# Some Physical Properties of the Ternary System Benzene-Cyclohexane-n-Hexane 

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#### Abstract

In the ternary system benzene-cyclohexane-n-hexane and its three associated binary systems the following physical properties have been measured over the complete composition range: refractive index, density, surface tension, viscosity, heat of mixing, solid-liquid equilibrium, and isobaric vapor-liquid equilibrium at 760 mm . of Hg . A six-stage equilibrium still unit was used to obtain distillation lines on the ternary diagram.


THE BENZENE-cyclohexane- $n$-hexane system has been studied in some detail because it is the simplest representative petroleum system, containing an aromatic, a cycloparaffin, and a straight-chain aliphatic hydrocarbon; furthermore, the three components are close-boiling, so that the isobaric and isothermal vapor-liquid equilibria are not very different. One object of the investigation was to provide a comprehensive set of experimental physical property measurements on a system, and these are presented here. A further object was to compare the measured data with predictions made on the basis of regular solution theory, using an interchange energy term obtained by direct calculation. This comparison has been made and will be published elsewhere. The calculation was based upon the known molecular shape and lattice; distances between pairs of atoms or active centers in adjacent molecules were calculated, and the interaction energy between them was computed using a Lennard-Jones potential.

## MATERIALS

Benzene ("for molecular weight determination") and cyclohexane ("special for spectroscopy") were obtained from the British Drug Houses, Ltd., and $n$-hexane from the Phillips Petroleum Co. The authors considered that these materials were sufficiently pure for the measurement of all properties except density and refractive index. Small quantities of each were purified as follows: Benzene was dried over sodium wire and fractionally distilled, rejecting the first and last $10 \%$. Cyclohexane and $n$-hexane were stirred with chlorosulfonic acid to remove branched-chain hydrocarbons, then washed with sulfuric acid, sodium carbonate solution, and water as recommended by Shepard and Henne (11). The properties of the materials as purchased and after purification are compared with literature values in Table I.

## DATA FOR BINARY SYSTEMS

The experimental method for all the measurements has been described (1).

Refractive Index. Mixtures were made up by weighing into glass-stoppered weighing bottles. Refractive indices at $25^{\circ} \mathrm{C}$. were determined with a Bellingham and Stanley Abbe-type refractometer to an accuracy of 0.0001 (Table II).

[^0]Table I. Physical Properties of Components

|  | As <br> Purchased | Purified | Literature <br> Value (6) |
| :---: | :---: | :---: | :---: |
|  | Benzene |  |  |
| Retractive index, $25^{\circ} \mathrm{C}$. | 1.4979 | 1.4979 | 1.49790 |
| Density, $25^{\circ} \mathrm{C} . \mathrm{g} . / \mathrm{ml} \text {. }$ | 0.87390 | 0.87358 | 0.87368 |
| Boiling point, ${ }^{\circ} \mathrm{C}$. | 80.10 | 80.10 | 80.103 |
|  | Cyclohexane |  |  |
| Refractive index, $25^{\circ} \mathrm{C}$. | 1.4229 | 1.4235 | 1.42354 |
| Density, $25^{\circ} \mathrm{C} . \mathrm{g} . / \mathrm{ml} .$ | 0.77210 | 0.77385 | 0.77389 |
| Boiling point, ${ }^{\circ} \mathrm{C}$. | 80.70 | 80.72 | 80.738 |
| $n$-Hexane |  |  |  |
| Refractive index, $25^{\circ} \mathrm{C}$. | 1.3725 | 1.3723 | 1.37226 |
| Density, $25^{\circ} \mathrm{C} . \mathrm{g} . / \mathrm{ml} .$ | 0.65490 | 0.65480 | 0.65480 |
| Boiling point, ${ }^{\circ} \mathrm{C}$. | 68.70 | 68.70 | 68.740 |

Density. Densities were measured by weighing made-up mixtures in Sprengel-Ostwald pycnometers. Weighing was carried out on a Stanton SM2 semimicrobalance, and densities were reproducible to 0.00006 gram per ml. on the average (Table III).
Surface Tension. A Cambridge torsion balance was used to determine the force necessary to detach a platinum ring, diameter 0.5 inch, from the liquid. Determinations were made in a controlled temperature room at $20^{\circ} \pm$ $1^{\circ} \mathrm{C}$. The results are given in Table IV and are accurate to about $\pm 0.2$ dyne per cm .

Viscosity. This was measured with an Ostwald viscometer. With the usual precautions, replicate efflux times usually agreed to 0.1 second, corresponding to a reproducibility in the viscosity of 0.001 centipoise (Table IV).

Heat of Mixing. Twenty-milliliter quantities of one pure component were added to 350 ml . of mixtures of various compositions in a Dewar vessel at $25^{\circ} \mathrm{C}$. The temperature change was measured. Because the added amount of pure component was small, differential heats of mixing could be obtained in calories per gram of added pure component.

Table II. Refractive Index in the Binary Systems

| Mole Fraction | Refractive Index, |
| :---: | :---: |
| Benzene | $25^{\circ} \mathrm{C}$. |

Benzene-Cyclohexane

| 0.0000 | 1.4235 |
| :--- | :--- |
| 0.1146 | 1.4291 |
| 0.2242 | 1.4350 |
| 0.4574 | 1.4501 |
| 0.6190 | 1.4623 |
| 0.7463 | 1.4728 |
| 0.8473 | 1.4824 |
| 1.0000 | 1.4979 |


| Benzene- $n$-Hexane |  |
| :--- | ---: |
| 0.0000 | 1.3723 |
| 0.2018 | 1.3893 |
| 0.3583 | 1.4045 |
| 0.4969 | 1.4204 |
| 0.6885 | 1.4454 |
| 0.8121 | 1.4645 |
| 0.9173 | 1.4824 |
| 1.0000 | 1.4979 |

