# Dynamic Viscosity of Gas Mixtures 

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#### Abstract

The viscosity of five binary gas mixtures-namely, oxygen-hydrogen, oxygen-nitrogen, oxygen-carbon dioxide, carbon dioxide-nitrogen, carbon dioxide-hydrogen-and two ternary mixtures-oxygen-nitrogen-carbon dioxide and oxygen-hydrogen-carbon dioxide-were determined at ambient temperature and pressure using an oscillating disk viscometer. The theoretical expressions of several investigators were in good agreement with the experimental results obtained with this viscometer. In the case of the ternary gas mixture oxygen-carbon dioxide-nitrogen, as long as the volumetric ratio of oxygen to carbon dioxide in the mixture was maintained at 11 to 8 , the viscosity of the ternary mixture at ambient temperature and pressure remained constant irrespective of the percentage of nitrogen present in the mixture.


IMANY engineering processes, particularly in transport phenomena, knowledge of the viscosity of gases and gas mixtures is necessary. Although viscosity data of pure gases are readily available, data for gas mixtures are lacking. An investigation was undertaken to determine the viscosity of binary and ternary gas mixtures resulting from the combination of hydrogen, oxygen, nitrogen, and carbon dioxide. Five binary mixtures, $\mathrm{O}_{2}-\mathrm{N}_{2}, \mathrm{O}_{2}-\mathrm{H}_{2}, \mathrm{O}_{2}-\mathrm{CO}_{2}, \mathrm{CO}_{2}-\mathrm{N}_{2}$, and $\mathrm{CO}_{2}-\mathrm{H}_{2}$, and two ternary mixtures, $\mathrm{CO}_{2}-\mathrm{O}_{2}-\mathrm{N}_{2}$ and $\mathrm{CO}_{2}-$ $\mathrm{O}_{2}-\mathrm{H}_{2}$, were studied at ambient temperature and pressure.

There are several methods available for the determination of viscosity of gases. Kestin et al. (4, 6, 9-12) have recently established that the oscillating disk viscometer is suitable for the determination of viscosity of gases at high pressures and ambient temperature. An oscillating disk viscometer was designed and developed for the present investigation. The design and operation of the viscometer is similar to that used by Kestin and Pilarczyk (9). The following equations derived according to Newell's theory (13) were used for evaluating the viscosity of gas mixtures.

$$
\begin{gather*}
C_{N}=\left[\frac{2 I}{\pi \rho b R^{4}}\left(\frac{\Delta}{\theta}-\Delta\right)+a \frac{\Delta}{\theta}\right] \beta^{2}+ \\
{\left[f \frac{3 \Delta^{2}-1}{\theta^{2}}\right] \beta^{4}+\left[h \frac{\Delta\left(\Delta^{2}-1\right)}{\theta}\right] \beta^{6}}  \tag{1}\\
\mu=\frac{2 \pi \rho b^{2}}{\beta^{2} T_{0}} \tag{2}
\end{gather*}
$$

## EXPERIMENTAL

Methods and Equipment. The oscillating disk viscometer employed in this investigation is shown in Figure 1. The main component parts are an oscillating disk, a fine suspension wire, two fixed plates, a stem with a mirror, and the container.

The oscillating disk, 1 , is a lapped glass disk of about $70-\mathrm{mm}$. diameter having a hole of $3-\mathrm{mm}$. diameter at the center. A miniature, specially made, two-jaw chuck, 2 , is fitted to the hole in the glass disk. Platinum-rhodium wire, 3 , having a diameter of 0.002 inch, is used as the suspension wire. The wire is gripped at one end by the two-jaw chuck of the glass disk, the other end being held in four-jaw chuck, 4. The lower portion of the two-jaw chuck carries a brass stem, 5 , of diameter $2-\mathrm{mm}$. and length $155-\mathrm{mm}$. A mirror $4 \times 4 \mathrm{~mm}$. is fixed at its lower end. This end of the brass stem is shaped to engage in an iron pole piece, 6 , for starting the oscillations by an external magnet.


