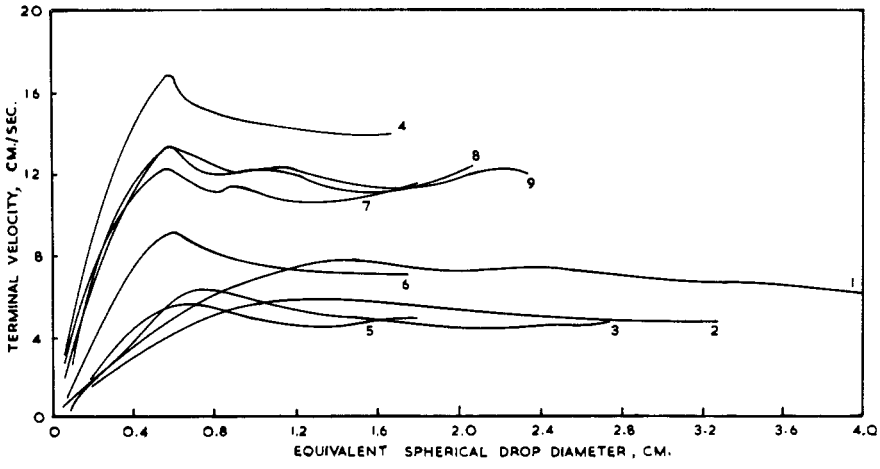

Figure 7. Terminal velocities of liquid drops

1. *n*-Amyl phthalate
2. *n*-Amyl phthalate, σ = 7.07
3. Aniline
4. Carbon disulfide
5. *m*-Cresol
6. Eugenol
7. Nitrobenzene, σ = 15.84
8. o-Nitrotoluene
9. 1,2-Dichloropropene

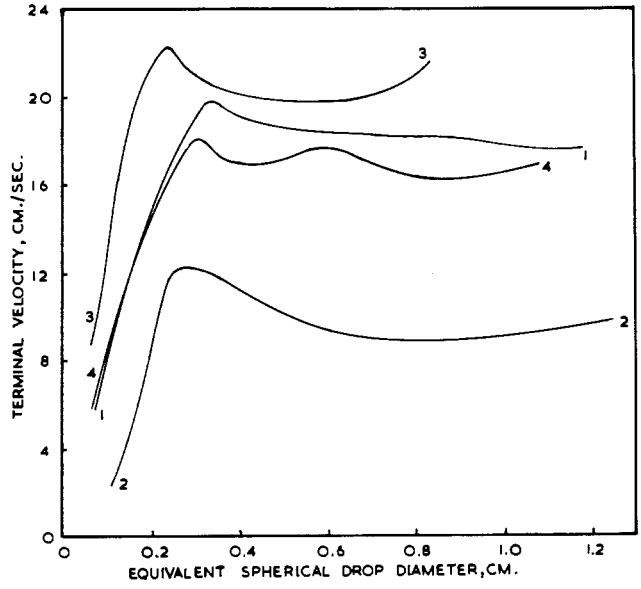

Figure 8. Terminal velocities of liquid drops

1. Carbon tetrachloride
2. Epichlorohydrin
3. 1,2-Dibromoethane
4. 1,1,2,2-Tetrachloroethane

ACKNOWLEDGMENT

The authors thank M. Narasinga Rao for his advice and suggestions.

LITERATURE CITED

(1) Bond, W. N., *Phil. Mag.* **4,** 889 (1927).
(2) Bond, W. N., Newton, D. A., *Ibid.*, **5,** 794 (1928).
(3) Boussinesq, J., *Compt. rend.* **156,** 1124 (1913); **157,** 313 (1913).
(4) Hadamard, J., *Ibid.*, **152,** 1735 (1911); **154,** 107 (1913).
(5) Harkins, W. D., Brown, F. E., *J. Am. Chem. Soc.* **41,** 499 (1919).
(6) Hu, S., Kintner, R. C., *A. I. Ch. E. Journal*, **1,** 42 (1955).
(7) Hughes, R. R., Gilliland, E. R., *Chem. Eng. Progr.* **48,** 497 (1952).
(8) Johnson, A. I., Braida, L., *Can. J. Chem. Eng.* **35,** 165 (1957).
(9) Klee, A. J., Treybal, R. E., *A. I. Ch. E. Journal*, **2,** 444 (1956).
(10) Licht, Wm., Narasimhamurty, G. S. R., *Ibid.*, **1,** 366 (1955).
(11) Rybzynski, W., *Bull. Acad. Sci., Cracovie* **1,** 40 (1911).
(12) Smirnov, N. I., Ruban, V. L., *J. Appl. Chem. (U.S.S.R.)* **22,** 1068, 1211 (1949); **24,** 57 (1951); **25,** 12 (1952).

