

# Solid-Liquid Phase Equilibria of Binary and Ternary Mixtures of Benzene and Polynuclear Aromatic Compounds

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Solid-liquid equilibrium phase diagrams for three binary mixtures of benzene with polynuclear aromatic compounds (fluorene, dibenzofuran, and dibenzothiophene) have been measured by a dynamic method in the temperature range from 0 to 80 °C. These binary data plus previously published data derived from calorimetric studies were used to determine experimental activity coefficients and interaction parameters using the Wilson and UNIQUAC models. The resulting set of interaction parameters gave a good representation of the experimental binary data and ternary data at 50 °C, but were not generally satisfactory for predicting solid-liquid equilibrium in ternary systems at lower temperatures.

## Introduction

Separation of organic close-boiling or close-melting components from a mixture is a challenging task in some chemical engineering processes, such as crystallization and extraction. With the present trend in the petroleum industry toward heavier feedstocks and coal-derived liquids, there is a strong need for expansion of the small database of thermodynamic data on heavier compounds that presently exists. The present work is a continuation of systematic studies on solid-liquid equilibrium (SLE) in binary and ternary systems of polynuclear aromatic compounds (1-7). There are very few data available for ternary aromatic systems consisting of solvent-solid-solid at different temperatures. Some results of this kind are reported in refs 8-10, for example. In our previous study (6), we examined the phase equilibria of two ternary systems containing *cis*-decalin, naphthalene, and biphenyl and tetralin, biphenyl, and dibenzofuran at 25 °C. The first component in each of these systems was a liquid, and the other two components were solids. These ternary systems included three pairs of binary eutectics. The ternary phase diagrams were also predicted using binary parameters obtained from regression of binary data using the UNIQUAC model.

The purpose of the present study was to select systems with a solid solution on one side of the phase diagram. In this paper we present solid-liquid phase equilibria of three ternary systems of this kind: benzene, fluorene, dibenzofuran; benzene, fluorene, dibenzothiophene; and benzene, dibenzothiophene, dibenzofuran, at temperatures from 0 to 80 °C. As indicated by Sediawan et al. (1), the fluorene-dibenzofuran system forms a solid solution at any composition, while the fluorene-dibenzothiophene and dibenzothiophene-dibenzofuran systems have solid-phase immiscibility gaps.

New SLE data are also reported for three binary systems: mixtures of benzene with fluorene, dibenzofuran, and dibenzothiophene.

An additional goal of this study is to predict solid-liquid equilibria (SLE) for ternary systems containing such polynuclear aromatic compounds by using binary Wilson or UNIQUAC parameters obtained from the regression of binary SLE data.

The first section is a discussion of the procedure for the solid-liquid equilibrium experiments. This is followed by a

brief discussion of the method of determining the Wilson and UNIQUAC constants and correlating results for binary systems. The results of the ternary system analysis are presented in the third section. The last section discusses some of the important conclusions observed in this study. We hope these data will be useful in industry for design purposes as well as in the development of solid-liquid equilibrium theories.

## Experimental Section

The chemicals used were from Aldrich Chemicals, 99% pure grade. The fluorene, dibenzofuran, and dibenzothiophene were recrystallized from toluene. They were analyzed by gas chromatography using a phenyl methyl silicone capillary column and flame ionization detector. Purities (area %) were 99.7% for fluorene, 99.8% for dibenzofuran, and 99.4% for dibenzothiophene. Benzene was purified by fractional distillation and stored over 4A molecular sieves.

Solubilities were determined by a dynamic (synthetic) method described in full by Domanska (7). Mixtures of solute and solvent in binary systems and of two solutes and solvent (benzene) in ternary systems, prepared by weighing, were heated very slowly (heating rate did not exceed 2 °C h<sup>-1</sup> near the equilibrium temperature) with stirring. The temperature at which the last crystals disappeared (decline of solution cloudiness) was taken as the temperature of the solution-crystal equilibrium. Measurements were performed over a wide range of solute concentration and over the temperature range from 0 °C to the boiling temperature of benzene (80.3 °C). The accuracy of temperature measurements was ±0.1 °C. The mixture composition was known within ±0.0003 mole fraction. The saturation temperature for a given mixture was reproducible within ±0.1 °C.

The characteristics of the compounds are collected in Table I. All the direct experimental data are shown in Tables II-V.

Table I. Thermodynamic Properties of Solutes

solute	$t_m/$ °C	$\Delta H_m/$ (J mol <sup>-1</sup> )	$\Delta C_p/$ (J mol <sup>-1</sup> )	$V_m^{20}/$ (cm <sup>3</sup> mol <sup>-1</sup> )	$t_{12}/$ °C	$\Delta H_{12}/$ (J mol <sup>-1</sup> )
benzene	5.50	9 866 <sup>a</sup>		89.4		
fluorene	114.75	19 200 <sup>b</sup>	1.98 <sup>b</sup>	163.7		
dibenzo- furan	82.15	18 600 <sup>c</sup>	9.60 <sup>c</sup>	170.0		
dibenzo- thiophene	98.80	21 000 <sup>d</sup>	31.40 <sup>c</sup>	170.1	88.8	1500

<sup>a</sup> Oliver et al. (11). <sup>b</sup> Sediawan et al. (1). <sup>c</sup> Coon et al. (4).

