Part II

Viscosity of Gaseous Air at Moderate and High Pressures

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A viscosity correlation for air has been developed for temperatures up to 600° C. and pressures to 1000 atm. Experimental viscosities available in the literature were utilized to establish the atmospheric viscosity behavior of air. A linear relationship resulted when, for each experimental high pressure value, the residual viscosity, $\mu - \mu^*$, was plotted against the corresponding value of P_T/T. The high pressure viscosity behavior was confirmed by the use of experimental data for nitrogen and oxygen. This information enabled viscosity values to be established for gaseous air for a complete range of temperatures and pressures.

THE VALUES of the viscosity of gases at moderate and high pressures are required in many important applications. Accurate experimental data are not available for air for the complete range of temperatures and pressures in the dense gaseous region. The only previous correlation for the viscosity of air is that of Granet and Kass (17) which was based on the approximate reduced state correlation of Uyehara and Watson (45), and which is applicable only for pressures to 2000 p.s.i.a. Therefore, in the present study, relationships have been developed for the viscosity of gaseous air at moderate and high pressures for temperatures to 600° C. and pressures to 1000 atm.

VISCOSITY BEHAVIOR AT NORMAL PRESSURES

Experimental values of the viscosity at atmospheric pressure, μ^*, reported by 37 investigators for air were obtained from the literature and plotted against the corresponding temperature. In Table I, the sources are listed along with the temperature ranges investigated. The data for most of the references were consistent with each other and permitted the moderate pressure viscosity behavior of air to be established with a high degree of accuracy for temperatures up to 2500° K. In Figure 1, the resulting relationship between viscosity and temperature is presented, with experimental points obtained from the 12 major sources

Table I. Source of Viscosity Data and Average Deviations for Air at Atmospheric Pressure

Source	Number of Points	Temperature Range, ° K.	Av. Dev., %
Bonilla, Brooks, and Walker (7)	24	200–2500	0.89
Braune, Basch, and Wentzel (8)	25	293–944	1.47
Bremond (10)	19	291–1407	1.27
Grindley and Gibson (18)	57	273–373	0.70
Johnston and McCloskey (24)	15	90–300	0.53
Shilling and Laxton (37)	36	342–1120	0.80
Sutherland and Maass (39)	8	79–294	1.85
Titani (40, 41)	11	293–550	0.36
Trautz and Ludewigs (43)	17	287–524	0.23
Trautz and Zink (44)	48	554–1102	0.46
Vasilesco (47)	36	273–1845	0.57
Williams (48)	135	462–1275	1.39
Others	58	250–720	0.86