Figure 1. Oscillating disk viscometer

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OSCILLATING DISK. (GLASS DISK)
TWO-JAW CHUCK.
Pt-rh SUSPENSION WIRE.
FOUR-JAW CHUCK.
BRASS STEM.
POLE PIECE.
FIXED PLATES (LAPPED ALUMINIUM-
TCI,TC 2-THERMOCOUPLES.
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The glass disk oscillates between the two lapped aluminium disks (fixed plates), 7. The gap between the oscillating disk and fixed aluminium disks is about 1 mm . Two calibrated copper-constantan thermocouples, $T C_{1}, T C_{2}$, located close to the fixed plates, are used to determine the temperature of the gas. The oscillations are observed with

## Table 1. Characteristics of the Suspension System

| Radius of the disk | 3.487 cm. |
| :--- | :---: |
| Moment of inertia of the suspension |  |
| system | $57.06 \mathrm{~g} . \mathrm{sq} . \mathrm{cm}$. |
| Total gap, $D$ | 0.2997 cm. |
| Upper separation, $b_{1}$ | 0.0980 cm. |
| Lower separation, $b_{2}$ | 0.1019 cm. |
| Thickness of the disk, $d$ | 0.0998 cm. |
| Damping decrement in vacuum, $\Delta_{0}$ | 0.0001 |
| Period of oscillation in vacuum, $T_{0}$ | 34.28 sec. |
| Distance of the scale from the mirror | 200.8 cm. |

the help of a standard lamp and scale arrangement. The characteristics of the suspension system are given in Table I.

The gas mixtures for investigation were mixed outside the viscometer in a separate mixing chamber fitted with a needle valve and a combined vacuum and pressure gage.
The residual gases were evacuated before filling the chamber with the test gas mixture. This mixture was allowed to remain thus for two days to ensure thorough mixing by diffiusion. The gas mixture was periodically analyzed during this period using an Orsat apparatus to assure a homogeneous composition of the mixture. At the end of two days, experiments were conducted for the determination of viscosity.

The diameter of the glass disk was determined within $0.01 \%$ with a toolmaker's microscope and slip gages. The