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**Table II. Experimental Mole Fraction Solubilities and Activity Coefficients for the Three Solutes in Benzene**

fluorene (1)-benzene (2)			dibenzofuran (1)-benzene (2)			dibenzothiophene (1)-benzene (2)		
$x_1$	$t/^\circ\text{C}$	$\gamma_1$	$x_1$	$t/^\circ\text{C}$	$\gamma_1$	$x_1$	$t/^\circ\text{C}$	$\gamma_1$
0.0000	5.5		0.0000	5.5		0.0000	5.5	
0.0202	4.3	1.002 <sup>a</sup>	0.0635	2.9	1.027 <sup>a</sup>	0.0230	4.8	1.013 <sup>a</sup>
0.0407	3.8	1.008 <sup>a</sup>	0.0856	0.8	1.017 <sup>a</sup>	0.0430	4.0	1.021 <sup>a</sup>
0.0535	2.6	1.014 <sup>a</sup>	0.0934	0.2	1.015 <sup>a</sup>	0.0532	3.4	1.022 <sup>a</sup>
0.0631	2.1	1.321	0.1122	-0.8	1.369	0.0598	2.8	1.898
0.0714	4.9	1.314	0.1239	1.6	1.328	0.0663	5.0	1.821
0.0826	9.9	1.306	0.1363	4.7	1.317	0.0784	9.9	1.764
0.0986	15.7	1.294	0.1463	7.0	1.311	0.0867	12.5	1.712
0.1040	17.5	1.290	0.1599	9.6	1.288	0.1010	17.3	1.670
0.1181	21.8	1.279	0.1978	17.1	1.264	0.1152	21.5	1.633
0.1301	25.2	1.271	0.2346	23.1	1.240	0.1330	26.0	1.586
0.1548	31.5	1.242	0.2480	25.1	1.232	0.1429	28.1	1.556
0.1723	35.5	1.221	0.2813	29.3	1.202	0.1573	31.1	1.522
0.2062	42.0	1.189	0.2995	32.1	1.206	0.1699	33.5	1.494
0.2609	51.3	1.160	0.3559	38.8	1.183	0.2066	39.7	1.425
0.3144	58.4	1.140	0.4486	46.8	1.119	0.2516	46.0	1.354
0.3565	64.3	1.113	0.5791	55.9	1.051	0.3038	52.4	1.295
0.3792	66.5	1.110	0.7045	64.5	1.023	0.3601	57.5	1.222
0.4253	71.3	1.099	0.7570	67.7	1.014	0.4131	62.2	1.178
0.4538	74.5	1.094	0.8346	73.2	1.020	0.4733	67.6	1.152
0.4688	76.0	1.000	1.0000	82.1	1.000	0.5351	73.2	1.142
1.0000	114.8					0.5858	77.1	1.129
						0.6423	80.0	1.090
						1.0000	98.8	1.000

<sup>a</sup> Activity coefficients for benzene as a solute.

## Results and Discussion

The solubility of solid 1 in a liquid may be expressed by (12)

$$\ln x_1 = \frac{\Delta H_{m1}}{R} \left( \frac{1}{T_{t1}} - \frac{1}{T} \right) - \frac{\Delta C_{pm1}}{R} \left( \ln \frac{T_{t1}}{T} - \frac{T_{t1}}{T} + 1 \right) + \frac{\Delta H_{tr1}}{R} \left( \frac{1}{T_{tr1}} - \frac{1}{T} \right) - \ln \gamma_1 \quad (1)$$

where  $x_1$ ,  $\gamma_1$ ,  $\Delta H_{m1}$ ,  $\Delta C_{pm1}$ ,  $T_{t1}$ , and  $T$  stand for the mole fraction, activity coefficient, enthalpy of fusion, solute heat capacity difference between the solid and the liquid at the melting point, triple point temperature of the solute, and equilibrium temperature, respectively, and  $\Delta H_{tr}$  and  $T_{tr}$  stand for enthalpy of transition and transition temperature of the solute.

For the organic compounds of this study it is an excellent approximation to substitute the normal melting point temperature for the triple point temperature. The term involving the transition enthalpy and temperature is only needed when the solution temperature is below a solid-phase transition temperature.

In this work the activity coefficient was calculated by eq 1 from the experimentally determined solution composition and temperature. Activity coefficients obtained in this way from binary data were used to determine Wilson (13) and UNIQUAC (14) interaction parameters. The Wilson and UNIQUAC equations were then used to predict the equilibrium data for the ternary mixtures.

The parameters were fitted to the binary data by an optimization technique. The objective function was as follows:

$$F(A_1, A_2) = \sum_{i=1}^n w_i^{-2} [\ln(x_{1i} \gamma_{1i}(T_i, x_{1i}, A_1, A_2)) - \ln a_{1i}(T_i)]^2 \quad (2)$$

where  $\ln a_{1i}$  denotes an experimental value of the logarithm of solute activity ( $x_{1i} \gamma_{1i}$ , from eq 1),  $w_i$  is the weight of an experimental point,  $A_1$  and  $A_2$  are the two adjustable parameters of the correlation equations,  $i$  denotes the  $i$ th experimental point, and  $n$  is the number of experimental