Minor sources (less than eight data points) are: (2, 3, 6, 9, 12-15, 19, 21-23, 25-27, 29, 30, 33-36, 42, 46, 49).
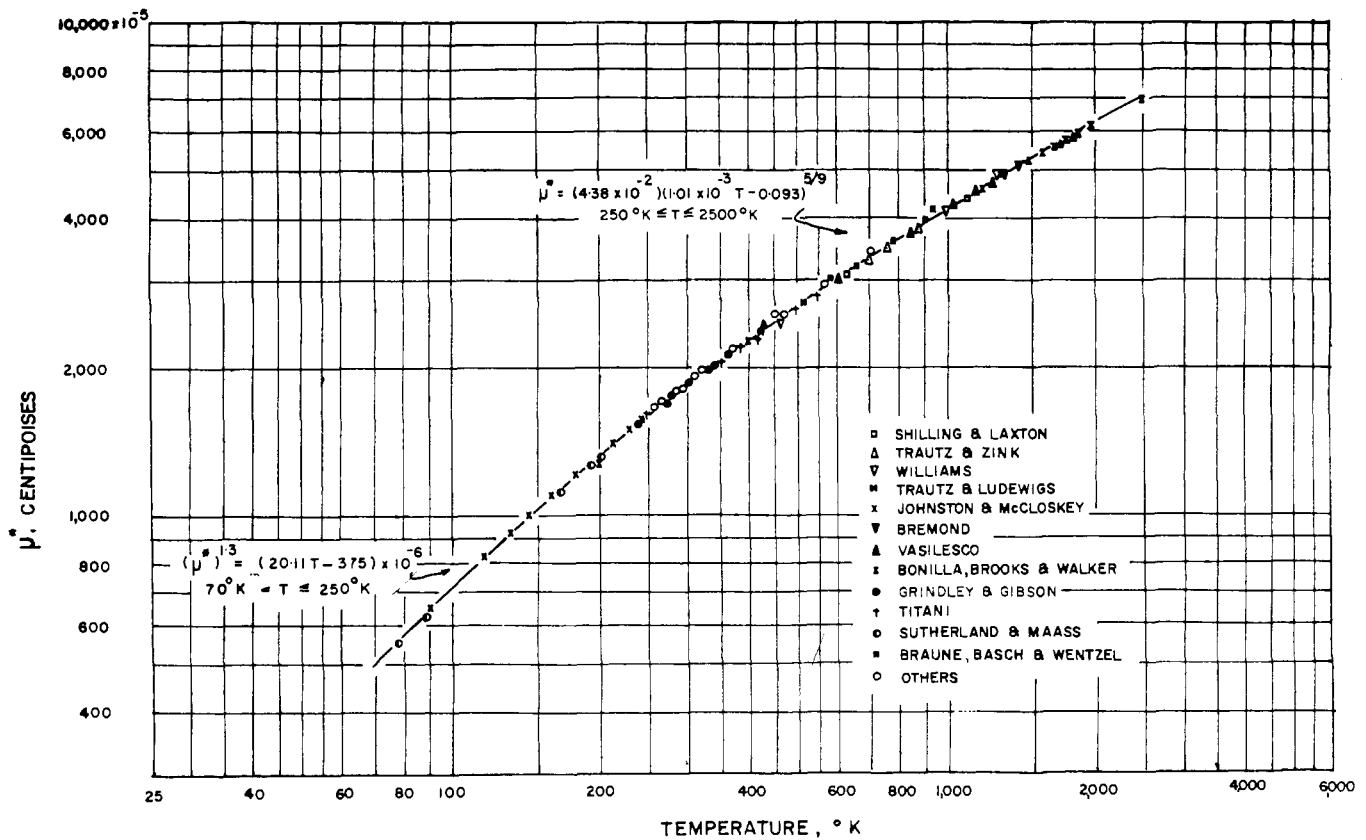
Figure 1. Relationship between viscosity and temperature for air at moderate pressure
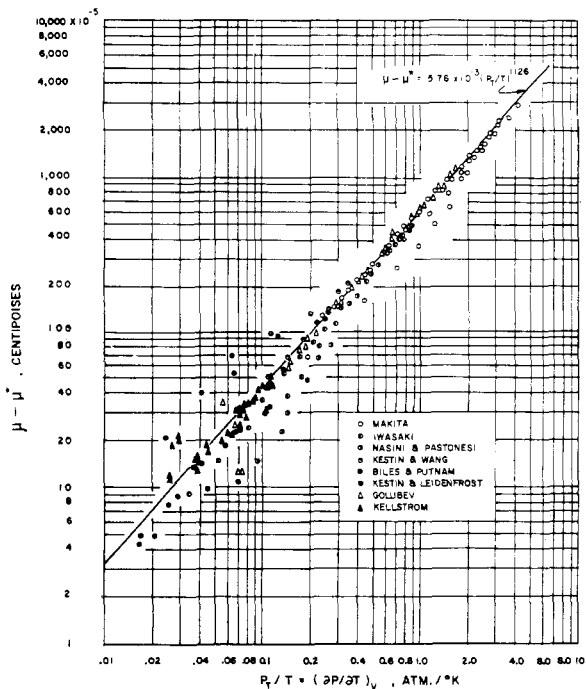

