

# Theory of Spin Triplet Ground States in $d^6$ Transition Metal Compounds of $D_{4h}$ and $D_{3d}$ Symmetries.

## I. Ground State Boundary Delineation

E. König and R. Schnakig

Received September 29, 1972

The formation of spin triplet ground states in the  $3d^6$  configuration of transition metal complexes is investigated within  $D_{4h}$  and  $D_{3d}$  symmetries. Employing as a basis the state functions resulting from the cubic  ${}^5T_{2g}(t_{2g}^4e_g^2)$ ,  ${}^1A_{1g}(t_{2g}^6)$ ,  ${}^3T_{1g}(t_{2g}^5e_g)$ , and  ${}^3T_{2g}(t_{2g}^5e_g)$  terms it is demonstrated that, in  $D_{4h}$  symmetry, the triplet states  ${}^3A_2$ ,  ${}^3B_2$ ,  ${}^3E({}^3T_1)$ , and  ${}^3E({}^3T_2)$  may be stabilized as ground states. On the other hand, only the  ${}^3E({}^3T_1)$  state may become ground state if the symmetry is  $D_{3d}$ . Diagrams showing the ground state boundaries, in  $D_{4h}$  symmetry, as function of the parameters  $Dq$ ,  $Dt$ , and  $\chi = Ds/Dt$  are presented. In  $D_{3d}$  symmetry,  $Dq$ ,  $D\tau$ , and  $\chi = D\sigma/D\tau$  are equally employed. The relevance of the present calculations to available experimental data is discussed.

## 1. Introduction

In the electronic configurations  $d^4$ ,  $d^5$ ,  $d^6$ , and  $d^7$  within octahedral symmetry, one of two possible electronic ground states may be stabilized, depending on the strength of the field set up by the ligands. These states which are known as high-spin and low-spin states are characterized by a different symmetry transformation property and a different value of the total spin  $S$ .<sup>1</sup> The configurations  $d^4$ ,  $d^5$ , and  $d^6$  are remarkable in that states of an additional, third, spin multiplicity are formed. As long as  $O_h$  symmetry is strictly preserved, it is only certain excited states to which the total spin  $S = 0$ ,  $S = 3/2$ , and  $S = 1$  may be assigned in  $d^4$ ,  $d^5$ , and  $d^6$ , respectively.

In the  $d^6$  configuration, in particular, the possible ground terms within a field of octahedral symmetry are  ${}^5T_{2g}(t_{2g}^4e_g^2)$  and  ${}^1A_{1g}(t_{2g}^6)$  and complexes of these ground states are well documented. In addition, there are numerous spin triplet excited states, although only two, *viz.*  ${}^3T_{1g}(t_{2g}^5e_g)$  and  ${}^3T_{2g}(t_{2g}^5e_g)$ , have reasonably low energies. Recently, the question has been raised, whether spin triplet ground states may be formed, provided the symmetry of the field is lower than cubic. The  $S = 1$  ground states expected would then possibly arise from the excited  ${}^3T_{1g}(t_{2g}^5e_g)$  and  ${}^3T_{2g}(t_{2g}^5e_g)$  terms of the octahedral field. The problem is stimulated by experimental

magnetic and spectroscopic results in certain complex compounds of iron(II) and cobalt(III). These data which will be briefly discussed below justify a theoretical study. Therefore, in this paper, we will investigate the conditions for spin triplet ground state formation within a limited basis set of the  $d^6$  configuration assuming that the effective ligand field is of tetragonal ( $D_{4h}$ ) or trigonal ( $D_{3d}$ ) symmetry.

## 2. Single d Electron Eigenfunctions and Energies

In the ligand field model of a transition metal complex, the complete Hamiltonian considered may be written as

$$H = \sum_i \left( -\frac{\nabla_i^2}{2m_i} - \frac{Ze^2}{r_i} \right) + V_{Lig} + \sum_{i>j} \frac{e^2}{r_{ij}} + \sum_i \zeta_i l_i \cdot s_i \quad (1)$$

If it is assumed that a central field solution to the first two terms of  $H$  has been obtained, the remaining three terms may be accounted for by a perturbation calculation. In what follows, we will employ the so-called strong-field approximation, *i.e.*

$$V_{Lig} > \sum_{i>j} \frac{e^2}{r_{ij}} > \sum_i \zeta_i l_i \cdot s_i \quad (2)$$

The ligand field potential  $V_{Lig}$  may be expanded into spherical harmonics and is frequently expressed as a cubic term and a perturbing term due to lower symmetry. Below, we will always use the combined potential which may be written, in  $D_{4h}$  symmetry,<sup>2</sup>

$$V_{Lig} = A(r)C_0^{(2)}(\theta, \varphi) + B(r)\{C_0^{(4)}(\theta, \varphi) - \sqrt{\frac{7}{10}}[C_4^{(4)}(\theta, \varphi) + C_{-4}^{(4)}(\theta, \varphi)]\} \quad (3)$$

where

$$C_m^{(k)}(\theta, \varphi) = \left( \frac{4\pi}{2k+1} \right)^{1/2} Y_{km}(\theta, \varphi) \quad (4)$$

(2) S. Sugano, Y. Tanabe, H. Kamimura, *Multiplets of Transition-Metal Ions in Crystals*, Academic Press, New York, 1970.

(3) B.R. Judd, *Operator Techniques in Atomic Spectroscopy*, McGraw-Hill, New York, 1953.

(1) E. König, S. Kramer) *Theoret. Chim. Acta* (Berlin) 23, 12 (1971).

are Racah's rationalized spherical harmonics.<sup>3</sup> On the other hand, in  $D_{3d}$  symmetry, it is

$$V_{\text{irig}} = A(r)C_0^{(2)}(\theta, \varphi) + D(r)\{C_0^{(4)}(\theta, \varphi) - \frac{1}{2}\sqrt{\frac{7}{10}}[C_3^{(4)}(\theta, \varphi) - C_{-3}^{(4)}(\theta, \varphi)]\} \quad (5)$$

In  $D_{4h}$  symmetry, the cubic single d electron orbitals

$$\begin{aligned} e\theta &= |20\rangle = d_{z^2} \\ e\epsilon &= \frac{1}{\sqrt{2}}[|22\rangle + |2-2\rangle] = d_{x^2-y^2} \\ t_2\eta &= -\frac{1}{\sqrt{2}}[|21\rangle - |2-1\rangle] = d_{xz} \\ t_2\xi &= \frac{1}{\sqrt{2}}[|21\rangle + |2-1\rangle] = d_{yz} \\ t_2\zeta &= -\frac{i}{\sqrt{2}}[|22\rangle - |2-2\rangle] = d_{yx} \end{aligned} \quad (6)$$

may be employed. These orbitals transform now, in the above sequence, according to  $a_1$ ,  $b_1$ ,  $e_a$ ,  $e_b$ , and  $b_2$ , respectively. The corresponding single electron energies may be expressed in terms of the splitting parameters for  $Y_{20}(\theta, \varphi)$  and  $Y_{40}(\theta, \varphi)$ ,  $D_s$  and  $D_t$  respectively, according to the definition of Ballhausen<sup>4</sup>

$$\begin{aligned} E(a_{1g}) &= \langle e\theta | V_{\text{irig}} | e\theta \rangle = 6Dq - 2D_s - 6D_t \\ E(b_{1g}) &= \langle e\epsilon | V_{\text{irig}} | e\epsilon \rangle = 6Dq + 2D_s - D_t \\ E(e_g) &= \langle t_2\xi | V_{\text{irig}} | t_2\xi \rangle = \langle t_2\eta | V_{\text{irig}} | t_2\eta \rangle = -4Dq - D_s + 4D_t \\ E(b_{2g}) &= \langle t_2\zeta | V_{\text{irig}} | t_2\zeta \rangle = -4Dq + 2D_s - D_t \end{aligned} \quad (7)$$

The resulting orbital energies are depicted in Figure 1. As is well known, the center of gravity holds for the  $D_s$  terms within the  $e_g$  and  $t_{2g}$  orbitals separately, while for the  $D_t$  terms this is true for the whole configuration only.