Table II. Viscosity Data of Binary Gas Mixtures ${ }^{\text {a }}$

| $x_{1}, \%$ | $x_{2}, \%$ | Period, $T_{1}$ Sec. | Damping Decrement, $\Delta$ | Temperature, ${ }^{\circ} \mathrm{C}$. | Density, G./Cc. | Experimental Viscosity, <br> $\mu \times 10^{6}$ Poise | Viscosity by Wilke's (17) Equation $\mu \times 10^{6}$ Poise | Viscosity by Saxena and Gambhir's (14) Equation, $\mu \times 10^{6}$ Poise |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{O}_{2}-\mathrm{N}_{2}$ Mixture |  |  |  |  |  |  |  |  |
| 0 | 100 |  |  | 25.02 |  | 177.96 |  |  |
| 13.2 | 86.8 | 34.49 | 0.0602 | 24.45 | 0.00117 | 178.50 | 181.20 |  |
| 25.6 | 74.4 | 34.50 | 0.0612 | 25.20 | 0.00114 | 184.50 | 185.50 |  |
| 41.0 | 59.0 | 34.50 | 0.0626 | 25.30 | 0.00117 | 188.55 | 190.18 | . . |
| 51.0 | 49.0 | 34.51 | 0.0636 | 25.02 | 0.00118 | 191.00 | 192.70 |  |
| 66.0 | 34.0 | 34.51 | 0.0649 | 25.02 | 0.00120 | 196.50 | 197.95 |  |
| 76.0 | 24.0 | 34.51 | 0.0655 | 25.02 | 0.00122 | 197.50 | 199.80 |  |
| 100 | 0 | 34.54 | 0.0692 | 29.45 | 0.00125 | 208.00 | ... | $\ldots$ |
| $\underset{\%}{\mathrm{O}_{2}-\mathrm{CO}_{2} \text { Mixture }} \underset{x_{2}=\%}{ } \text { volume } \mathrm{CO}_{2}$ |  |  |  |  |  |  |  |  |
| 0 | 100 | 34.49 | 0.0500 | 27.55 | 0.00162 | 149.90 |  |  |
| 8.3 | 91.7 | 34.49 | $0.0520^{\circ}$ | 24.40 | 0.00158 | 154.20 | 152.40 | 153.15 |
| 20.0 | 80.0 | 34.49 | 0.0534 | 24.40 | 0.00164 | 159.50 | 157.70 | 159.30 |
| 29.0 | 71.0 | 34.50 | 0.0557 | 25.00 | 0.00161 | 166.00 | 162.20 | 165.70 |
| 44.0 | 56.0 | 34.50 | 0.0574 | 25.00 | 0.00153 | 177.10 | 169.90 | 174.32 |
| 66.1 | 33.9 | 34.50 | 0.0614 | 25.00 | 0.00142 | 184.50 | 182.40 | 187.37 |
| 69.4 | 30.6 | 34.50 | 0.0619 | 25.00 | 0.00141 | 186.00 | 184.70 | 189.04 |
| 80.5 | 19.5 | 34.50 | 0.0634 | 24.20 | 0.00136 | 189.50 | 191.90 | 195.16 |
| 100 | 0 | 34.54 | 0.0692 | 29.45 | 0.00125 | 208.00 |  |  |
| $\mathrm{N}_{2}-\mathrm{CO}_{2}$ Mixture <br> $\%$ volume $\mathrm{N}_{2} \quad x_{2}=\%$ volume $\mathrm{CO}_{2}$ |  |  |  |  |  |  |  |  |
| 0 | 100 | 34.49 | 0.0500 | 27.55 | 0.00162 | 149.90 |  |  |
| 20.0 | 80.0 | 34.50 | 0.0521 | 23.80 | 0.00162 | 152.70 | 153.60 | 159.48 |
| 25.0 | 75.0 | 34.50 | 0.0524 | 24.78 | 0.00158 | 156.90 | 155.10 | 163.60 |
| 42.0 | 58.0 | 34.57 | 0.0542 | 24.00 | 0.00185 | 161.00 | 159.40 | 168.45 |
| 67.4 | 32.6 | 34.50 | 0.0562 | 23.45 | 0.00131 | 167.20 | 167.50 | 176.90 |
| 72.3 | 27.7 | 34.51 | 0.0567 | 24.00 | 0.00128 | 169.20 | 168.60 | 178.20 |
| 77.4 | 22.6 | 34.52 | 0.0566 | 23.80 | 0.00126 | 170.10 | 169.80 | 177.70 |
| 100 | 0 |  |  | 25.02 | . . | 177.96 |  | ... |
| $\mathrm{H}_{2}-\mathrm{CO}_{2} \text { Mixture }$ <br> \% volume $\mathrm{H}_{2} \quad x_{2}={ }_{5}$ volume $\mathrm{CO}_{2}$ |  |  |  |  |  |  |  |  |
| 0 | 100 | 34.49 | 0.0500 | 27.55 | 0.00162 | 149.90 |  | . . |
| 10.0 | 90.0 | 34.50 | 0.0493 | 23.80 | 0.00158 | 148.52 | 150.80 | $\ldots$ |
| 22.0 | 78.0 | 34.50 | 0.0507 | 24.00 | 0.00138 | 150.42 | 150.50 | . . . |
| 44.0 | 56.0 | 34.50 | 0.0515 | 23.80 | 0.00101 | 150.70 | 151.90 | . . |
| 61.6 | 38.4 | 34.50 | 0.0508 | 24.30 | 0.00072 | 150.00 | 149.10 | . . |
| 63.0 | 37.0 | 34.50 | 0.0504 | 24.25 | 0.00070 | 149.00 | 148.80 | . . |
| $\begin{gathered} \mathrm{O}_{2}-\mathrm{H}_{2} \text { Mixture } \\ =\sigma_{6} \text { volume } \mathrm{O}_{2} \quad x_{2}=x_{\%} \text { volume } \mathrm{H}_{2} \end{gathered}$ |  |  |  |  |  |  |  |  |
| 25.0 | 75.0 | 34.49 | 0.0520 | 24.00 | 0.00038 | 156.00 | 170.00 | 158.25 |
| 36.7 | 63.3 | 34.49 | 0.0567 | 24.20 | 0.00052 | 170.00 | 184.20 | 174.30 |
| 57.5 | 42.5 | 34.49 | 0.0619 | 24.20 | 0.00077 | 186.00 | 197.40 | 191.05 |
| 65.0 | 35.0 | 34.49 | 0.0638 | 24.00 | 0.00086 | 192.20 | 199.82 | 195.35 |
| 74.5 | 25.5 | 34.49 | 0.0642 | 24.20 | 0.00096 | 195.30 | 202.74 | 199.75 |
| 81.7 | 18.3 | 34.50 | 0.0652 | 24.20 | 0.00105 | 196.00 | 203.81 | 201.73 |
| 100 | 0 | 34.54 | 0.0692 | 29.45 | 0.00125 | 208.00 | . . | . . |
| ${ }^{\circ}$ All data at 1 | .a. |  |  |  |  |  |  |  |

mass of the glass disk, the two-jaw chuck, and the brass stem with mirror was determined with a precision balance within $0.001 \%$, and the moment of inertia of the oscillating system was known with an uncertainty of $0.01 \%$. The harmonic mean separation, $b$, was known with an uncertainty better than $0.6 \%$ from careful measurements of upper separation, total gap between the two lapped aluminium disks, and thickness of the glass disk.

