# Molar Heat Capacity at Constant Volume for Isobutane at Temperatures from (114 to 345) K and at Pressures to $35 \mathrm{MPa}^{\dagger, \%}$ 

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#### Abstract

Molar heat capacities at constant volume $\left(C_{V}\right)$ were measured with an adiabatic calorimeter for pure isobutane. The high purity of the samples was verified by chemical analysis. Temperatures ranged from the triple point of isobutane near 114 K to the upper temperature limit of the calorimeter at 345 K , whereas pressures ranged up to 35 MPa . Measurements were conducted on liquid in equilibrium with its vapor and on compressed liquid samples along isochores. Heat capacity results are reported for two-phase ( $C_{V}^{(2)}$ ), saturated liquid $\left(C_{\sigma}\right)$, and single-phase $\left(C_{V}\right)$ isochores. Vapor pressure data are reported that are based on measurements of $C_{V}{ }^{(2)}$ along a two-phase isochore. Measurements were also made to determine the triple-point temperature of $(113.707 \pm 0.030) \mathrm{K}$ and enthalpy of fusion of $(4494 \pm 20) \mathrm{J} \cdot \mathrm{mol}^{-1}$ for isobutane near its triple point. The principal sources of uncertainty are the temperature rise measurement and the change-of-volume work adjustment. The expanded uncertainty (i.e., a coverage factor $k=2$ and thus a two-standard deviation estimate) for values of $C_{V}^{(2)}$ is estimated to be $0.5 \%$, for $C_{\sigma}$ it is $0.7 \%$, and for $C_{V}$ it is $0.7 \%$.


## Introduction

Isobutane is a common hydrocarbon fuel and is a key component of liquefied petroleum gas (LPG) and natural gas mixtures; it is also widely used as a component in mixtures for refrigeration cycles. The isochoric heat capacity of a fluid may be calculated from knowledge of its ideal gas properties and an equation of state that represents the relationship between temperature $(T)$, pressure $(P)$, and density $(\rho)$. The isochoric heat capacity is given by

$$
\begin{equation*}
C_{V}-C_{V}^{0}=-T \int_{0}^{\rho}\left(\frac{\partial^{2} P}{\partial T^{2}}\right)_{\rho} \frac{\mathrm{d} \rho}{\rho^{2}} \tag{1}
\end{equation*}
$$

where $C_{V}{ }^{0}$ is the ideal gas heat capacity. The isochoric derivative $\left(\partial^{2} P / \partial T^{2}\right)_{\rho}$ is known to possess small absolute values, except in the vicinity of the critical point, and is difficult to measure directly. Equation 1 provides a means to check for consistency between independent measurements of density and heat capacity, usually through an equation of state.

Molar heat capacity was measured with an adiabatic calorimeter at saturated liquid and single-phase compressed liquid state conditions. These measurements cover temperatures from the triple point to 345 K , with compressed liquid isochores at pressures up to 35 MPa . The triple-point temperature, enthalpy of fusion, and vapor pressures are reported on the basis of the evaluation of two-phase heat capacity measurements at temperatures from below the triple point to 345 K .

## Experimental Section

Materials. The isobutane sample used in this work was specified by the supplier as having a purity of $99.99 \%$ (molar

[^0]basis), as determined by gas chromatography - mass spectrometry analysis. As shown by its certificate of chemical analysis, the principal impurities are isobutylene ( 25.1 ppm ), 1-butene ( 15.4 ppm ), and $n$-butane ( 15.2 ppm ). Measurements of the triple-point temperature described here on this isobutane sample are consistent with this purity.

Measurements. The adiabatic calorimeter that was used for these measurements was originally described by Goodwin ${ }^{1}$ to study hydrogen heat capacity and was used to measure the heat capacity of light hydrocarbons including methane, ${ }^{2}$ ethane, ${ }^{3}$ (methane + ethane) mixtures, ${ }^{4}$ and propane. ${ }^{5}$ The calorimeter was improved by Magee ${ }^{6}$ prior to measurements on butane. ${ }^{7}$ Recent measurements on a very high purity sample of propane ${ }^{8}$ allowed the volume of the calorimeter to be recalibrated with an uncertainty of $0.012 \%$, which reduces the uncertainty in the present results for both heat capacity and density. The twophase heat capacity data are used to generate vapor pressure data down to the triple-point temperature, and the triple-point temperature and the associated enthalpy of fusion are reported here.

Details of the calorimeter cell are described in detail by Goodwin ${ }^{1}$ and Magee. ${ }^{6}$ The present automated data acquisition and control systems are discussed in detail by Magee. ${ }^{6}$ The calorimeter consists of a nearly adiabatic cell (also called a calorimetric bomb) that is spherical and has been operated at temperatures from ( 14 to 345 ) K at pressures up to 35 MPa . During a measurement, a sample of well-established mass is confined within the cell with a well-known volume (approximately $77 \mathrm{~cm}^{3}$ ); as shown by Magee ${ }^{6}$ and Perkins et al., ${ }^{8}$ the exact volume varies with both temperature and pressure. After a precisely measured quantity of electrical energy ( $Q$ ) pulse is applied and the cell temperature equilibrates, the resulting temperature increase $\left(\Delta T=T_{1}-T_{2}\right)$ is measured. We found the molar heat capacity at constant volume $\left(C_{V}\right)$ of the sample by subtracting the energy required to heat the empty cell $\left(Q_{0}\right)$ and the pressure-volume work ( $W_{\mathrm{PV}}$ ) as

$$
\begin{equation*}
C_{V}=\left(\frac{\partial U}{\partial T}\right)_{V} \cong \frac{Q-Q_{0}-W_{\mathrm{PV}}}{n \Delta T} \tag{2}
\end{equation*}
$$

To fill the cell, sample gas was condensed into the cold sample cell until both the cell temperature and pressure reached plateau values when the cell was completely filled. Then, the charge valve was closed, and the cell and its contents were cooled to the starting temperature. A small quantity of gas was slowly vented to an exhaust stack until the pressure fell to the starting value. A series of heating cycles was started and continued until the upper temperature limit of 345 K was reached or until the maximum pressure of 35 MPa was reached for single-phase samples. At the completion of an isochore, some of the sample was discharged into a collection cylinder to obtain the next filling density. The increment of mass discharged was determined from difference weighings of the collection cylinder after a buoyancy correction was applied. Following the last isochore, the sample remaining in the cell was collected and weighed. For each isochoric run, the mass of sample was determined from the sum of all mass increments that were measured after that run. A series of such isochores at different densities $(\rho)$ completed the $C_{V}(\rho, T)$ surface for the sample.