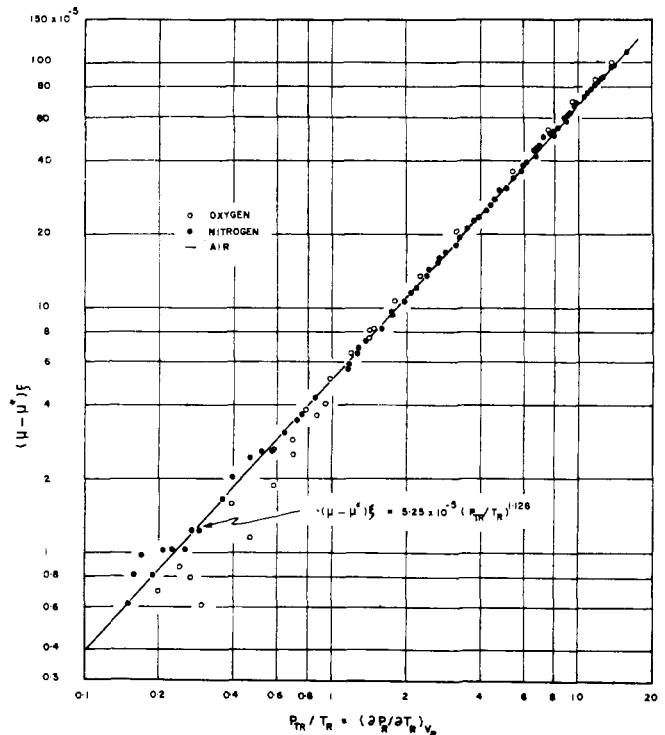Figure 2. Relationship between $\mu - \mu^*$ and P_T/T for air

Figure 3. Relationship between $(\mu - \mu^*)\xi$ and $(\partial P_R/\partial T_R)_{V_R}$ for nitrogen, oxygen, and air

listed in Table I. Representative data points from the other sources are also included.

For temperatures in the range $70°$ K. $\leqq T \leqq 250°$ K., the curve of Figure 1 can be expressed analytically as follows:

$$(\mu^*)^{1.3} = (20.11T - 375) \times 10^{-6} \qquad (1)$$

where the viscosity is in centipoises. Similarly, for $250°$ K.

$\leqq T \leqq 2500°$ K., the viscosity dependence on temperature can be expressed as

$$\mu^* = (4.38 \times 10^{-2})(1.01 \times 10^{-3}\,T - 0.093)^{5/9} \qquad (2)$$

The relationship of Figure 1 has been terminated at $2500°$ K., at which point dissociation becomes appreciable. Theoretical values of the viscosity of dissociated air have

been calculated by Bade, Mason, and Yun (1) and are tabulated for temperatures from 3000° to 8000° K. and pressures from one to 1000 atm.

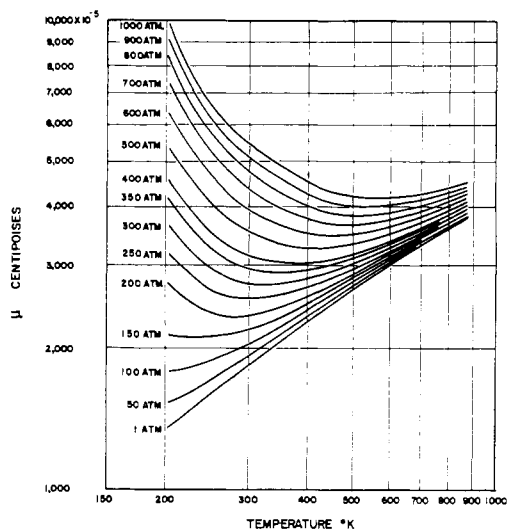Viscosity values calculated with Equations 1 and 2 agree very well with those tabulated by Hilsenrath (20) for air for temperatures below 100° K. For temperatures below 200° K., the data of Johnston and McCloskey (24) are slightly higher than those of Sutherland and Maass (39). To establish the viscosity behavior in this region, values of the group $\mu^*\xi$ (where $\xi = T_c^{1/6}/M^{1/2}P_c^{2/3}$) for nitrogen, oxygen, and air were plotted against the corresponding reduced temperature. Stiel and Thodos (38) have shown that a unique relationship should result for all nonpolar gases when plotted in this manner. The points of Johnston and McCloskey fell on the curve resulting from the data for oxygen and nitrogen, and, therefore, these data were accepted for use in this study.

The average deviation between experimental viscosities and those calculated from Equations 1 and 2 are presented in Table I. The resulting over-all average deviation is 0.94% for 489 points.