The period of oscillation in vacuum, $T_{0}$, was determined by an indirect method suggested by Kestin and Moszynski (7). The period of oscillation in the gas mixture, $T$, was measured with a precision stop watch, and two sets of readings for five complete oscillations were taken and averaged. The uncertainty in the measured time for five complete oscillations in any experiment was better than $0.06 \%$.

The damping decrement in vacuum, $\Delta_{0}$, for platinumrhodium wire ( 8 ) is 0.0001 . The damping decrement, $\Delta$, in gas was calculated from the equation

$$
\begin{equation*}
\Delta=\frac{1}{2 \pi m} \ln \frac{\left(\alpha_{n-m}\right)+\left(\beta_{n-m}\right)}{\alpha_{n}+\beta_{n}} \tag{3}
\end{equation*}
$$

where $\alpha_{n}, \beta_{n}$ are the angular amplitudes to the right and to the left, respectively, at the commencement of measurement for $\Delta$ and $\alpha_{n-m}, \beta_{n+m}$ are the corresponding angular amplitudes to the right and to the left after the completion of $m$ oscillations. The angular amplitudes were evaluated from linear amplitudes to the right and to the left measured from the zero postion on the scale situated at a distance, $L$, from the mirror. The number of oscillations, $m$, was so chosen that the ratio $\left(\alpha_{n+m}+\beta_{n+m} / \alpha_{n}+\beta_{n}\right)$ was as near to the Napierian base $e$ as possible. This improves the precision of the determination of $\Delta$. Depending on the rate of damping, four or five values of $\Delta$ were obtained during each run and averaged. The average value of 18 to 20 decrements was taken as the final value of the decrement for the calculation of the viscosity.

The temperature of the gas mixture was measured with the two calibrated copper-constantan thermocouples; the e.m.f. developed was measured with a precision L and N potentiometer of accuracy 0.01 mv . The pressure of the gas mixture in the viscometer was kept slightly above atmospheric and was measured by a manometer. The absolute pressure of the gas was determined within $0.07 \%$.

The density of the gas mixture was estimated from the known values of pressure, temperature, and percentage composition. The uncertainties in the density estimates varied in extreme cases from 0.2 to $0.8 \%$ for the gas mixtures $\mathrm{O}_{2}-\mathrm{N}_{2}, \mathrm{O}_{2}-\mathrm{CO}_{2}, \mathrm{CO}_{2}-\mathrm{N}_{2}$, and $\mathrm{CO}_{2}-\mathrm{O}_{2}-\mathrm{N}_{2}$. The uncertainties in the density estimates of the remaining gas mixtures $\mathrm{O}_{2}-\mathrm{H}_{2}, \mathrm{CO}_{2}-\mathrm{H}_{2}$, and $\mathrm{CO}_{2}-\mathrm{O}_{2}-\mathrm{H}_{2}$ were larger owing to the presence of hydrogen and varied in extreme cases from 3 to $7 \%$.

Calibration of the Viscometer. The apparatus was calibrated using nitrogen. The viscosity of nitrogen was taken from the published data indicated below:

Kestin and Pilarczyk (9), $175.55 \times 10^{-6}$ poise at $21^{\circ} \mathrm{C}$.
Temperature correction factor taken from (9), $0.455 \times$ $10^{-6}$ poise per ${ }^{\circ} \mathrm{C}$.
Viscosity of $\mathrm{N}_{2}$ corrected to $30.56^{\circ} \mathrm{C}$. is $179.99 \times 10^{-6}$ poise (average)

Temperature inside viscometer $=30.56^{\circ} \mathrm{C}$.
Pressure in the viscometer $=14.20$ p.s.i.a.
Period of oscillation in gas, $T_{1}=34.35$ seconds
Damping decrement in gas

$$
\begin{aligned}
د & =0.0595 \\
a & =0.66690 \\
f & =0.02227 \\
h & =0.00851
\end{aligned}
$$

Substituting the relevant quantities in Equation 1 and neglecting $\Delta^{3}$, the calibrated instrument constant was obtained as $C_{N}=1.489$.

## TEST RESULTS AND DISCUSSION

The viscosities of air, oxygen, and carbon dioxide were determined at ambient temperature and pressure to check the accuracy of the viscometer. The viscosity thus determined is compared with the data from other sources.