The energy required to heat the empty cell $\left(Q_{0}\right)$ from the initial temperature $\left(T_{1}\right)$ to the final temperature $\left(T_{2}\right)$ was found from previous experiments with the cell thoroughly evacuated. ${ }^{6}$ The fit for these empty cell results is described by Magee ${ }^{6}$ and is the basis for evaluation of $Q_{0}$ in the analysis of the present measurements with eq 2 . The pressure-volume work ( $W_{\mathrm{PV}}$ ) accounts for the work done by the fluid on the thin-walled cell as the pressure rises from $P_{1}$ to $P_{2}$. Calculation of the pressure-volume work is discussed by Goodwin and Weber ${ }^{9}$ and is given by

$$
\begin{equation*}
W_{\mathrm{PV}}=\left(T_{2}\left(\frac{\partial P}{\partial T}\right)_{V_{2}}-\frac{\Delta P}{2}\right) \Delta V \tag{3}
\end{equation*}
$$

where $\Delta P=P_{2}-P_{1}$ is the pressure rise and $\Delta V=V_{2}-V_{1}$ is the volume increase. The pressure derivative in eq 3 has been calculated with the equation of state of Bücker and Wagner. ${ }^{10}$
The amount (mol) of substance, $n$, contained in the cell varies slightly because of the noxious volume, which is about $0.2 \%$ of the cell volume. ${ }^{6}$ This noxious volume is the sum of the volumes of the pressure transducer, charging valve, and interconnecting capillary tubing. The pressure transducer and valve are contained in an isothermal aluminum block that is controlled at a temperature near 320 K such that the temperature gradient occurs along a thin capillary (ID $=0.13 \mathrm{~mm}$ ) tube between the cell and the pressure transducer + valve assembly. The amount (mol) of the sample in this noxious volume is obtained from the measured volumes and the density calculated with the equation of state ${ }^{10}$ at the measured pressure and temperature of the pressure transducer plus valve assembly and by assuming a quadratic temperature profile along the length of the capillary. The molar mass for isobutane of 58.122 $\mathrm{g} \cdot \mathrm{mol}^{-1}$, based on the atomic weights of the elements, ${ }^{11}$ was used throughout this work.

## Results

Triple-Point Temperature. Measurements of the triple point of isobutane provide a check of the purity of the sample because an impure sample will exhibit freezing point depression. The triple-point temperature of isobutane has been studied by Aston et al., ${ }^{12}$ who reported a value of (113.747


Figure 1. Measured calorimeter temperatures as isobutane was slowly melted near its triple point: + , measured temperature; line at $T=113.707$ K , measured triple-point temperature of this sample.
$\pm 0.05) \mathrm{K}$ converted from the IPTS-27 temperature scale to the ITS-90 scale. Aston et al. ${ }^{12}$ observed a depression in the freezing point that indicated molar impurities on the order of 0.0015 mol percent in their isobutane sample. Parks et al. ${ }^{13}$ reported $(113.207 \pm 0.7) \mathrm{K}$, Das et al. ${ }^{14}$ reported 113.56 K, Rossini et al. ${ }^{15}$ reported 113.57 K , and Francis ${ }^{16}$ reported 113.75 K , where all values have been converted to the ITS90 scale. The value of $(113.730 \pm 0.010) \mathrm{K}$ was recently measured by Glos et al. ${ }^{17}$ for the triple-point temperature of a 99.97 \% pure sample of isobutane. It is the value of Glos et al. ${ }^{17}$ that has been adopted for the equation of state of Bücker and Wagner ${ }^{10}$ for isobutane.
The present isobutane sample is $99.99 \%$ pure relative to the $99.9985 \%$ pure sample studied by Aston et al., ${ }^{12}$ so there is more freezing point depression in the measurements reported here. Figure 1 shows the measured temperature as a function of time that we observed for isobutane near its triple point. The temperature increases with elapsed heating time as more of the sample melts. This temperature increase is due only in part to freezing point depression; most of the temperature increase is due to temperature gradients across the liquid layer between the heated calorimeter vessel and the remaining solid sample. The experiment shown in Figure 1 was optimized for accurate determination of the enthalpy of fusion reported below. However, the temperature rise can be extrapolated to the start of melting to provide an estimate of the triple-point temperature of isobutane that corresponds to a very small melt fraction, where freezing point depression is expected to be the largest. We measured a temperature of ( $113.707 \pm 0.005$ ) K for the initial melting temperature of this 99.99 \% pure sample of isobutane that does not account for freezing point depression. Temperature measurement noise is about 0.003 K . Our initial melting temperature is lower than the measurements of Aston et al. ${ }^{12}$ and Glos et al., ${ }^{17}$ who report values of $(113.747 \pm 0.05) \mathrm{K}$ and $(113.730$ $\pm 0.010) \mathrm{K}$, respectively. It is likely that a difference of $(0.02$ to 0.04 ) K between the present initial melting temperature and these published values for the triple-point temperature is primarily due to freezing point depression. Raw data from our scanning experiments simply does not lend itself to characterizing the melt temperature as a function of fraction melted, such as Aston et al. ${ }^{12}$ were able to do for a purer sample of isobutane. Because our result is an extrapolation, its uncertainty is estimated to be $\pm 0.030 \mathrm{~K}$. Therefore, we report a value of $(113.707 \pm 0.030) \mathrm{K}$ for our determination