VISCOSITY BEHAVIOR IN THE DENSE GASEOUS REGION

The experimental data available for the viscosity of air at elevated pressures are those of Kestin and Wang (27) for 298° K. and pressures from 2 to 100 atm., Kestin and Leidenfrost (26) for temperatures from 293° to 298° K. and pressures from 4 to 70 atm., Biles and Putnam (5) for 297° to 543° K. and 7 to 61 atm., Iwasaki (23) for 323° to 423° K. and 10 to 200 atm., Kellstrom (25) for 293° K. and 8 to 30 atm., Golubev (15) for 273° to 373° K. and 20 to 300 atm., Makita (30) for 298° to 473° K. and 97 to 775 atm., Nasini and Pastonesi (32) for 287° K.

Figure 4. Viscosity correlation for air in gaseous region

Table II. Viscosity Values of Gaseous Air (Centipoises $\times 10^{-5}$)

Pressure, Atm.	-70° C.	-50° C.	-25° C.	0° C.	25° C.	50° C.	75° C.	100° C.	150° C.
1	1350	1455	1590	1710	1820	1925	2050	2160	2360
20	1401	1500	1628	1743	1850	1953	2075	2183	2380
50	1526	1604	1700	1810	1912	2008	2126	2227	2419
75	1654	1726	1769	1873	1966	2054	2160	2266	2454
100	1784	1856	1871	1951	2033	2115	2218	2312	2496
150	2145	2122	2140	2132	2188	2253	2327	2410	2584
200	2743	2540	2368	2330	2349	2384	2460	2504	2661
250	3192	2918	2646	2564	2551	2554	2611	2635	2778
300	3633	3297	2933	2795	2741	2742	2753	2761	2853
350	4084	3654	3226	3033	2954	2896	2923	2897	2952
400	4544	4016	3505	3285	3163	3117	3061	3042	3054
450	4988	4385	3789	3521	3375	3254	3224	3183	3166
500	5450	4737	4065	3761	3577	3429	3381	3245	3271
550	5895	5093	4335	4004	3777	3608	3534	3463	3377
600	6368	5442	4619	4250	3987	3788	3702	3607	3482
700	7270	6150	5172	4717	4390	4145	4007	3889	3717
800	8176	6788	5667	5158	4794	4491	4342	4169	3939
900	9022	7363	6078	5539	5164	4829	4665	4448	4165
1000	9756	7910	6458	5878	5510	5152	4943	4721	4409

	200° C.	250° C.	300° C.	350° C.	400° C.	450° C.	500° C.	550° C.	600° C.
1	2570	2740	2920	3090	3255	3400	3550	3690	3830
20	2588	2756	2935	3104	3267	3410	3559	3698	3837
50	2621	2786	2961	3126	3286	3429	3576	3714	3851
75	2648	2804	2982	3145	3303	3444	3591	3728	3866
100	2679	2835	3006	3165	3320	3459	3605	3742	3878
150	2742	2893	3058	3205	3356	3494	3636	3772	3908
200	2807	2949	3113	3256	3413	3538	3671	3782	3915
250	2893	3024	3172	3308	3456	3584	3703	3826	3952
300	2974	3096	3228	3354	3497	3572	3742	3859	3987
350	3063	3179	3288	3408	3546	3667	3784	3916	4030
400	3148	3251	3351	3459	3590	3709	3819	3946	4064
450	3232	3320	3413	3512	3642	3752	3859	3976	4101
500	3327	3387	3472	3567	3693	3804	3902	4023	4144
550	3414	3456	3540	3626	3744	3834	3946	4059	4182
600	3500	3537	3604	3690	3793	3896	3989	4112	4217
700	3679	3687	3743	3806	3897	3927	4079	4183	4296
800	3867	3855	3879	3934	4010	4094	4173	4279	4375
900	4070	4025	4027	4066	4124	4191	4272	4365	4453
1000	4272	4209	4173	4205	4243	4298	4377	4458	4524

and 35 to 200 atm., and Moulton and Beuschlein (31) for 303° K. and 9 to 300 atm. The data of Moulton and Beuschlein (31) scattered considerably and therefore were not included in this study. The only data available for pressures greater than 300 atm. are those of Makita (30) which were obtained by the rolling-ball method.