Mole Fraction
Cyclohexane
Cyclohexane-n-Hexane

| 0.0000 | 1.3723 |
| :--- | :--- |
| 0.1611 | 1.3793 |
| 0.3660 | 1.3885 |
| 0.4747 | 1.3935 |
| 0.6247 | 1.4013 |
| 0.7624 | 1.4089 |
| 0.8903 | 1.4169 |
| 1.0000 | 1.4235 |

Table III. Densities of the Binary Systems

| Mole Fraction | Density, |
| :---: | :---: |
| Benzene | $25^{\circ}$ C., G. $/$ Ml. |

Benzene-Cyclohexane

| 0.0000 | 0.77385 |
| :--- | :--- |
| 0.1260 | 0.78249 |
| 0.2463 | 0.79144 |
| 0.4552 | 0.80946 |
| 0.6153 | 0.82554 |
| 0.7090 | 0.83602 |
| 0.8609 | 0.85460 |
| 1.0000 | 0.87358 |


| Benzene-n-Hexane |  |
| :--- | ---: |
| 0.0000 | 0.65480 |
| 0.1358 | 0.67524 |
| 0.2431 | 0.69264 |
| 0.3733 | 0.71584 |
| 0.5307 | 0.74785 |
| 0.6914 | 0.78561 |
| 0.7950 | 0.81358 |
| 0.9145 | 0.84762 |
| 1.0000 | 0.87358 |

Mole Fraction
Cyclohexane
Cyclohexane- $n$-Hexane

| 0.0000 | 0.65480 |
| :--- | :--- |
| 0.1930 | 0.67453 |
| 0.3330 | 0.68957 |
| 0.4805 | 0.70607 |
| 0.6502 | 0.72640 |
| 0.7809 | 0.74325 |
| 0.8455 | 0.75196 |
| 1.0000 | 0.77385 |

Table IV. Surface Tension and Viscosity in the Binary Systems

| Mole Fraction <br> Benzene | Surface Tension <br> $20^{\circ} \mathrm{C} .$, Dynes $/ \mathrm{Cm}$. | Viscosity $25^{\circ} \mathrm{C} .$, |
| :---: | :---: | :---: |
| Centipoise |  |  |


| Benzene-Cyclohexane |  |  |
| :---: | :---: | :---: |
| 0.0000 | 24.84 | 0.8690 |
| 0.1282 | 25.00 | 0.7622 |
| 0.2174 | 25.14 | 0.7122 |
| 0.3364 | 25.43 | 0.6599 |
| 0.4874 | 25.84 | 0.6124 |
| 0.6470 | 26.54 | 0.5879 |
| 0.7814 | 27.20 | 0.5830 |
| 0.9033 | 27.88 | 0.5926 |
| 1.0000 | 28.87 | 0.6059 |
| Benzene-n-Hexane |  |  |
| 0.0000 | 18.95 | 0.3008 |
| 0.1281 | 19.53 | 0.3134 |
| 0.2665 | 20.22 | 0.3270 |
| 0.4050 | 21.05 | 0.3471 |
| 0.5704 | 22.16 | 0.3822 |
| 0.7216 | 23.43 | 0.4254 |
| 0.8811 | 25.97 | 0.5139 |
| 1.0000 | 28.87 | 0.6059 |

Mole Fraction
Cyclohexane

| Cyclohexane- $n$-Hexane |  |  |
| :--- | :---: | :---: |
| 0.0000 | 18.95 | 0.3008 |
| 0.1714 | 19.39 | 0.3405 |
| 0.2742 | 19.90 | 0.3670 |
| 0.498 | 20.40 | 0.4234 |
| 0.5873 | 21.30 | 0.4846 |
| 0.7520 | 22.54 | 0.5887 |
| 0.9034 | 23.81 | 0.7347 |
| 1.0000 | 24.84 | 0.8690 |

Cyclohexane-n-Hexane

By plotting the differential heat of mixing against composition for each component, the integral heat of mixing was obtained at regular intervals of composition. These values have a precision of about $1 \%$. The differential (or partial) and the total heats of mixing at rounded values of mole fractions are given in Table V.

Solid-Liquid Equilibrium. Cooling curves were obtained by connecting a copper-constantan thermocouple, immersed in the liquid sample, to a Kent recorder. The liquid sample, held in a boiling tube, was suspended within a larger diameter boiling tube, and the outer tube immersed in liquid nitrogen. With this arrangement, the sample cooled at the convenient rate of about $3^{\circ} \mathrm{C}$. per minute. From the arrest-points on the cooling curves the phase diagrams could be plotted, and the three binary diagrams are shown in Figure 1.

Vapor-Liquid Equilibrium. Measurements were made at 760mm . pressure, using six Ellis-type equilibrium stills (3) connected in series to improve accuracy and to reduce the time needed to obtain the data points required to characterize a system. At equilibrium in a series type of experiment, the vapor of one stage is identical in composition to the liquid in the next stage; this can be observed as a cross-correlation in Table VI.

## DATA FOR THE TERNARY SYSTEM

The triangular diagram was covered by making up a number of stock mixtures of two of the components. The compositions of these mixtures were evenly spaced along one of the sides of the triangle. Addition of known amounts of the third component then gave mixtures lying on lines crossing the triangle and converging on the third component apex (Tables VII, VIII, and IX).

The ternary heats of mixing were determined by adding small amounts ( 20 ml .) of the pure components to large