Table 1. Isochoric Heat Capacities, $C_{V}$, of Isobutane at Temperatures from (120 to 345) K at Pressures to 35 MPa

| $T$ | $\rho$ | $P$ | $C_{V}$ | $T$ | $\rho$ | $P$ | $C_{V}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| K | $\mathrm{mol} \cdot \mathrm{L}^{-1}$ | MPa | $\mathrm{J} \cdot \mathrm{mol}^{-1} \cdot \mathrm{~K}^{-1}$ | K | $\mathrm{mol} \cdot \mathrm{L}^{-1}$ | MPa | $\mathrm{J} \cdot \mathrm{mol}^{-1} \cdot \mathrm{~K}^{-1}$ |
| 119.824 | 12.679 | 6.500 | 71.22 | 294.138 | 10.171 | 24.048 | 99.10 |
| 124.969 | 12.662 | 15.325 | 72.14 | 298.278 | 10.165 | 26.943 | 99.99 |
| 130.018 | 12.643 | 25.536 | 73.32 | 302.393 | 10.159 | 29.793 | 100.74 |
| 122.330 | 12.671 | 10.732 | 71.61 | 306.498 | 10.153 | 32.612 | 102.04 |
| 127.431 | 12.652 | 20.667 | 72.63 | 266.606 | 10.211 | 4.054 | 92.14 |
| 132.429 | 12.636 | 28.973 | 73.82 | 270.879 | 10.205 | 7.235 | 92.99 |
| 141.365 | 12.331 | 4.922 | 73.32 | 275.117 | 10.199 | 10.363 | 94.06 |
| 146.265 | 12.315 | 13.401 | 74.25 | 279.338 | 10.192 | 13.450 | 95.20 |
| 151.090 | 12.299 | 21.305 | 75.18 | 283.538 | 10.186 | 16.491 | 96.18 |
| 155.838 | 12.285 | 28.673 | 76.28 | 287.716 | 10.180 | 19.487 | 97.22 |
| 143.716 | 12.323 | 9.088 | 73.74 | 291.871 | 10.174 | 22.438 | 98.44 |
| 148.585 | 12.307 | 17.196 | 74.72 | 296.012 | 10.168 | 25.350 | 99.94 |
| 153.376 | 12.293 | 24.872 | 75.69 | 300.136 | 10.162 | 28.222 | 100.48 |
| 162.300 | 11.989 | 4.549 | 75.71 | 304.247 | 10.156 | 31.058 | 101.52 |
| 167.029 | 11.976 | 11.587 | 76.59 | 283.657 | 9.842 | 2.372 | 95.67 |
| 171.692 | 11.963 | 18.296 | 77.43 | 287.933 | 9.836 | 5.135 | 96.68 |
| 176.294 | 11.951 | 24.825 | 78.43 | 292.183 | 9.831 | 7.863 | 98.07 |
| 180.830 | 11.939 | 31.062 | 79.80 | 296.421 | 9.825 | 10.563 | 98.75 |
| 164.597 | 11.983 | 8.126 | 76.26 | 300.638 | 9.820 | 13.229 | 100.03 |
| 169.293 | 11.970 | 14.798 | 77.08 | 304.846 | 9.814 | 15.868 | 101.01 |
| 173.926 | 11.957 | 21.461 | 78.06 | 309.040 | 9.809 | 18.478 | 102.38 |
| 178.500 | 11.945 | 27.869 | 78.99 | 313.228 | 9.803 | 21.063 | 102.98 |
| 183.242 | 11.645 | 4.049 | 78.07 | 317.393 | 9.798 | 23.611 | 103.51 |
| 187.834 | 11.633 | 9.994 | 79.02 | 321.547 | 9.793 | 26.133 | 104.78 |
| 192.378 | 11.622 | 15.769 | 79.81 | 325.699 | 9.787 | 28.633 | 106.08 |
| 196.863 | 11.611 | 21.369 | 81.16 | 329.842 | 9.782 | 31.107 | 106.78 |
| 201.308 | 11.601 | 26.822 | 81.98 | 333.968 | 9.777 | 33.555 | 107.88 |
| 205.687 | 11.591 | 32.085 | 83.13 | 285.763 | 9.839 | 3.730 | 96.28 |
| 185.448 | 11.639 | 6.869 | 78.63 | 290.038 | 9.834 | 6.483 | 97.37 |
| 190.011 | 11.628 | 12.743 | 79.62 | 294.296 | 9.828 | 9.205 | 98.00 |
| 194.515 | 11.617 | 18.415 | 80.61 | 298.534 | 9.822 | 11.894 | 98.97 |
| 198.974 | 11.607 | 23.939 | 81.69 | 302.754 | 9.817 | 14.551 | 100.42 |
| 203.378 | 11.596 | 29.282 | 82.49 | 306.975 | 9.811 | 17.187 | 101.25 |
| 203.616 | 11.304 | 3.700 | 80.96 | 311.165 | 9.806 | 19.783 | 102.14 |
| 208.089 | 11.294 | 8.785 | 82.04 | 315.352 | 9.801 | 22.355 | 103.37 |
| 212.530 | 11.285 | 13.758 | 83.00 | 319.529 | 9.795 | 24.899 | 104.29 |
| 216.918 | 11.276 | 18.597 | 83.89 | 323.688 | 9.790 | 27.412 | 105.09 |
| 221.262 | 11.266 | 23.313 | 85.00 | 327.840 | 9.785 | 29.902 | 106.30 |
| 225.572 | 11.258 | 27.916 | 85.85 | 331.979 | 9.779 | 32.367 | 107.91 |
| 229.830 | 11.249 | 32.388 | 86.73 | 303.811 | 9.423 | 2.271 | 99.83 |
| 205.821 | 11.299 | 6.192 | 81.19 | 308.123 | 9.418 | 4.631 | 100.80 |
| 210.261 | 11.290 | 11.205 | 82.36 | 312.423 | 9.413 | 6.973 | 101.82 |
| 214.660 | 11.280 | 16.095 | 83.42 | 316.705 | 9.408 | 9.293 | 102.97 |
| 219.020 | 11.271 | 20.866 | 84.44 | 320.980 | 9.403 | 11.596 | 104.60 |
| 223.328 | 11.262 | 25.507 | 85.63 | 325.247 | 9.398 | 13.880 | 105.73 |
| 227.602 | 11.253 | 30.036 | 86.44 | 329.508 | 9.393 | 16.147 | 106.45 |
| 224.207 | 10.949 | 3.209 | 84.09 | 333.759 | 9.388 | 18.392 | 107.16 |
| 228.594 | 10.940 | 7.563 | 85.18 | 337.998 | 9.383 | 20.617 | 108.51 |
| 232.950 | 10.932 | 11.833 | 86.15 | 342.233 | 9.378 | 22.826 | 110.23 |
| 237.259 | 10.924 | 16.001 | 87.31 | 306.067 | 9.420 | 3.504 | 100.71 |
| 241.536 | 10.916 | 20.084 | 88.35 | 310.381 | 9.415 | 5.860 | 101.98 |
| 245.774 | 10.908 | 24.074 | 89.40 | 314.682 | 9.410 | 8.197 | 102.64 |
| 249.980 | 10.900 | 27.977 | 90.18 | 318.978 | 9.405 | 10.517 | 103.88 |
| 254.151 | 10.893 | 31.797 | 91.18 | 323.266 | 9.400 | 12.819 | 104.79 |
| 226.505 | 10.944 | 5.490 | 84.53 | 327.539 | 9.395 | 15.099 | 105.76 |
| 230.859 | 10.936 | 9.783 | 85.57 | 331.807 | 9.390 | 17.362 | 106.57 |
| 235.183 | 10.928 | 13.991 | 86.65 | 336.062 | 9.385 | 19.603 | 107.61 |
| 239.464 | 10.920 | 18.103 | 87.79 | 340.310 | 9.381 | 21.824 | 108.42 |
| 243.715 | 10.912 | 22.131 | 88.62 | 344.554 | 9.376 | 24.029 | 109.59 |
| 247.921 | 10.904 | 26.062 | 89.81 | 312.622 | 9.223 | 2.143 | 102.01 |
| 252.106 | 10.897 | 29.918 | 91.01 | 316.946 | 9.218 | 4.329 | 103.28 |
| 244.913 | 10.582 | 2.940 | 87.70 | 321.275 | 9.213 | 6.508 | 104.19 |
| 249.240 | 10.575 | 6.667 | 88.84 | 325.586 | 9.209 | 8.668 | 104.99 |
| 253.542 | 10.567 | 10.334 | 89.74 | 329.891 | 9.204 | 10.815 | 106.10 |
| 257.801 | 10.560 | 13.926 | 90.58 | 334.190 | 9.199 | 12.947 | 107.66 |
| 262.027 | 10.553 | 17.452 | 91.98 | 338.485 | 9.195 | 15.066 | 108.51 |
| 266.232 | 10.546 | 20.919 | 93.11 | 342.772 | 9.190 | 17.169 | 109.53 |
| 270.404 | 10.539 | 24.318 | 94.15 | 314.809 | 9.220 | 3.247 | 102.19 |
| 274.555 | 10.533 | 27.659 | 95.07 | 319.151 | 9.216 | 5.436 | 103.12 |
| 278.680 | 10.526 | 30.942 | 95.99 | 323.482 | 9.211 | 7.612 | 104.96 |
| 246.876 | 10.579 | 4.626 | 88.13 | 327.822 | 9.206 | 9.782 | 105.71 |
| 251.174 | 10.571 | 8.311 | 89.23 | 332.136 | 9.201 | 11.928 | 106.39 |
| 255.440 | 10.564 | 11.931 | 90.33 | 336.448 | 9.197 | 14.061 | 107.61 |
| 259.679 | 10.557 | 15.488 | 91.30 | 340.748 | 9.192 | 16.177 | 108.82 |