Isotherms of the viscosity of a gas against density are parallel, indicating that a unique relationship should exist between $\mu - \mu^*$ and density, where μ is the viscosity of a gas at a certain temperature and pressure and μ^* is the viscosity at the same temperature and atmospheric pressure. When values of $\mu - \mu^*$ for each experimental point for air were plotted in this manner against the corresponding density, a single curve did not result for the data of Makita for the high pressure region, indicating that these data may be in error. Since these are the only values available for this region, an alternate procedure was utilized to establish the high pressure viscosity behavior of air.

Golubev (16) has shown through theoretical arguments that the residual viscosity of a gas, $\mu - \mu^*$, should be a unique function of P_T/T, where $P_T = T[(\partial P)/(\partial T)]_V$ is the thermal pressure. Golubev (16) plotted $\mu - \mu^*$ for the dense gaseous and liquid regions vs. P_T/T for a number of substances and obtained linear relationships. These linear relationships were confirmed by Lennert and Thodos (28) for the inert gases for the dense gaseous region considered in this paper. The use of the thermal pressure therefore has the advantage that the line resulting from accurate low pressure data can be extrapolated to determine the high pressure values. To obtain values of P_T/T for air, thermal pressures were calculated from the derived data of Deming and Shupe (11) for nitrogen. Values of $P_{TR}/T_R = P_T/P_cT_R$ were calculated and plotted against reduced temperature for parameters of reduced pressure. A crossplot was then prepared of P_{TR}/T_R against P_R for values of T_R corresponding to the temperatures for which experimental data for air were obtained. Values of P_T/T for each experimental point for air were then calculated and plotted vs. $\mu - \mu^*$, as shown in Figure 2.

Figure 2 shows that most of the data fall on a single line. The deviations of the experimental points from this line for $P_T/T < 0.3$ are not significant, since $\mu - \mu^*$ represents a viscosity difference. The maximum deviation from the line for this region is about 50 centipoises, which corresponds to a maximum per cent error of approximately 2.5% between the experimental viscosity and that calculated from the line. However, the experimental points of Makita (30) for 423° and 473° K. at high pressures exhibit significant deviations, with a maximum of approximately 10%. The relationship of Figure 2 can be expressed analytically as follows:

$$\mu - \mu^* = 5.76 \times 10^{-3}(P_T/T)^{1.126} \tag{3}$$

Lennert and Thodos (28) have also shown that a single line should result when the group $(\mu - \mu^*)\xi$ is plotted vs. P_{TR}/T_R for substances having a similar nature. Therefore, to confirm the high pressure viscosity behavior of air represented by Equation 3, values of $(\mu - \mu^*)\xi$ for nitrogen and oxygen calculated from the data of Golubev and Petrov (16) were plotted against P_{TR}/T_R, as shown in Figure 3. The nitrogen data covered temperatures to 523° K. Equation 3 for air was expressed in the following reduced form by the use of the critical constants of air:

$$(\mu - \mu^*)\xi = 5.25 \times 10^{-5} (P_{TR}/T_R)^{1.126} \tag{4}$$

Equation 4 was also included in Figure 3. Figure 3 shows that Equation 4 represents well the viscosity behavior of oxygen, nitrogen, and air.

Equation 3 was used in conjunction with the atmospheric viscosity relationship of Figure 1 and the plots for P_{TR}/T_R to establish viscosity values for air for temperatures

to 873° K. and pressures to 1000 atm. These viscosity values were plotted against the corresponding temperatures and pressures to produce the viscosity correlation for air presented in Figure 4. Viscosity values for even intervals of temperature and pressure are presented in Table II. Viscosity values for air can also be calculated from Equation 3 by the use of an appropriate equation of state such as the Beattie-Bridgeman equation (4), to establish values of P_T/T.