In  $D_{3d}$  symmetry, the trigonally oriented single d electron orbitals

$$\begin{aligned} eu_+ &= -\frac{1}{\sqrt{3}}[|2-2\rangle - \sqrt{2}|21\rangle] \\ eu_- &= \frac{1}{\sqrt{3}}[|22\rangle + \sqrt{2}|2-1\rangle] \\ t_2x_+ &= -\frac{1}{\sqrt{3}}[\sqrt{2}|2-2\rangle + |21\rangle] \\ t_2x_- &= \frac{1}{\sqrt{3}}[\sqrt{2}|22\rangle - |2-1\rangle] \\ t_2x_0 &= |20\rangle \end{aligned} \quad (8)$$

are used to advantage. In Eq. (8), the first two orbitals transform now according to  $e(e)$ , the following

(\*) It should be observed that  $D_s$  and  $D_t$  parameters (and  $D\sigma$  and  $D\tau$  as well) differing in sign from those used here [4] have been defined by Perumareddi [5,6].

(4) J.C. Ballhausen, *Introduction to Ligand Field Theory*, McGraw-Hill, New York, 1962.

(5) J.R. Perumareddi, *J. Phys. Chem.*, **71**, 3144 (1967).

(6) J.R. Perumareddi, *Coord. Chem. Rev.*, **4**, 73 (1969).

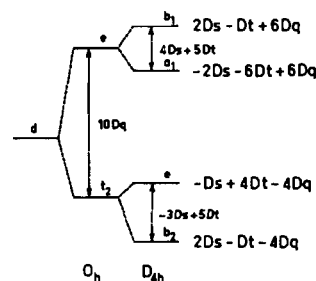


Figure 1. Single electron orbital energies in  $D_{4h}$  symmetry in terms of  $Dq$ ,  $D_s$ , and  $D_t$ .

two as  $e(t_2)$ , and  $t_2x_0$  according to  $a_1$ . In terms of the parameters  $D\sigma$  and  $D\tau$  of the  $D_{3d}$  field,<sup>4</sup> the corresponding single electron energies result as

$$\begin{aligned} E[e(e)] &= \langle eu_{\pm} | V_{\text{irig}} | eu_{\pm} \rangle = 6Dq + \frac{7}{3}D\tau \\ E[e(t_2)] &= \langle t_2x_{\pm} | V_{\text{irig}} | t_2x_{\pm} \rangle = -4Dq + D\sigma + \frac{2}{3}D\tau \end{aligned} \quad (9)$$

$$E(a_1) = \langle t_2x_0 | V_{\text{irig}} | t_2x_0 \rangle = -4Dq - 2D\sigma - 6D\tau$$

However, in contrast to the situation in a  $D_{4h}$  field, the trigonal field mixes the two  $e$  orbitals by way of the off-diagonal matrix element

$$\langle eu_{\pm} | V_{\text{irig}} | t_2x_{\pm} \rangle = \frac{\sqrt{2}}{3}(3D\sigma - 5D\tau) \quad (10)$$

which has to be taken into account in calculating  $E[e]$  according to Eq. (9).

### 3. Total Six-Electron States

Total  $d^6$  electron states transforming according to the irreducible representation  $\Gamma$  (in one of the point groups considered,  $D_{4h}$  or  $D_{3d}$ ), component  $\gamma$ , and possessing a total spin  $S$  with  $z$  component  $M_S$  may be constructed from the one-electron orbitals Eq. (6) and (8) by the application of

$$|^{2S+1}\Gamma M_S \gamma\rangle = \sum_{\substack{m_1, m_2, \\ \gamma_1, \gamma_2}} |\Gamma_1 m_1 \gamma_1\rangle |\Gamma_2 m_2 \gamma_2\rangle \langle s_1 m_{s1} s_2 m_{s2} | S M_S \rangle \times \langle \Gamma_1 \gamma_1 \Gamma_2 \gamma_2 | \Gamma \gamma \rangle \quad (11)$$

taking two orbitals at one time. In Eq. (11),  $\langle s_1 m_{s1} s_2 m_{s2} | S M_S \rangle$  are Wigner coefficients for the electron spin and  $\langle \Gamma_1 \gamma_1 \Gamma_2 \gamma_2 | \Gamma \gamma \rangle$  are coupling coefficients for the symmetry group concerned. In fact, only total wavefunctions of the  $d^4$  electron problem are required, if the equivalency between electrons and holes is utilized. Previous calculations indicate that it is essentially only the lowest multiplets which provide a major contribution to the electronic ground state. Therefore, we limit our calculations, in the present study, to those states arising from the octahedral terms  $^5T_{2g}(t_{2g}^4 e_g^2)$ ,  $^1A_{1g}(t_{2g}^6)$ ,  $^3T_{1g}(t_{2g}^5 e_g)$ , and  $^3T_{2g}(t_{2g}^5 e_g)$  comprising altogether 34 electron levels. In this case, the interelectronic repulsion energies are diagonal to

first order and may thus be written as

$$\begin{aligned} E_{\text{Coulomb}}(^3T_{2g}) &= 0 \\ E_{\text{Coulomb}}(^3T_{1g}) &= 5B + 5C \\ E_{\text{Coulomb}}(^3T_{2g}) &= 13B + 5C \\ E_{\text{Coulomb}}(^1A_{1g}) &= 5B + 8C \end{aligned} \quad (12)$$

(a) *Tetragonal Symmetry Case.* In  $D_{4h}$  symmetry, the splitting of the parent octahedral terms is according to  ${}^5T_{2g} \rightarrow {}^5B_2 + {}^5E$ ,  ${}^3T_{1g} \rightarrow {}^3A_2 + {}^3E$ , and  ${}^3T_{2g} \rightarrow {}^3B_2 + {}^3E$ , all resulting states being g on the basis of parity. In addition, the  ${}^1A_{1g}$  state is not affected by the change of symmetry. Since there is a one-to-one correspondence of states in  $O_h$  and  $D_{4h}$ , the required  $d^4$  state functions may be easily set up as<sup>7</sup>

$$\begin{aligned} |{}^1A_{1g}0a_1\rangle &= |\theta^2\epsilon^2\rangle \\ |{}^3B_22b_2\rangle &= |{}^3T_22\zeta\rangle = |\theta^+\epsilon^+\xi^+\eta^+\rangle \\ |{}^3E2e_a(e_b)\rangle &= |{}^3T_22\eta(\xi)\rangle = |\theta^+\epsilon^+\zeta^+\eta^+(\xi^+)\rangle \\ |{}^3A_21a_2\rangle &= |{}^3T_11z\rangle = |\theta^+\epsilon^+\zeta^+\rangle \\ |{}^3E1e_a(e_b)\rangle &= |{}^3T_11y(x)\rangle = -\frac{1}{2}\theta^2\epsilon^+\eta^+(\xi^+) \pm \frac{\sqrt{3}}{2}|\theta^+\epsilon^2\eta^+(\xi^+)\rangle \\ |{}^3B_21b_2\rangle &= |{}^3T_21\zeta\rangle = |\theta^+\epsilon^2\zeta^+\rangle \\ |{}^3E1e_a(e_b)\rangle &= |{}^3T_21\eta(\xi)\rangle = \mp \frac{\sqrt{3}}{2}|\theta^2\epsilon^+\eta^+(\xi^+)\rangle - \frac{1}{2}|\theta^+\epsilon^2\eta^+(\xi^+)\rangle \end{aligned} \quad (13)$$

In Eq. (13), only functions characterized by the maximum  $M_S$  value are listed. The remaining functions may be easily generated by application of the step-down operator  $S_-$ . These functions, however, are needed in calculations including spin-orbit interaction, although complex functions may be preferred for convenience.