Table 1. Continued

| $T$ | $\rho$ | $P$ | $C_{V}$ | $T$ | $\rho$ | $P$ | $C_{V}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| K | $\mathrm{mol} \cdot \mathrm{L}^{-1}$ | MPa | $\mathrm{J} \cdot \mathrm{mol}^{-1} \cdot \mathrm{~K}^{-1}$ | K | $\mathrm{mol} \cdot \mathrm{L}^{-1}$ | MPa | $\mathrm{J} \cdot \mathrm{mol}^{-1} \cdot \mathrm{~K}^{-1}$ |
| 263.881 | 10.550 | 18.974 | 92.40 | 322.431 | 8.989 | 2.027 | 103.88 |
| 268.064 | 10.543 | 22.404 | 93.80 | 326.805 | 8.984 | 4.037 | 105.22 |
| 272.220 | 10.537 | 25.772 | 94.90 | 331.168 | 8.980 | 6.040 | 106.74 |
| 276.358 | 10.530 | 29.087 | 95.35 | 335.531 | 8.975 | 8.034 | 107.69 |
| 280.463 | 10.523 | 32.342 | 96.46 | 339.887 | 8.971 | 10.016 | 108.49 |
| 264.623 | 10.214 | 2.580 | 91.58 | 344.235 | 8.966 | 11.989 | 109.21 |
| 268.908 | 10.208 | 5.781 | 92.42 | 324.786 | 8.986 | 3.113 | 105.07 |
| 273.170 | 10.201 | 8.940 | 93.07 | 329.145 | 8.982 | 5.110 | 105.78 |
| 277.400 | 10.195 | 12.047 | 94.20 | 333.489 | 8.977 | 7.102 | 106.91 |
| 281.615 | 10.189 | 15.113 | 95.82 | 337.839 | 8.973 | 9.085 | 108.12 |
| 285.810 | 10.183 | 18.136 | 97.40 | 342.194 | 8.968 | 11.067 | 108.69 |
| 289.990 | 10.177 | 21.118 | 98.16 |  |  |  |  |

of the triple-point temperature of isobutane, which is 0.04 K lower than Aston et al. ${ }^{12}$ primarily because of freezing point depression.

