Accommodation Coefficients of Hydrogen, Helium, Air, and Argon for Chrome Surfaces at Reduced Pressures

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The purpose of this investigation is to measure the accommodation coefficients of chrome surfaces for gases at reduced pressures. An apparent thermal conductivity is determined for various reduced pressures, from which the accommodation coefficient and the true thermal conductivity can be calculated.

In 1942 Boelter (*1*) and associates found a scarcity of accurate data on the thermal conductivity of gases. Among 19 reports on the thermal conductivity of air, the deviation from the mean value was as much as 7%. The apparent thermal conductivity was measured and accommodation coefficients and true thermal conductivities of the gas were calculated from these data.

Applying the kinetic theory of gases to the concept of thermal conductivity leads to the conclusion that the heat conductivity should be proportional to the product of the specific heat and the viscosity. As the coefficient of viscosity and the specific heat are substantially independent of the pressure of the gas in the low pressure range, thermal conductivity should also be independent of the pressure. Experiments performed by Stefan (*12*) confirmed that the thermal conductivity of a gas is independent of pressure at a range of 428 to 760 mm. of mercury. Kundt and Warburg (*6*) showed independence for the thermal conductivity of air at pressures of 0.5 to 19.5 mm. of mercury. All the many subsequent investigators have experimentally confirmed the law.

However, Kundt and Warburg (*6*) first noticed that the thermal conductivity varied with pressure at pressures lower than 0.5 mm. of mercury. Quantitative measurements of this variation were made by Brush (*2*) and Smoluchowski (*9*). Careful experimental measurements show that at very low pressures the thermal conductivity becomes proportional to pressure, decreasing linearly with it (*3*). In the pressure range over which thermal conductivity shows this variation, the mean free path of the gas molecules is of the same order of magnitude as the dimensions of the system. In this range the mechanism of energy transfer between the heat transfer surfaces and the rebounding molecules (accommodation coefficient) becomes important, depending not only on the mean free path of the molecules and the system's dimensions but also on the physical nature of the heat transfer surfaces.

CONDUCTION OF HEAT IN A RAREFIED GAS

The theory of heat conduction assumes a special form when the mean free path of the molecule is larger than the dimensions of the apparatus. Molecules then transport heat across the whole apparatus at a single bound, so that the whole gas must be assumed to be at a uniform temperature. Thus, there is no longer a temperature gradient. A mathematical theory for this case has been developed by Knudsen (*5*). He states that the heat transferred through contact with a rarefied gas will be jointly proportional to the temperature difference and to the pressure.

Experiments have confirmed the proportionality of this theory, but not the multiplying factor in the formula. The actual heat transferred was found to be less than that predicted. Smoluchowski (*10*) in 1898 suggested, as an explanation, that the exchange of energy from the solid surface to the absorbing molecule was far from complete. Soddy and Berry (*11*) also proposed this theory.

This led Knudsen (*5*) to introduce a quantity a which he described as a "coefficient of accommodation." He assumed that, when a molecule strikes the surface, its temperature is not adjusted through the whole temperature difference range, but only through a fractional amount, a, of this range.

The value of a depends very largely on the temperature and other physical conditions, such as surface conditions, and may vary from 0 to 1. For carbon dioxide contact with platinum heavily coated with platinum black, Knudsen found a to be 0.975. Experiments performed by Roberts (*8*) illustrated the variance of a with temperature. A study of the relation between a and the temperature suggested that a would vanish at the absolute zero.

Jeans (*4*) developed a simple formula for a based on the mass of the gas molecule and the mass of the surface molecules.

Various attempts have been made to develop a formula for a which agrees with experimental values, but none has been developed. The problem seems to involve forces which cause absorption of the molecules on the surface.

The theoretical calculations have been based on the supposition that the only energy which the impinging molecule can transfer to the wall is its kinetic energy of motion. Molecules have many other forms of energy, and the question arises as to what happens to this other energy when the molecule impinges on a solid. Knudsen (*5*) has devised methods for probing this question and has reached the conclusion that the internal molecular energy also has an accommodation coefficient, which, within the limits of experimental error, is the same as the accommodation coefficient for the kinetic energy of translation.