It should be observed that, due to the use of  $d^4$  functions Eq. (13), the sign of the ligand field parameters  $Dq$ ,  $Ds$ , and  $Dt$  (as well as that of the spin-orbit coupling constant  $\zeta$  to be introduced below) has to be changed. The calculation of matrix elements of the ligand field potential  $V_{\text{tet}}$  of Eq. (3) then yields the term energies

$$\begin{aligned} E({}^1A_1) &= -24Dq + 14Dt + 5B + 8C \\ E({}^3A_2) &= -14Dq + 14Dt + 5B + 5C \\ E({}^3E({}^3T_1)) &= -14Dq + Ds + \frac{13}{2}Dt - 9B + 5C - R_1 \\ E({}^3B_2) &= -14Dq - 4Ds + 9Dt + 13B + 5C \\ E({}^3E({}^3T_2)) &= -14Dq + Ds + \frac{13}{2}Dt + 9B + 5C + R_1 \\ E({}^3B_2) &= -4Dq + 2Ds - Dt \\ E({}^3E) &= -4Dq - Ds + 4Dt \end{aligned} \quad (14)$$

where

$$R_1 = \frac{1}{2} \left\{ (2Ds + \frac{10}{4}Dt + 8B)^2 + \frac{3}{4}(4Ds + 5Dt)^2 \right\}^{1/2} \quad (15)$$

(7) J.S. Griffith, *The Theory of Transition Metal Ions*, Cambridge University Press 1961.

In Eq. (14), the appropriate interelectronic repulsion energies of Eq. (12) have been included which are not affected by a lowering of symmetry. The square root Eq. (15) results from a non-zero ligand field interaction between the  ${}^3E$  states, viz.

	${}^3E({}^3T_1)$	${}^3E({}^3T_2)$
${}^3E({}^3T_1)$	$-14Dq + \frac{21}{4}Dt + 5B + 5C$	$\frac{\sqrt{3}}{4}(4Ds + 5Dt)$
${}^3E({}^3T_2)$		$-14Dq + 2Ds + \frac{31}{4}Dt + 13B + 5C$

(16)

Spin-orbit matrix elements have been calculated by application of the operator  $\sum_i \zeta_i l_i \cdot s_i$  to the complex functions corresponding to Eq. (13) as well as to the functions of lower  $M_S$  values.\* The resulting  $34 \times 34$  spin-orbit interaction matrix then factors into a matrix of dimension 10 and three  $8 \times 8$  matrices, two of which are identical. The matrices are listed in Appendix I. Since basis functions transforming according to the irreducible representations of  $\Gamma \times \Gamma_s$  have not been constructed, the individual matrices may be attributed to two representations (including spin) at a time, viz.  $\Gamma_{11}, \Gamma_{12}; \Gamma_{13}, \Gamma_{14}$ ; two times  $\Gamma_{15a}, \Gamma_{15b}$ .\*\*

(b) *Trigonal Symmetry Case.* In  $D_{3d}$  symmetry, the parent octahedral terms are split as indicated by  ${}^5T_{2g} \rightarrow {}^5A_1 + {}^5E$ ,  ${}^3T_{1g} \rightarrow {}^3A_2 + {}^3E$ , and  ${}^3T_{2g} \rightarrow {}^3A_1 + {}^3E$ , whereas the  ${}^1A_{1g} \rightarrow {}^1A_1$  and is not split. Again, all resulting states are g on the basis of parity. State functions within the  $d^4$  configuration transforming according to the resulting irreducible representations may be set up on the basis of Eq. (11) producing

$$\begin{aligned} |{}^1A_10a_1\rangle &= |u_+^2u_-^2\rangle \\ |{}^3A_12a_1\rangle &= -|u_+^+u_-^+x_+^+x_-^+\rangle \\ |{}^3E2u_\pm\rangle &= -|u_+^+u_-^+x_\pm^+x_0\rangle \\ |{}^3A_21a_2\rangle &= \frac{i}{\sqrt{2}} \{ |u_+^2u_-^+x_-^+\rangle - |u_-^2u_+^+x_+^+\rangle \} \\ |{}^3E({}^3T_1)1u_\pm\rangle &= \mp \frac{i}{\sqrt{2}} \{ |u_-^2u_+^+x_0^+\rangle - |u_-^2u_-^+x_\mp^+\rangle \} \\ |{}^3A_11a_1\rangle &= -\frac{1}{\sqrt{2}} \{ |u_+^2u_-^+x_-^+\rangle + |u_-^2u_+^+x_+^+\rangle \} \\ |{}^3E({}^3T_2)1u_\pm\rangle &= \mp \frac{1}{\sqrt{2}} \{ |u_-^2u_+^+x_0^+\rangle + |u_+^2u_-^+x_\mp^+\rangle \} \end{aligned} \quad (17)$$

Again we list in Eq. (17) only functions characterized by the maximum value of  $M_S$ , functions of lower  $M_S$  being easily generated by application of  $S_-$ . The corresponding energies then result from the matrix elements of  $V_{\text{trig}}$ , cf. Eq. (5), and the interelectronic interaction, Eq. (12)

(\*) If the real functions Eq. (13) are used, much lower splitting of the overall spin-orbit interaction matrix is produced, viz. one  $18 \times 18$  ( $\Gamma_{11}, \Gamma_{12}, \Gamma_{13}, \Gamma_{14}$ ) and one  $16 \times 16$  matrix ( $\Gamma_{15a}, \Gamma_{15b}$ ).

(\*\*) In what follows we employ the Bethe nomenclature of irreducible representations to denote states including spin-orbit coupling. In particular,  $\Gamma_i$  refers to representations within  $D_{4h}$  and  $\Gamma^r$  to those within  $D_{3d}$  symmetry.

$$\begin{aligned}
 E({}^1A_1) &= -24Dq - \frac{28}{3}D\tau + 5B + 8C \\
 E({}^3A_2) &= -14Dq - D\sigma - \frac{23}{3}D\tau + 5B + 5C \\
 E({}^1E({}^3T_1)) &= -14Dq + \frac{1}{2}D\sigma - \frac{13}{3}D\tau + 9B + 5C - R_2 \\
 E({}^3A_1) &= -14Dq - D\sigma - \frac{23}{3}D\tau + 13B + 5C \quad (18) \\
 E({}^2E({}^3T_2)) &= -14Dq + \frac{1}{2}D\sigma - \frac{13}{3}D\tau + 9B + 5C + R_2 \\
 E({}^3A_1) &= -4Dq - 2D\sigma - 6D\tau \\
 E({}^3E) &= -4Dq + D\sigma + \frac{2}{3}D\tau
 \end{aligned}$$

where

$$R_2 = \frac{1}{2} \left\{ 4 \left( \frac{3}{2}D\sigma + \frac{10}{3}D\tau \right)^2 + 64B^2 \right\}^{1/2} \quad (19)$$

The square root Eq. (19) stems from a ligand field interaction between the two  ${}^3E$  states, *viz.*

	${}^3E({}^3T_1)u_{\pm}$	${}^3E({}^3T_2)u_{\pm}$
${}^3E({}^3T_1)u_{\pm}$	$-14Dq + \frac{1}{2}D\sigma - \frac{13}{3}D\tau + 5B + 5C$	$\mp \frac{3}{2}D\sigma + \frac{10}{3}D\tau$
${}^3E({}^3T_2)u_{\pm}$		$-14Dq + \frac{1}{2}D\sigma - \frac{13}{3}D\tau + 13B + 5C$

(20)

Spin-orbit interaction may be included, in the same way as shown in section 3a above, by application of  $\Sigma_i \zeta_i \mathbf{l}_i \cdot \mathbf{s}_i$  to the complex functions corresponding to Eq. (17) including those with  $M_S < S$ . The total  $34 \times 34$  matrix then factors into one matrix  $12 \times 12$  and two identical matrices  $11 \times 11$  which may be found in Appendix II. The matrices may be labeled by the  $\Gamma \times \Gamma_S$  representations  $\Gamma_1^T, \Gamma_2^T$ ; and two times  $\Gamma_3^T u_{\mp}$ .