Enthalpy of Fusion. The frozen sample was heated at a constant power of 0.0810 W while the temperature was recorded, as shown in Figure 1. We measured the enthalpy of fusion, $\Delta_{\text {fus }} H$, by integrating the applied heater power over the period of melting and applying corrections for heating of the calorimeter and parasitic heat losses. The onset of melting is indicated by a sudden decrease in the rate of temperature increase at fixed heating power. The measured temperature starts out near the triple-point temperature when melting begins but slowly increases as the thickness of the liquid layer increases between the cell wall and the melting solid. The temperature again rises more rapidly when the last solid has melted. The results and expanded uncertainties $(k=2)$ for $\Delta_{\text {fus }} H$ obtained from this experiment were (4494 $\pm 20) \mathrm{J} \cdot \mathrm{mol}^{-1}$ at the triple point. These values agree well with those reported by Parks et al. ${ }^{13}$ of $(4498.6 \pm 3) \mathrm{J} \cdot \mathrm{mol}^{-1}$ but deviate by $47 \mathrm{~J} \cdot \mathrm{~mol}^{-1}(1 \%)$ from the values of Aston et al. ${ }^{12}$ of $(4541.3 \pm 2) \mathrm{J} \cdot \mathrm{mol}^{-1}$, whose claimed error of $\pm 2$ $\mathrm{J} \cdot \mathrm{mol}^{-1}$ represents their observed repeatability between multiple determinations rather than the uncertainty.

Isochoric Heat Capacity. The values measured for the isochoric heat capacity, $C_{V}$, of isobutane are given in Table 1 at 171 single-phase states. The average of the initial and


Figure 2. Relative deviations $100\left(\rho-\rho_{\text {calcd }}\right) / \rho_{\text {calcd }}$ between the experimental densities in Table 1 and the densities calculated with the equation of state of Bücker and Wagner ${ }^{10}$ at temperatures from (120 to 345) K and at pressures up to 35 MPa . The densities were calculated from the mass of sample and the calorimeter volume, which had been calibrated during separate measurements on propane. ${ }^{8}$ The present data agree with the equation of state with a relative expanded uncertainty $u=0.017 \%(k=2)$, shown as dashed lines.
final temperatures for each heating interval is given for the temperature (ITS-90) of each data point. The tabulated pressures are calculated from a least-squares fit of the $(P, T)$ data by use of a seven-term function given by

$$
\begin{equation*}
P=\frac{c_{1}}{T_{\mathrm{r}}^{4}}+\frac{c_{2}}{T_{\mathrm{r}}^{3}}+\frac{c_{3}}{T_{\mathrm{r}}^{2}}+\frac{c_{4}}{T_{\mathrm{r}}}+c_{5}+c_{6} \sqrt{T_{\mathrm{r}}}+c_{7} T_{\mathrm{r}} \tag{4}
\end{equation*}
$$

where the $c_{i}$ are the fit coefficients for the isochore and $T_{\mathrm{r}}=$ $T / T_{\mathrm{c}}$, where $T_{\mathrm{c}}=407.81 \mathrm{~K}$ is the critical temperature of isobutane. ${ }^{10}$ The density is calculated from the corrected


Figure 3. Measured isochoric heat capacities $\left(C_{V}\right)$ of isobutane at temperatures from ( 120 to 345 ) K and at pressures up to 35 MPa .


Figure 4. Relative deviations between measured isochoric heat capacities $\left(C_{V}\right)$ of isobutane and values calculated with the equation of state of Bücker and Wagner ${ }^{10}$ at temperatures from (120 to 345 ) K and at pressures up to $35 \mathrm{MPa}: \mathbf{\Delta}$, present results.

Table 2. Saturated Liquid Heat Capacities, $\boldsymbol{C}_{\boldsymbol{\sigma}}$, of Isobutane ( 0.70101 mol sample) at Temperatures from ( 117 to 316) $\mathrm{K}^{\boldsymbol{a}}$

