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Improved Formulas for Heat Transfer Calculation in Multipass Exchangers

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Problem of 1–2 exchangers has been re-studied, assuming that overall coefficient of heat transfer varies as a linear function of temperature. Graphs similar to those of Ten Broeck have been drawn, taking into consideration this variation. Differential heat balance and rate equations were changed to appropriate dimensionless form and solved by an iterative procedure using a digital computer. A solution for 2–4 exchangers using the 1–2 charts also discussed.

IN CONVENTIONAL analysis of multipass exchangers, the heat transfer area is calculated from the equation:

$$Q = UAF_T\,(LMTD) \tag{1}$$

Solutions of this equation depend upon whether U is assumed constant or variable. Formulas for the $LMTD$ correction factor F_T were derived by Bowman, Mueller, and Nagle (*1*) under the assumption of a constant over-all coefficient U along the exchanger. Values of F_T for multipass exchangers are conveniently presented in form of charts (*4, 5*), as a function of dimensionless parameters dependent on terminal temperatures.

Assuming linear variation of U with temperature, Colburn (*2*) derived the following expression for the heat exchanged in a countercurrent 1-1 exchanger:

$$\frac{Q}{A} = \frac{U_1\Delta T_2 - U_2\Delta T_1}{\ln(U_1\Delta T_2/U_2\Delta T_1)} \tag{2}$$

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Equation 2 involves a "mixed mean" of the coefficient, U, and the temperature difference, ΔT. Colburn's procedure is valid only when $F_T = 1$.

For 1-2 exchangers with variable coefficients of heat transfer, Bowman, Mueller, and Nagle (*1*) combined their results for a 1-2 exchanger involving constant U with those of Colburn for a 1-1 exchanger where U was assumed to be linear with temperature. The resulting equation took the form:

$$Q = AF_T\,\frac{U_1\Delta T_2 - U_2\Delta T_1}{\ln(U_1\Delta T_2/U_2\Delta T_1)} \tag{3}$$

Although of great practical importance and fair accuracy, Equation 3 involves the inconsistency of calculating F_T on the basis of constant U and the mixed mean on the basis of variable U. This procedure will be referred to as the "F_T method" in the remainder of this paper.

Using numerical methods, the authors have restudied the problem of 1-2 exchangers assuming that U varies as a linear function of the temperature. Graphs similar to the ones developed by Ten Broeck (6) were drawn, but taking also in consideration the variation of the over-all coefficient of heat transfer. In addition to previous parameters, the ratio U_2/U_1 enters into the charts. In the procedure utilized in this work, the differential heat balance and rate equations were changed to appropriate dimensionless form and solved by an iterative procedure utilizing a digital computer. Variations of as much as 25% were found between values calculated by the F_T method and those presented in this paper.

A solution for 2-4 exchangers utilizing the charts for the 1-2 exchangers is also discussed; as an extension of this work, charts for 2-4, 3-6, 4-8, etc., exchangers are being prepared.

DERIVATIONS

Mathematical Derivation—1-2 Exchangers. Consider the temperature profiles for a 1-2 exchanger shown in Figure 1. The following assumptions will be made:

1. The variation of U is given by $U = U_0(1 + bt)$, as suggested by Colburn.
2. Surface areas are equal for each pass.
3. Shell fluid temperatures are constant at any cross section.
4. Rates of flow and specific heats are constant for both fluids.
5. There are no phase changes.
6. Heat losses are negligible.

Heat balances over the infinitesimal area dA for the first and second tube passes yield, respectively:

$$wcdt' = U'(T - t')\frac{dA_x}{2} \qquad (4)$$

$$-wcdt'' = U''(T - t'')\frac{dA_x}{2} \qquad (4a)$$

For the first pass select the following dimensionless groups:

$$R' = \frac{T_1 - T}{t_2 - t_1}\ ;\ (t_2 - t_1)\,dR' = -dT \qquad (5)$$

$$S' = \frac{T_1 - t'}{t_2 - t_1}\ ;\ (t_2 - t_1)\,dS' = -dt' \qquad (6)$$

$$E = \frac{t_2 - t_1}{T_1 - t_1} \qquad (7)$$

$$N = \frac{U_1 A_x}{wc}\ ; \qquad dN = \frac{U_1}{wc}\,dA_x \qquad (8)$$

where E is known as the thermal efficiency of the exchanger. The coefficients are related to the temperature by:

$$U = U_0(1 + bt) \qquad (9)$$

$$U_1 = U_0(1 + bt_1) \qquad (9a)$$

$$U_2 = U_0(1 + bt_2) \qquad (9b)$$

The following identities can be derived:

$$S' - R' = \frac{T - t'}{t_2 - t_1} \qquad (10)$$

$$\frac{1}{E} - S' = \frac{t' - t_1}{t_2 - t_1} \qquad (11)$$

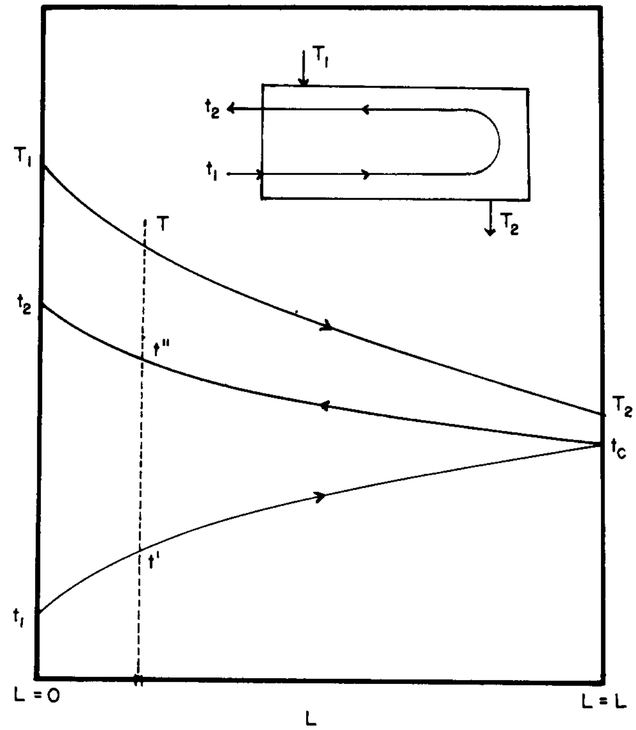

Figure 1. Temperature profile for a 1–2 exchanger

The coefficient in the first pass can be represented by:

$$U' = U_0(1 + bt') = U_0(1 + bt_1) + \frac{t' - t_1}{t_2 - t_1}\left[U_0(1 + bt_2) - U_0(1 + bt_1)\right] \qquad (12)$$

Utilizing Equations 9a, 9b, and 11:

$$U' = U_1 + \left(\frac{1}{E} - S'\right)(U_2 - U_1) \qquad (13)$$

With the differentials given in Equations 5, 6, and 8, the value of $(T - t')$ from Equation 10, and U' from Equation 13, Equation 4 may be placed in the following dimensionless form:

$$dS' = -\tfrac{1}{2}\left[1 + \left(\frac{U_2}{U_1} - 1\right)\left(\frac{1}{E} - S'\right)\right](S' - R')\,dN \qquad (14)$$

where the variables t, T, and A_x have been replaced, respectively, by S'. R'. and N.

For the second pass, it is possible to derive a similar equation:

$$dS'' = -\tfrac{1}{2}\left[1 + \left(\frac{U_2}{U_1} - 1\right)\left(\frac{1}{E} - S''\right)\right](S'' - R')\,dN \qquad (15)$$

$$S'' = \frac{T_1 - t''}{t_2 - t_1} \qquad (16)$$

The differential heat balance:

$$WCdT = wc\,(dt'' - dt') \qquad (17)$$

can be replaced by:

$$dR' = R[dS'' - dS'] \qquad (18)$$

Where $R = wc/WC$.