**Table III. Equilibrium Data for the Ternary System Benzene (1)-Fluorene (2)-Dibenzofuran (3)<sup>a</sup>**

$x_1$	$t/^\circ\text{C}$	$x_1$	$t/^\circ\text{C}$	$x_1$	$t/^\circ\text{C}$	$x_1$	$t/^\circ\text{C}$	$x_1$	$t/^\circ\text{C}$
$x_2^0 = 0.0866$		$x_2^0 = 0.1469$		$x_2^0 = 0.2046$		$x_2^0 = 0.2437$		$x_2^0 = 0.2520$	
0.8220	12.0	0.8915	11.0	0.8178	10.4	0.8152	11.9	0.8003	15.2
0.7905	16.8	0.7857	16.9	0.7984	13.8	0.8008	15.0	0.7452	24.5
0.7555	22.0	0.7495	22.4	0.7773	18.1	0.7792	18.9	0.7193	28.6
0.7248	27.4	0.6487	35.8	0.7516	22.2	0.7288	27.5	0.6824	33.6
0.7027	29.8	0.5723	43.1	0.6795	32.1	0.6390	40.0	0.6423	38.8
0.6799	32.7	0.5051	50.0	0.6189	38.9	0.6008	43.4	0.5728	46.8
0.6549	35.6	0.3910	60.0	0.5775	44.1	0.5351	50.0	0.5251	51.7
0.6179	39.6			0.5281	49.4	0.4350	60.0	0.4310	61.2
0.5010	50.0			0.4750	54.6				
0.4351	55.7			0.4080	61.2				
0.3720	60.0								
$x_2^0 = 0.2939$		$x_2^0 = 0.3397$		$x_2^0 = 0.3736$		$x_2^0 = 0.3979$		$x_2^0 = 0.4537$	
0.8198	10.4	0.8042	15.3	0.8158	13.7	0.8311	11.9	0.8172	16.7
0.7652	20.4	0.7849	18.4	0.7943	18.0	0.8124	15.7	0.7825	23.3
0.7168	27.7	0.7384	26.4	0.7621	23.1	0.7940	19.1	0.7604	27.2
0.6972	30.0	0.7164	29.2	0.7306	30.0	0.7786	21.9	0.7092	35.1
0.6345	38.1	0.6676	36.3	0.6781	35.8	0.7612	24.8	0.6663	40.6
0.6050	42.0	0.6294	40.8	0.6368	40.2	0.7357	28.8		
0.5601	47.0	0.5477	50.8			0.6929	34.6		
0.4922	53.9					0.6270	42.8		
0.4220	60.6					0.5710	48.8		
$x_2^0 = 0.5003$		$x_2^0 = 0.5566$		$x_2^0 = 0.6005$		$x_2^0 = 0.6719$		$x_2^0 = 0.7262$	
0.8367	14.7	0.8632	11.4	0.8573	14.0	0.8742	13.2	0.8730	15.5
0.8235	17.3	0.8257	19.9	0.8383	18.5	0.8574	18.1	0.8519	20.9
0.8059	21.4	0.8040	24.1	0.8193	22.5	0.8410	21.1	0.8410	23.2
0.7703	27.5	0.7830	27.8	0.7945	27.4	0.8051	27.8	0.8146	29.0
0.7405	32.1	0.7601	31.4	0.7654	32.2	0.7680	34.5	0.7850	35.0
0.6105	49.7	0.7260	36.8	0.7394	36.3	0.7498	37.4	0.7281	43.1
		0.6830	42.6	0.7041	41.5			0.6876	51.0
		0.6118	50.7	0.6494	48.4			0.6003	61.7
				0.5689	57.6			0.4734	80.0
$x_2^0 = 0.7600$		$x_2^0 = 0.8086$		$x_2^0 = 0.8681$		$x_2^0 = 0.9138$			
0.8784	14.5	0.8808	15.8	0.8821	16.0	0.9113	9.5		
0.8373	24.6	0.8646	20.2	0.8585	24.9	0.8796	20.1		
0.8051	31.1	0.8501	23.8	0.8352	30.3	0.8452	29.1		
0.7654	39.2	0.8372	26.8	0.8271	32.5	0.8121	35.8		
0.7167	46.0	0.8202	30.6	0.8002	37.5	0.7656	44.1		
0.6781	51.2	0.8093	33.0	0.7890	41.5	0.6660	56.2		
0.6314	56.6	0.7775	38.5	0.7631	43.4	0.5871	66.2		
0.5990	61.4	0.7642	41.1	0.6903	53.4	0.4801	79.1		
0.5085	70.9	0.6757	53.3	0.6170	62.5				
		0.5842	64.0						

<sup>a</sup>  $x_2^0$  = mole fraction of the fluorene in the benzene-free binary solid mixtures.

points. The weights were calculated by means of the error propagation formula:

$$w_i^2 = \left( \frac{\partial \ln x_1 \gamma_1 - \partial \ln a_i}{\partial T} \right)_{T=T_i}^2 (\Delta T_i)^2 + \left( \frac{\partial \ln x_1 \gamma_1}{\partial x_1} \right)_{x_1=x_{1i}}^2 (\Delta x_{1i})^2 \quad (3)$$

where  $\Delta T$  and  $\Delta x_1$  are the estimated errors of  $T$  and  $x_1$ , respectively.

According to the above formulation, the objective function is consistent with the maximum likelihood principle, provided that the first-order approximation (eq 3) is valid. Neau and Peneloux (15) called this procedure the observed deviation method.

The experimental errors of temperature and solute mole fraction were fixed for all cases and set to  $\Delta T = 0.1$  K and  $\Delta x_1 = 0.001$ .

The root mean square deviation of temperature defined by eq 4, where  $T_i^{\text{calcd}}$  and  $T_i$  are, respectively, the calculated and experimental temperatures of the  $i$ th point and  $n$  is the number of experimental points, was used as a measure of the

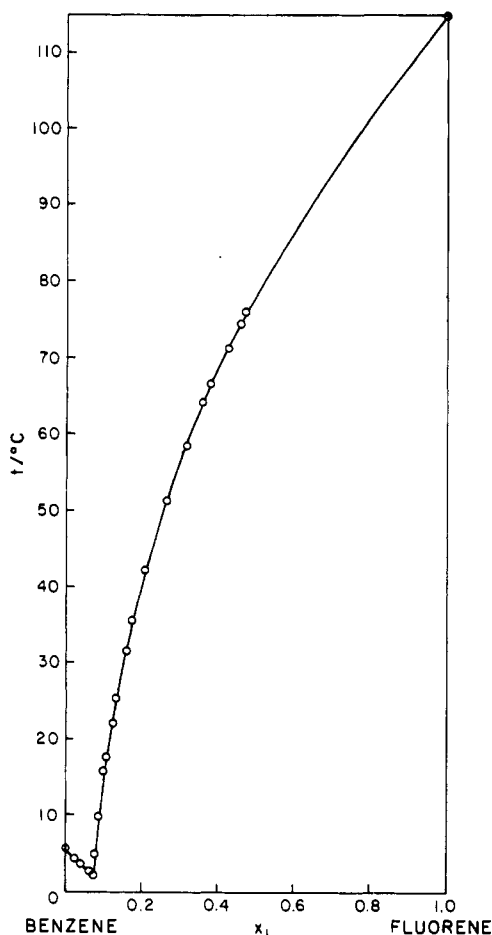
$$\sigma_T = \left[ \frac{\sum_{i=1}^n (T_i^{\text{calcd}} - T_i)^2}{(n-2)} \right]^{1/2} \quad (4)$$