| $T$ | $\rho_{\text {calcd }, \sigma}$ | $P_{\text {calcd }, \sigma}$ | $C_{V}{ }^{(2)}$ | $C_{\sigma}$ | $T$ | $\rho_{\text {calcd } \sigma}$ | $P_{\text {calcd } \sigma}$ | $C_{V}{ }^{(2)}$ | $C_{\sigma}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| K | $\mathrm{mol} \cdot \mathrm{L}^{-1}$ | Pa | $\mathrm{J} \cdot \mathrm{mol}^{-1} \cdot \mathrm{~K}^{-1}$ | $\mathrm{J} \cdot \mathrm{mol}^{-1} \cdot \mathrm{~K}^{-1}$ | K | $\mathrm{mol} \cdot \mathrm{L}^{-1}$ | Pa | $\mathrm{J} \cdot \mathrm{mol}^{-1} \cdot \mathrm{~K}^{-1}$ | $\mathrm{J} \cdot \mathrm{mol}^{-1} \cdot \mathrm{~K}^{-1}$ |
| 116.795 | 12.688 | 0.049607 | 99.974 | 99.973 | 119.574 | 12.643 | 0.096332 | 100.39 | 100.39 |
| 122.226 | 12.600 | 0.17596 | 100.86 | 100.86 | 124.959 | 12.555 | 0.317878 | 101.19 | 101.19 |
| 127.546 | 12.513 | 0.542256 | 101.83 | 101.83 | 130.235 | 12.470 | 0.921516 | 102.10 | 102.10 |
| 132.769 | 12.428 | 1.48619 | 102.54 | 102.54 | 135.417 | 12.385 | 2.39759 | 103.17 | 103.17 |
| 137.901 | 12.345 | 3.68596 | 103.69 | 103.68 | 140.511 | 12.302 | 5.68689 | 103.94 | 103.94 |
| 142.953 | 12.263 | 8.39707 | 104.41 | 104.41 | 145.522 | 12.221 | 12.4545 | 104.92 | 104.91 |
| 147.922 | 12.181 | 17.7517 | 105.31 | 105.30 | 150.467 | 12.140 | 25.4937 | 105.75 | 105.75 |
| 152.826 | 12.101 | 35.2272 | 106.17 | 106.16 | 155.336 | 12.060 | 49.0947 | 106.51 | 106.50 |
| 157.664 | 12.022 | 66.0854 | 107.02 | 107.02 | 160.145 | 11.981 | 89.7403 | 107.18 | 107.18 |
| 162.439 | 11.944 | 117.967 | 107.79 | 107.78 | 164.887 | 11.903 | 156.448 | 108.28 | 108.27 |
| 167.155 | 11.866 | 201.522 | 108.62 | 108.62 | 169.580 | 11.826 | 261.926 | 109.09 | 109.08 |
| 171.820 | 11.789 | 331.201 | 109.18 | 109.17 | 174.213 | 11.749 | 422.336 | 110.10 | 110.08 |
| 176.428 | 11.712 | 525.355 | 110.09 | 110.08 | 178.798 | 11.673 | 659.024 | 110.99 | 110.97 |
| 180.987 | 11.636 | 807.611 | 111.27 | 111.25 | 183.328 | 11.597 | 997.596 | 112.15 | 112.12 |
| 185.503 | 11.561 | 1207.3 | 112.03 | 112.00 | 187.820 | 11.522 | 1471.13 | 112.82 | 112.78 |
| 189.970 | 11.486 | 1758.51 | 112.85 | 112.81 | 192.258 | 11.447 | 2115.5 | 113.26 | 113.22 |
| 194.391 | 11.411 | 2501.96 | 113.77 | 113.72 | 196.656 | 11.373 | 2976.15 | 114.26 | 114.21 |
| 198.771 | 11.337 | 3485.33 | 114.82 | 114.75 | 201.014 | 11.299 | 4103.54 | 115.12 | 115.05 |
| 203.107 | 11.263 | 4760.96 | 115.69 | 115.61 | 205.337 | 11.225 | 5556.2 | 115.98 | 115.90 |
| 207.405 | 11.189 | 6389.87 | 116.45 | 116.35 | 209.614 | 11.151 | 7392.85 | 116.89 | 116.79 |
| 211.667 | 11.116 | 8438.9 | 117.50 | 117.40 | 213.859 | 11.078 | 9688.07 | 117.64 | 117.52 |
| 215.898 | 11.042 | 10983.4 | 118.25 | 118.12 | 218.066 | 11.005 | 12513.7 | 118.64 | 118.51 |
| 220.086 | 10.969 | 14093 | 118.94 | 118.79 | 222.244 | 10.932 | 15956.8 | 119.78 | 119.63 |
| 224.246 | 10.896 | 17861.2 | 120.01 | 119.86 | 226.389 | 10.859 | 20100.7 | 120.78 | 120.61 |
| 228.375 | 10.823 | 22374.8 | 121.25 | 121.07 | 230.505 | 10.785 | 25040.8 | 121.80 | 121.62 |
| 232.477 | 10.750 | 27732.3 | 122.12 | 121.93 | 234.587 | 10.712 | 30865.5 | 122.50 | 122.30 |
| 236.553 | 10.677 | 34034.9 | 122.82 | 122.62 | 238.645 | 10.639 | 37688.5 | 123.96 | 123.75 |
| 240.594 | 10.604 | 41368.3 | 123.92 | 123.70 | 242.678 | 10.566 | 45614.1 | 124.53 | 124.31 |
| 244.612 | 10.531 | 49857.7 | 124.95 | 124.72 | 246.680 | 10.493 | 54735.8 | 125.38 | 125.15 |
| 248.600 | 10.457 | 59595.8 | 126.11 | 125.88 | 250.661 | 10.419 | 65185.4 | 126.61 | 126.37 |
| 252.572 | 10.383 | 70729.5 | 127.01 | 126.78 | 254.617 | 10.345 | 77065.9 | 128.02 | 127.78 |
| 256.519 | 10.309 | 83350.7 | 128.04 | 127.81 | 258.554 | 10.271 | 90511.5 | 129.15 | 128.92 |
| 260.446 | 10.235 | 97591.5 | 128.91 | 128.69 | 262.471 | 10.196 | 105639 | 129.84 | 129.62 |
| 264.352 | 10.160 | 113568 | 130.07 | 129.86 | 266.369 | 10.121 | 122574 | 130.67 | 130.48 |
| 268.239 | 10.085 | 131408 | 131.66 | 131.47 | 270.244 | 10.046 | 141418 | 131.87 | 131.70 |
| 272.113 | 10.009 | 151268 | 132.60 | 132.45 | 274.098 | 9.970 | 162298 | 133.58 | 133.44 |
| 275.959 | 9.933 | 173190 | 133.68 | 133.56 | 277.943 | 9.894 | 185409 | 134.34 | 134.24 |
| 279.802 | 9.856 | 197445 | 134.77 | 134.70 | 281.765 | 9.817 | 210792 | 135.41 | 135.37 |
| 283.622 | 9.779 | 224040 | 135.70 | 135.69 | 285.575 | 9.739 | 238646 | 136.12 | 136.15 |
| 287.429 | 9.701 | 253168 | 137.63 | 137.70 | 289.369 | 9.660 | 269071 | 137.38 | 137.50 |
| 291.232 | 9.621 | 285042 | 138.68 | 138.84 | 293.154 | 9.581 | 302257 | 138.93 | 139.14 |
| 295.011 | 9.541 | 319623 | 139.10 | 139.36 | 296.926 | 9.500 | 338306 | 139.80 | 140.12 |
| 298.780 | 9.460 | 357164 | 140.46 | 140.84 | 300.683 | 9.419 | 377328 | 140.69 | 141.15 |
| 302.543 | 9.378 | 397849 | 142.07 | 142.60 | 308.174 | 9.253 | 465072 | 143.08 | 143.87 |
| 306.297 | 9.295 | 441792 | 143.08 | 143.77 | 311.910 | 9.168 | 514113 | 145.02 | 146.00 |
| 310.036 | 9.211 | 489057 | 143.99 | 144.87 | 315.634 | 9.081 | 566717 | 146.20 | 147.42 |
| 313.767 | 9.125 | 539869 | 145.52 | 146.62 |  |  |  |  |  |