Viscosity of air: $\mu=185.6 \times 10^{-6}$ poise at $30.56^{\circ} \mathrm{C}$. (present investigation). This value of the viscosity of air is compared with data of other investigators. All values are corrected to a temperature of $30.56^{\circ} \mathrm{C} .: \mu=186.47$ $\times 10^{-6}$ poise, Bearden (1); $\mu=186.80 \times 10^{-6}$ poise, Hirschfelder, Bird, and Spotz (3); $\mu=187.28 \times 10^{-6}$ poise, Kestin and Leidenfrost (6); $\mu=186.56 \times 10^{-6}$ poise, Kestin and Wang (10).


Figure 2. Viscosity data of $\mathrm{O}_{2}-\mathrm{N}_{2}$ (left), $\mathrm{O}_{2}-\mathrm{CO}_{2}$ (center), and $\mathrm{N}_{2}-\mathrm{CO}_{2}$ (right) mixtures
A I.I.Sc DATA.
$X$ BUDDENBERG \& WILKE $25^{\circ} \mathrm{C}$ (2)
0 TRAUTZ \& MELSTER $26.9{ }^{\circ} \mathrm{C}$ (15)

- KESTIN, KOBAYASHI \& WOOD $24.55^{\circ} \mathrm{C}$ (5)

Viscosity of oxygen: $\mu=208.00 \times 10^{-6}$ poise at $29.18^{\circ} \mathrm{C}$. (present investigation). This value of the viscosity of oxygen is compared with the results of the following investigators. All values are corrected to a temperature of $29.18^{\circ} \mathrm{C} .: \mu$ $=208.50 \times 10^{-6}$ poise, Hirschfelder, Bird, and Spotz (3); $\mu=208.97 \times 10^{-6}$ poise, Kestin and Leidenfrost (6).

Viscosity of carbon dioxide: $\mu=149.90 \times 10^{-6}$ poise at $27.55^{\circ} \mathrm{C}$. (present investigation). The present results are compared with the data of Kestin and Leidenfrost (6) and Hirschfelder, Bird, and Spotz (3), the values being corrected to $27.55^{\circ} \mathrm{C}$.: $\mu=150.70 \times 10^{-6}$ poise, Kestin and Leidenfrost (6); $\mu=149.80 \times 10^{-6}$ poise, Hirschfelder, Bird, and Spotz (3).

Data obtained in the present investigation agreed within $\pm 1^{\%}$ of the data of earlier investigators ( $1,3,6,10$ ). This indicated the suitability of the apparatus for determining the viscosity of gas mixtures at ambient temperature and pressure.

Viscosity of Binary Gas Mixtures. The viscosity of binary gas mixtures investigated are given in Table II.

Oxygen-Nitrogen. The data for the oxygen-nitrogen mixture are shown in Figure 2 and in Table II. Most of the experimental values agree within $1 \%$ of the equation given by Wilke (17). Also shown in the figure are the data of Buddenberg and Wilke (2) at $25^{\circ} \mathrm{C}$., and Trautz and Melster (15) at $26.9^{\circ} \mathrm{C}$. The present authors' results are in close agreement with the earlier data.

Oxygen-Carbon Dioxide. The present experimental results are compared with the equations of Wilke (17) and Saxena and Gambhir (14) (Figure 2 and Table II). At lower percentages of oxygen, the Sexena and Gambhir's equation gives results which are in close agreement with the experimental results. Their equation, however, predicts slightly higher viscosity for percentages of oxygen between 30 and $90 \%$, whereas Wilke's equation (17) is closer to the experimental data.

Nitrogen-Carbon Dioxide. The results of this gas mixture are shown in Figure 2 and Table II. Wilke's equa-
tion (17) is within about $\pm 1 \%$ of the experimental results. The Saxena and Gambhir's equation (14) predicts viscosities which are higher than the experimental values. Also shown in Figure 2 are the experimental data of Kestin, Kobayashi, and Wood (5) at $24.55^{\circ} \mathrm{C}$.

Hydrogen-Carbon Dioxide. The results are presented in Table II and Figure 3. The small increase in viscosity with the addition of moderate amounts of hydrogen to carbon dioxide was followed by the instrument. The present experimental data are in good agreement with the equation of Wilke (17) and the results of Hirschfelder, Bird, and Spotz (3).

Oxygen-Hydrogen. The experimental results are shown in Figure 3 and Table II. The experimental results of Trautz and Melster (15) at $26.9^{\circ} \mathrm{C}$., and Van Itterbeek, Van Paemel, and Vanlierde (16) at $20.6^{\circ} \mathrm{C}$. are also shown on the same figure. Wilke's equation (17) gives values of viscosity which are higher than the experimental values, whereas those of Saxena and Gambhir (14) are in good agreement with the present results.