goodness of fit of the solubility curves. The calculated values



**Table VI. Values of the Parameters and of the Root Mean Square Deviations of Temperature Obtained by Wilson and UNIQUAC Equations in Binary Eutectic Systems**

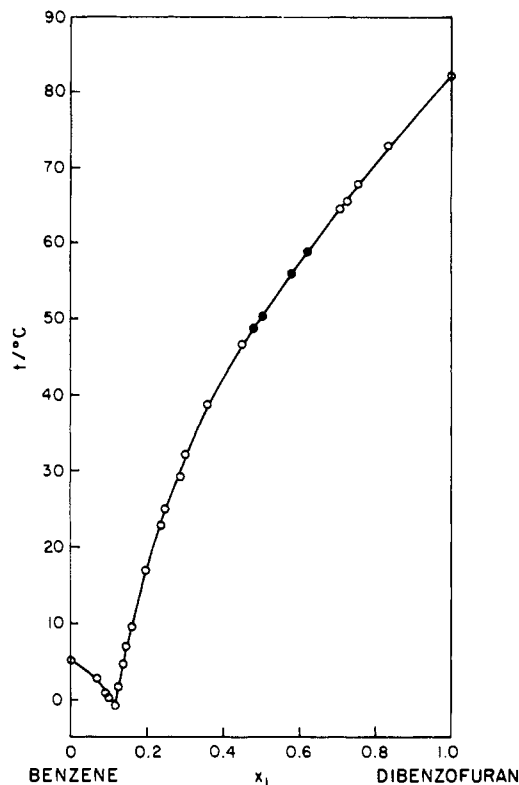
system	parameter		$\sigma_T/^\circ\text{C}$	
	Wilson $(g_{12} - g_{22})/(\text{J mol}^{-1})$	UNIQUAC $\Delta U_{12} (\Delta U_{21})/(\text{J mol}^{-1})$	Wilson	UNI- QUAC
fluorene (1)- benzene (2)	-1264.27 (2210.32)	1462.73 (-738.38)	0.32	0.37
dibenzofuran (1)- benzene (2)	-1373.34 (2354.80)	1640.56 (-852.64)	0.40	0.41
dibenzothiophene (1)- benzene (2)	-1175.65 (2828.56)	1918.68 (-830.52)	1.67	1.78
benzene (1)- fluorene (2)	78 770.36 (3053.71)	5443.90 (-2877.67)	0.35	0.25
benzene (1)- dibenzofuran (2)	-2327.28 (100 837.6)	6112.95 (-2910.86)	0.13	0.09
benzene (1)- dibenzothiophene (2)	8 432 769.6 (5077.38)	1525.97 (2 336 122.1)	0.34	0.41

**Figure 1. Solid-liquid phase diagram for fluorene (1)-benzene (2). The lines are predicted by the Wilson equation.**

$$x_2^* \gamma_2^* = x_2 \gamma_2 \exp \left[ \frac{\Delta H_{m2}}{R} \left( \frac{1}{T} - \frac{1}{T_{t2}} \right) + \frac{\Delta C_{pm2}}{R} \left( \ln \frac{T_{t2}}{T} - \frac{T_{t2}}{T} + 1 \right) \right] \quad (6)$$

where the mole fraction and activity coefficient of the solid phase are shown with an asterisk. Summing up the mole fractions leads to the two equations

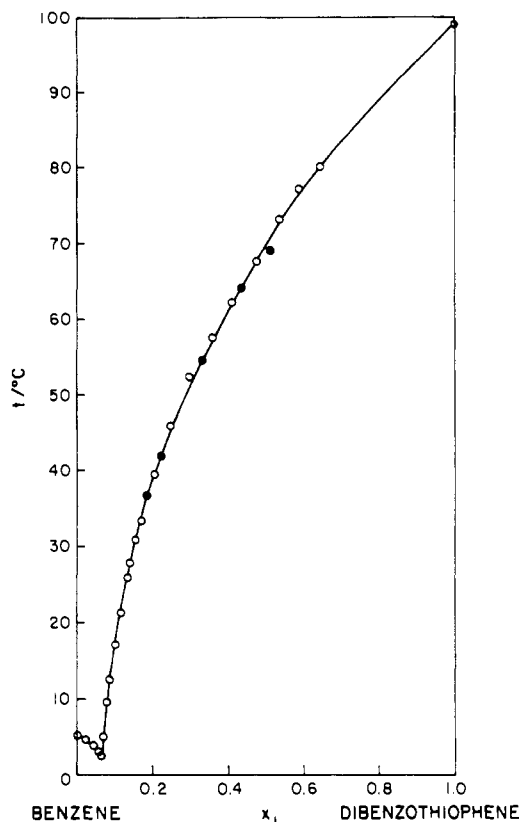
$$x_1 + x_2 = (\gamma_1^*/\gamma_1)x_1^* \exp \left[ \frac{\Delta H_{m1}}{R} \left( \frac{1}{T} - \frac{1}{T_{t1}} \right) - \frac{\Delta C_{pm1}}{R} \left( \ln \frac{T_{t1}}{T} - \frac{T_{t1}}{T} + 1 \right) \right] + (\gamma_2^*/\gamma_2)x_2^* \times \exp \left[ \frac{\Delta H_{m2}}{R} \left( \frac{1}{T} - \frac{1}{T_{t2}} \right) - \frac{\Delta C_{pm2}}{R} \left( \ln \frac{T_{t2}}{T} - \frac{T_{t2}}{T} + 1 \right) \right] \quad (7)$$