Experimental determinations of thermal conductivities of gases have been confined to three principal methods: the cooling thermometer method, the hot wire method, and the plate method. The latter method was used in this investigation.

DEVELOPMENT OF THEORY

When heat is transferred by conduction between two parallel solid surfaces, there is a discontinuity in the heat transfer process at the solid interface. A gas molecule, leaving a solid surface, will have the surface temperature only if it has been absorbed on that surface. At pressures where little gas is absorbed, a molecule striking a hot surface may rebound from the surface with practically no increase in temperature.

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Heat transfer per square foot of solid surface may be expressed as

$$q = h_a(T_g - T_s)$$

where T_g is the gas temperature next to the solid surface, T_s is the solid surface temperature, and h_a is an accommodation coefficient. $1/h_a$ would be the resistance to heat transfer at the surface.

Most accommodation coefficients are defined as:

$$a = \frac{T_{\text{entering surface}} - T_{\text{leaving surface}}}{T_{\text{entering surface}} - T_{\text{surface}}}$$

The coefficient as used in these results is related to a by the equation:

$$a = \frac{1}{\dfrac{k}{dha} + 1}$$

It is much more convenient to express the results in terms of h_a but values of a can be calculated from the results.

The total resistance to heat transfer between two parallel surfaces could be expressed as:

$$\frac{d}{k_1} = \frac{2}{h_a} + \frac{d}{k} \tag{1}$$

where $1/h_a$ is the resistance at each surface and d/k is the gas resistance. k_1 is the apparent thermal conductivity and is defined by the equation

$$q = \frac{k_1}{d}(T_1 - T_2) \tag{2}$$

where T_1 and T_2 are the two surface temperatures, d is the distance between the surface, and q is the heat transferred per hour per square foot.

Thus, if $1/k_1$, the experimentally determined apparent thermal conductivity (Equation 2), is plotted against reciprocal of the distance between surfaces $1/d$ for a constant pressure, the intercept would be $1/k$ and the slope would be $2/h_a$ (Equation 1).

The accommodation coefficient varies with the surface composition, the surface conditions, the gas, and the gas pressure.

In a plot of $1/k_1$ vs. $1/d$, the slope $2/h_a$ approaches zero as the pressure of the gas increases from 0 to 1000 microns. At pressures higher than 1000 microns, the slope essentially can be considered zero. Therefore, under normal pressure conditions, the term $2/h_a$ can be dropped from the heat transfer equation.

As the mean free path increases, the accommodation coefficient becomes more important. Finally, when L (mean free path) is larger than the distance between the heat transfer surfaces, collisions between molecules are few and the temperature gradient in the gas disappears. In this case (free molecule conduction), the effect of the accommodation coefficient is great.

When L is larger than the distance between surfaces, there are few collisions between gas molecules and there is no mechanism for molecular temperature change other than collision with the surfaces. In this case, there is no temperature difference in the gas phase and all the resistance is at the surfaces. At these pressures, the heat flow is almost independent of thermal conductivity of the gas. Where d is much less than L, k approaches infinity as the Δt across the gas approaches zero. There is a finite heat transfer and the only temperature jump is at the surface. At this low heat flux it is hard to obtain accurate data but results indicate a large increase in k.

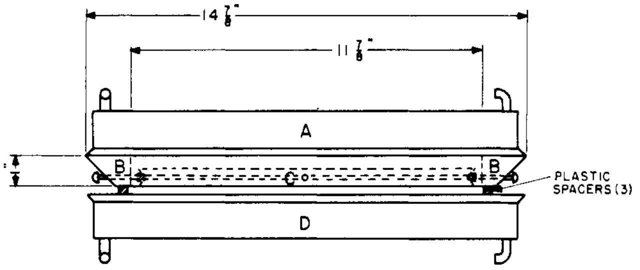

Figure 1. Heat transfer assembly

APPARATUS

A complete description of the equipment was given by Lokay (7). The main features which were vital to the results are shown in Figure 1. C is the electrically heated sandwich, A and B are the guard plate and ring, and D is the cold plate.