Viscosity of Ternary Gas Mixtures. Oxygen-HydrogenCarbon Dioxide. The experimental results are given in Table III. The data calculated from•Wilke's (17) equation are shown for comparison. In general, the agreement is good. No experimental data are available in the literature for this combination.
Oxygen-Nitrogen-Carbon Dioxide. The results of this ternary mixture are shown in Table III and compared with Wilke's equation. The agreement between experimental and theoretically predicted data is good. An interesting observation was made during this experimentation.

Wilke's (17) equation for a ternary mixture is:

$$
\begin{align*}
\mu_{n 2}=\frac{\mu_{2}}{1+\left(x_{2} / x_{1}\right) \phi_{12}+\left(x_{13} / x_{1}\right) \phi_{12}} & +\frac{\mu_{2}}{1+\left(x_{3} / x_{2}\right) \phi_{23}+\left(x_{1} / x_{2}\right) \phi_{21}}
\end{align*}+
$$



Figure 3. Viscosity data of $\mathrm{H}_{2}-\mathrm{CO}_{2}$ (left) and $\mathrm{O}_{2}-\mathrm{H}_{2}$ (right) mixtures
$\triangle$ LI.Sc. DATA

- HIRSCHFELDER etal (3)

O TRAUTZ \& MELSTER $26.0^{\circ} \mathrm{C}$ (15)
$\nabla$ VAN ITTEREEEK $20.6^{\circ} \mathrm{C}$ (16)

Table III. Viscosity Data of Ternary Gas Mixtures ${ }^{a}$

| $x_{1} \%$ | $x_{2}, \%$ | $x_{3}, \%$ | Period, $T_{1}$ Sec. | Damping Decrement, $\Delta$ | Temperature, ${ }^{\circ} \mathrm{C}$. | Density, G. / Cc. | Experimental Viscosity, $\mu \times 10^{6}$ Poise | Viscosity by Wilke's (17) Equation, $\mu \times 10^{6}$ Poise |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $x_{1}=\%$ volume $\mathrm{O}_{2}$ |  | $\begin{aligned} & \mathrm{O}_{2}-\mathrm{H}_{2}-\mathrm{CO}_{2} \text { Mixture } \\ & x_{2}=\% \text { volume } \mathrm{H}_{2} \quad x_{3}=\% \text { volume } \end{aligned}$ |  |  |  |  |
| 12.3 | 74.9 | 12.8 | 34.50 | 0.0541 | 25.30 | 0.00044 | 158.5 | 155.50 |
| 15.3 | 14.7 | 70.0 | 34.50 | 0.0530 | 25.80 | 0.00142 | 157.8 | 157.65 |
| 16.0 | 6.0 | 78.0 | 34.50 | 0,0534 | 26.10 | 0.00156 | 158.0 | 156.20 |
| 21.7 | 69.1 | 9.2 | 34.48 | 0.0557 | 24.80 | 0.00049 | 166.1 | 167.30 |
| 23.7 | 58.9 | 17.4 | 34.50 | 0.0562 | 24.95 | 0.00065 | 167.2 | 168.95 |
| 26.7 | 14.3 | 59.0 | 34.51 | 0.0558 | 24.78 | 0.00142 | 166.7 | 163.58 |
| 30.6 | 50.0 | 19.4 | 34.50 | 0.0577 | 23.80 | 0.00077 | 171.2 | 174.60 |
| 39.2 | 18.8 | 42.0 | 34.50 | 0.0575 | 24.50 | 0.00124 | 171.0 | 171.46 |
| 44.2 | 22.0 | 33.8 | 34.50 | 0.0598 | 23.80 | 0.00117 | 178.6 | 176.07 |
| 44.7 | 22.0 | 33.3 | 34.50 | 0.0587 | 23.10 | 0.00116 | 168.5 | 177.16 |
| 48.3 | 40.3 | 11.4 | 34.51 | 0.0608 | 23.80 | 0.00090 | 181.9 | 186.90 |
| 49.3 | 36.0 | 14.7 | 34.51 | 0.0610 | 23.68 | 0.00091 | 181.8 | 176.60 |
|  |  |  |  |  |  | $\mathrm{CO}_{2}$ |  |  |
| 3.3 | 88.3 | 8.4 | 34.50 | 0.0587 | 24.30 | 0.00117 | 174.5 | 175.28 |
| 9.0 | 81.2 | 9.8 | 34.50 | 0.0592 | 24.68 | 0.00118 | 175.5 | 176.00 |
| 11.4 | 79.6 | 9.0 | 34.50 | 0.0590 | 22.78 | 0.00118 | 179.0 | 180.28 |
| 11.8 | 73.6 | 14.6 | 34.50 | 0.0592 | 25.30 | 0.00114 | 176.0 | 176.44 |
| 20.3 | 50.0 | 29.7 | 34.50 | 0.0583 | 24.65 | 0.00134 | 174.0 | 171.38 |
| 21.3 | 28.0 | 50.7 | 34.50 | 0.0568 | 24.65 | 0.00146 | 168.0 | 164.60 |
| 23.0 | 68.0 | 9.0 | 34.50 | 0.0610 | 23.79 | 0.00120 | 180.5 | 179.10 |
|  |  |  | lume $\mathrm{O}_{2}$ | $\begin{aligned} & \mathrm{O}_{2}-\mathrm{N}_{2}-\mathrm{CO}_{2} \mathrm{Mi} \\ & x_{2}=\% \text { volume } \end{aligned}$ | $\mathrm{N}_{2} \quad x_{3}=\%$ | me $\mathrm{CO}_{2}$ |  |  |
| 8.0 | 85.8 | 6.2 | 34.50 | 0.0585 | 24.30 | 0.00128 | 177.8 | 177.89 |
| 16.9 | 70.3 | 12.8 | 34.50 | 0.0597 | 24.70 | 0.00122 | 176.5 | 179.90 |
| 26.8 | 52.0 | 21.2 | 34.50 | 0.0602 | 24.30 | 0.00128 | 176.5 | 177.40 |
| 33,4 | 40.0 | 26.6 | 34.50 | 0.0594 | 24.15 | 0.00133 | 176.5 | 176.80 |