${ }^{a}$ Tabulated values for saturated liquid density and vapor pressure were calculated with the equation of state of Bücker and Wagner. ${ }^{10}$
amount (mol) of sample and the calibrated cell volume (L) that was obtained earlier with an uncertainty of $0.012 \%$ from measurements on propane. ${ }^{8}$ The densities reported in Table 1 are consistent with the equation of state of Bücker and Wagner. ${ }^{10}$ The experimental densities deviate from this equation of state with a standard deviation of $0.0085 \%$, illustrated in Figure 2 as the dashed lines that represent two standard deviations ( $\pm 0.017 \%$ ).

Values of the single-phase heat capacity of the liquid are shown in Figure 3 at temperatures from (120 to 345) K with pressures up to 35 MPa . The data were measured along isochores with increasing temperature and pressure. Most of the isochores overlap in their temperature ranges, except at the lowest temperatures (highest densities), where the slope $\mathrm{d} C_{V} /$ dT is observed to increase with increasing density. Figure 4 shows deviations between these measured $C_{V}$ data relative to calculated values from the equation of state of Bücker and Wagner. ${ }^{10}$ Scatter in the $C_{V}$ data, primarily due to thermally induced noise, increases with temperature but remains within the estimated uncertainty of $0.7 \%$. The $C_{V}$ data are (1 to 2 ) \%
higher than the equation of state calculations of Bücker and Wagner ${ }^{10}$ over the temperature region from (120 to 300) K, decreasing for temperatures above 300 K . Because Bücker and Wagner ${ }^{10}$ developed their model by using isobaric heat capacity data in a limited range of pressures up to 0.7 MPa and no isochoric heat capacity data, the observed (1 to 2) \% deviations are entirely reasonable. All observed deviations are within the $\pm(2$ to 5$) \%$ range of tolerances given by Bücker and Wagner ${ }^{10}$ in their Figure 40, which apply to calculations of both isobaric and isochoric heat capacities.
Heat Capacity of the Saturated Liquid. The values measured for the saturated liquid heat capacity, $C_{\sigma}$, of isobutane are given in Table 2 at 94 two-phase states. The average of the initial and final temperatures for each heating interval is given for the temperature (ITS-90) of each data point. Directly measured vapor pressures are not reported in Table 2 because they fall in a low-accuracy range of the pressure transducer. Furthermore, saturated liquid densities were not measured in this work. Therefore, it made the most sense to report thermodynamically consistent vapor pressure and saturated liquid density values in


Figure 5. Heat capacities of saturated liquid isobutane $\left(C_{\sigma}\right)$ from (117 to 316) $\mathrm{K}: \Delta$, present results for $n=0.70101 \mathrm{~mol} ;-$, calculated with the equation of state of Bücker and Wagner. ${ }^{10}$

Table 2. Those values were calculated with the Bücker and Wagner equation of state,,$^{10}$ which those authors had shown to reproduce high-accuracy vapor pressure and density data faithfully. Values of the two-phase heat capacity at constant volume $\left(C_{V}{ }^{(2)}\right)$ are presented as well as values of the saturated-liquid heat capacity $C_{\sigma}$. We obtained values of $C_{\sigma}$ by adjusting the $C_{V}{ }^{(2)}$ data with the equation given by Rowlinson ${ }^{18}$

$$
\begin{equation*}
C_{\sigma}=C_{V}^{(2)}-\frac{T}{\rho_{\sigma}^{2}}\left(\frac{\mathrm{~d} \rho_{\sigma}}{\mathrm{d} T}\right)\left(\frac{\mathrm{d} P_{\sigma}}{\mathrm{d} T}\right)+T\left(\frac{1}{\rho_{\sigma}}-\frac{1}{\rho}\right) \frac{\mathrm{d}^{2} P_{\sigma}}{\mathrm{d} T^{2}} \tag{5}
\end{equation*}
$$

where $P_{\sigma}$ and $\rho_{\sigma}$ are the pressure and density of the saturated liquid and $\rho$ is the bulk density of the sample residing in the cell. The derivative quantities were all calculated with the equation of state of Bücker and Wagner. ${ }^{10}$

The saturated-liquid heat capacity, $C_{\sigma}$, depends only on the temperature. Figure 5 shows the present data for $C_{\sigma}$ at temperatures from (116 to 316) K along with values calculated with the equation of state of Bücker and Wagner. ${ }^{10}$ Figure 6 shows relative deviations between the data for $C_{\sigma}$ at temperatures from (116 to 316) K and the values calculated with the equation of state of Bücker and Wagner. ${ }^{10}$ Low-temperature $C_{P}$ (less than


Figure 6. Relative deviations between data for heat capacities of saturated liquid isobutane $\left(C_{\sigma}\right)$ and values calculated with the equation of state of Bücker and Wagner ${ }^{10}$ at temperatures from $(117$ to 316$) \mathrm{K}: \Delta$, present results for $n=0.70101 \mathrm{~mol}$. Deviations for low-temperature $C_{P}(<0.1 \mathrm{MPa})$ values that are nearly equal to $C_{\sigma}$ are also shown: © Aston et al.; ${ }^{12} \bullet$, Parks et al. ${ }^{13}$
0.1 MPa ) data that are nearly equal to $C_{\sigma}$ are also compared with the present measurements in Figure 6. The $C_{P}$ data of Aston et al. ${ }^{12}$ agree with the present $C_{\sigma}$ data to within $1 \%$, whereas the $C_{P}$ data of Parks et al. ${ }^{13}$ are $0.5 \%$ to $1 \%$ lower than the present $C_{\sigma}$ data. Values calculated with the equation of state of Bücker and Wagner ${ }^{10}$ agree with the experimental data to within $1 \%$ from the triple-point temperature to 316 K .