To determine the distance between the plates, a hardened steel ball bearing was held against each surface and a series of readings of ball surface was taken with a cathetometer. The contour of each surface was obtained in this fashion, and the plate spacing was calculated. The plate spacing was measured to the nearest thousandth of an inch.

Thermocouples were installed on the surface of the plates by drilling two holes through the back of each plate. A fine groove was drilled between the holes and the thermocouples were placed in the groove. The residual space was filled with solder, and the plate was resurfaced.

The temperature contours were measured for the hot plate, the cold plate, the guard plate, and the guard ring. Temperatures were measured to the nearest 0.05° F.

The apparatus shown in Figure 1 was placed in a vacuum chamber where the pressure could be reduced to less than 10^{-2} micron.

The greatest error in results probably would be caused by heat loss to the guard ring or plate. The upper plate on the heating sandwich has to be at a higher temperature than the lower surface. This deviation is caused by heat flux through the bottom copper plate. Thus, it is impossible to hold the temperature of B, Figure 1, at the sandwich temperature. The top plate of C was always held at the temperature of the guard plate, A, for the experimental results. Some runs were made where temperature at A was above and below the temperature on top of C, but 0.1° did not affect the results a great deal.

Another source of error was convection. The apparatus was shielded by glass plate and glass wool which cut down any stray currents that originated in the equipment. The top plate was always used at a higher temperature, which tended to dampen any thermal convection inside the unit. Calculated thermal conductivity was independent of the temperature difference between the plates.

Radiation was determined by dropping the pressure to less than 10^{-2} micron. The radiation convection should be almost independent of spacing, and this was found to be the case.

All results were based on the geometric measurements of the apparatus; however, the results for air checked the best results that have been reported in the literature.

CALCULATIONS

Determining the apparent thermal conductivity from the experimental data required only calculation of the radiation losses and measurement of the physical dimensions.

Once the apparent thermal conductivities were determined, they were plotted against pressure to give a curve