${ }^{a}$ All data at 14.2 p.s.i.a.
where subscript 1 refers to oxygen, 2 refers to nitrogen, and 3 refers to carbon dioxide.

Let:

$$
\left.\begin{array}{l}
F_{1}=1+\left(x_{2} / x_{1}\right) \phi_{12}+\left(x_{3} / x_{1}\right) \phi_{13}  \tag{5}\\
F_{2}=1+\left(x_{1} / x_{3}\right) \phi_{31}+\left(x_{2} / x_{3}\right) \phi_{32}
\end{array}\right\}
$$

Now, if the percentage of nitrogen, $x_{2}$, is held fixed at a certain value-e.g., $90 \%$-then, because $x_{1}+x_{2}+x_{3}$ $=100, x_{1}+x_{3}=10$. Hence, $x_{1}$ and $x_{3}$ can take various values such that $x_{1}+x_{3}=10$.
Thus,

$$
\begin{array}{lll}
x_{1}=1, & x_{3}=9, & x_{1} / x_{3}=1 / 9 \\
x_{1}=2, & x_{3}=8, & x_{1} / x_{3}=1 / 4 \\
x_{1}=3, & x_{3}=7, & x_{1} / x_{3}=3 / 7 \\
x_{1}=9, & x_{3}=1, & x_{1} / x_{3}=9
\end{array}
$$

In all cases, $x_{1}+x_{2}+x_{3}=100$ with $x_{2}$ fixed at 90 .
If $F_{1}$ is calculated for all the various values of $x_{1} / x_{3}$ as parameter with a fixed value of $x_{2}$, a set of values for $F_{1}$ is obtained. If these values of $F_{1}$ are now plotted against $x_{1} / x_{3}$ on semilog paper as shown in Figure 4, curve $A_{1}$ is obtained. If this procedure is repeated for $F_{2}$ for the same values of $x_{2}$, a set of values for $F_{2}$ is obtained. If these values of $F_{2}$ are plotted on the same graph against $x_{1} / x_{3}$, the curve $A_{2}$ is obtained. These two curves, $A_{1}$ and $A_{2}$, intersect at some point.


Figure 4. Variation of $F_{1}$ and $F_{2}$ with $x_{1} / x_{3}$ for constant values of $x_{2}$ for the gas mixture $\mathrm{O}_{2}-\mathrm{N}_{2}-\mathrm{CO}_{2}$

If now $x_{2}$ is held fixed at $80 \%$ then $x_{1}+x_{3}=20$ and if $F_{1}$ and $F_{2}$ are calculated as indicated above and plotted on the graph, two curves $B_{1}$ and $B_{2}$ are obtained. The intersection of $B_{1}$ and $B_{2}$ is at the same value of $x_{1} / x_{3}$ as that of $A_{1}$ and $A_{2}$. This is true for all percentages of nitrogen-say, $70,50 \%$, etc.-as long as the above procedure is followed. The graph shows that all such curves intersect at the value of $x_{1} / x_{3}=1.375=11 / 8$. This ratio is in the inverse ratio of the molecular weights of the individual gases, oxygen and carbon dioxide.