RECEIVED for review October 1, 1958. Accepted April 10, 1959.

Material supplementary to this article has been deposited as Document No. 6035 with the ADI Auxiliary Publications Project, Photoduplication Service, Library of Congress, Washington 25, D. C. A copy may be secured by citing the document number and by remitting $15.00 for photoprints or $4.75 for 35-mm. microfilm. Advance payment is required. Make checks or money orders payable to Chief, Photoduplication Service, Library of Congress.

Fall of Liquid Drops in Water

Drag Coefficients, Peak Velocities, and Maximum Drop Sizes

P. M. KRISHNA [1], D. VENKATESWARLU, and G. S. R. NARASIMHAMURTY [2]
Indian Institute of Technology, Kharagpur, India

The equations for the drag coefficients of rigid spheres given by Stokes (*9*), Oseen (*6*), and Goldstein (*1*) are valid up to Reynolds numbers of about 4. The drag coefficients of rigid spheres and a few other regular shapes at higher Reynolds numbers have been experimentally determined by many investigators (*8*).

Licht and Narasimhamurty (*5*) correlated their data on the fall of liquid drops of diameter greater than the peak velocity diameter by an equation involving C_R and B groups. Hu and Kintner (*2*) correlated their experimental data by an equation involving Re, C, We, and P groups. They correlated the peak velocity and maximum drop size data also. Hughes and Gilliland (*3*) used Wt, Tv, and Sd groups in their study of the mechanics of drops. Peebles and Garber (*7*) used a plot of C *vs.* G in correlating their data on gas bubbles. In their work, Sd group is used as Sd values of liquids changing from 727 to 5773 seem to give a physical sense and to characterise a system. Also, the Sd group is simple to calculate.

Dimensional Analysis. The terminal velocity of a liquid drop in water may be assumed to depend upon a number of variables and is given by the equation,

$$U = f(D, d, \rho_0, \rho, \mu_0, \mu, \sigma, g) \quad (1)$$

Of these, d may be omitted if wall effect is considered negligible. μ_0 may also be dropped, because terminal velocity studies have shown its effect not to be significant (*4*). A variety of dimensionless groups may be obtained from the other variables, and a suitable combination of these variables gives the equation,

[1] Present address, University of Minnesota, Minneapolis, Minn.
[2] Present address, Indian Institute of Technology, Bombay, India.

$$C = f(\mathrm{Re}, \mathrm{We}, Sd) \qquad (2)$$

Plots of Dimensionless Groups. Plots of the following groups are attempted in correlating the terminal velocity data:

C *vs.* Re, C *vs.* G, C_R *vs.* Re, C_R *vs.* B, We *vs.* Re, and Tv *vs.* Wt

Figures 1 to 5 are plots of drag coefficients as a function of Reynolds numbers for all the systems studied (*4*). The drag coefficients of rigid spheres and disks are shown by the unnumbered full and broken lines. For all liquids except aniline, epichlorohydrin, ethyl chloroacetate, and *m*-nitrotoluene, the drag coefficients are about the same as those of rigid spheres up to a certain Re characteristic of each liquid, beyond which they rise up rapidly. In the higher Reynolds ranges, they are higher than those of disks which are commonly considered as shapes of maximum drag (form drag plus friction drag). As each liquid gives its own characteristic plot, the drag coefficient of a liquid drop is not a unique function of Reynolds number.

Figures 6 to 8, which are typical plots of the other dimensionless groups, show that the experimental data of all the systems cannot be correlated into a single curve by any of these plots.