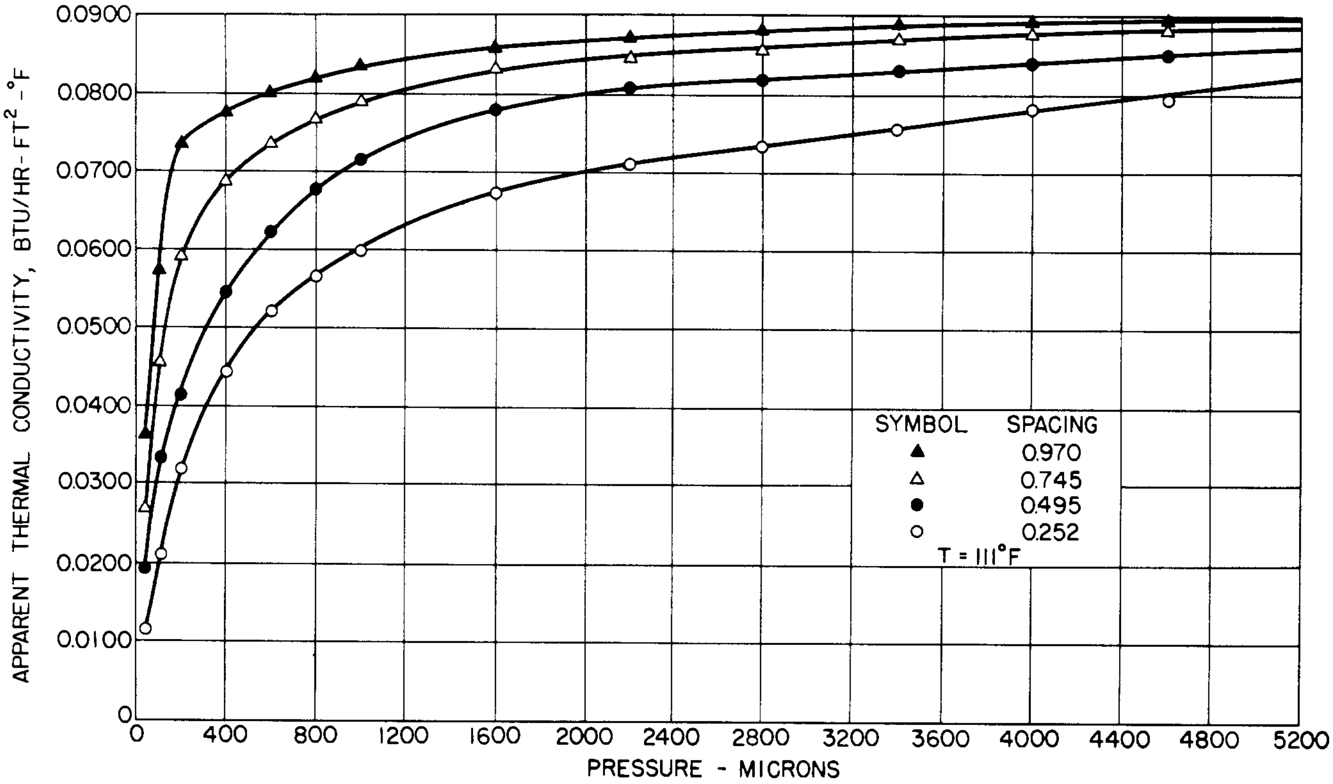

Figure 2. Apparent thermal conductivity of helium vs. pressure

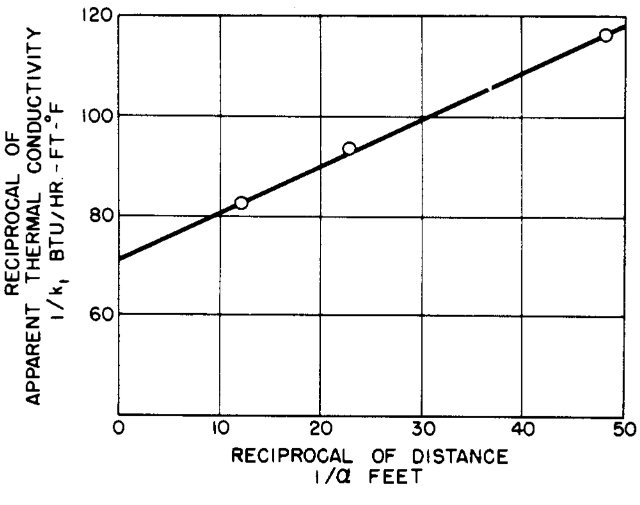

Figure 3. Calculation of slope and intercept

Air pressure, 50 microns
Intercept, $1/K$, 71.3
Slope, 21 h_a, 0.928

for each plate spacing (see Figure 2 for an example). Values of the apparent thermal conductivity, k_1, were then obtained from these curves for various pressures between 1 and 1000 microns (0.001 and 1.00 mm.). The reciprocals of these apparent thermal conductivities for a constant pressure and temperature were then plotted against the reciprocals of the spacing distance (see Figure 3 for an example).