**Figure 2. Solid-liquid phase diagram for dibenzofuran (1)-benzene (2). The lines are predicted by the UNIQUAC equation. The solid circles represent experimental points published earlier (4).****Table VII. Values of the Calculated Temperatures and Activity Coefficients Obtained by Wilson and UNIQUAC Methods and Ideal Solubilities for the Dibenzofuran (1)-Benzene (2) System**

Wilson		UNIQUAC		ideal solubility $t/^\circ\text{C}$
$t_{\text{calcd}}/^\circ\text{C}$	$\gamma_1^{\text{calcd}}$	$t_{\text{calcd}}/^\circ\text{C}$	$\gamma_1^{\text{calcd}}$	
5.5		5.5		5.5 <sup>a</sup>
2.8	1.023 <sup>a</sup>	2.9	1.025 <sup>a</sup>	1.3 <sup>a</sup>
0.9	1.019 <sup>a</sup>	1.0	1.019 <sup>a</sup>	-0.2 <sup>a</sup>
0.2	1.016 <sup>a</sup>	0.1	1.015 <sup>a</sup>	-0.8 <sup>a</sup>
-1.4	1.293	-1.5	1.296	-11.3
1.8	1.285	1.7	1.288	-8.1
4.8	1.276	4.8	1.279	-4.9
7.2	1.269	7.1	1.271	-2.5
10.1	1.259	10.1	1.262	0.2
17.3	1.234	17.3	1.236	8.2
23.1	1.211	23.2	1.213	14.6
25.1	1.203	25.2	1.204	16.8
29.5	1.184	29.6	1.185	21.7
31.8	1.174	31.8	1.174	24.3
38.0	1.145	38.0	1.145	31.5
46.6	1.105	46.5	1.103	41.6
56.6	1.060	56.4	1.057	53.6
64.9	1.029	64.7	1.027	63.3
68.1	1.019	68.0	1.018	67.2
72.7	1.009	72.6	1.008	72.2
82.1	1.000	82.1	1.000	82.1

<sup>a</sup> Activity coefficients and ideal solubilities for benzene as a solute.

$$x_1^* + x_2^* = (\gamma_1/\gamma_1^*)x_1 \exp \left[ \frac{\Delta H_{m1}}{R} \left( \frac{1}{T} - \frac{1}{T_{t1}} \right) + \frac{\Delta C_{pm1}}{R} \left( \ln \frac{T_{t1}}{T} - \frac{T_{t1}}{T} + 1 \right) \right] + (\gamma_2/\gamma_2^*)x_2 \times \exp \left[ \frac{\Delta H_{m2}}{R} \left( \frac{1}{T} - \frac{1}{T_{t2}} \right) + \frac{\Delta C_{pm2}}{R} \left( \ln \frac{T_{t2}}{T} - \frac{T_{t2}}{T} + 1 \right) \right] \quad (8)$$



**Figure 3.** Solid-liquid phase diagram for dibenzothiophene (1)-benzene (2). The lines are predicted by the Wilson equation. The solid circles represent experimental points published earlier (4).

Remembering that

$$x_1 + x_2 = 1 \quad (9)$$

$$x_1^* + x_2^* = 1 \quad (10)$$

it is possible to minimize the following objective function:

$$\Omega = \sum [x_i^{\text{exptl}} - x_i^{\text{calcd}}(T, A_1, A_2, A_3, A_4)]^2 + [x_i^* \text{exptl} - x_i^* \text{calcd}(T, A_1, A_2, A_3, A_4)]^2 \quad (11)$$

For an ideal solid phase ( $\gamma_1^* = \gamma_2^* = 1$ ) the problem reduces to the minimization of  $\Omega$  with two parameters ( $A_1, A_2$ ), using composition data from the liquidus curve. Using the experimental data of Sediawan et al. (1), with liquid-phase activity coefficients derived from the UNIQUAC and Wilson equations, the interaction parameters were fitted by this optimization technique. The effect of the  $\alpha/\beta$  phase transition of dibenzothiophene had to be added for the dibenzothiophene-dibenzofuran system. The resulting parameters, calculated from the liquidus curves, are presented in Table VIII.

Table IX shows the experimental data of Sediawan (1) in the form of  $x_1$  and  $x_1^*$  as functions of temperature for the solid solution region. The table also shows calculated temperatures:  $T^{\text{calcd}}$  (nonideal solid solution, four UNIQUAC parameters) and  $T^{\text{calcd, id}}$  (ideal solid solution, two UNIQUAC parameters). Values of the four parameters and standard deviations of temperature, obtained by the UNIQUAC model in every binary solid-solid system are shown in Table X.