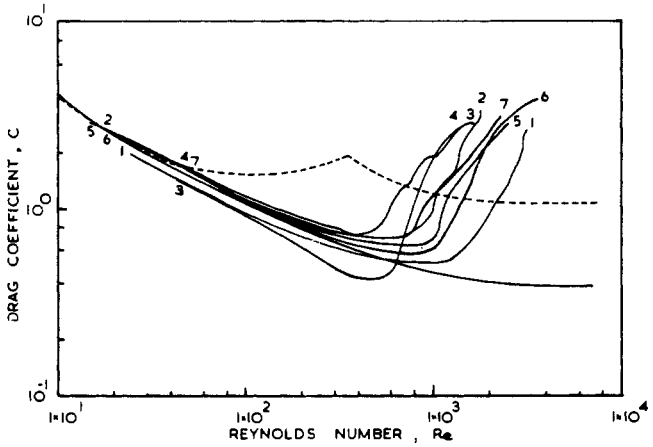

Figure 1. Drag coefficients of liquid drops

1. *n*-Amyl phthalate
2. *n*-Amyl phthalate, σ = 7.07
3. Aniline
4. Bromoform
5. *n*-Butyl phthalate
6. Carbon disulfide
7. Carbon tetrachloride

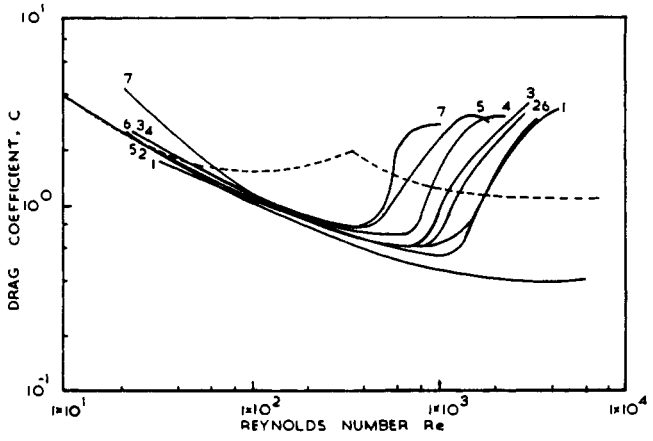

Figure 2. Drag coefficients of liquid drops

1. Chlorobenzene, σ = 36.02
2. Chlorobenzene, σ = 24.54
3. Chlorobenzene, σ = 19.56
4. Chlorobenzene, σ = 14.07
5. Chlorobenzene, σ = 9.14
6. 1-Chloronaphthalene
7. *m*-Cresol

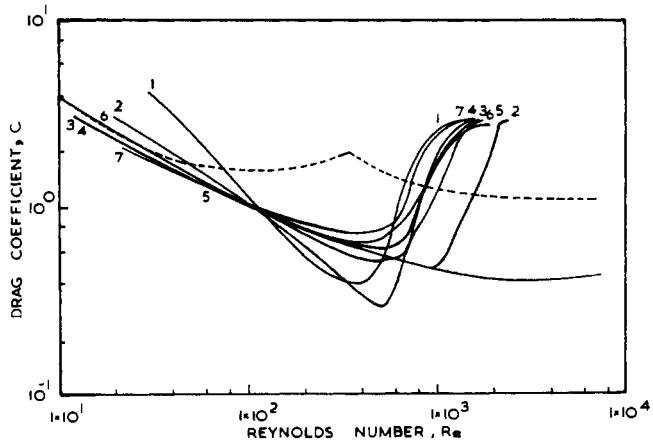

Figure 3. Drag coefficients of liquid drops

1. Epichlorohydrin
2. Ethyl chloroacetate
3. Ethyl cinnamate
4. Ethyl phthalate
5. 1,2-Dibromoethylene
6. Eugenol
7. Isoeugenol