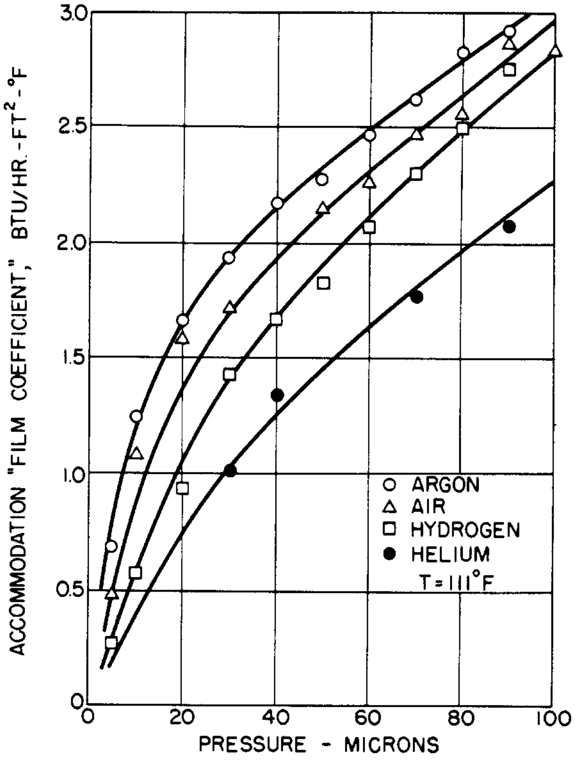

Figure 4. Accommodation film coefficient vs. pressure

Table I. Calculated Data on Conductivity and Accommodation Coefficient

Pressure (Mercury), Microns	Apparent Thermal Conductivity, B.t.u./Hour Foot °F., at Spacing of				Slope, $\frac{2}{h_a}$	Intercept, $\frac{1}{k}$	Accommodation "Film Coefficient," B.t.u./Hour Foot °F., h_a	Thermal Conductivity, B.t.u./Hour Foot °F., k
					For Air			
	0.250 in.	0.525 in.	0.995 in.					
5	0.00228	0.00407	0.00500		4.16	164	0.481	0.00610
10	0.00423	0.00603	0.00683		1.84	127	1.09	0.00787
20	0.00610	0.00798	0.00902		1.26	100	1.59	0.0100
30	0.00733	0.00933	0.0105		1.16	81.3	1.72	0.0123
40	0.00824	0.0102	0.0115		0.926	77.0	2.16	0.0130
50	0.00863	0.0109	0.0122		0.928	71.3	2.16	0.0140
60	0.00908	0.0114	0.0127		0.882	68.2	2.27	0.0147
70	0.00943	0.0118	0.0132		0.808	67.1	2.48	0.0149
80	0.00976	0.0122	0.0135		0.780	65.3	2.56	0.0153
90	0.0101	0.0125	0.0138		0.692	65.3	2.89	0.0153
100	0.0103	0.0127	0.0141		0.704	63.2	2.84	0.0158
250	0.0128	0.0146	0.0156		0.328	62.7	6.10	0.0159
500	0.0142	0.0155	0.0159		0.164	61.5	12.20	0.0163
750	0.0149	0.0157	0.0160		0.125	61.2	16.13	0.0163
1000	0.0153	0.0158	0.0160		0.089	61.6	22.4	0.0162
					For Helium			
	0.252 in.	0.495 in.	0.745 in.	0.970 in.				
5	0.0052	0.0090	0.0113	0.0125	3.185	37.60	0.63	0.0266
10	0.0068	0.0113	0.0155	0.0200	2.640	21.60	0.76	0.0463
20	0.0089	0.0143	0.0200	0.0273	1.780	19.00	1.12	0.0526
30	0.0105	0.0170	0.0236	0.0322	1.960	14.90	1.02	0.0670
40	0.0120	0.0193	0.0268	0.0365	1.485	11.65	1.35	0.0858
70	0.0160	0.0253	0.0350	0.0465	1.130	11.30	1.78	0.0885
90	0.0185	0.0290	0.0405	0.0518	0.962	11.00	2.08	0.0910
110	0.0213	0.0328	0.0460	0.0573	0.820	10.70	2.44	0.0935
200	0.0320	0.0415	0.0595	0.0735	0.475	9.82	4.21	0.1002