Taking into consideration that the results of the solid solubility correlation obtained by using the Wilson and UNIQUAC equations were comparable with each other, calculations of ternary system phase diagrams have been continued with the UNIQUAC equation only. Using the parameters derived from the binary data, it is possible to

**Table VIII.** Values of the Parameters and of the Root Mean Square Deviations of Temperature Obtained by Wilson and UNIQUAC Equations in Binary Systems (Liquidus Curves)<sup>a</sup>

system	parameter		$\sigma_T/^\circ\text{C}$	
	Wilson $(g_{12} - g_{22})/$ (J mol <sup>-1</sup> )	UNIQUAC $\Delta U_{12} (\Delta U_{21})/$ (J mol <sup>-1</sup> )	Wilson	UNIQUAC
fluorene (1)- dibenzofuran (2)	2617.12 (3420.17)	1081.57 (153.70)	1.9	2.2
fluorene (1)- dibenzothiophene (2)	8009.28 (4381.22)	129.57 (1796.20)	1.5	3.0
dibenzothiophene (1)- fluorene (2)	28 721 337.2 (8346.55)	3046.89 (51 075.6)	0.3	0.2
dibenzothiophene (1)- dibenzofuran (2)	65 651.02 (2453.41)	-185.45 (1827.92)	0.7	0.4
dibenzofuran (1)- dibenzothiophene (2)	1 795 145.9 (7693.86)	5683.57 (-2227.56)	2.8	0.1

<sup>a</sup> For  $\gamma_1^* = \gamma_2^* = 1$  (ideal solid phase).

**Table IX.** Equilibrium Data for the Binary System Dibenzothiophene (1)-Dibenzofuran (2) and Temperatures Calculated by the UNIQUAC Equation with Four and Two Parameters (Ideal Solid Phase)

$x_1$	$x_1^*$	$t/^\circ\text{C}$	$t^{\text{calcd}}/^\circ\text{C}$	$t^{\text{calcd, id}}/^\circ\text{C}$
0.000	0.000	82.1	82.1	82.1
0.395	0.475	81.3	81.4	80.3
0.405	0.499	81.4	81.5	80.4
0.445	0.535	81.8	81.8	80.9
0.499	0.591	82.4	82.3	81.7
0.531	0.625	82.8	82.7	82.3
0.565	0.671	83.3	83.2	83.1
0.600	0.694	83.8	83.8	84.0
0.650	0.751	84.9	84.8	85.4
0.721	0.824	86.8	86.7	87.7
0.751	0.850	87.5	87.7	88.8
0.805	0.882	88.8	89.7	91.0
0.865	0.928	92.7	92.4	93.4
0.880	0.930	93.2	93.1	94.0
0.890	0.940	93.8	93.6	94.4
0.915	0.951	95.1	94.8	95.4
0.929	0.960	95.7	95.5	96.0
0.951	0.972	96.8	96.5	96.8
0.962	0.981	97.3	97.0	97.3
1.000	1.000	98.7	98.7	98.7

**Table X.** Values of the Parameters and of the Root Mean Square Deviations of Temperature Obtained by the UNIQUAC Equation in Binary Systems (Liquid and Solidus Curves)<sup>a</sup>

system	parameter		$\sigma_T/^\circ\text{C}$
	$\Delta U_{12} (\Delta U_{21})/$ (J mol <sup>-1</sup> )	$\Delta U_{12}^b (\Delta U_{21}^b)/$ (J mol <sup>-1</sup> )	
fluorene (1)- dibenzofuran (2)	2788.68 (-1921.60)	-131.37 (453.08)	0.4
fluorene (1)- dibenzothiophene (2)	-1630.57 (3388.50)	-1516.13 (3567.87)	0.1
dibenzothiophene (1)- dibenzofuran (2)	1208.92 (-1827.76)	3450.25 (-2509.60)	0.3

<sup>a</sup> Standard deviation  $\sigma_T = [\sum_{i=1}^n [(T_i^{\text{calcd}} - T_i)^2 / n]]^{1/2}$ . <sup>b</sup> Solid phase.

predict the solidus and liquidus curves in ternary, two-phase mixtures (solid solution-liquid). Taking into account 12 parameters (4 for every binary system), the equilibrium temperature can be predicted by solution of three equations similar to eqs 5 and 6 with  $\sum x_i = 1$  and  $\sum x_i^* = 1$ . In this way it is possible to calculate the temperature and composition of the solid phase for the given composition of the liquid phase. In every ternary system tested in this paper, we have assumed the four solid-phase parameters equal to zero for the two binary eutectic systems. The results of calculation at 50 °C with values of binary eutectic parameters (aromatic compound-benzene liquidus curve) given in Table VI, parameters for solid-solid systems given in Table X, and four

**Table XI. Values of the Temperatures Predicted by the UNIQUAC Equation for the System Benzene (1)–Fluorene (2)–Dibenzofuran (3) at 50 °C**

$x_2$	$x_3$	$t_{\text{calcd}}/^\circ\text{C}$	$x_2$	$x_3$	$t_{\text{calcd}}/^\circ\text{C}$
0.0432	0.4558	51.3	0.1860	0.2239	48.7
0.0727	0.4222	51.4	0.1967	0.1964	48.3
0.0982	0.3817	50.8	0.2127	0.1694	48.5
0.1133	0.3518	50.1	0.2174	0.1446	48.0
0.1154	0.3426	49.7	0.2351	0.1148	48.8
0.1381	0.3319	50.9	0.2382	0.0898	48.5
0.1529	0.2971	50.0	0.2356	0.0744	48.0
0.1642	0.2754	49.7	0.2426	0.0574	48.5
0.1749	0.2646	49.9	0.3253	0.0307	56.2

**Table XII. Values of the Temperatures Predicted by the UNIQUAC Equation for the System Benzene (1)–Fluorene (2)–Dibenzothiophene (3) at 50 °C**

$x_2$	$x_3$	$t_{\text{calcd}}/^\circ\text{C}$	$x_2$	$x_3$	$t_{\text{calcd}}/^\circ\text{C}$
0.0080	0.2671	50.6	0.1186	0.2004	51.4
0.0237	0.2623	50.2	0.1347	0.1853	51.6
0.0391	0.2809	52.3	0.1511	0.1549	51.0
0.0442	0.2738	51.9	0.1631	0.1349	50.7
0.0572	0.2709	52.4	0.1784	0.1166	50.9
0.0663	0.2637	52.3	0.1851	0.1049	50.7
0.0671	0.2549	51.7	0.2033	0.0807	50.7
0.0936	0.2363	51.9	0.2193	0.0427	49.6
0.1025	0.2300	52.2	0.2291	0.0290	49.5
0.1096	0.2124	51.5			