#### 4. Conditions of Ground State Formation

Obviously, the formation of possible ground states within the problem at hand will be dependent on the parameters  $Dq$ ,  $Ds$ ,  $Dt$  (or  $D\sigma$ ,  $D\tau$ ),  $B$ , and  $C$  if, at first, spin-orbit interaction is disregarded. Instead of  $Ds$  (or  $D\sigma$ ) it is convenient to introduce a new parameter  $\chi$  such that

$$\begin{aligned}
 \chi &= Ds/Dt \text{ in } D_{4h} \text{ symmetry} \\
 \chi &= D\sigma/D\tau \text{ in } D_{3d} \text{ symmetry}
 \end{aligned}$$

For convenience of presentation we consider below only states resulting from the  ${}^5T_{2g}$ ,  ${}^3T_{1g}$ , and  ${}^1A_{1g}$  terms. It may then be shown that, in  $D_{4h}$  symmetry, four different combinations of possible ground states should be distinguished. These four cases are characterized by  $Dt > 0$  or  $Dt < 0$  in conjunction with  $\chi > 5/3$  or  $\chi < 5/3$ . The resulting ground states are list-

ed below.

	$Dt < 0$	$Dt > 0$
$\chi < \frac{5}{3}$	${}^3E, {}^3A_2, {}^1A_1$	${}^3B_2, {}^3E, {}^1A_1$
$\chi > \frac{5}{3}$	${}^3B_2, {}^3A_2, {}^1A_1$	${}^3E, {}^3E, {}^1A_1$

(21)

Employing the energy expressions Eq. (14), conditions for the formation of the various ground states in Eq. (21) may be set up. In general, there are two conditions which have to be met for each single ground state. These conditions are given below. Where one of the equations occurs repeatedly, only the number of the equation is listed where it shows up for the second time. If a change of the unequal sign is involved, this is indicated by a minus sign preceding the corresponding equation number.

(1)  $Dt < 0, \chi < \frac{5}{3}$ .

$${}^1A_1 \quad \left\{ \begin{array}{l} Ds + 10Dt \leq 20Dq - 5B - 8C \end{array} \right. \quad (22)$$

$$0 \leq 10Dq - 3C \quad (23)$$

$${}^3E \quad \left\{ \begin{array}{l} Ds + 10Dt \geq 10Dq - 5B - 5C \\ -(22) \end{array} \right. \quad (24)$$

$${}^3A_2 \quad \left\{ \begin{array}{l} -(23) \\ -(24) \end{array} \right.$$

(2)  $Dt > 0, \chi < \frac{5}{3}$ .

$${}^1A_1 \quad \left\{ \begin{array}{l} -2Ds + 15Dt \leq 20Dq - 5B - 8C \end{array} \right. \quad (25)$$

$$\frac{35}{4}Dt \leq 10Dq - 3C \quad (26)$$

$${}^3B_2 \quad \left\{ \begin{array}{l} -2Ds + \frac{25}{4}Dt \geq 10Dq - 5B - 5C \\ -(25) \end{array} \right. \quad (27)$$

$${}^3E \quad \left\{ \begin{array}{l} -(26) \\ -(27) \end{array} \right.$$

(3 and 4)  $Dt \geq 0, \chi > \frac{5}{3}$ .

	$Dt < 0$		$Dt > 0$
${}^1A_1$	$\left\{ \begin{array}{l} (25) \\ (23) \end{array} \right.$	${}^1A_1$	$\left\{ \begin{array}{l} (22) \\ (26) \end{array} \right.$

${}^3B_2$	$\left\{ \begin{array}{l} -(25) \\ (29) \end{array} \right.$	${}^3E$	$\left\{ \begin{array}{l} -(22) \\ (28) \end{array} \right.$
-----------	--	---------	--

${}^3A_2$	$\left\{ \begin{array}{l} -(23) \\ -(29) \end{array} \right.$	${}^3E$	$\left\{ \begin{array}{l} -(26) \\ -(28) \end{array} \right.$
-----------	---	---------	---

where

$$Ds + \frac{5}{4}Dt \geq 10Dq - 5B - 5C \quad (28)$$

$$2Ds + 15Dt \leq 10Dq - 5B - 5C \quad (29)$$

If the states originating in the octahedral  ${}^3T_{2g}$  term are taken into account, essentially six different combinations of possible ground states have to be distinguished. These cases may be characterized, in good approximation, as shown below.

	Dt < 0	Dt > 0
$x > \frac{5}{3}$	${}^3B_2, {}^1A_1, {}^3E$	${}^3E, {}^1A_1, {}^3B_2$
$ x  < \frac{5}{3}$	${}^3E, {}^1A_1, {}^3A_2$	${}^3B_2, {}^1A_1, {}^3E$
$x < -\frac{5}{3}$	${}^3E, {}^1A_1, {}^3B_2$	${}^3B_2, {}^1A_1, {}^3E$

In addition, for certain specific values of the parameters  $x$  and Dt in the transition regions, more complicated ground state combinations may arise. In Eq. (30), the notation  ${}^3E$  and  ${}^3E$  refers to the states  ${}^3E({}^3T_{1g})$  and  ${}^3E({}^3T_{2g})$ , respectively. As a consequence of the non-zero ligand field interaction between these two  ${}^3E$  states, *viz.* Eq. (16), simple conditions for the formation of ground states similar to those given above in absence of the  ${}^3T_{2g}$ -based states cannot be set up. However, in a Dt versus Dq plot, the distribution of quintet, singlet, and triplet states is not significantly changed except for the symmetry transformation property of the states involved, *cf.* Eq. (21) and Eq. (30).

In  $D_{3d}$  symmetry, two combinations of possible ground states have to be distinguished if, at first, again states originating in the octahedral terms  ${}^5T_{2g}$ ,  ${}^1A_{1g}$ , and  ${}^3T_{1g}$  only are considered. These combinations may be characterized by  $x \geq -20/9$  as shown below.

$$\frac{x > -\frac{20}{9}}{{}^3E, {}^3E, {}^1A_1} \quad \frac{x < -\frac{20}{9}}{{}^3A_1, {}^3A_2, {}^1A_1} \quad (31)$$

The notation in the following conditions is the same as above.

$$(1) \quad x > -\frac{20}{9}$$

$${}^1A_1 \quad \left\{ \begin{array}{l} -\frac{1}{2}D\sigma - 5D\tau \leq 10Dq - 3C \\ -\frac{1}{2}D\sigma - 5D\tau \leq 10Dq - \frac{1}{2}(5B + 8C) \end{array} \right. \quad (32)$$

$${}^3E \quad \left\{ \begin{array}{l} -\frac{1}{2}D\sigma - 5D\tau \geq 10Dq - 5B - 5C \\ \text{---(33)} \end{array} \right. \quad (34)$$

$${}^3E \quad \left\{ \begin{array}{l} \text{---(34)} \\ \text{---(32)} \end{array} \right.$$

$$(2) \quad x < -\frac{20}{9}$$

$${}^1A_1 \quad \left\{ \begin{array}{l} D\sigma - \frac{5}{3}D\tau \leq 10Dq - 3C \\ D\sigma - \frac{5}{3}D\tau \leq 10Dq - \frac{1}{2}(5B + 8C) \end{array} \right. \quad (35)$$

$$\text{---(36)}$$

$${}^3A_1 \quad \left\{ \begin{array}{l} D\sigma - \frac{5}{3}D\tau \geq 10Dq - 5B - 5C \\ \text{---(36)} \end{array} \right. \quad (37)$$

$${}^3A_2 \quad \left\{ \begin{array}{l} \text{---(35)} \\ \text{---(37)} \end{array} \right.$$

However, the situation in  $D_{3d}$  symmetry is considerably more involved than if the symmetry is  $D_{4h}$ . Thus, in case (1), it is easily visualized that Eq. (33) is sufficient to specify a  ${}^1A_1$  ground state and, similarly, Eq. ---(33) to establish a  ${}^3E$  ground state. In addition, the conditions Eq. ---(34) and Eq. ---(32) for a  ${}^3E$  ground state (valid in absence of  ${}^3T_{2g}$  interaction) cannot be met at the same time. Likewise, in case (2), Eq. (36) and Eq. ---(36) are sufficient to characterize the ground states  ${}^1A_1$  and  ${}^3A_1$ , respectively. Again, the conditions Eq. ---(35) and Eq. ---(37) for a  ${}^3A_2$  ground state cannot be satisfied simultaneously. It follows that, in  $D_{3d}$  symmetry, spin triplet ground states are not formed if only the terms arising from  ${}^5T_{2g}$ ,  ${}^1A_{1g}$ , and  ${}^3T_{1g}$  are taken into account.