400	0.0445	0.0545	0.0690	0.0775	0.358	10.62	5.60	0.0941
600	0.0520	0.0620	0.0730	0.0800	0.179	10.54	11.18	0.0949
800	0.0568	0.0673	0.0765	0.0820	0.148	10.65	13.50	0.0940
1000	0.0603	0.0715	0.0792	0.0835	0.133	10.65	15.05	0.0948
1600	0.0675	0.0780	0.0828	0.0855	0.086	10.70	23.30	0.0935
2200	0.0710	0.0805	0.0845	0.0870	0.072	10.60	27.80	0.0945
2800	0.0735	0.0185	0.0858	0.0883	0.065	10.55	30.80	0.0948
3400	0.0758	0.0828	0.0870	0.0890	0.057	10.63	35.10	0.0940
4000	0.0778	0.0840	0.0878	0.0895	0.050	10.67	40.00	0.0938
4600	0.0795	0.0850	0.0885	0.0895	0.040	10.68	50.00	0.0937
					For Hydrogen			
	0.525 in.	0.959 in.						
5	0.0060	0.0115			7.38	0	0.271	..[illegible]
10	0.0125	0.0235			3.46	0	0.578	..[illegible]
20	0.0185	0.0320			2.11	5.9	0.948	0.169
30	0.0240	0.0375			1.39	9.9	1.44	0.101
40	0.0270	0.0415			1.19	9.8	1.68	0.102
50	0.0285	0.0430			1.09	10.2	1.83	0.0980
60	0.0308	0.0452			0.962	10.5	2.08	0.0950
70	0.0335	0.0487			0.866	10.1	2.31	0.0991
80	0.0355	0.0510			0.796	10.0	2.51	0.100
90	0.0378	0.0536			0.725	9.9	2.76	0.101
100	0.0403	0.0565			0.657	9.8	3.04	0.102
250	0.0590	0.0770			0.361	8.3	5.54	0.120
500	0.0730	0.0870			0.204	9.0	9.80	0.111
750	0.0830	0.0950			0.139	8.8	14.4	0.113
1000	0.0910	0.101			0.102	8.7	19.6	0.115
					For Argon			
	0.250 in.	0.525 in.						
5	0.00244	0.00297			2.90	270.3	0.69	0.00370
10	0.00370	0.00435			1.60	193.4	1.25	0.00517
20	0.00494	0.00580			1.20	145.1	1.67	0.00689
30	0.00567	0.00664			1.03	126.9	1.94	0.00788
40	0.00622	0.00727			0.917	118.4	2.18	0.00845
50	0.00664	0.00777			0.790	107.1	2.28	0.00934
60	0.00701	0.00818			0.810	103.8	2.47	0.00963
70	0.00732	0.00852			0.763	100.0	2.62	0.0100
80	0.00760	0.00881			0.707	98.1	2.83	0.0102
90	0.00784	0.00903			0.685	94.4	2.92	0.0106
100	0.00810	0.00932			0.643	92.6	3.11	0.0108

