure (for understanding which polyhedral skeletal electron-counting rules are extremely useful) also suggest another, stable and semiconducting phase, BeB_{3.23}.

For the simpler stoichiometries BeB_x with $x = 2, 3$, and 4, we suggest structural candidates which we find to be more stable than those previously discussed in the literature; in particular, this is the case for the BeB₂ phase. The optimum BeB₂ structures are not the AlB₂ structure type but adopt several Zintl phase structures with four-coordinated boron networks.

We believe we have made some sense, on electronic and structural grounds, of a complicated segment of the Be–B phase diagram.

■ ASSOCIATED CONTENT

Supporting Information

Details of the computational method, crystal structure information on all discussed phases, and symmetry relations between several of these phases. This material is available free of charge via the Internet at <http://pubs.acs.org>.

■ AUTHOR INFORMATION

Corresponding Author

*E-mail: ah736@cornell.edu.

Notes

The authors declare no competing financial interest.

■ ACKNOWLEDGMENTS

We wish to thank Prasad LVK Dasari for helpful discussions and three reviewers for knowledgeable suggestions. Support for our work comes from EFree, an Energy Frontier Research Center funded by the U.S. Department of Energy (Award Number DESC0001057 at Cornell), and the National Science Foundation through grants CHE-0910623 and DMR-0907425. Computational resources provided by the Cornell NanoScale Facility (supported by the National Science Foundation through grant ECS-0335765) and the XSEDE network (provided by the National Center for Supercomputer Applications through grant TG-DMR060055N) are gratefully acknowledged.

■ REFERENCES

- (1) Markovskii, L. Y.; Kondrashev, Y. A.; Kaputovskaia, G. V. *Zh. Obshch. Khim. USSR* **1955**, *25*, 1045.
- (2) Sands, D. E.; Cline, C. F.; Zalkin, A.; Hoenig, C. L. *Acta Crystallogr.* **1961**, *14*, 309–310.
- (3) Mattes, R.; Tebbe, K.-F.; Neidhard, H.; Rethfeld, H. *J. Less-Common Met.* **1976**, *47*, 29–32.
- (4) Chan, J. Y.; Fronczek, F. R.; Young, D. P.; DiTusa, J. F.; Adams, P. W. *J. Solid State Chem.* **2002**, *163*, 385–389.
- (5) Young, D.; Goodrich, R.; Adams, P.; Chan, J.; Fronczek, F.; Drymiotis, F.; Henry, L. *Phys. Rev. B, Rapid Commun.* **2002**, *65*, 180518.
- (6) Okamoto, H.; Tanner, L. E. In *Binary Alloy Phase Diagrams*; Massalski, T. B., Okamoto, H., Subramanian, P. R., Kacprzak, L., Eds.; ASM International: Materials Park, OH, 1990; p 460.
- (7) Borovikova, M. S.; Fesenko, V. V. *J. Less-Common Met.* **1986**, *117*, 287–291.
- (8) Hirsch, J. E. *Phys. Lett. A* **2001**, *282*, 392–398.
- (9) Felner, I. *Physica C* **2001**, *353*, 11–13.
- (10) Nagamatsu, J.; Nakagawa, N.; Muranaka, T.; Zenitani, Y.; Akimitsu, J. *Nature* **2001**, *410*, 63–4.
- (11) Singh, P. *Phys. Rev. Lett.* **2001**, *87*, 87004.
- (12) Medvedeva, N.; Ivanovskii, A.; Medvedeva, J.; Freeman, A. *Phys. Rev. B, Rapid Commun.* **2001**, *64*, 020502.
- (13) Kresse, G.; Furthmüller, J. *Phys. Rev. B* **1996**, *54*, 11169–11186.
- (14) Blöchl, P. E. *Phys. Rev. B* **1994**, *50*, 17953–17979.
- (15) Kresse, G.; Joubert, D. *Phys. Rev. B* **1999**, *59*, 1758–1775.
- (16) Alfé, D. *Comput. Phys. Commun.* **2009**, *180*, 2622–2633.
- (17) Lonie, D. C.; Zurek, E. *Comput. Phys. Commun.* **2011**, *182*, 372.
- (18) Perdew, J. P.; Burke, K.; Ernzerhof, M. *Phys. Rev. Lett.* **1996**, *77*, 3865–3868.
- (19) Momma, K.; Izumi, F. *J. Appl. Crystallogr.* **2011**, *44*, 1272–1276.
- (20) Kokalj, A. *Comput. Mater. Sci.* **2003**, *28*, 155–168.
- (21) Rivillo, T.; Wallace, W. E. *J. Solid State Chem.* **1980**, 309.
- (22) Cordier, G.; Czech, E.; Schäfer, H. *Z. Naturforsch. B* **1982**, *37*, 1442.
- (23) Iandelli, A. *Z. Anorg. Allg. Chem.* **1964**, *330*, 221–232.
- (24) Nuspl, G.; Polborn, K.; Evers, J.; Landrum, G. A.; Hoffmann, R. *Inorg. Chem.* **1996**, *35*, 6922–6932.
- (25) Smith, G. S.; Mucker, K. F.; Johnson, Q.; Wood, D. H. *Acta Crystallogr. B* **1969**, *25*, 549–553.
- (26) Zintl, E.; Dullenkopf, W. *Z. Phys. Chem. B* **1932**, *16*, 195–205.
- (27) Schäfer, H.; Eisenmann, B.; Müller, W. *Angew. Chem., Int. Ed.* **1973**, *12*, 694–712.