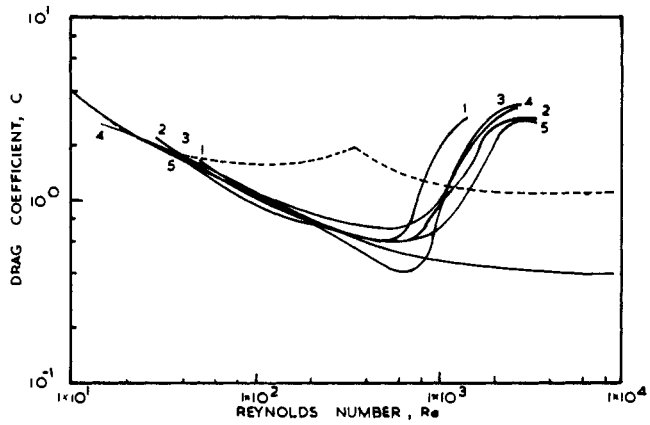

Figure 4. Drag coefficients of liquid drops

1. Methyl phthalate
2. Nitrobenzene
3. Nitrobenzene, σ = 15.84
4. *m*-Nitrotoluene
5. o-Nitrotoluene

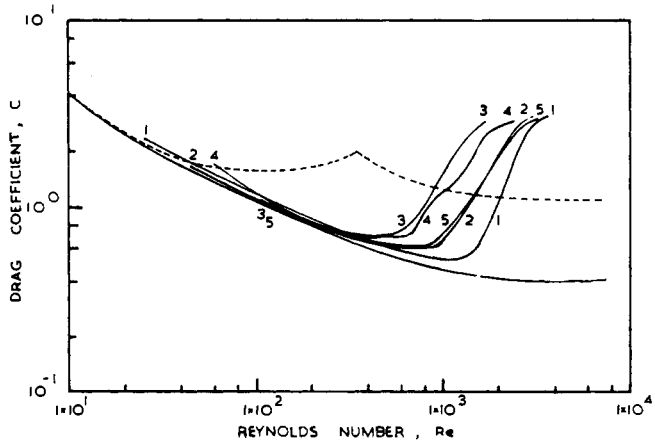

Figure 5. Drag coefficients of liquid drops

1. Diphenyl ether
2. 1,2-Dichloropropene
3. 1,1,2,2-Tetrabromoethane
4. 1,1,2,2-Tetrachloroethone
5. Tetrachloroethylene

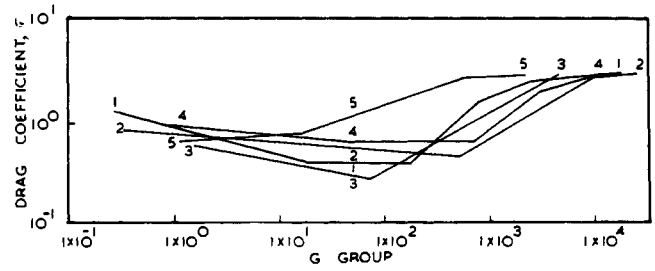

Figure 6. Drag coefficient as a function of G group

1. Epichlorohydrin
2. Ethyl cinnamate
3. Ethyl chloroacetate
4. Ethyl phthalate
5. 1,2-Dibromoethylene

Table I. Peak Velocity and Maximum Drop Size Data

Liquid	Sd	Peak Velocity Data					Maximum Drop Size Data		
		Re_P	C_P	We_P	$(Re/We)_P$	$(Re_P/Sd)^{0.50}$	C_M	$(C/We)_M$	$(Re/We)_M$
n-Amyl phthalate	4214	1347	0.571	4.26	316.3	20.75	2.67	20.78	399.7
n-Amyl phthalate (σ reduced)	1475	881	0.855	3.63	243.0	22.94	3.52	34.87	183.7
Aniline	1365	569	0.443	4.52	125.8	15.40	2.80	27.13	164.8
Bromoform	1578	502	0.783	2.74	183.0	12.64	2.80	18.66	200.7
Butyl phthalate	3021	1010	0.598	3.98	253.6	18.38	3.98	44.60	314.7
Carbon disulfide	3250	1023	0.686	3.59	284.7	17.94	2.90	20.78	343.7
Carbon tetrachloride	2462	730	0.713	3.12	234.0	14.71	3.19	27.58	256.6
Chlorobenzene	4261	1263	0.596	3.80	332.1	19.35	3.22	35.39	388.3
Chlorobenzene (σ = 19.56)	2542	936	0.649	4.16	224.8	18.56	3.35	37.56	264.6
Chlorobenzene (σ = 24.54)	2988	1133	0.656	4.55	249.0	20.72	2.84	26.86	299.8
Chlorobenzene (σ = 14.07)	1829	750	0.752	4.09	183.3	17.54	3.03	30.82	206.7
Chlorobenzene (σ = 9.14)	1182	511	0.812	3.70	138.2	14.85	2.89	37.76	138.7
1-Chloronaphthalene	4296	1293	0.676	3.73	346.4	19.73	2.90	24.72	405.1
m-Cresol	726.9	447	0.817	5.03	88.9	16.56	2.91	30.24	101.7
Epichlorohydrin	1113	424	0.415	3.85	110.2	12.71	2.99	31.31	140.4
Ethyl chloroacetate	1752	556	0.308	3.86	143.9	13.28	2.68	21.82	205.3
Ethyl cinnamate	4122	1045	0.485	4.02	160.0	16.28	2.84	19.38	354.4
Ethyl phthalate	1252	694	0.675	5.60	123.9	19.62	3.03	32.83	153.2
1,2-Dibromoethylene	1658	584	0.739	3.25	179.5	14.36	2.75	28.77	186.1
Eugenol	1721	674	0.542	4.21	160.3	16.24	2.76	18.90	213.0
Isoeugenol	974.3	626	0.810	5.90	106.2	20.05	2.81	32.93	122.2
Methyl phthalate	1262	657	0.668	5.16	127.4	18.49	2.84	27.25	156.4
Nitrobenzene	2396	736	0.640	3.21	229.0	15.03	2.89	33.29	242.3
Nitrobenzene (σ = 15.84)	1596	863	0.791	5.56	155.1	21.60	2.91	41.93	169.0
m-Nitrotoluene	2370	768	0.429	3.95	194.6	15.78	3.11	32.39	250.8
o-Nitrotoluene	2825	997	0.683	4.01	248.9	18.76	2.79	35.30	265.5
Phenyl ether	5773	1509	0.564	3.63	416.3	19.86	3.35	23.92	548.3
1,2-Dichloropropene	3424	985	0.637	3.32	296.5	16.83	3.29	34.76	334.7
1,1,2,2-Tetrabromoethane	1575	615	0.728	3.69	166.7	15.50	2.70	22.96	187.6
1,1,2,2-Tetrachloroethane	1964	678	0.694	3.43	197.7	15.30	2.90	30.54	212.9
Tetrachloroethylene	2997	933	0.637	3.54	263.8	17.04	3.01	29.65	301.6