Table V. Partial and Total Heats of Mixing at $25^{\circ} \mathrm{C}$. in the Binary Systems

| Mole Fraction 3enzene | Partial Heat B in CH , ${ }^{\text {a }}$ Cal./G. | Partial Heat CH in B , Cal./G. | Total Heat, Cal. /G. Mole |
| :---: | :---: | :---: | :---: |
| Benzene-Cyclohexane |  |  |  |
| 0.1 | 8.28 | 0.11 | 72.92 |
| 0.2 | 6.40 | 0.32 | 121.30 |
| 0.3 | 4.86 | 0.73 | 156.75 |
| 0.4 | 3.64 | 1.38 | 183.10 |
| 0.5 | 2.60 | 2.20 | 193.65 |
| 0.6 | 1.74 | 3.20 | 188.95 |
| 0.7 | 1.07 | 4.37 | 168.45 |
| 0.8 | 0.49 | 5.89 | 129.38 |
| 0.9 | 0.14 | 7.64 | 74.03 |
| Benzene-n-Hexane |  |  |  |
| 0.1 | 7.86 | 0.06 | 65.96 |
| 0.2 | 6.52 | 0.20 | 115.59 |
| 0.3 | 5.28 | 0.53 | 155.70 |
| 0.4 | 4.27 | 1.10 | 189.85 |
| 0.5 | 3.15 | 1.92 | 205.60 |
| 0.6 | 2.18 | 3.04 | 206.80 |
| 0.7 | 1.31 | 4.59 | 190.20 |
| 0.8 | 0.63 | 6.78 | 156.30 |
| 0.9 | 0.20 | 9.54 | 96.28 |
| Mole Fraction Cyclohexane |  |  |  |
| Cyclohexane-n-Hexane |  |  |  |
| 0.1 | 1.68 | 0.00 | 14.13 |
| 0.2 | 1.43 | 0.02 | 25.43 |
| 0.3 | 1.18 | 0.05 | 32.98 |
| 0.4 | 0.95 | 0.12 | 38.21 |
| 0.5 | 0.71 | 0.27 | 41.49 |
| 0.6 | 0.49 | 0.55 | 43.70 |
| 0.7 | 0.30 | 0.94 | 41.98 |
| 0.8 | 0.10 | 1.53 | 33.02 |
| 0.9 | -0.07 | 2.28 | 14.34 |

amounts ( 350 ml .) of ternary mixtures of various compositions. Sufficient determinations were carried out to enable a contour plot of the differential heat of addition of each pure compound to be made. From these plots the total or integral heat of mixing could be found. The results of the determinations of the differential heats of mixing are given in Table X.

The integral heats of mixing for the ternary system are shown in the triangular plot in Figure 2.

Solid-Liquid Equilibrium. The ternary diagram was covered by adding benzene to stock solutions of cyclohexane- $n$ hexane and obtaining a cooling curve for each mixture. The results are presented as a triangular plot in Figure 3.

Vapor-Liquid Equilibrium. The six-stage unit was employed and has the advantage that the results give the course of a distillation line on the ternary diagram-that is, the alternate liquid and vapor compositions are those which would be obtained in an ideal plate-type distillation column operating at total reflux. A few such lines give a better idea of the general pattern of the vapor-liquid equilibrium for the system than would a much larger number of liquidvapor tie lines of the type obtained by making measurements with a single equilibrium still. The results for 18 distillation lines are given in Table XI. Because the vapor of one stage and the liquid of the next have the same composition, the same correlation in adjacent columns of the table is found as occurs in Table VI for the binary systems.


Figure 1. Solid-liquid equilibrium in the three binary systems

## agreement with other workers

For the majority of the measurements reported here, there are no data in the literature with which comparison can be made. This is particularly true of the ternary system, where the only property which has previously been examined is the heat of mixing, and here the results obtained by Mathieson and Thynne (9) agree in all cases to within $5 \%$ of values interpolated from Figure 2.

A more detailed comparison of the authors' results with literature values for the binary systems is given in the doctoral thesis (1) from which this paper is derived. No cases of divergence arose.

Thermodynamic Consistency Tests. Herington (4) and Redlich and Kister (10) have described a test for binary vapor-liquid equilibrium data in which the logarithm of the ratio of the activity coefficients, $\ln \left(\gamma_{1} / \gamma_{2}\right)$, is plotted against composition. The data are consistent if

$$
\int_{0}^{1} \ln \left(\gamma_{1} / \gamma_{2}\right) \cdot d x_{1}=0
$$

in the sense that they satisfy the Gibbs-Duhem equation. A small correction is recommended for isobaric data, the test equation being

$$
\int_{0}^{1} \frac{-\Delta H^{M}}{R T^{2}} \quad \frac{d T}{d x} d x_{1}+\int_{0}^{1} \ln \left(\gamma_{1} / \gamma_{2}\right) d x_{1}=0
$$

where $\Delta H^{M}$ is the heat of mixing. In close-boiling systems such as those investigated by the present authors, the first

Table VI. Vapor-Liquid Equilibrium at 760 Mm . in the Binary Systems Mole

| Fraction Benzene |  | Boiling$\text { Pt., }{ }^{\circ} \mathrm{C} .$ |
| :---: | :---: | :---: |
| Liquid | Vapor |  |
| Benzene-Cyclohexane |  |  |
| 0.077 | 0.096 | 79.80 |
| 0.096 | 0.118 | 79.50 |
| 0.118 | 0.145 | 79.25 |
| 0.145 | 0.176 | 79.05 |
| 0.176 | 0.208 | 78.85 |
| 0.208 | 0.243 | 78.70 |
| 0.245 | 0.280 | 78.50 |
| 0.280 | 0.314 | 78.25 |
| 0.314 | 0.346 | 78.10 |
| 0.346 | 0.375 | 77.95 |
| 0.375 | 0.402 | 77.85 |
| 0.402 | 0.426 | 77.75 |
| 0.507 | 0.512 | 77.60 |
| 0.512 | 0.516 | 77.60 |
| 0.516 | 0.519 | 77.60 |
| 0.519 | 0.522 | 77.55 |
| 0.522 | 0.524 | 77.55 |
| 0.524 | 0.526 | 77.55 |
| 0.628 | 0.612 | 77.65 |
| 0.645 | 0.628 | 77.70 |
| 0.663 | 0.645 | 77.75 |
| 0.685 | 0.663 | 77.80 |
| 0.712 | 0.685 | 77.90 |
| 0.742 | 0.712 | 78.00 |
| 0.775 | 0.744 | 78.05 |
| 0.806 | 0.775 | 78.30 |
| 0.836 | 0.806 | 78.55 |
| 0.865 | 0.836 | 78.80 |
| 0.892 | 0.865 | 79.05 |
| 0.916 | 0.892 | 79.25 |