Similar to  $D_{4h}$  symmetry, the inclusion of the states resulting from the  ${}^3T_{2g}$  term complicates and modifies the conditions listed above. However, Eq. (31) remains approximately valid if the  ${}^3A_2$  term under  $x < -20/9$  is replaced by a  ${}^3E$  state and if always  ${}^3E = {}^3E$ . The most important change thus introduced is that now a  ${}^3E({}^3T_{1g})$  ground state may be stabilized, whereas, as before, the  ${}^3A_2$  state (and the  ${}^3A_1$  resulting from  ${}^3T_{2g}$  as well) cannot become ground state.

No closed expressions concerning formation of ground states can be given if spin-orbit interaction is included.

## 5. Results: Parameter Dependence of Ground State Regions

(a) *Without Spin-Orbit Coupling.* Disregarding the  ${}^3T_{2g}$  term, the conditions for ground state formation in section 4 have been written in form of a linear equation, *viz.*

$$f(Ds(x) + Dt) = aDq + bB + cC \quad (38)$$

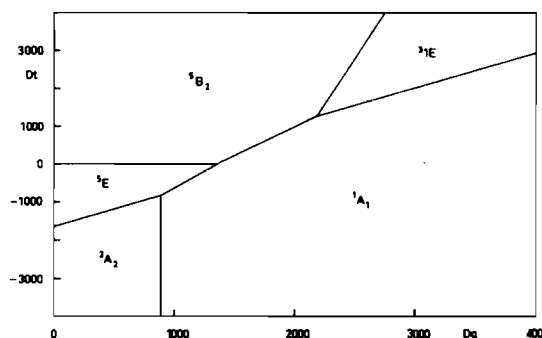


Figure 2. Ground state regions for the  $d^6$  configuration in  $D_{4h}$  symmetry without spin-orbit coupling as function of the tetragonal and cubic field strengths Dt and Dq, respectively ( $B = 730 \text{ cm}^{-1}$ ,  $C = 4B$ ,  $x = 1.0$ ).

In Eq. (38),  $Dq$  may be considered as the independent variable with  $bB+cC$  as the intercept on the ordinate. Consequently, if  $f(Ds(x)+Dt)$  is plotted vs.  $Dq$ , existence regions of the various ground states are obtained for fixed values of  $B$  and  $C$ .

If, on the other hand, the  ${}^3T_{2g}$  term is included, numerical solutions of the resulting equations are required. The results are illustrated in Figure 2 for  $D_{4h}$  symmetry and for  $\kappa = 1.0$  where  $B = 730 \text{ cm}^{-1}$  and  $C = 4B$  has been assumed.

(b) *The Effect of Spin-Orbit Coupling.* If spin-orbit interaction is taken into account, there may be non-zero contributions of various spin multiplicities to each state in question. In addition to spin

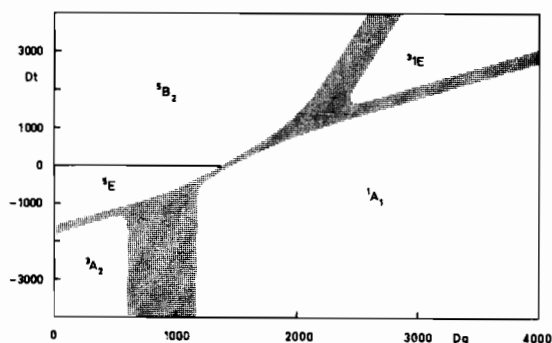


Figure 3. Ground state regions for the  $d^6$  configuration in  $D_{4h}$  symmetry with spin-orbit coupling ( $B = 730 \text{ cm}^{-1}$ ,  $C = 4B$ ,  $\zeta = 420 \text{ cm}^{-1}$ ) assuming  $\kappa = 1.0$ .

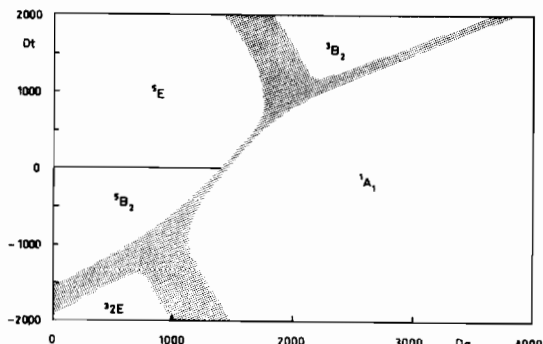


Figure 4. Ground state regions for the  $d^6$  configuration in  $D_{4h}$  symmetry with spin-orbit coupling ( $B = 730 \text{ cm}^{-1}$ ,  $C = 4B$ ,  $\zeta = 420 \text{ cm}^{-1}$ ) assuming  $\kappa = 3.0$ .

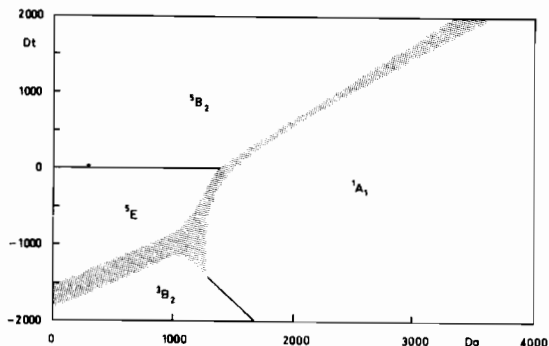


Figure 5. Ground state regions for the  $d^6$  configuration in  $D_{4h}$  symmetry with spin-orbit coupling ( $B = 730 \text{ cm}^{-1}$ ,  $C = 4B$ ,  $\zeta = 420 \text{ cm}^{-1}$ ) assuming  $\kappa = -3.0$ .

singlet, triplet, and quintet ground states, substantially spin-mixed ground states are thus expected. We therefore arbitrarily define a pure spin ground state as one having less than 2% admixture of any other spin multiplicity (blank areas in Figures 3, 4, 5, 6, and 7) and all other ground states as spin-mixed (shaded areas in Figures 3, 4, 5, 6, and 7). In all calculations comprising spin-orbit coupling, the states  ${}^5T_{2g}$ ,  ${}^1A_{1g}$ ,  ${}^3T_{1g}$ , and  ${}^3T_{2g}$  have been included.

Figures 3, 4, and 5 show the results for  $D_{4h}$  symmetry with  $\kappa = 1.0$ ,  $\kappa = 3.0$ , and  $\kappa = -3.0$ , respectively. Similarly, the results for  $D_{3d}$  symmetry with  $\kappa = 1.0$  and  $\kappa = 3.0$  are displayed in Figures 6 and 7, respectively.

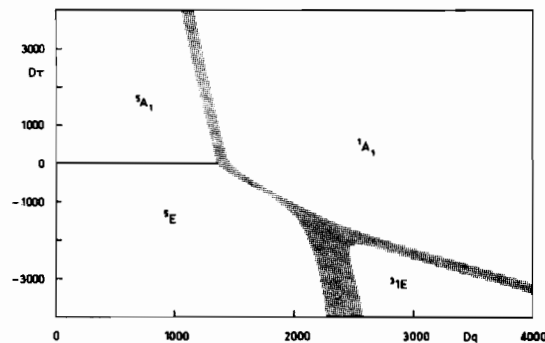


Figure 6. Ground state regions for the  $d^6$  configuration in  $D_{3d}$  symmetry with spin-orbit coupling ( $B = 730 \text{ cm}^{-1}$ ,  $C = 4B$ ,  $\zeta = 420 \text{ cm}^{-1}$ ) assuming  $\kappa = 1.0$ .