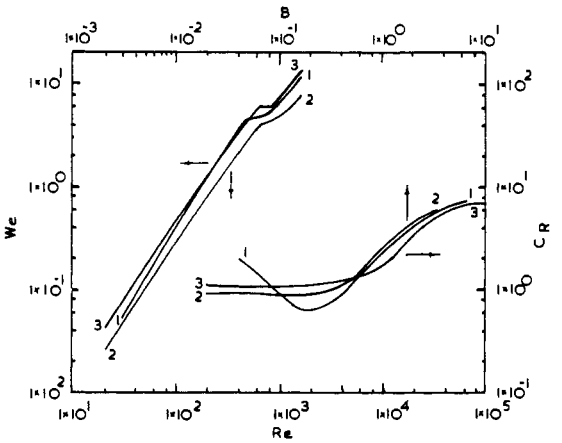

Figure 7. Plot of dimensionless groups
1. Epichlorohydrin
2. Eugenol
3. Isoeugenol

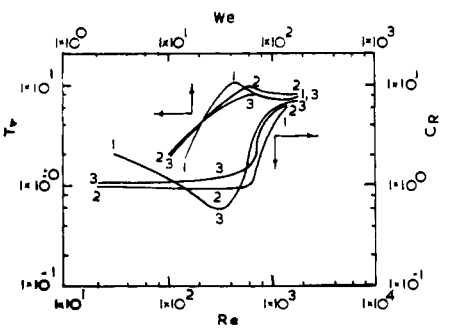

Figure 8. Plot of dimensionless groups
1. Epichlorohydrin
2. Eugenol
3. Isoeugenol

CORRELATIONS

Drag Coefficient Correlations. It appears from dimensional analysis and plots of dimensionless groups, that a combination of C, Re, We, and Sd groups may correlate the experimental data. A combination of these groups, C We $Sd^{0.50}$ *vs.* Re/$Sd^{0.50}$