| Benzene- $n$-Hexane |  |  |
| :---: | :---: | :---: |
| 0.080 | 0.078 | 68.70 |
| 00.082 | 0.080 | 68.75 |
| 0.085 | 0.082 | 68.75 |
| 0.088 | 0.085 | 68.80 |
| 0.091 | 0.088 | 68.80 |
| 0.095 | 0.091 | 68.80 |
| 0.144 | 0.137 | 68.85 |
| 0.152 | 0.144 | 68.90 |
| 0.160 | 0.152 | 68.95 |
| 0.170 | 0.160 | 68.95 |
| 0.183 | 0.170 | 69.00 |
| 0.199 | 0.183 | 69.00 |
| 0.252 | 0.226 | 69.00 |
| 0.281 | 0.252 | 69.15 |
| 0.318 | 0.281 | 69.35 |
| 0.366 | 0.318 | 69.65 |
| 0.430 | 0.366 | 70.05 |
| 0.512 | 0.430 | 70.65 |
| 0.583 | 0.489 | 71.35 |
| 0.690 | 0.583 | 72.70 |
| 0.797 | 0.690 | 74.55 |
| 0.881 | 0.797 | 76.65 |
| 0.940 | 0.881 | 78.10 |
| 0.972 | 0.940 | 79.10 |


| Mole <br> Fraction Cyclohexane |  |  |
| :---: | :---: | :---: |
| Liquid | Vapor |  |
| Cyclohexane-n-Hexane |  |  |
| 0.123 | 0.096 | 70.05 |
| 0.160 | 0.123 | 70.40 |
| 0.203 | 0.160 | 70.85 |
| 0.260 | 0.203 | 71.40 |
| 0.329 | 0.260 | 72.05 |
| 0.409 | 0.329 | 72.90 |
| 0.477 | 0.398 | 73.70 |
| 0.562 | 0.477 | 74.70 |
| 0.616 | 0.532 | 75.40 |
| 0.647 | 0.562 | 75.70 |
| 0.700 | 0.616 | 76.40 |
| 0.730 | 0.647 | 76.75 |
| 0.774 | 0.700 | 77.35 |
| 0.833 | 0.774 | 78.20 |
| 0.879 | 0.833 | 79.00 |
| 0.918 | 0.879 | 79.50 |

Table VII. Refractive Index in the Ternary System Benzene-Cyclohexane-n-Hexane at $25^{\circ} \mathrm{C}$.

| Mole Fraction Benzene | Mole Fraction Cyclohexane | Mole Fraction $n$-Hexane | Refractive Index $n_{\mathrm{D}}^{25}$ |
| :---: | :---: | :---: | :---: |
| 0.1146 | - 0.8854 | 0.1070 | 1.4249 |
|  |  | 0.2325 | 1.4132 |
|  |  | 0.4216 | 1.4008 |
|  |  | 0.5264 | 1.3959 |
|  |  | 0.6470 | 1.3891 |
|  |  | 0.7015 | 1.3866 |
|  |  | 0.7806 | 1.3824 |
|  |  | 0.9444 | 1.3750 |
| 0.2242 | 0.7758 | 0.0989 | 1.4269 |
|  |  | 0.2379 | 1.4168 |
|  |  | 0.3410 | 1.4096 |
|  |  | 0.4965 | 1.3996 |
|  |  | 0.6446 | 1.3907 |
|  |  | 0.7093 | 1.3872 |
|  |  | 0.7888 | 1.3830 |
|  |  | 0.8375 | 1.3805 |
|  |  | 0.9098 | 1.3766 |
| 0.4574 | 0.5426 | 0.1875 | 1.4315 |
|  |  | 0.3976 | 1.4134 |
|  |  | 0.5938 | 1.3985 |
|  |  | 0.6943 | 1.3914 |
|  |  | 0.8166 | 1.3831 |
|  |  | 0.8719 | 1.3797 |
|  |  | 0.9182 | 1.3768 |
| 0.6190 | 0.3810 | 0.1789 | 1.4414 |
|  |  | 0.3018 | 1.4281 |
|  |  | 0.4085 | 1.4180 |
|  |  | 0.5107 | 1.4085 |
|  |  | 0.5857 | 1.4018 |
|  |  | 0.6852 | 1.3941 |
|  |  | 0.7241 | 1.3905 |
|  |  | 0.8181 | 1.3826 |
| 0.7463 | 0.2537 | 0.1165 | 1.4562 |
|  |  | 0.1945 | 1.4464 |
|  |  | 0.4182 | 1.4213 |
|  |  | 0.5890 | 1.4045 |
|  |  | 0.7306 | 1.3922 |
|  |  | 0.7572 | 1.3900 |
|  |  | 0.8208 | 1.3849 |
|  |  | 0.9176 | 1.3777 |
| 0.8473 | 0.1527 | 0.1545 | 1.4580 |
|  |  | 0.3663 | 1.4311 |
|  |  | 0.4637 | 1.4198 |
|  |  | 0.5155 | 1.4141 |
|  |  | 0.6108 | 1.4042 |
|  |  | 0.7131 | 1.3950 |
|  |  | 0.8253 | 1.3849 |

Table VIII. Density in the Ternary System
Benzene-Cyclohexane-n-Hexane at $25^{\circ} \mathrm{C}$.

| Mole Fraction <br> Cyclohexane | Mole Fraction <br> $n$-Hexane | Mole Fraction <br> Benzene | Density at $25^{\circ} \mathrm{C}$., <br> G. $/ \mathrm{Ml}$. |
| :---: | :---: | :---: | :---: |
| 0.1930 | 0.8070 | 0.2643 | 0.71309 |
|  |  | 0.4720 | 0.74931 |
|  |  | 0.6139 | 0.77918 |
| 0.3358 | 0.6642 | 0.7127 | 0.80086 |
|  |  | 0.2744 | 0.72720 |
|  |  | 0.5201 | 0.76787 |
|  |  | 0.6598 | 0.79500 |
| 0.4805 | 0.5195 | 0.7752 | 0.81930 |
|  |  | 0.3072 | 0.74511 |
|  |  | 0.5455 | 0.78232 |
|  |  | 0.6485 | 0.80095 |
|  |  | 0.7789 | 0.82567 |
|  |  | 0.2215 | 0.75087 |
|  |  | 0.4976 | 0.78661 |
|  |  | 0.7424 | 0.82483 |
|  |  | 0.8650 | 0.84698 |
|  |  | 0.1834 | 0.76132 |
|  |  | 0.4913 | 0.79611 |
|  |  | 0.7018 | 0.82499 |
|  |  | 0.8315 | 0.84496 |
|  |  | 0.2179 | 0.77141 |
|  |  | 0.4175 | 0.79210 |
|  |  | 0.6399 | 0.81918 |
|  |  | 0.7856 | 0.83965 |