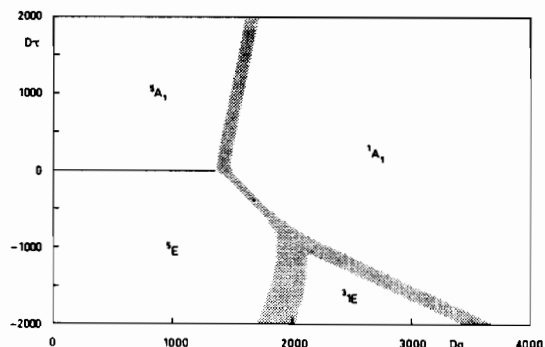


Figure 7. Ground state regions for the  $d^6$  configuration in  $D_{3d}$  symmetry with spin-orbit coupling ( $B = 730 \text{ cm}^{-1}$ ,  $C = 4B$ ,  $\zeta = 420 \text{ cm}^{-1}$ ) assuming  $\kappa = 3.0$ .

## 6. Discussion

The results for  $D_{4h}$  symmetry demonstrate that, if  $\kappa = 1.0$  (cf. Figure 2), a  ${}^3A_2$  ground state indeed appears at low values of  $Dq$ , i.e. specifically, if  $Dq \leq 876 \text{ cm}^{-1}$  and  $Dt \leq -863 \text{ cm}^{-1}$  where  $Dt$  is dependent on  $Dq$ , viz. section 4. On the other hand, a  ${}^3E$  ground state is formed at high values of  $Dq$ , i.e. if  $Dq \geq 2200 \text{ cm}^{-1}$  and  $Dt \geq 1300 \text{ cm}^{-1}$  where, in this case,  $Dq$  and  $Dt$  are dependent on each other. If spin-orbit interaction is included, the ground state regions arise approximately for the same parameter values as without spin-orbit coupling. However, the clear boundaries between these regions are now replaced by spin-mixed areas (viz. Figure 3), some of these extending over a considerable range of values

of one or two specific parameters. Thus, if  $\kappa = 1.0$ , the  ${}^3A_2$ - ${}^1A_1$  spin-mixed area covers a range from  $Dq = 600 \text{ cm}^{-1}$  to  $Dq = 1170 \text{ cm}^{-1}$  and the  ${}^3E$ - ${}^5B_2$  area extends over about  $300 \text{ cm}^{-1}$  in  $Dq$ .

With  $\kappa$  increasing, *i.e.*  $1 < \kappa < 5/3$ , the  ${}^3A_2$  changes little, the  ${}^3E$  slowly moving to lower  $Dq$ . If  $\kappa = 5/3$ , the possible ground states change as demonstrated in section 4 above. Thus, if  $\kappa = 3.0$  (*cf.* Figure 4), a  ${}^3E$  ground state is now encountered with low values of  $Dq$  and  $Dt < 0$ , whereas if  $Dt > 0$  in conjunction with high values of  $Dq$ , a  ${}^3B_2$  ground state may form. With still larger  $\kappa$ , the  ${}^3E$  and the  ${}^3B_2$  are shifted to higher and to lower  $Dq$ , respectively. At  $\kappa \sim 10.0$ , the two triplet regions are distributed almost symmetrically to the singlet-quintet crossover. Let us consider a decrease of  $\kappa$  below 1.0, *i.e.*  $1 > \kappa > 0$ . In this case, the  ${}^3E$  moves slowly to higher  $Dq$ , the  ${}^3A_2$  changing little. The behavior at  $0 > \kappa > -5/3$  is similar to that at  $5/3 > \kappa > 0$ , except that below  $\kappa < -1/2$  the  ${}^3E$  is shifted out of the region considered. At  $\kappa = -5/3$ , the  ${}^3B_2$  arises at low values of  $Dq$  and  $Dt < 0$  and moves to higher  $Dq$  if  $\kappa < -5/3$  (*cf.* Figure 5). Finally, at  $\kappa < -3.0$ , the  ${}^3E$  appears at  $Dt > 0$  and high  $Dq$  slowly shifting to lower  $Dq$  as  $\kappa$  becomes more negative.

In  $D_{3d}$  symmetry, the relation between the results obtained with spin-orbit coupling having not and having been taken into account is similar to that in  $D_{4h}$  symmetry discussed above. If  $\kappa = 1.0$  (*cf.* Figure 6), a  ${}^3E$  ground state is formed if  $Dq \geq 2500 \text{ cm}^{-1}$  and  $D\tau \leq -2000 \text{ cm}^{-1}$ ,  $Dq$  and  $D\tau$  being mutually dependent to a certain extent. Again a  ${}^3E$ - ${}^5E$  spin-mixed region of about  $300 \text{ cm}^{-1}$  in  $Dq$  is found. With increasing  $\kappa$ , the  ${}^3E$  region is shifted to lower  $Dq$  (*viz.* Figure 7) arriving with  $\kappa \sim 10.0$  at the singlet-quintet crossover. If negative values of  $\kappa$  are considered, the  ${}^3E$  disappears below  $\kappa = -1.0$ , and below  $\kappa = -20/9$  a new region of  ${}^3E$  is encountered at  $D\tau > 0$  and small  $Dq$ . This  ${}^3E$  slowly shifts to larger  $Dq$  as  $\kappa$  decreases further.

It should be observed that there is, in general, a considerable mixing involved between the  ${}^3E$  ( ${}^3T_{1g}$ ) =  ${}^3E$  and  ${}^3E$  ( ${}^3T_{2g}$ ) =  ${}^3E$  states, the actual amount of the admixture being dependent on  $Dt$  or  $D\tau$  in  $D_{4h}$  and  $D_{3d}$  symmetry, respectively. For the lowest values of  $Dq$  and  $Dt$  or  $D\tau$  applicable, the contribution of the  ${}^3E$  to the  ${}^3E$  is specifically 13% in  $D_{4h}$  and 33% in  $D_{3d}$  symmetry if  $\kappa = 1.0$ . On the other hand, the  ${}^3E$  contributes about 40% to the  ${}^3E$  in  $D_{4h}$  symmetry if  $\kappa = 3.0$ .

The effect of including a limited number of additional states originating in the next higher excited states of  $O_h$  symmetry has been considered, especially with respect to  ${}^1T_{1g}$  ( $t_{2g}^5 e_g$ ) and  ${}^5E_g$  ( $t_{2g}^3 e_g^3$ ). These states are split in a field of  $D_{4h}$  symmetry according to  ${}^1T_{1g} \rightarrow {}^1A_2 + {}^1E$  and  ${}^5E \rightarrow {}^5A_1 + {}^5B_1$ . However, it is found that the  ${}^1T_{1g}$  is always by  $2C$  higher in energy than the  ${}^3T_{1g}$  and the  ${}^5B_1$  ( ${}^5E_g$ ) is by  $10 Dq$  higher than the  ${}^5B_2$  ( ${}^5T_{2g}$ ). Consequently both the  ${}^1T_{1g}$  or states originating therefrom and the  ${}^5B_1$  ( ${}^5E_g$ ) are unlikely to become ground state. The only additional state to be considered, in this approximation, is  ${}^5A_1$  ( ${}^5E_g$ ). Indeed, it may be demonstrated that for higher values of  $Dt > 0$  a  ${}^5A_1$  ground state replaces the  ${}^5B_2$  ground

state in the diagrams Figures 3 and 4. Since, in this paper, we are interested in spin triplet rather than in spin quintet ground states, the  ${}^5A_1$  state will not be considered further.