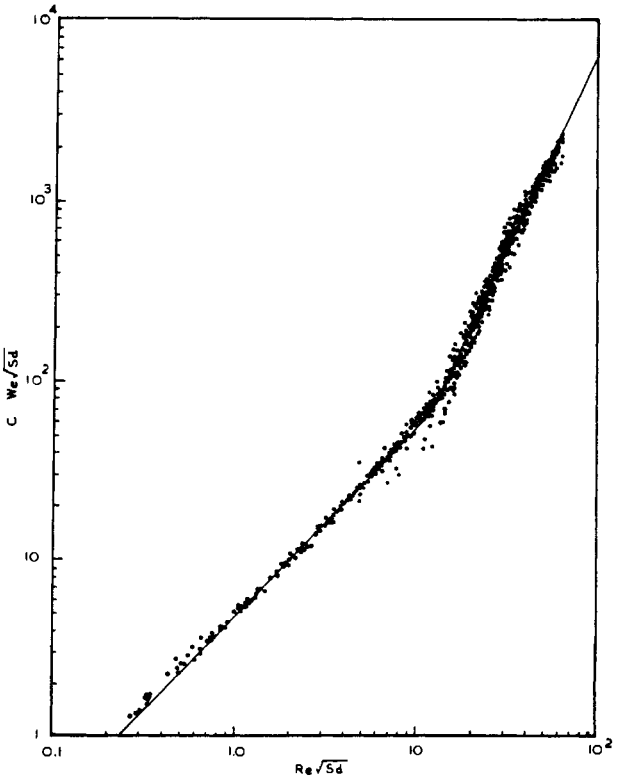

Figure 9. Drag coefficient correlation

shown in Figure 9 correlates the data for all the liquids. The equations for the lines best fitting the data are

$$C\ \mathrm{We}\ Sd^{0.50} = 5.0(\mathrm{Re}/Sd^{0.50})^{1.049} \text{ for } (\mathrm{Re}/Sd^{0.50}) < 12 \quad (3)$$

and

$$C\ \mathrm{We}\ Sd^{0.50} = 0.308(\mathrm{Re}/Sd^{0.50})^{2.162} \text{ for } (\mathrm{Re}/Sd^{0.50}) > 20 \quad (4)$$

The interval $12 < (\mathrm{Re}/Sd^{0.5}) > 20$ corresponds to the peak velocity region.

Peak Velocity Correlation. Peak velocity data shown in Table I is correlated by a plot of $(\mathrm{Re/We})_P$ *vs.* *Sd*, as shown in Figure 10. The equation for the line is

$$(\mathrm{Re/We})_P = 0.306\ Sd^{1.220} \quad (5)$$

which gives the following equation for peak velocity:

$$U_P = 0.306\ \sigma/\mu\ Sd^{1.220} \quad (6)$$

Table I shows that $\mathrm{Re}/Sd^{0.50}$ varies between 12.64 and 22.94. Using an average for all the systems, peak velocity diameter is given by

$$D_P = 17.32\ \mu\ Sd^{0.50}/U_P\rho \quad (7)$$

Maximum Drop Size Correlation. The maximum drop size data given in Table I are correlated by a plot of $(\mathrm{Re/We})_M$ *vs.* *Sd*, as shown in Figure 11. The equation for the lines is

$$(\mathrm{Re/We})_M = 1.761\ Sd^{1.305} \quad (8)$$

which on simplifying, gives

$$U_M = 0.568\ \sigma/\mu\ Sd^{1.305} \quad (9)$$

The drag coefficients for the maximum size drops range between 2.67 and 3.98. An average value of 3.0 seems to be a good approximation for most of the systems. Using this value, the equation

$$D_M = 2.25\ U_M^2/g(\rho_0 - \rho) \quad (10)$$

may be used to calculate the maximum drop diameter.