Table IX. Surface Tension and Viscosity in the Ternary System Benzene-Cyclohexane-n-Hexane at $20^{\circ} \mathrm{C}$.

| Mole Fraction Cyclohexane | Mole <br> Fraction <br> $n$-Hexane | Mole Fraction Benzene | Surface <br> Tension at $20^{\circ} \mathrm{C}$., Dynes/Cm. | Viscosity at $25^{\circ}$ C., Centipoise |
| :---: | :---: | :---: | :---: | :---: |
| 0.1714 | 0.8286 | 0.1998 | 20.30 | 0.3523 |
|  |  | 0.3764 | 21.48 | 0.3720 |
|  |  | 0.5285 | 22.50 | 0.3991 |
|  |  | 0.6781 | 23.38 | 0.4397 |
|  |  | 0.8394 | 25.62 | 0.5073 |
|  |  | 0.9584 | 27.76 | 0.5774 |
| 0.2742 | 0.7258 | 0.1444 | 20.72 | 0.3746 |
|  |  | 0.3488 | 21.51 | 0.3897 |
|  |  | 0.6080 | 23.74 | 0.4390 |
|  |  | 0.7850 | 25.19 | 0.4957 |
|  |  | 0.8978 | 27.00 | 0.5489 |
| 0.4498 | 0.5502 | 0.1385 | 21.06 | 0.4250 |
|  |  | 0.3447 | 22.15 | 0.4359 |
|  |  | 0.5081. | 22.98 | 0.4498 |
|  |  | 0.6515 | 23.89 | 0.4794 |
|  |  | 0.7510 | 25.37 | 0.5100 |
|  |  | 0.9163 | 27.24 | 0.5680 |
| 0.5873 | 0.4127 | 0.1436 | 22.11 | 0.4743 |
|  |  | 0.3017 | 22.97 | 0.4714 |
|  |  | 0.4836 | 23.50 | 0.4786 |
|  |  | 0.6676 | 24.44 | 0.5025 |
|  |  | 0.8251 | 26.15 | 0.5394 |
|  |  | 0.9229 | 27.80 | 0.5754 |
| 0.7520 | 0.2480 | 0.1461 | 23.19 | 0.5570 |
|  |  | 0.2681 | 23.50 | 0.5380 |
|  |  | 0.4231 | 24.02 | 0.5283 |
|  |  | 0.5664 | 24.67 | 0.5303 |
|  |  | 0.7122 | 25.55 | 0.5396 |
|  |  | 0.8756 | 27.09 | 0.5694 |
| 0.9034 | 0.0966 | 0.1231 | 24.10 | 0.6731 |
|  |  | 0.2662 | 24.40 | 0.6223 |
|  |  | 0.4094 | 25.00 | 0.5913 |
|  |  | 0.5700 | 25.61 | 0.5720 |
|  |  | 0.7122 | 26.08 | 0.5685 |
|  |  | 0.8941 | 27.70 | 0.5860 |



Figure 2. Integral heats of mixing for the ternary system benzene-cyclohexane- $n$-hexane at $25^{\circ} \mathrm{C}$.

Contour figures are cal. per mole of mixture
term is negligible. The maximum value for $\left(\Delta H^{M} / R T^{2}\right)$ - (dT/dx) was 0.008 or $0.5 \%$ of the end value of $\ln \left(\gamma_{1} /\right.$ $\gamma_{2}$ ). Expressing the area differences as a percentage of the total area, the discrepancies for the binary systems were benzene-cyclohexane $3.1 \%$, benzene $-n$-hexane $5.5 \%$, and cyclohexane- $n$-hexane $1.3 \%$.

Colburn and Schoenborn (2) and Herington (5) have proposed graphical integration methods for testing ternary data, but they involve much graphical interpolation and extrapolation. A more easily applied test is that of Li

Table X. Differential Heats of Mixing in the Ternary System Benzene-Cyclohexane-n-Hexane