The viscosity of the mixture is now calculated with $x_{1} / x_{3}$ (oxygen to carbon dioxide) held fixed at $11 / 8$ and varying $x_{2}$ such that $x_{1}+x_{2}+x_{3}=100$. The viscosity of all
mixtures which had $x_{1} / x_{3}$ in the ratio $11 / 8$ was the same irrespective of the percentage of nitrogen present in the mixture. The experimental data, with $x_{1} / x_{3}$ ranging from 1.25 to 1.33 and nitrogen percentage varying widely (Table III), indicate that the viscosity was practically constant at ambient temperature and pressure as mentioned above.

## CONCLUSIONS

The viscosity values obtained from Wilke's equation for the binary gas mixtures oxygen-nitrogen, oxygen-carbon dioxide, nitrogen-carbon dioxide, and hydrogen-carbon dioxide agree within $1 \%$ with the experimental data obtained in this investigation. However, for oxygen-hydrogen mixture, Wilke's equation predicts values of viscosity which are higher by as much as $9 \%$.

In the case of oxygen-carbon dioxide and oxygenhydrogen gas mixtures, Saxena and Gambhir's equations predict viscosities which agree closely with the present experimental results. However, in the case of nitrogencarbon dioxide mixture, the equations of Saxena and Gambhir predict viscosities which are about $6 \%$ higher than those of the present investigation.

For the ternary gas mixtures oxygen-nitrogen-carbon dioxide and oxygen-hydrogen-carbon dioxide, the viscosity values predicted by Wilke's equation agree well with the present experimental results. This equation can be used for the prediction of viscosity of ternary gas mixtures.

When the volumetric ratio of oxygen to carbon dioxide is about 11 to 8 in a ternary mixture of oxygen-nitrogencarbon dioxide, the viscosity of this mixture at ambient temperature and pressure remains constant irrespective of the percentage of nitrogen in the mixture.

## NOMENCLATURE

$a=$ constant in Equation 1
$A_{1}, A_{2}=$ curves in Figure 3, plots of $F_{1}$ and $F_{2}$ against $x_{1} / x_{3}$ for the same percentage of $\mathrm{N}_{2}\left(x_{2}=90 \%\right)$
$b_{1}=$ upper separation
$b_{2}=$ lower separation
$b=$ harmonic mean separation, $2 b_{1} b_{2} /\left(b_{1}+b_{2}\right)$
$B_{1}, B_{2}=$ curves in Figure 3, plots of $F_{1}$ and $F_{2}$ against $x_{1} / x_{3}$ for the same percentage of $\mathrm{N}_{2}\left(x_{2}=80 \%\right)$
$C_{1}, C_{2}=$ curves in Figure 3, plots of $F_{1}$ and $F_{2}$ against $x_{1} / x_{3}$ for the same percentage of $\mathrm{N}_{2}\left(x_{2}=50 \%\right)$
$C_{v}=$ calibrated instrument constant
$f=$ constant in Equation 1
$F_{1}, F_{2}=$ functions given by Equation 5
$h=$ constant in Equation 1
$I=$ moment of inertia of the suspension system
$L=$ distance of the scale from the mirror
$R=$ radius of the disk
$T=$ period of oscillation of the disk in vacuum
$T_{1}=$ period of oscillation of the disk in the gas
$x_{1}, x_{2}, x_{3}=$ molar fractions of gases in the mixture
$\alpha_{n}, \alpha_{n+m}=$ angular amplitudes to the right
$B_{n}, B_{n+m}=$ angular amplitudes to the left
$\beta=$ dimensionless harmonic mean separation, $b / j$
$\delta=$ boundary layer thickness
$\lambda_{1}=$ damping decrement in vacuum
$\Delta=$ damping decrement in the gas
$\mu_{, \mu_{1}, \mu_{2}, \mu_{3}, \mu_{m}}=$ dynamic viscosity
$\rho=$ density of the fluid
${ }_{A}=$ ratio of periods of oscillation in the gas and in vacuum, $T_{1} / T_{0}$
$\phi_{12}, \phi_{13}, \phi_{22}, \phi_{23}, \phi_{31}, \phi_{32}=$ constants in Equation 4

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