Within a field of  $D_{3d}$  symmetry, the splitting is according to  ${}^1T_1 \rightarrow {}^1A_2 + {}^1E$ , whereas the  ${}^5E$  is not split. There arises now a non-zero interaction between  ${}^5E$  ( ${}^5E_g$ ) and  ${}^5E$  ( ${}^5T_{2g}$ ) by way of the off-diagonal matrix element  $+\sqrt{2/3}(3D\sigma - 5D\tau)$ , although the results (*cf.* Figures 6 and 7) are not affected to any significant extent.

## 7. Experimental Evidence for Triplet Ground States

The first compound of iron(II) in which a spin triplet ground state was suggested is, to the authors' knowledge, ferrous phthalocyanine,  $\text{Fe}(\text{pc})$ .<sup>8</sup> On the basis of magnetic measurements between 292.5 and  $1.25^\circ\text{K}$ <sup>9</sup> and the study of magnetic anisotropy,<sup>10</sup> the ground state is assumed to be  ${}^3B_2$  with a zero-field splitting  $D = 64 \text{ cm}^{-1}$ . From section 4 it then follows that  $\kappa > 5/3$  and  $Dt > 0$ . Recently, a  ${}^3B_2$  ground state has likewise been suggested in the planar bis(biuretato)cobalt(III) complexes from magnetic and optical spectral studies.<sup>11</sup> In addition, spin triplet ground states were tentatively assigned to a series of bis(1,10-phenanthroline)iron(II) complexes and several related bis(diimine)iron(II) compounds.<sup>12,13</sup> In this case it was demonstrated by extending magnetic measurements and Mössbauer effect studies to 1.2 and  $4.2^\circ\text{K}$ , respectively, that the ground state is  ${}^3A_2$ .<sup>14</sup> Again the  ${}^3A_2$  is slightly split by 2.0 to  $3.0 \text{ cm}^{-1}$  and is considerably spin-mixed in addition.<sup>15</sup> It may then be concluded that here,  $\kappa < 5/3$  and  $Dt < 0$  is required. There are various other occasions where triplet ground states have been observed in iron(II). One of the most conspicuous encounters of this sort is the formation of an  $S = 1$  ground state on reduction from iron(III) to iron(II) under high pressure. This has been observed recently in biological compounds like hemin, hematin, and imidazole protoheme.<sup>16</sup> As it stands, there has been no report on a  ${}^3E$  ground state in iron(II), although, as shown above, this state should likewise be stable. Finally, there are numerous inorganic compounds and biological materials containing iron(II) reported in literature where the value of the effective magnetic moment suggests that triplet ground states might be involved. In a forthcoming publication,<sup>17</sup> the known physical properties of the compounds listed above will be compared in detail with calculated values and predictions concerning unknown data will be made.

(8) A.B.P. Lever, *J. Chem. Soc.*, 1821 (1965).

(9) B.W. Dale, R.J.P. Williams, C.E. Johnson, and T.L. Thorp, *J. Chem. Phys.*, 49, 3441 (1968).

(10) C.G. Barraclough, R.L. Martin, S. Mitra, and R.C. Sherwood, *J. Chem. Phys.*, 53, 1643 (1970).

(11) J.J. Bour, P.T. Beurskens, J.J. Steggerda, *Chem. Commun.*, 221 (1972).

(12) E. König, K. Madeja, *J. Am. Chem. Soc.*, 88, 4528 (1966).

(13) E. König, K. Madeja, *Inorg. Chem.*, 7, 1848 (1968).

(14) E. König, B. Kanellakopoulos, *Chem. Phys. Lett.*, 12, 485 (1972).

(15) E. König, G. Ritter, B. Kanellakopoulos, *J. Chem. Phys.*, 58, 3001 (1973).

(16) D.C. Grenoble, C.W. Frank, C.B., Barger, H.G. Drickamer, *J. Chem. Phys.*, 55, 1633 (1971).

(17) E. König, R. Schnakig, to be published.

Appendix I. Complete ligand field, interelectronic repulsion, and spin-orbit coupling matrices in  $D_{6h}$  symmetry including the cubic parent terms  ${}^3T_{2g}(t_{2g}^4 e_g^2)$ ,  ${}^1A_{1g}(t_{2g}^3 e_g^3)$ ,  ${}^3T_{1g}(t_{2g}^3 e_g^3)$ , and  ${}^3T_{2g}(t_{2g}^3 e_g^3)$ .

Matrix A.  $\Gamma_{11}, \Gamma_{12} (= A_1, A_2)$ .

${}^5E-11$	${}^5E1-1$	${}^5B_22b_2$	${}^5B_2-2b_2$	${}^3E-11$	${}^3E1-1$	${}^3A_20a_2$	${}^3E1-1$	${}^3E-11$	${}^1A_10a_1$
$\frac{1}{4}q$ $-4Dq$ $-2S+4Dc$			$\frac{1}{4}12q$	$-\frac{1}{2}13q$		$\frac{1}{2}13q$		$\frac{1}{2}q$	
	$\frac{1}{4}q$ $-4Dq$ $-2S+4Dc$	$\frac{1}{4}12q$			$\frac{1}{2}13q$	$-\frac{1}{2}13q$	$-\frac{1}{2}q$		
		$-4Dq$ $+2Dc-2c$			$\frac{1}{2}16q$		$\frac{1}{2}12q$		
			$-4Dq$ $+2Dc-2c$	$-\frac{1}{2}16q$				$-\frac{1}{2}12q$	
				$\frac{1}{4}q$ $-14Dq+214Dc$ $+5B+5C$		$-\frac{1}{4}q$		$-\frac{1}{2}13q$ $+(3Dc-54Dc)$	$-12q$
					$\frac{1}{4}q$ $-14Dq+214Dc$ $+5B+5C$	$-\frac{1}{4}q$	$-\frac{1}{4}13q$ $+(3Dc-54Dc)$		$-12q$
						$-14Dq+14Dc$ $+5B+5C$	$-\frac{1}{4}13q$	$-\frac{1}{4}13q$	$12q$
							$-\frac{1}{4}q-14Dq$ $+2Dc+314Dc$ $+13B+5C$		
								$-\frac{1}{4}q-14Dq$ $+2Dc+314Dc$ $+13B+5C$	
									$-24Dq+14Dc$ $+5B+5C$

Matrix B.  $\Gamma_{13}, \Gamma_{14} (= B_1, B_2)$ .

${}^5E-1-1$	${}^5E11$	${}^5B_20b_2$	${}^3E-1-1$	${}^3E11$	${}^3E-1-1$	${}^3B_20b_2$	${}^3E11$
$-\frac{1}{4}q-4Dq$ $-2S+4Dc$		$\frac{1}{4}13q$	$\frac{1}{2}13q$			$-\frac{1}{2}q$	$-\frac{1}{2}q$
	$-\frac{1}{4}q-4Dq$ $-2S+4Dc$	$\frac{1}{4}13q$			$-\frac{1}{2}13q$	$\frac{1}{2}q$	$\frac{1}{2}q$
		$-4Dq$ $+2Dc-2c$	$\frac{1}{2}q$	$-\frac{1}{2}q$	$\frac{1}{6}13q$		$-\frac{1}{6}13q$
			$-\frac{1}{4}q$ $-14Dq+214Dc$ $+5B+5C$			$\frac{1}{4}13q$ $+B(2Dc-54Dc)$	$\frac{1}{4}13q$
				$-\frac{1}{4}q$ $-14Dq+214Dc$ $+5B+5C$			$\frac{1}{4}13q$ $+B(2Dc-54Dc)$
					$\frac{1}{4}q-14Dq$ $+2Dc+314Dc$ $+13B+5C$	$-\frac{1}{4}q$	
						$-14Dq$ $-4Dc+9Dc$ $+13B+5C$	$-\frac{1}{4}q$
							$\frac{1}{4}q-14Dq$ $+2Dc+314Dc$ $+13B+5C$



Matrix C.  $\Gamma_{15a}, \Gamma_{15b} (=E \neq 1)$ .