| Mean Mole Fractions |  |  | Temperature Change, ${ }^{\circ} \mathrm{C}$. | Heat of Mixing, Cal./G. |
| :---: | :---: | :---: | :---: | :---: |
| Benzene | Cyclohexane | $n$-Hexane |  |  |
| Addition of Benzene |  |  |  |  |
| 0.036 | 0.779 | 0.185 | -0.991 | 8.78 |
| 0.039 | 0.450 | 0.511 | -0.992 | 8.65 |
| 0.142 | 0.171 | 0.687 | -0.756 | 6.65 |
| 0.200 | 0.499 | 0.301 | -0.683 | 5.88 |
| 0.209 | 0.372 | 0.419 | -0.669 | 5.75 |
| 0.234 | 0.294 | 0.472 | -0.656 | 5.62 |
| 0.317 | 0.583 | 0.100 | -0.504 | 4.35 |
| 0.373 | 0.135 | 0.492 | $-0.480$ | 4.08 |
| 0.415 | 0.383 | 0.202 | -0.396 | 3.46 |
| 0.471 | 0.138 | 0.391 | -0.368 | 3.18 |
| 0.516 | 0.232 | 0.252 | -0.305 | 2.68 |
| 0.559 | 0.326 | 0.115 | -0.242 | 2.09 |
| 0.692 | 0.103 | 0.205 | -0.123 | 1.08 |
| 0.758 | 0.164 | 0.078 | $-0.077$ | 0.67 |
| Addition of Cyclohexane |  |  |  |  |
| 0.658 | 0.113 | 0.229 | -0.399 | 3.96 |
| 0.357 | 0.123 | 0.520 | -0.164 | 1.62 |
| 0.460 | 0.127 | 0.413 | -0.238 | 2.37 |
| 0.712 | 0.175 | 0.113 | -0.452 | 4.51 |
| 0.149 | 0.176 | 0.675 | -0.119 | 1.26 |
| 0.491 | 0.236 | 0.273 | -0.235 | 2.37 |
| 0.238 | 0.284 | 0.478 | -0.118 | 1.17 |
| 0.324 | 0.312 | 0.364 | -0.135 | 1.34 |
| 0.564 | 0.314 | 0.122 | -0.286 | 2.85 |
| 0.405 | 0.381 | 0.214 | -0.180 | 1.75 |
| 0.121 | 0.400 | 0.479 | -0.087 | 0.85 |
| 0.177 | 0.499 | 0.324 | -0.074 | 0.74 |
| 0.305 | 0.586 | 0.109 | -0.103 | 0.91 |
| 0.070 | 0.756 | 0.174 | -0.017 | 0.17 |
| Addition of $n$-Hexane |  |  |  |  |
| 0.327 | 0.578 | 0.095 | -0.214 | 2.50 |
| 0.751 | 0.153 | 0.096 | -0.569 | 6.58 |
| 0.593 | 0.300 | 0.107 | -0.378 | 4.46 |
| 0.067 | 0.742 | 0.191 | -0.138 | 1.61 |
| 0.427 | 0.360 | 0.231 | -0.200 | 2.28 |
| 0.687 | 0.097 | 0.216 | $-0.414$ | 4.80 |
| 0.520 | 0.218 | 0.262 | -0.239 | 2.78 |
| 0.187 | 0.495 | 0.318 | -0.077 | 0.91 |
| 0.340 | 0.302 | 0.358 | -0.115 | 1.34 |
| 0.478 | 0.131 | 0.391 | -0.175 | 2.01 |
| 0.235 | 0.275 | 0.472 | -0.054 | 0.63 |
| 0.127 | 0.394 | 0.479 | -0.038 | 0.44 |
| 0.388 | 0.126 | 0.486 | -0.094 | 1.09 |
| 0.158 | 0.161 | 0.681 | -0.018 | 0.21 |



Figure 3. Solid-liquid equilibrium in the ternary system benzene-cyclohexane-n-hexane

Table XI. Vapor-Liquid Equilibrium in the Ternary System Benzene-Cyclohexane-n-Hexane