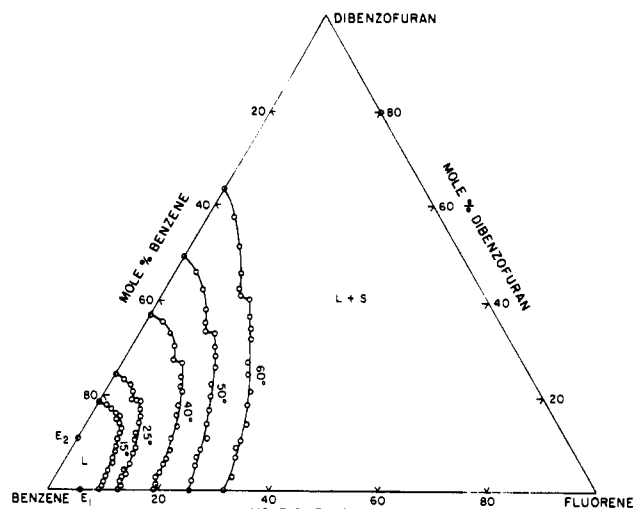
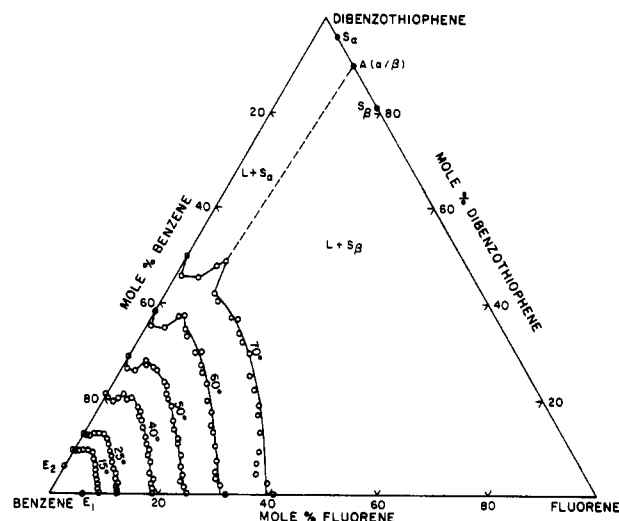
**Table XIII. Values of the Temperatures Predicted by the UNIQUAC Equation for the System Benzene (1)–Dibenzothiophene (2)–Dibenzofuran (3) at 50 °C**

$x_2$	$x_3$	$t_{\text{calcd}}/^\circ\text{C}$	$x_2$	$x_3$	$t_{\text{calcd}}/^\circ\text{C}$
0.0188	0.4500	48.3	0.2113	0.2687	52.2
0.0211	0.4439	48.0	0.2357	0.2294	52.6
0.0407	0.4493	49.3	0.2450	0.2091	53.2
0.0444	0.4477	49.3	0.2604	0.1995	53.4
0.0681	0.4240	49.4	0.2576	0.0803	50.4
0.0825	0.4125	49.7	0.2562	0.0488	49.7
0.1036	0.3944	50.2	0.2831	0.0449	51.7
0.1406	0.3575	50.9	0.2657	0.0252	50.0
0.1752	0.3208	51.7	0.2766	0.0134	50.7
0.1935	0.2945	51.9			

**Table XIV. Values of the Standard Deviations of Temperature Obtained by the UNIQUAC Equation for Prediction of Isotherms in Ternary Systems**

system	$t/^\circ\text{C}$	$\sigma_T/^\circ\text{C}$
benzene (1)–fluorene (2)– dibenzofuran (3)	15	17.3
	25	11.3
	40	3.7
	50	1.9
	60	4.2
benzene (1)–fluorene (2)– dibenzothiophene (3)	15	16.2
	25	13.0
	40	5.5
	50	1.5
	60	3.8
benzene (1)–dibenzothiophene (2)– dibenzofuran (3)	15	18.4
	25	13.3
	40	5.5
	50	1.7
	60	3.3
70	6.7	

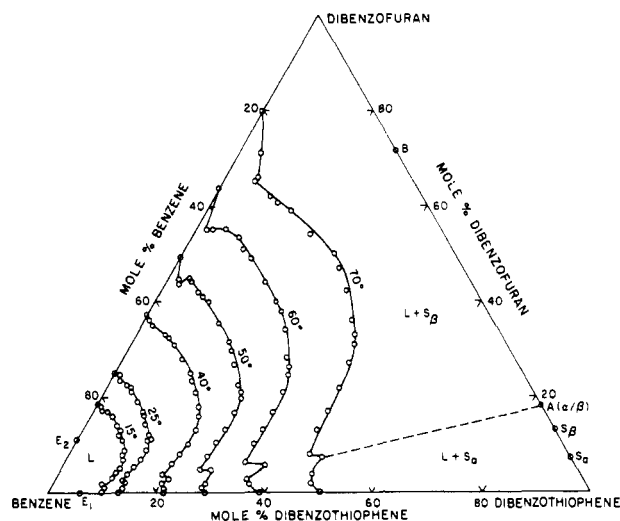
parameters equal to zero are shown in Tables XI–XIII. Columns 1 and 2 of Tables XI–XIII list the experimental values of solution composition at 50 °C, and column 3 presents the predicted temperatures. The standard deviation of temperature was from 1.5 to 1.9 °C in these ternary systems at 50 °C. Overly large values of the standard deviation were observed at higher and lower temperatures, especially at 15 °C ( $\bar{\sigma} = 17.3$  °C) as shown in Table XIV. Figures 4–6 show the isothermal lines constructed from data on compositions