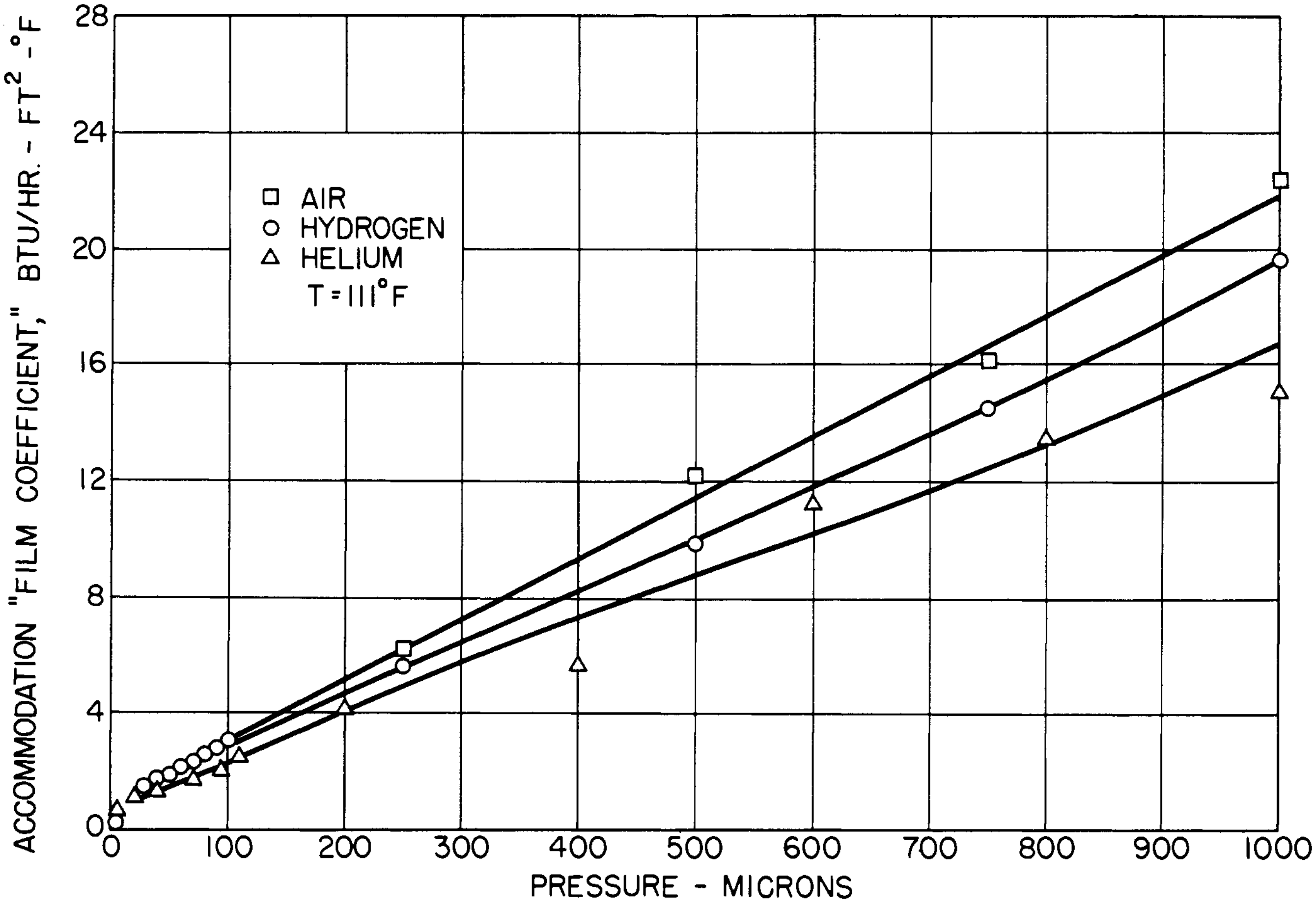

Figure 5. Accommodation film coefficient vs. pressure

As can be seen from the derived Equation 2, the slope of this straight line will be $2/h_a$, and the intercept (infinite spacing) will be $1/k$, the reciprocal of the true thermal conductivity. The values of h_a and k for air, helium, hydrogen, and argon are given in Table I and Figures 4, 5, and 6. The data (7) show that the thermal conductivity decreases with pressure down to where the mean free path equals the plate spacings. At this point the data show a rise going off toward infinity. The thermal conductivity should go toward infinity for this dimension of apparatus as the temperature gradient in the gas approaches zero. All the curves were then extrapolated to zero when the limit of the experimental apparatus was reached.

DISCUSSION

There are, unfortunately, no values of the accommodation coefficients in the literature with which the values given in Table I may be compared. This quantity is a function both of the gas and the heat transfer surfaces. One general fault with most of the literature concerning accommodation coefficients is that the preparation of the surfaces and the gases is not described. This makes comparison of values very difficult. The surface in this investigation was a chromium-plated surface and was cleaned with a jeweler's rouge polishing cloth.

The results indicate that helium and hydrogen have an apparent thermal conductivity which is a function of pressure. If these results are corrected for accommodation, these gases have conductivities which are independent of pressure down to under 500 microns, as is the case with other gases.

CONCLUSIONS

The experimental data confirmed the relationship among the apparent thermal conductivity, the true thermal conductivity, and the accommodation coefficient for the pressure range of 10 to 1000 microns of (mercury) pressure.

The thermal conductivity of gases at a low reduced pressure (100 microns or less) varies with pressure. The conductivity decreases with a decrease in pressure. This relationship is not a linear one except in the very low pressure range (1 micron or less).

The thermal conductivity is essentially constant for the pressure range of 200 microns (mercury) up to atmospheric pressure.