| Run | Liquid Mole Fraction |  |  | Vapor Mole Fraction |  |  | Boiling Pt., ${ }^{\circ} \mathrm{C}$. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| No. | Benzene | Cyclohexane | $n$-Hexane | Benzene | Cyclohexane | $n$-Hexane |  |
| 1 | 0.861 | 0.066 | 0.073 | 0.806 | 0.067 | 0.127 | 77.45 |
|  | 0.806 | 0.067 | 0.127 | 0.732 | 0.068 | 0.200 | 76.05 |
|  | 0.732 | 0.068 | 0.200 | 0.637 | 0.070 | 0.293 | 74.55 |
|  | 0.637 | 0.070 | 0.293 | 0.544 | 0.071 | 0.383 | 73.15 |
|  | 0.544 | 0.071 | 0.383 | 0.461 | 0.067 | 0.472 | 71.95 |
|  | 0.461 | 0.067 | 0.472 | 0.409 | 0.052 | 0.539 | 71.05 |
| 2 | 0.512 | 0.110 | 0.378 | 0.443 | 0.096 | 0.461 | 72.15 |
|  | 0.443 | 0.096 | 0.461 | 0.395 | 0.079 | 0.526 | 71.25 |
|  | 0.395 | 0.079 | 0.526 | 0.342 | 0.064 | 0.594 | 70.70 |
|  | 0.342 | 0.064 | 0.594 | 0.304 | 0.053 | 0.643 | 70.25 |
|  | 0.304 | 0.053 | 0.643 | 0.279 | 0.037 | 0.684 | 69.85 |
|  | 0.279 | 0.037 | 0.684 | 0.250 | 0.025 | 0.725 | 69.55 |
| 3 | 0.157 | 0.503 | 0.340 | 0.176 | 0.414 | 0.410 | 74.35 |
|  | 0.176 | 0.414 | 0.410 | 0.180 | 0.347 | 0.473 | 73.30 |
|  | 0.180 | 0.347 | 0.473 | 0.183 | 0.267 | 0.550 | 72.55 |
|  | 0.183 | 0.267 | 0.550 | 0.183 | 0.214 | 0.603 | 71.80 |
|  | 0.183 | 0.214 | 0.603 | 0.176 | 0.179 | 0.645 | 71.10 |
|  | 0.176 | 0.179 | 0.645 | 0.171 | 0.138 | 0.691 | 70.60 |
| 4 | 0.078 | 0.289 | 0.633 | 0.081 | 0.231 | 0.688 | 71.75 |
|  | 0.081 | 0.231 | 0.688 | 0.079 | 0.173 | 0.748 | 71.05 |
|  | 0.079 | 0.173 | 0.748 | 0.077 | 0.122 | 0.801 | 70.50 |
|  | 0.077 | 0.122 | 0.801 | 0.075 | 0.097 | 0.828 | 70.15 |
|  | 0.075 | 0.097 | 0.828 | 0.073 | 0.074 | 0.853 | 69.80 |
|  | 0.073 | 0.074 | 0.853 | 0.070 | 0.059 | 0.871 | 69.50 |
| 5 | 0.024 | 0.708 | 0.268 | 0.030 | 0.635 | 0.335 | 76.55 |
|  | 0.030 | 0.635 | 0.335 | 0.036 | 0.540 | 0.424 | 75.50 |
|  | 0.036 | 0.540 | 0.424 | 0.041 | 0.458 | 0.501 | 74.40 |
|  | 0.041 | 0.458 | 0.501 | 0.048 | 0.363 | 0.589 | 73.25 |
|  | 0.048 | 0.363 | 0.589 | 0.055 | 0.290 | 0.655 | 72.35 |
|  | 0.055 | 0.290 | 0.655 | 0.057 | 0.227 | 0.716 | 71.60 |
| 6 | 0.081 | 0.843 | 0.076 | 0.109 | 0.786 | 0.105 | 78.70 |
|  | 0.109 | 0.786 | 0.105 | 0.128 | 0.730 | 0.142 | 78.00 |
|  | 0.128 | 0.730 | 0.142 | 0.152 | 0.655 | 0.193 | 77.25 |
|  | 0.152 | 0.655 | 0.193 | 0.169 | 0.579 | 0.252 | 76.35 |
|  | 0.169 | 0.579 | 0.252 | 0.182 | 0.506 | 0.312 | 75.30 |
|  | 0.182 | 0.506 | 0.312 | 0.195 | 0.420 | 0.385 | 74.30 |
| 7 | 0.315 | 0.648 | 0.037 | 0.345 | 0.605 | 0.050 | 77.80 |
|  | 0.345 | 0.605 | 0.050 | 0.375 | 0.554 | 0.071 | 77.35 |
|  | 0.375 | 0.554 | 0.071 | 0.397 | 0.511 | 0.092 | 77.00 |
|  | 0.397 | 0.511 | 0.092 | 0.411 | 0.457 | 0.132 | 76.45 |
|  | 0.411 | 0.475 | 0.132 | 0.413 | 0.415 | 0.172 | 75.80 |
|  | 0.413 | 0.415 | 0.172 | 0.404 | 0.365 | 0.231 | 75.15 |
| 8 | 0.425 | 0.451 | 0.125 | 0.423 | 0.408 | 0.169 | 76.00 |
|  | 0.423 | 0.408 | 0.169 | 0.413 | 0.364 | 0.223 | 75.35 |
|  | 0.413 | 0.364 | 0.223 | 0.398 | 0.313 | 0.289 | 74.65 |
|  | 0.398 | 0.313 | 0.289 | 0.381 | 0.270 | 0.349 | 73.80 |
|  |  | 0.270 | 0.349 | 0.356 | 0.216 | 0.428 | 72.95 |
|  | 0.356 | 0.216 | 0.428 | 0.330 | 0.180 | 0.490 | 72.15 |
| $\begin{array}{r}9 \\ \\ \hline\end{array}$ | 0.254 | 0.276 | 0.470 | 0.248 | 0.220 | 0.532 | 72.25 |
|  | 0.248 | 0.220 | 0.532 | 0.238 | 0.163 | 0.599 | 71.55 |
|  | 0.238 | 0.163 | 0.599 | 0.227 | 0.130 | 0.643 | 71.05 |
|  | 0.277 | 0.130 | 0.643 | 0.216 | 0.100 | 0.684 | 70.55 |
|  | 0.216 | 0.100 | 0.684 | 0.200 | 0.078 | 0.722 | 70.10 |
|  | 0.200 | 0.078 | 0.722 | 0.190 | 0.060 | 0.750 | 69.75 |
| 10 | 0.120 | 0.123 | 0.757 | 0.118 | 0.091 | 0.791 | 70.15 |
|  | 0.118 | 0.091 | 0.791 | 0.116 | 0.064 | 0.820 | 69.85 |
|  | 0.116 | 0.064 | 0.820 | 0.114 | 0.045 | 0.841 | 69.60 |
|  | 0.114 | 0.045 | 0.841 | 0.111 | 0.036 | 0.853 | 69.45 |
|  | 0.111 | 0.036 | 0.853 | 0.103 | 0.026 | 0.871 | 69.30 |
|  | 0.103 | 0.026 | 0.871 | 0.097 | 0.023 | 0.880 | 69.15 |
| 11 | 0.310 | 0.577 | 0.113 | 0.336 | 0.505 | 0.159 | 76.50 |
|  | 0.336 | 0.505 | 0.159 | 0.345 | 0.440 | 0.215 | 75.85 |
|  | 0.345 | 0.440 | 0.215 | 0.344 | 0.386 | 0.270 | 75.10 |
|  | 0.344 | 0.386 | 0.270 | 0.333 | 0.334 | 0.333 | 74.30 |
|  | 0.333 | 0.334 | 0.333 | 0.318 | 0.278 | 0.404 | 73.50 |
|  | 0.318 | 0.278 | 0.404 | 0.298 | 0.224 | 0.478 | 72.65 |
| 12 | 0.685 | 0.237 | 0.078 | 0.642 | 0.230 | 0.128 | 76.50 |
|  | 0.642 | 0.230 | 0.128 | 0.591 | 0.219 | 0.190 | 75.65 |
|  | 0.591 | 0.219 | 0.190 | 0.539 | 0.200 | 0.261 | 74.75 |
|  | 0.539 | 0.200 | 0.261 | 0.480 | 0.180 | 0.340 | 73.70 |
|  | 0.480 | 0.180 | 0.340 | 0.431 | 0.158 | 0.411 | 72.75 |
|  | 0.431 | 0.158 | 0.411 | 0.383 | 0.130 | 0.487 | 71.95 |
| 13 | 0.898 | 0.080 | 0.022 | 0.849 | 0.112 | 0.039 | 78.75 |
|  | 0.849 | 0.112 | 0.039 | 0.810 | 0.128 | 0.062 | 78.20 |
|  | 0.810 | 0.128 | 0.062 | 0.749 | 0.138 | 0.113 | 77.40 |
|  | 0.749 | 0.138 | 0.113 | 0.681 | 0.135 | 0.184 | 76.20 |
|  | 0.681 | 0.135 | 0.184 | 0.610 | 0.129 | 0.261 | 74.95 |
|  | 0.610 | 0.129 | 0.261 | 0.539 | 0.115 | 0.356 | 73.70 |
| 14 | 0.652 | ${ }^{0.332}$ | 0.016 | 0.629 | 0.333 | 0.038 | 77.50 |
|  | 0.629 | 0.333 | 0.038 | 0.600 | 0.329 | 0.071 | 77.15 |