- (28) Nowotny, H. In *Electronic structure and alloy chemistry of the transition elements*; Beck, P. A., Ed.; Interscience Publishers: New York, 1963; pp 179–220.
- (29) Shein, I. R.; Ivanovskii, A. L. *J. Phys.: Condens Matter* **2008**, *20*, 415218.
- (30) Haarmann, F.; Koch, K.; Grüner, D.; Schnelle, W.; Pecher, O.; Cardoso-Gil, R.; Borrmann, H.; Rosner, H.; Grin, Y. *Chem.—Eur. J.* **2009**, *15*, 1673–84.
- (31) Spina, L.; Jia, Y. Z.; Ducourant, B.; Tillard, M.; Belin, C. Z. *Kristallogr.* **2003**, *218*, 740–746.
- (32) Šimůnek, A. *Phys. Rev. B* **2007**, *75*, 172108.
- (33) Andrievski, R. A. *Int. J. Refract. Met. Hard Mater.* **2001**, *19*, 447–452.
- (34) Cutler, R. A. In *Engineered Materials Handbook*; Schneider, S. J., Ed.; ASM International: Materials Park, OH, 1991; Vol. 4 (Ceramics and Glasses), pp 787–803.
- (35) Chamberlain, A. L.; Fahrenholtz, W. G.; Hilmas, G. E.; Ellerby, D. T. *J. Am. Ceram. Soc.* **2004**, *87*, 1170–1172.
- (36) Chung, H.-Y.; Weinberger, M. B.; Levine, J. B.; Cumberland, R. W.; Kavner, A.; Yang, J.-M.; Tolbert, S. H.; Kaner, R. B. *Science* **2007**, *316*, 436–9.
- (37) Šimůnek, A. *Phys. Rev. B, Rapid Commun.* **2009**, *80*, 060103.
- (38) Naslain, R.; Guette, A.; Barret, M. *J. Solid State Chem.* **1973**, *8*, 68–85.
- (39) Schmitt, R.; Blaschkowski, B.; Eichele, K.; Meyer, H.-J. *Inorg. Chem.* **2006**, *45*, 3067–73.
- (40) Bruzzone, G. *Acta Crystallogr.* **1965**, *18*, 1081–1082.
- (41) Bruzzone, G.; Fornasini, M. L.; Merlo, F. *J. Less-Common Met.* **1989**, *154*, 67–77.
- (42) Ben Yahia, M.; Reckeweg, O.; Gautier, R.; Bauer, J.; Schleid, T.; Halet, J.-F.; Saillard, J.-Y. *Inorg. Chem.* **2008**, *47*, 6137–43.
- (43) Gundersen, G.; Hedberg, L.; Hedberg, K. *J. Chem. Phys.* **1973**, *59*, 3777.
- (44) Marynick, D. S.; Lipscomb, W. N. *Inorg. Chem.* **1972**, *11*, 820–823.
- (45) Gaines, D. F.; Coleson, K. M.; Calabrese, J. C. *J. Am. Chem. Soc.* **1979**, *101*, 3979–3980.
- (46) Hofmann, K.; Rocquefelte, X.; Halet, J.-F.; Bächtz, C.; Albert, B. *Angew. Chem., Int. Ed.* **2008**, *47*, 2301–3.
- (47) Chepulskii, R.; Curtarolo, S. *Phys. Rev. B* **2009**, *79*, 134203.
- (48) Falge, R. L., Jr. *Phys. Lett. A* **1967**, *24*, 579–580.
- (49) Wade, K. *J. Chem. Soc. D: Chem. Commun.* **1971**, 792.
- (50) Tillard-Charbonnel, M.; Manteghetti, A.; Belin, C. *Inorg. Chem.* **2000**, *39*, 1684–96.
- (51) Prasad, D. L. V. K.; Balakrishnarajan, M. M.; Jemmis, E. D. *Phys. Rev. B* **2005**, *72*, 195102.
- (52) Longuet-Higgins, H. C.; de V. Roberts, M. *Proc. Roy. Soc. A* **1955**, *230*, 110–119.
- (53) Fun, H.-K.; Yang, P.; Sasaki, M.; Inoue, M.; Kadomatsu, H. *Powder Diffr.* **1999**, *14*, 284.
- (54) Knorr, R.; Müller, U. *Z. Anorg. Allg. Chem.* **1995**, *621*, 541–545.
- (55) Chabot, B.; Cenzual, K.; Parthé, E. *Acta Crystallogr. B* **1980**, *36*, 7–11.
- (56) Weirich, T. E.; Ramlau, R.; Simon, A.; Hovmöller, S.; Zou, X. *Nature* **1996**, *382*, 144–146.
- (57) Kontio, A.; Stevens, E. D.; Coppens, P.; Brown, R. D.; Dwight, A. E.; Williams, J. M. *Acta Crystallogr. B* **1980**, *36*, 435–436.
- (58) Amerioun, S.; Häussermann, U. *Inorg. Chem.* **2003**, *42*, 7782–8.
- (59) Sichevych, O.; Prots, Y.; Grin, Y. Z. *Kristallogr. NCS* **2006**, *221*, 265–266.
- (60) Haarmann, F.; Prots, Y.; Göbel, S.; von Schnering, H. G. Z. *Kristallogr. NCS* **2006**, *221*, 257–258.
- (61) Smith, G. S.; Johnson, Q.; Wood, D. N. *Acta Crystallogr. B* **1969**, *25*, 554–557.