**Figure 4. Equilibria for benzene (1)–fluorene (2)–dibenzofuran (3). The second and last of these form solid solutions at any composition with point A characteristic for a rapidly changed liquidus curve and two binary eutectic points  $E_1$  and  $E_2$  for 1–2 and 1–3 systems, respectively.****Figure 5. Benzene (1)–fluorene (2)–dibenzothiophene (3). Two different crystal structures  $\alpha$  and  $\beta$  and the solid immiscibility gap are present between points  $S_\alpha$  and  $S_\beta$  and a minimum on the liquidus curves, changing the  $\alpha$  for the  $\beta$  form at point A. The binary eutectics are at points  $E_1$  and  $E_2$  for 1–2 and 1–3, respectively.**

of mixtures having the same saturation temperature. These diagrams are the contour diagrams of the equilibrium surface of the ternary systems in the triangular prism with temperature as the vertical axis. The isotherms are drawn for 15, 25, 40, 50, 60, and 70 °C. In all the major regions high-temperature isotherms are spread over a large area as compared to low-temperature isotherms, indicating that the slope of the surface at lower temperature is steeper than that at high temperature.

In the first phase diagram (Figure 4) the isotherms represent the varying compositions of the liquid solution in equilibrium with solid solution for the benzene–fluorene–dibenzofuran system at the given temperature. At point A a rapid change in slope of the liquidus curve occurs in the binary system fluorene–dibenzofuran. This may be the reason for existence of the valley on the ternary surface, which is observed as a minimum of the isotherm curve.

Figure 5 shows isotherms for the benzene–fluorene–dibenzothiophene system. Since pure fluorene and diben-



**Figure 6.** Benzene (1)–dibenzothiophene (2)–dibenzofuran (3). Two different crystal structures  $\alpha$  and  $\beta$  and the solid immiscibility gap are present between points  $S_\alpha$  and  $S_\beta$ . Point A is characteristic for changing the  $\alpha$  form for the  $\beta$  form, and point B is a minimum of the  $T$ - $x$  diagram on the liquidus curve. The binary eutectics are at points  $E_1$  and  $E_2$  for 1–2 and 1–3, respectively.

zothiophene have different crystal structures (16, 17), it was observed (1) that the fluorene–dibenzothiophene system had a solid immiscibility gap between points  $S_\alpha$  and  $S_\beta$  on the solidus curve, and point A was observed as a minimum on the liquidus curves, changing  $\alpha$  for  $\beta$  crystal structures on the liquidus curve. We observed two minima, connected with the eutectic points of fluorene–benzene and dibenzothiophene–benzene systems and the maximum of the isotherm connected with point A—changing of crystal structure  $\alpha$  to  $\beta$ . The same explanation can be applied to the third system, presented in Figure 6, where point A represents the change of solid crystal structure from  $\alpha$  to  $\beta$  on the liquidus curve and point B is a minimum of the  $T$ - $x$  diagram. The eutectic valleys were found on both sides of the isotherms.

In summary, results of calculation show that the prediction of ternary system isotherms using the UNIQUAC binary system parameters is possible at moderate temperatures (40–60°C) with average standard deviation  $\bar{\sigma}_T = 3.4^\circ\text{C}$ . At higher temperatures, the liquidus surfaces of the ternary systems are difficult to describe and the prediction of SLE, especially in the region of the eutectic valleys, is not generally satisfactory. It should be expected that the application of more parameters for two eutectic liquidus lines or two parts of liquidus characteristic for the  $\alpha/\beta$  phase transition of the fluorene–dibenzothiophene system, for example, will make a better description of isotherm curves of such polynuclear aromatic systems possible. Additionally the effect of the immiscibility gap in the solid phase should be added to eqs 5 and 6. At lower temperatures the standard deviation of prediction is unacceptable. Perhaps the ternary systems under study have a tendency to form binary eutectics in equilibrium with solid solution and liquid at lower temperature. Generally, we may expect better predictions for ternary systems which have three pairs of binary eutectics or binary solid solutions.

## Glossary

$a_1$	activity of the solute
$A_1, A_2, A_3, A_4$	Wilson or UNIQUAC adjustable parameters
$\Delta C_{pm1}$	difference between heat capacities of the solute in the solid and liquid states
$F$	objective function for optimization
$g_{ij}$	molar energy of interaction between $i$ and $j$
$\Delta H_{m1}$	molar enthalpy of fusion of the solute
$\Delta H_{tr1}$	molar enthalpy of transition of the solute
$n$	number of experimental points
$R$	universal gas constant
$S_\alpha, S_\beta$	points of solid solution crystal structures changing
$T/t$	experimental equilibrium temperature, K/ $^\circ\text{C}$
$T_m/t_m$	melting point temperature, K/ $^\circ\text{C}$
$T_v/t_t$	triple point temperature, K/ $^\circ\text{C}$
$T_{tr1}/t_{tr1}$	transition point temperature of the pure solute, K/ $^\circ\text{C}$
$\Delta T$	estimated error of temperature
$T^{\text{calcd}}$	calculated equilibrium temperature
$\Delta u_{ij}$	binary interaction parameter
$V$	molar volume
$x_1$	mole fraction of the solute
$x_1^{\text{calcd}}$	calculated equilibrium mole fraction
$\Delta x_1$	estimated error of the solute mole fraction
$x_i^\circ$	mole fraction of the first named component in the solvent-free binary solid mixture
$w$	weight of experimental point

## Greek Letters

$\alpha$	crystal structures
$\beta$	crystal structures
$\gamma_i$	activity coefficient of component $i$
$\sigma_T$	root mean square deviation of temperature
$\sigma_x$	mean absolute deviation of composition
$\Omega$	objective function for the least-squares optimization

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