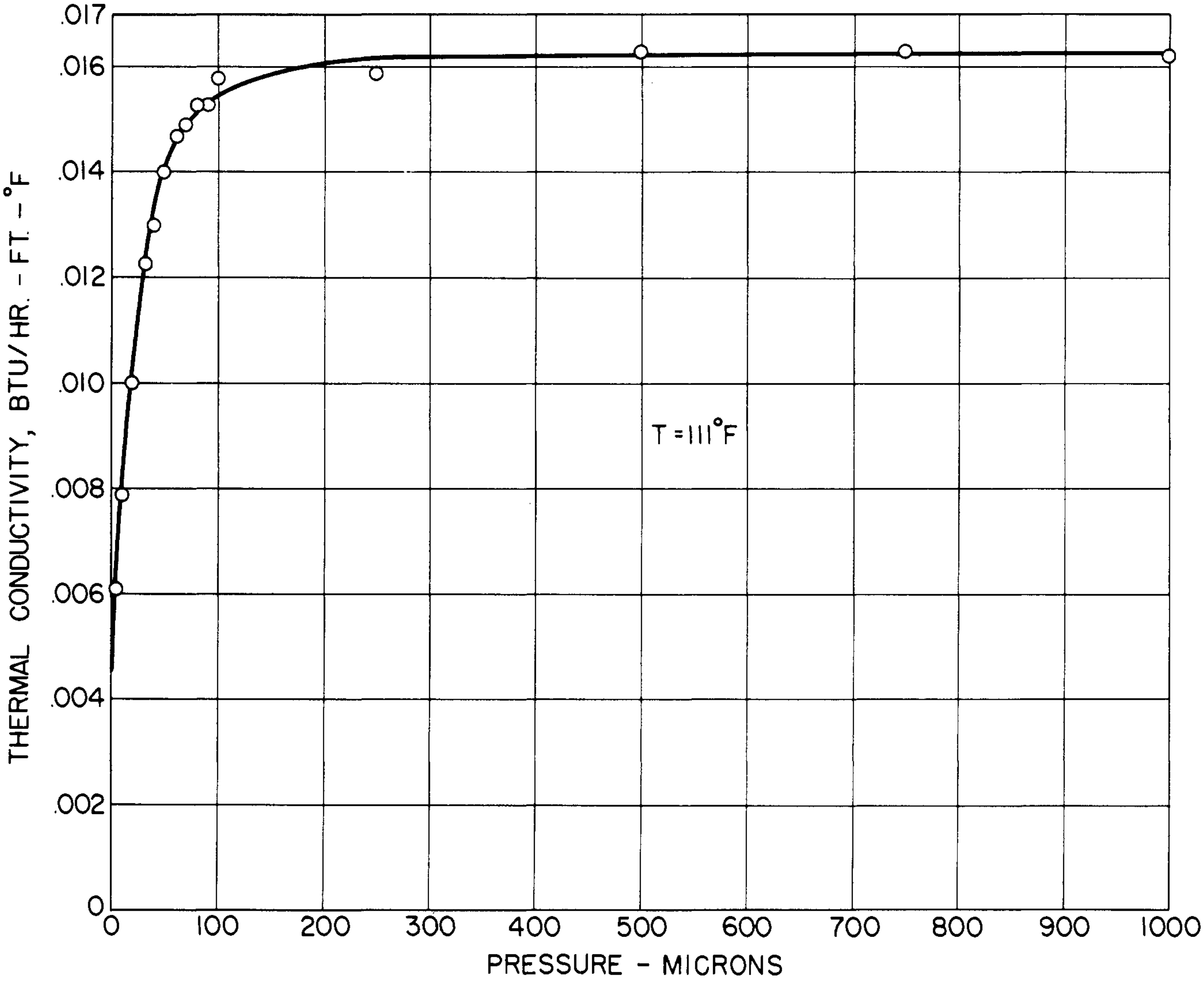

Figure 6. Thermal conductivity of air vs. pressure

The value of the accommodation coefficient increases with pressure toward a magnitude which causes it to vanish from the heat transfer equation for a gas at atmospheric pressure.

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