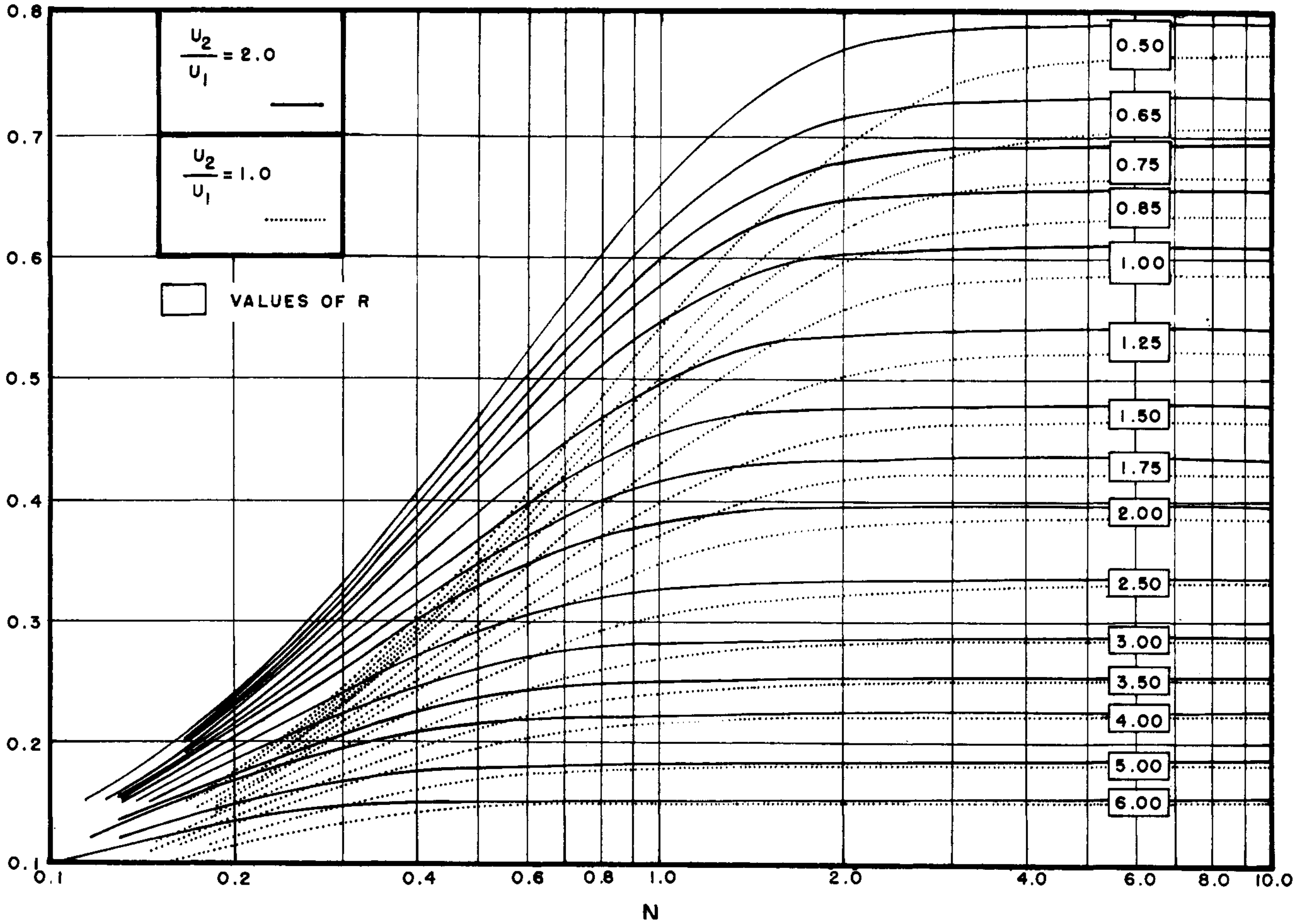

Figure 2. Comparison of thermal efficiency charts for 1–2 exchangers for values of U_2/U_1 = 2.0 and 1.0

Iterative Procedure for 1-2 Exchanger Equation. Equations 14, 15, and 18 represent simultaneous equations in R', S', S'', and N. Boundary conditions at the inlet to the exchanger yield the following values:

$$A_x = 0 \qquad N = 0 \tag{19}$$

$$t' = t_1 \qquad S' = \frac{T_1 - t_1}{t_1 - t_2} = \frac{1}{E} \tag{20}$$

$$t'' = t_2 \qquad S'' = \frac{T_1 - t_2}{t_1 - t_2} = 1 - \frac{1}{E} \tag{21}$$

$$T = T_1 \qquad R' = 0 \tag{22}$$

Assuming values of E and R, increments were given to dN (ΔN) in Equations 14 and 15 and successive increments of S' and S'' were calculated. By use of Equation 18, values of R' were obtained for use in successive iterations involving Equations 14 and 15. The numerical process was continued until S' equaled S''. If the values of S' and S'' never reached the same values, an impossible choice of E and R had been made, and the operation was discontinued.

