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Time in Terms of the Half Life for Integer Order Decombination Reactions

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The rate law for a decombination reaction according to the law of mass action [1] is $dN/dt = kN^m$ where dN/dt is the rate of decombination, k is the rate constant, N is the amount of reactant remaining after decombination and m is the order of decombination. The order for the most part is a natural number (positive integer) but it also is zero, a negative integer or a fraction either positive or negative.

The definite integral [2], used in solvent $dN/dt = kN$ for a first order process which applies to all nuclear disintegrations [3] and other decombinations [4] so that $\ln N/N_0 = kt$ and $\ln N/N_0 = (t/t_{1/2}) \ln 1/2$ or $N/N_0 = (1/2)^{t_{1/2}}$ — a logarithm property [5], is useful also in solving other integer order decombination rate equations and advantage was taken of this to obtain the results below for various orders which are negative integers, zero and positive integers.

Order	Time, t, in terms of the half life
–6	$\frac{128}{127} t_{1/2} \left(1 - \frac{N^7}{N_0^7}\right)$
–5	$\frac{64}{63} t_{1/2} \left(1 - \frac{N^6}{N_0^6}\right)$
–4	$\frac{32}{31} t_{1/2} \left(1 - \frac{N^5}{N_0^5}\right)$
–3	$\frac{16}{15} t_{1/2} \left(1 - \frac{N^4}{N_0^4}\right)$
–2	$\frac{8}{7} t_{1/2} \left(1 - \frac{N^3}{N_0^3}\right)$
–1	$\frac{4}{3} t_{1/2} \left(1 - \frac{N^2}{N_0^2}\right)$
0	$2\, t_{1/2} \left(1 - \frac{N}{N_0}\right)$
2	$t_{1/2} \left(\frac{N_0}{N} - 1\right)$
3	$\frac{t_{1/2}}{3} \left(\frac{N_0^2}{N^2} - 1\right)$
4	$\frac{t_{1/2}}{1} \left(\frac{N_0^3}{N^3} - 1\right)$
5	$\frac{t_{1/2}}{15} \left(\frac{N_0^4}{N^4} - 1\right)$
6	$\frac{t_{1/2}}{31} \left(\frac{N_0^5}{N^5} - 1\right)$
7	$\frac{t_{1/2}}{63} \left(\frac{N_0^6}{N^6} - 1\right)$
8	$\frac{t_{1/2}}{127} \left(\frac{N_0^7}{N^7} - 1\right)$
9	$\frac{t_{1/2}}{255} \left(\frac{N_0^8}{N^8} - 1\right)$
10	$\frac{t_{1/2}}{511} \left(\frac{N_0^9}{N^9} - 1\right)$

It should be evident now that for all zero order decombinations and for all negative order decombinations, the general expression for time in terms of the half life is $(2^{m+1} t/2^{m+1} - 1)(1 - N^{m+1}/N_0^{m+1})$ and that for all natural number order decombinations beginning at two, the general expression is $(t_{1/2}/2^{m-1} - 1)(N_0^{m-1}/N^{m-1} - 1)$. The amount that decomposes and the amount that remains can be found for any t when $t_{1/2}$ is given and t can be found for any N in terms of N_0 just as in all first order cases.

References

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