${}^5E_0 \neq 1$	${}^5E \neq 2=1$	${}^5E \neq 2=1$	${}^5B_2 \neq 1b_2$	${}^3E_0 \neq 1$	${}^3A_2 \neq 1a_2$	${}^3B_2 \neq 1b_2$	${}^3E_0 \neq 1$
$-4Dq$ $-D_5 + 4D_6$			$\frac{1}{4}F_3 \psi$	$= \psi$	$\pm \frac{1}{2} \psi$	$\pm \frac{1}{6} F_3 \psi$	$\pm \frac{1}{3} F_3 \psi$
	$-\frac{1}{2} \psi - 4Dq$ $-D_5 + 4D_6$		$\frac{1}{4} F_2 \psi$			$\pm \frac{1}{2} F_2 \psi$	
		$\frac{1}{2} \psi - 4Dq$ $-D_5 + 4D_6$			$\pm \frac{1}{2} F_6 \psi$		
			$-4Dq$ $+2D_5 - D_6$	$\pm \frac{1}{2} F_3 \psi$			$= \frac{1}{2} \psi$
				$-4Dq + 2 \frac{1}{4} D_6$ $+5B - 5C$	$- \frac{1}{4} \psi$	$\frac{1}{4} F_3 \psi$	$F_3(2D_5 + 9 \frac{1}{4} D_6)$
					$-4Dq + 4D_6$ $+5B + 5C$		$- \frac{1}{4} F_3 \psi$
						$-4Dq$ $-4D_5 + 9D_6$ $+13B + 5C$	$- \frac{1}{4} \psi$
							$-4Dq$ $+2D_5 - 3 \frac{1}{4} D_6$ $+13B + 5C$

Appendix II. Complete ligand field, interelectronic repulsion, and spin-orbit coupling matrices in  $D_{3d}$  symmetry including the cubic parent terms  ${}^5T_{2g}(t_{2g}^4 e_g^2)$ ,  ${}^1A_{1g}(t_{2g}^6)$ ,  ${}^3T_{1g}(t_{2g}^5 e_g)$ , and  ${}^3T_{2g}(t_{2g}^5 e_g)$ .

Matrix A.  $\Gamma^1, \Gamma^2 (=A_1, A_2)$ .

${}^5E_{1u}$	${}^5E_{-2u}$	${}^5A_1 O_{a_1}$	${}^5E_{2u}$	${}^5E_{-1u}$	${}^3E_{1u}$	${}^3E_{-1u}$	${}^3A_2 O_{a_2}$	${}^3E_{1u}$	${}^3E_{-1u}$	${}^3A_2 O_{a_1}$	${}^1A_1 O_{a_1}$
$-44\psi - 4Dq$ $+2D_5 + \frac{2}{3} D_6$		$\frac{1}{4} F_3 \psi$			$- \frac{1}{2} \psi$		$- \frac{1}{2} \psi$	$\frac{1}{2} \psi$		$- \frac{1}{2} \psi$	
	$\frac{1}{2} \psi - 4Dq$ $+2D_5 + 2 \frac{2}{3} D_6$				$F_2 \psi$						
		$-4Dq$ $-2D_5 - 6D_6$	$\frac{1}{4} F_3 \psi$	$- \frac{1}{6} F_3 \psi$	$- \frac{1}{6} F_3 \psi$	$- \frac{2}{3} F_3 \psi$	$- \frac{1}{6} F_3 \psi$	$\frac{1}{6} F_3 \psi$			
			$\frac{1}{2} \psi - 4Dq$ $+2D_5 + 4 \frac{2}{3} D_6$	$- \sqrt{2} \psi$							
$-24Dq$ $-28 \frac{2}{3} D_6$ $+5B + 8C$				$-14\psi - 4Dq$ $+2D_5 + 2 \frac{2}{3} D_6$	$- \frac{1}{2} \psi$	$- \frac{1}{2} \psi$		$- \frac{1}{2} \psi$	$\frac{1}{2} \psi$		
	$-14Dq$ $-2D_5 - 2 \frac{2}{3} D_6$ $+13B + 5C$				$\frac{1}{4} \psi - 14Dq$ $+ \frac{1}{2} D_5 - 13 \frac{2}{3} D_6$ $+5B + 5C$		$\frac{1}{4} \psi$	$\frac{1}{4} \psi$ $+3 \frac{2}{3} D_5 - 13 \frac{2}{3} D_6$	$- \frac{1}{4} \psi$	$- \sqrt{2} \psi$	
		$\frac{1}{4} \psi$ $\frac{1}{4} \psi - 14Dq$ $+ \frac{1}{2} D_5 - 13 \frac{2}{3} D_6$ $+13B + 5C$				$\frac{1}{4} \psi - 14Dq$ $+ \frac{1}{2} D_5 - 13 \frac{2}{3} D_6$ $+5B + 5C$	$\frac{1}{4} \psi$		$- \frac{1}{4} \psi$ $+ \frac{1}{2} D_5 - 13 \frac{2}{3} D_6$	$\frac{1}{4} \psi$	$- \sqrt{2} \psi$
			$\frac{1}{4} \psi$ $\frac{1}{4} \psi - 14Dq$ $+ \frac{1}{2} D_5 - 13 \frac{2}{3} D_6$ $+13B + 5C$				$-14Dq$ $-2D_5 - 2 \frac{2}{3} D_6$ $+5B + 5C$	$- \frac{1}{4} \psi$	$\frac{1}{4} \psi$		$- \sqrt{2} \psi$

Matrix B.  $\Gamma_{3u\pm}(=Eu\pm)$ .

${}^5E+2u_{\pm}$	${}^3E+1u_{\pm}$	${}^6EOu_{\pm}$	${}^5A_1+1a_1$	${}^6A_1+2a_1$	${}^3E+1u_{\pm}$	${}^3EOu_{\pm}$	${}^3A_2+1a_2$	${}^3E+1u_{\pm}$	${}^3EOu_{\pm}$	${}^3A_2+1a_2$
$-1/2\psi - 4Dq$ $+2B + 1/2DC$			$1/4\sqrt{2}\psi$				$-1/2\sqrt{2}\psi$			$\mp 1/2\sqrt{2}\psi$
	$1/4\psi - 4Dq$ $+2B - 2/3DC$			$1/4\sqrt{2}\psi$	$-1/2\psi$	$\pm\psi$		$\pm 1/2\psi$		
		$-4Dq$ $+2B + 2/3DC$	$1/4\sqrt{3}\psi$		$\mp 1/3\sqrt{3}\psi$	$-2/3\sqrt{3}\psi$	$-1/6\sqrt{3}\psi$		$\mp 1/3\sqrt{3}\psi$	$\pm 1/6\sqrt{3}\psi$
			$-4Dq$ $-2B - 6DC$			$-1/2\psi$	$-\psi$		$\pm 1/2$	
				$-4Dq$ $-2B - 6DC$	$-1/2\sqrt{2}\psi$			$\mp 1/2\sqrt{2}\psi$		
					$-1/4\psi - 14Dq$ $+1/2B - 1/3DC$ $+5B+5C$			$\mp 1/4\psi$ $\mp 3/2B = 10/3DC$	$-1/2\psi$	
						$-14Dq$ $+1/2B - 1/3DC$ $+5B+5C$	$1/4\psi$	$-1/2\psi$	$\pm 3/2B = 10/3DC$	$\pm 1/4\psi$
							$-14Dq$ $-2B - 23/3DC$ $+5B+5C$		$\pm 1/4\psi$	$\mp 1/2\psi$
								$-1/4\psi - 14Dq$ $+1/2B - 1/3DC$ $+13B+5C$		
									$-14Dq$ $+1/2B - 1/3DC$ $+13B+5C$	$1/4\psi$
										$-14Dq$ $-2B - 23/3DC$ $+13B+5C$

**Acknowledgments.** The authors appreciate financial support by the Deutsche Forschungsgemeinschaft,

the Fonds der Chemischen Industrie, and the Stiftung Volkswagenwerk. Discussions with Dr. S. Kremer are gratefully acknowledged.