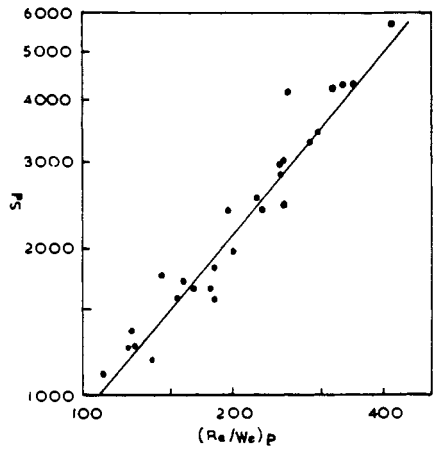

Figure 10. Correlation of peak velocity data

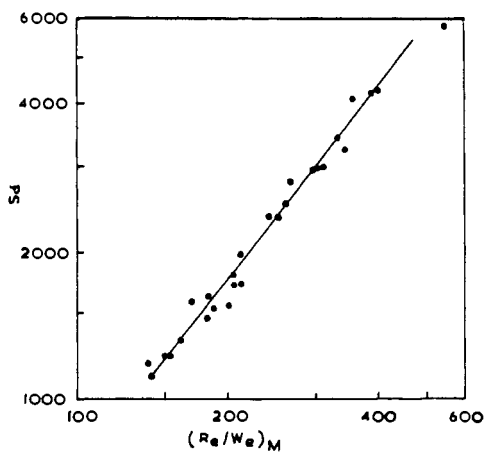

Figure 11. Correlation of maximum drop size data

NOMENCLATURE

B = B group, $\Delta\rho g D^2/4\sigma$, dimensionless
C = drag coefficient of drops, $\frac{4g}{3}\frac{\Delta\rho}{\rho}\frac{D}{U^2}$, dimensionless
C_R = ratio of drag coefficient of drops and rigid spheres, C/C_{RS}
C_{RS} = drag coefficient of a rigid sphere, dimensionless
d = diameter of container, cm.
D = equivalent spherical drop diameter, cm.
Fr = Froude number, U^2/gD, dimensionless
g = local acceleration due to gravity, 978 grams per cm.
g_c = conversion factor, 980.6 (mass) (cm.)/(grams force) (sec.^{2})
G = G group, $gD^4U^4\rho^3/\sigma^3$, dimensionless
P = P group, $\rho^2\sigma^3/\Delta\rho g\mu^4$, dimensionless
Re = Reynolds number, $DU\rho/\mu$, dimensionless
Sd = physical group, $\frac{g_c\sigma}{\mu}\left[\frac{3}{4g}\times\frac{\rho^2}{\mu\Delta\rho}\right]^{1/3}$, dimensionless
Tv = terminal velocity group, $U\left[\frac{3}{4g}\times\frac{\rho^2}{\mu\Delta\rho}\right]^{1/3}$, dimensionless
U = terminal velocity of drop, cm. per second
We = Weber group, $D\rho U^2/\sigma$, dimensionless
Wt = gravity group, $D\left[\frac{4g}{3}\times\frac{\rho\Delta\rho}{\mu^2}\right]^{1/3}$, dimensionless
μ = viscosity of water, grams per cm. sec.
μ_0 = viscosity of drop liquid, grams per cm. sec.
ρ = density of water, grams per ml.
ρ_0 = density of drop liquid, grams per ml.
$\Delta\rho$ = difference in densities of drop liquid and water, grams per ml.
σ = interfacial tension between drop liquid and water, dynes per cm.

Subscripts

M = refers to maximum size drop
P = refers to peak velocity drop

ACKNOWLEDGMENT

The authors thank M. Narasinga Rao for his advice and suggestions.

LITERATURE CITED

(1) Goldstein, S., "Modern Developments in Fluid Dynamics," vol. **I**, p. 104, vol. **II**, pp. 492, 504, Oxford University Press, London, 1938.
(2) Hu, S., Kintner, R. C., *A.I.Ch.E. Journal* **1**, 42 (1955).
(3) Hughes, R. R., Gilliland, E. R., *Chem. Eng. Progr.* **48**, 497 (1952).
(4) Krishna, P. M., Venkateswarlu, D., Narasimhamurty, G. S. R., J. Chem. Eng. Data **4**, 336 (1959).
(5) Licht, Wm., Narasimhamurty, G. S. R., *A.I.Ch.E. Journal* **1**, 366 (1955).
(6) Oseen, C. W., "Neure Methoden und Ergebnisse in der Hydrodynamik," Akademische Verlagsgesellschaft, Leipzig, 1927.
(7) Peebles, F. N., Garber, H. J., *Chem. Eng. Progr.* **49**, 88 (1953).
(8) Perry, J. H., "Chemical Engineers Handbook," 3rd ed., p. 1018, McGraw-Hill, New York, 1950.
(9) Stokes, G. G., "Mathematical and Physical Papers," vol. **III**, Cambridge Univ. Press, Cambridge, England, 1901.

Received for review October 1, 1958. Accepted April 10, 1959.

Material supplementary to this article has been deposited as Document No. 6035 with the ADI Auxiliary Publications Project, Photoduplication Service, Library of Congress, Washington 25, D. C. A copy may be secured by citing the document number and by remitting $15.00 for photoprints or $4.75 for 35-mm. microfilm. Advance payment is required. Make checks or money orders payable to Chief, Photoduplication Service, Library of Congress.