NOMENCLATURE

M = molecular weight
P = pressure, atm.
P_c = critical pressure, atm.
P_T = thermal pressure, $T[(\partial P)/(\partial T)]_V$, atm.
P_{TR} = reduced thermal pressure, $T_R[(\partial P_R)/(\partial T_R)]_V$
T = temperature, ° K.
T_c = critical temperature, ° K.
T_R = reduced temperature, T/T_c

Greek Letters

μ = viscosity, centipoises
μ^* = viscosity at moderate pressures (0.2–5 atm.), centipoises
ξ = viscosity, parameter, $T_c^{1/6}/M^{1/2}P_c^{2/3}$

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Latent Heat of Vaporization

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IN A PREVIOUS PAPER (*3*), a correlation was presented for calculating latent heats of vaporization at the normal boiling point. The authors at that time missed the excellent papers by Fishtine (*1*). It is the purpose of the present work to show that the correlation presented earlier is comparable to the modified Kistiakowsky relation proposed by Fishtine.

The equation proposed earlier (*3*) is

$$\lambda_b = \frac{4.7\, T_c\, (1.0 - Pr_b)^{0.69} \log Pr_b}{(1 - 1/Tr_b)} \qquad (1)$$

Earlier, this equation was used to predict λ_b for about 90 substances with an average deviation of about 1.8%. Equation 1 is used to predict λ_b values for substances considered by Fishtine and the results are shown in Table I. The input data used are those listed by Fishtine. In Table I, the first values after each substance give the calculated value of λ_b, and the second give per cent deviation defined as $(\lambda_F - \lambda_b)\, 100/\, \lambda_F$ where λ_F is the value given by Fishtine in his final paper under the column "Author's Choice".

In a recent paper (*2*), Fishtine has compared the different methods of predicting latent heat at the normal boiling point, but the author has not used the present method—Equation 1—in such a comparison. From Table I of the previous paper (*3*), the following table is prepared. In the case of a total of 143 substances tested, the present method gives an average deviation of 1.75%.

Table I. Comparison of Calculated λ_b Values

Compound	λ_b Calcd., Cal./G.	% Dev.
Acetaldehyde	145.5	−0.20
Acetic anhydride	94.5	−1.40
Benzaldehyde	96.0	−1.16
Benzyl alcohol	115.2	−2.58
Bromobenzene	58.0	−0.70
iso-Butyl *iso*-butyrate	62.8	0.80
iso-Butyl *n*-butyrate	65.3	−1.24
Carvacrol	82.0	0.61
Chlorobenzene	77.0	0.80
o-Chlorotoluene	73.0	−0.55
p-Chlorotoluene	73.5	−0.50
Di-*iso*-butylamine	71.0	−2.60
Diethyloxalate	73.7	0.0
Dimethylamine	137.2	2.14
Dimethylaniline	85.3	−1.91
Dimethyloxalate	85.6	1.39
Diphenylsulfide	67.5	−1.51
Ethyl phenyl ether	80.5	0.88
Furfural	106.0	1.4?
1-Heptanol	101.0	3.71
Hydroquinone	140.0	3.09
Methylamine	198.0	0.25
Methylmercaptan	121.0	0.82
Methyl sulfide	104.5	−0.87
Napthalene	80.5	0.38
o-Nitrobenzaldehyde	89.5	4.20
Nitrobenzene	90.2	−1.80
Nitromethane	141.0	−4.50
o-Nitrophenol	83.2	−2.60
1-Octanol	94.0	−3.60
Phenylacetonitrile	98.6	−0.61
Phthalic anhydride	89.2	−1.02
α-Picolene	91.5	1.92
Piperdine	96.5	−1.47
Pipernol	86.0	−2.74
Pyridine	104.5	2.60
Salicylaldehyde	89.2	3.24
2,3,4,6 Tetrachlorophenol	58.0	−0.20
1,2,3 Trichlorobenzene	59.2	−1.71
1,3,5 Trichlorobenzene	58.0	−3.57
2,3,5 Trimethylacetophenone	74.5	2.10
Trimethylamine	95.0	−2.48
2,4 Xyaldehyde	83.2	−1.58

Average deviation of the 43 compounds = 1.7%.

Table II. Comparison of Latent Heat Correlations at T_b

Compound Group	No. of Substances Tested	Present Method	Riedel's Method	Giacolone's Method
Inorganic	8	1.45	2.20	2.00
Hydrocarbons	18	2.45	1.92	2.44
All substances	51	1.70	2.27	2.20

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