Results of the calculations are shown in Figure 2 where E is plotted against N for constant R at values of U_2/U_1 = 1 and U_2/U_1 = 2. The curves for U_2/U_1 = 1 are equivalent to those constructed by Ten Broeck (*6*). Additional graphs for intermediate values of U_2/U_1 are available from the American Documentation Institute or from the authors.

Table I. Comparison of Heat Transfer Obtained by the Iterative and F_T Methods

	Iterative	F_T Method	% Dev.
Area, sq. ft.	314	339	+7.95
F_T factor	0.950	0.880 (true F_T)	−7.38
Area, sq. ft.	356	381	+7.03
F_T factor	0.945	0.880 (true F_T)	−6.89
Area, sq. ft.	447	469	+4.92
F_T factor	0.930	0.880 (true F_T)	−5.38
Area, sq. ft.	215	228	+6.05
F_T factor	0.995	0.945 (true F_T)	−5.05
Area, sq. ft.	302	316	+4.64
F_T factor	0.990	0.945 (true F_T)	−4.55
Area sq. ft.	530	574	+8.30
F_T factor	0.865	0.800 (true F_T)	−7.50

DISCUSSION OF RESULTS

1-2 Exchangers. Table I shows a comparison of heat transfer areas for several 1-2 exchangers determined by the F_T method and from the charts constructed by the proposed method. In order to obtain the true F_T factor, the value of the total area, obtained by the proposed procedure was introduced in Equation 3, which was then solved for F_T.

A more general comparison between the two methods has been made by Kahl (*3*). If $(Q/AU_1\Delta t_2)_{F_T}$ is the value of the ratio $Q/AU_1\Delta t_2$ calculated with the use of the F_T factor, and $(Q/AU_1\Delta t_2)_{TRUE}$ is the value of the same ratio

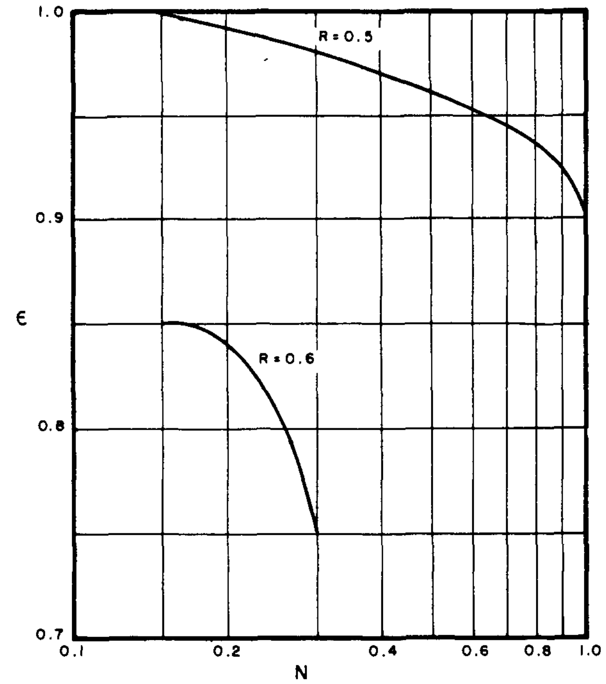

Figure 3. Variation of ϵ with N for $U_2/U_1 = 2$

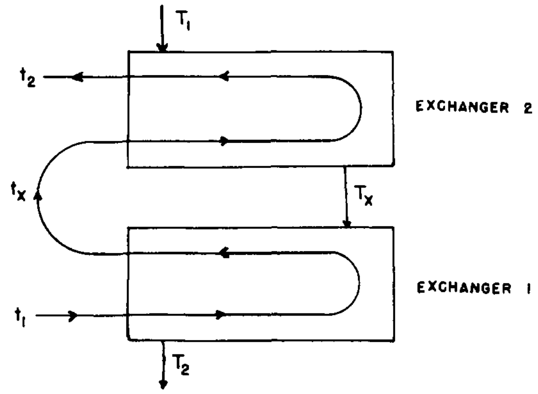

Figure 4. 2–4 exchanger

calculated by the iterative procedure it may be shown that Equation 3 becomes:

$$\epsilon = \frac{\left(\frac{Q}{AU_1\Delta t_2}\right)_{F_T}}{\left(\frac{Q}{AU_1\Delta t_2}\right)_{TRUE}} = \frac{F_T \dfrac{1 - \frac{U_2}{U_1}\left(\frac{1-RE}{1-E}\right)}{\ln \frac{U_1}{U_2}\left(\frac{1-E}{1-RE}\right)}}{\dfrac{1}{N\left(\frac{1}{E} - 1\right)}} \quad (23)$$

It can be observed that ϵ represents a correction factor for the area of an exchanger calculated by the F_T method. A plot of ϵ *vs.* N is presented in Figure 3, for a value of $U_2/U_1 = 2.0$ and for values of R equal to 0.5 and 6.0. Correction factors as significant as 0.75 were encountered, thus emphasizing the lack of rigor in the treatment of the 1-2 exchangers where the F_T method is used.

Design of 2-4 Exchangers. A procedure for calculation of heat transfer area for 2-4 exchangers utilizing the charts based on the iterative method for 1-2 exchangers has been developed. The detailed procedure and examples are available from the authors.

NOMENCLATURE

A = total heat transfer surface, sq. ft.
A_x = heat transfer surface until section x, sq. ft.
b = constant in Equation $U = U_0\,(1 + bt)$, ° F.$^{-1}$
C = specific heat of hot fluid, B.t.u./(lb.)(° F.)
c = specific heat of cold fluid, B.t.u./(lb.)(° F.)
E = temperature group, efficiency, $(t_2 - t_1)/(T_1 - t_1)$, dimensionless
F_T = *LMTD* correction factor, dimensionless
LMTD = log mean of the temperature differences between the hot and the cold streams at the two ends of the exchanger, (° F.).
N = Group (U_1A/wc), dimensionless
Q = exchanger duty, B.t.u./hr.
R = temperature group, $(T_1 - T_2)/(t_2 - t_1) = (wc/WC)$, dimensionless
R' = temperature group, $(T_1 - T)/(t_2 - t_1)$, dimensionless
S = temperature group, $(T_1 - t')/(t_2 - t_1)$, dimensionless
S'' = temperature group, $(T_1 - t'')/(t_2 - t_1)$, dimensionless
T, T_1, T_2, T_x = temperature, in general, of hot fluid (Figures 1 and 4), ° F.
t, t_1, t_2, t_x = temperature, in general, of cold fluid (Figures 1 and 4), ° F.
$\Delta T_1 = T_2 - t_1$ = temperature difference between the hot and cold streams at "cold end" of exchangers, ° F.
$\Delta T_2 = T_1 - t_2$ = temperature difference between the hot and cold streams at "hot end" of exchanger, ° F.
t', t'' = temperatures of cold fluid in first and second passes, ° F.
t_c = temperature at the end of first pass, ° F.
U = over-all coefficient of heat transfer, B.t.u./(hr.)(sq. ft.)(° F.)
U_1 = over-all coefficient of heat transfer at temperature t_1 (Figure 1 or 4), B.t.u./(hr.)(sq. ft.)(° F.)
U_2 = same at t_2 (Figure 1 or 4), B.t.u./(hr.)(sq. ft.)(° F.)
U_x = same at t_x (Figure 4), B.t.u./(hr.)(sq. ft.)(° F.)
U', U'' = over-all coefficients of heat transfer in first and second passes, B.t.u./(hr.)(sq. ft.)(° F.)
U_0 = constant in Equation $U = U_0\,(1 + bt)$, B.t.u./(hr.)(sq. ft.)(° F.)
W = mass flow of hot fluid, lb./hr.
w = mass flow of cold fluid, lb./hr.
ϵ = ratio defined by Equation 23, dimensionless

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