(Continued)

| Table XI. Vapor-Liquid Equilibrium in the Ternary System Benzene-Cyclohexane-n-Hexane (Continued) |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Run No. | Liquid Mole Fraction |  |  | Vapor Mole Fraction |  |  | Boiling Pt., |
|  | Benzene | Cyclohexane | $n$-Hexane | Benzene | Cyclohexane | $n$-Hexane | ${ }^{\circ} \mathrm{C}$. |
|  | 0.600 | 0.329 | 0.071 | 0.573 | 0.320 | 0.107 | 76.75 |
| 15 | 0.573 | 0.320 | 0.107 | 0.543 | 0.310 | 0.147 | 76.15 |
|  | 0.543 | 0.310 | 0.147 | 0.511 | 0.289 | 0.200 | 75.40 |
|  | 0.511 | 0.289 | 0.200 | 0.472 | 0.258 | 0.270 | 74.60 |
|  | 0.516 | 0.474 | 0.010 | 0.518 | 0.464 | 0.018 | 77.55 |
|  | 0.518 | 0.464 | 0.018 | 0.519 | 0.452 | 0.029 | 77.45 |
|  | 0.519 | 0.452 | 0.029 | 0.516 | 0.439 | 0.045 | 77.30 |
| 16 | 0.516 | 0.439 | 0.045 | 0.512 | 0.419 | 0.069 | 77.00 |
|  | 0.512 | 0.419 | 0.069 | 0.505 | 0.395 | 0.100 | 76.55 |
|  | 0.505 | 0.395 | 0.100 | 0.492 | 0.366 | 0.142 | 76.05 |
|  | 0.250 | 0.674 | 0.076 | 0.277 | 0.613 | 0.110 | 77.35 |
|  | 0.277 | 0.613 | 0.110 | 0.299 | 0.551 | 0.150 | 76.70 |
|  | 0.299 | 0.551 | 0.150 | 0.315 | 0.482 | 0.203 | 76.05 |
|  | 0.315 | 0.482 | 0.203 | 0.320 | 0.420 | 0.260 | 75.15 |
| 17 | 0.320 | 0.420 | 0.260 | 0.316 | 0.358 | 0.326 | 74.30 |
|  | 0.316 | 0.358 | 0.326 | 0.308 | 0.301 | 0.391 | 73.50 |
|  | 0.225 | 0.657 | 0.118 | 0.253 | 0.586 | 0.161 | 76.90 |
|  | 0.253 | 0.586 | 0.161 | 0.272 | 0.514 | 0.214 | 76.15 |
|  | 0.272 | 0.514 | 0.214 | 0.284 | 0.446 | 0.270 | 73.35 |
| 18 | 0.284 | 0.446 | 0.270 | 0.289 | 0.380 | 0.331 | 74.45 |
|  | 0.289 | 0.380 | 0.331 | 0.284 | 0.313 | 0.403 | 73.55 |
|  | 0.284 | 0.313 | 0.403 | 0.273 | 0.246 | 0.481 | 72.65 |
|  | 0.093 | 0.712 | 0.195 | 0.112 | 0.630 | 0.258 | 76.85 |
|  | 0.112 | 0.630 | 0.258 | 0.129 | 0.545 | 0.326 | 74.85 |
|  | 0.129 | 0.545 | 0.326 | 0.139 | 0.469 | 0.392 | 74.85 |
|  | 0.139 | 0.469 | 0.392 | 0.150 | 0.380 | 0.470 | 73.85 |
|  | 0.150 | 0.380 | 0.470 | 0.153 | 0.307 | 0.540 | 72.90 |
|  | 0.153 | 0.307 | 0.540 | 0.156 | 0.232 | 0.612 | 72.00 |

and Lu (7) which involves numerical integration round a loop of experimental points. The loops may be closed or open-ended, but there is a restriction that the points should not be widely separated, otherwise large errors are introduced by the numerical integration process. The condition for consistency is

$$
\sum_{n=1}^{N} \sum_{i} x_{i(n)}\left[\ln \gamma_{i(n-1:}-\ln \gamma_{i, n+1}\right]=0
$$

where $n$ refers to the $n$th point in a loop of $N$ points. For an open loop, $(n-1)$ is put equal to 1 for the first point, and ( $n+1$ ) is put equal to $N$ when the last point is reached. The test was applied to the open loops of seven points obtained from each experimental run of the six-stage still. The average deviation on 20 runs was $19.9 \%$. Li and Lu give no indication of what value is allowable, but two runs had deviations of 66.8 and $51.9 \%$ and were discarded. The mean deviation of the remaining 18 runs reported in Table XI is $15.5 \%$. The $\mathrm{Li}-\mathrm{Lu}$ test was refined by McDermott and Ellis (8), who suggested that the equation

$$
D=\sum\left[x_{i n i}+x_{i|n+1|}\right]\left[\ln \gamma_{i, n+1}-\ln \gamma_{i n}\right]=0
$$

should be the consistency condition. The points are taken in pairs, and $D$ could be allowed to be 0.01 for a sample composition measurement accurate to $\pm 0.001$ mole fraction. For 17 out of 100 pairs of points, $D$ was greater than 0.01 . Judgment is required as to which points to reject. Of the two loops rejected by the $\mathrm{Li}-\mathrm{Lu}$ test, only one would be inconsistent on the McDermott-Ellis test. The other had no $D$ value above 0.01 , but four of its values were above 0.005 . A rather more complicated test for consis-
tency over a composition path has been suggested by Tao (12). Over the 20 composition paths, the average deviation was $-1.9 \%$. Excluding the two inconsistent runs reduced the average deviation to $+1.5 \%$. No criterion is suggested by Tao of how small the deviation must be for acceptability. One disadvantage of this test is that it cannot be used if the liquid compositions for one component are the same at both ends of a composition path, as in run 3 of Table XI.

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