Derived Vapor Pressure. Accurate measurement of the vapor pressure of a liquid becomes increasingly difficult at temperatures well below the normal boiling point. Glos et al. ${ }^{17}$ report the normal boiling point temperature of isobutane to be 261.394 K. Very dilute but volatile impurities in a sample can introduce errors in direct measurements of vapor pressure that are difficult to quantify when the partial pressure of an impurity becomes significant relative to the vapor pressure of the sample. Dynamic techniques such as ebulliometry, effusion, and gas saturation that purge volatile impurities from the gas phase are effective at reducing such errors. It remains difficult to measure pressure accurately for samples that are at a pressure much less than ambient.

Vapor pressure data have been reported at temperatures below 260 K by three groups. The recent work of Glos et al. ${ }^{17}$ covered the temperature range from ( 120 to 340 ) K with very accurate direct measurement of the vapor pressure. Tickner and Lossing ${ }^{19}$ covered the temperature range from ( 122 to 187) K with vapor pressures determined from isobutane flow through a calibrated orifice quantified by the ion peak intensity from a mass spectrometer. Finally, Aston et al. ${ }^{12}$ covered the temperature range from (188 to 262) K by direct measurement of the vapor pressure.

Duarte-Garza and Magee ${ }^{20}$ have shown that vapor pressure values can be derived from measurements of the two-phase heat capacity $C_{V}^{(2)}$ that are described above. This works because $C_{V}^{(2)}$ values have an excellent internal consistency and have their lowest uncertainty below the normal boiling point because the necessary adjustments to the $C_{V}^{(2)}$ measurements for vaporization and $P V$ work are less than $0.1 \%$ of the resulting $C_{V}^{(2)}$ value. Similarly, vapor pressure values derived from these $C_{V}^{(2)}$ data have a low uncertainty from the triple point of 113.73 K up to the vicinity of the reported normal boiling point of 261.394 K .

In the present work, only a brief summary is given for the technique used to derive vapor pressures from isochoric internalenergy measurements in the two-phase region. A detailed discussion is presented in the original work. ${ }^{20}$ The method is based on the expression relating the two-phase internal energy $U^{(2)}(T, V)$ to the vapor pressure

$$
\begin{equation*}
\left(\frac{\partial U^{(2)}}{\partial V^{(2)}}\right)_{T}=T\left(\frac{\mathrm{~d} P_{\sigma}}{\mathrm{d} T}\right)-P_{\sigma}=T^{2}\left(\frac{\mathrm{~d}\left(P_{\sigma} / T\right)}{\mathrm{d} T}\right) \tag{6}
\end{equation*}
$$

where the subscript $\sigma$ signifies a quantity evaluated along the liquid-vapor saturation boundary. Because of the linear dependence of $U^{(2)}$ on the molar volume $V$, the derivative on the left side of eq 6 can be replaced exactly with a finite difference expression

$$
\begin{equation*}
\left(\frac{\partial U^{(2)}}{\partial V^{(2)}}\right)_{T}=\left(\frac{U_{2}^{(2)}-U_{1}^{(2)}}{V_{2}^{(2)}-V_{1}^{(2)}}\right)_{T} \tag{7}
\end{equation*}
$$

where subscripts 1 and 2 indicate any two state points within the two-phase region, including the liquid or vapor phase at saturation, and the superscript (2) indicates the bulk property.

In this work, a bulk property is any extensive thermodynamic property of the vapor in equilibrium with the liquid combined in proportion to their phase ratios on a molar basis.

After computing $\left(\partial U^{(2)} / \partial V^{(2)}\right)_{T}$ at different temperatures between the triple point and the normal boiling point with eq 7, we can fit eq 6 to these data using a nonlinear regression analysis of the parameters in an equation for $P_{\sigma}(T)$. As indicated by eq 6 , the regression analysis must fit the adjustable parameters in the difference expression $T\left(\mathrm{~d} P_{\sigma} / \mathrm{d} T\right)-P_{\sigma}$. Once an expression for $P_{\sigma}(T)$ is selected that can accurately represent the vapor pressure over a wide range of temperature, its analytic derivative $\left(\mathrm{d} P_{\sigma} / \mathrm{d} T\right)$ is determined so that the difference expression $T\left(\mathrm{~d} P_{\sigma} /\right.$ $\mathrm{d} T)-P_{\sigma}$ depends only on $T$ and the parameters in the equation for $P_{\sigma}(T)$. In principle, we could have used nonlinear regression to fit a vapor pressure equation to both $P_{\sigma}$ and $U^{(2)}$ data simultaneously, but we chose the simpler approach to fit only the $U^{(2)}(T, V)$ data for four reasons, namely: $U^{(2)}$ data vary slowly with temperature, they are only weakly affected by volatile impurities, they extend to temperatures close to the triple point, and we avoid the often difficult problem of assigning optimal fitting weights for the different thermodynamic data. The problem of weights would have been aggravated by the fact that $P_{\sigma}$ data typically pass through orders of magnitude in the temperature range of interest.
Experimental values for $U^{(2)}$ at precisely known densities are required to carry out the calculations with eq 7 . We use experimental energy-increment data from isochoric (constant $V^{(2)}$ ) measurements that were performed in this work. Values for $U^{(2)}$ at two or more densities are needed to calculate the change of the bulk internal energy with respect to the bulk molar volume at constant temperature. Because the calorimetric measurements provide the change of internal energy from one density to another, we need additional information at a reference temperature to determine the change of internal energy between two densities. This reference temperature is selected near the normal boiling point, where accurate, direct measurements of the vapor pressure are available.
The value of $\left(\partial U^{(2)} / \partial V^{(2)}\right)_{T}$ at the reference temperature can be calculated with eq 6 and vapor pressure data around the reference temperature. Then, the change of internal energy from density 1 to density 2 at that reference temperature can be determined from

$$
\begin{equation*}
U_{2}^{(2)}-U_{1}^{(2)}=\left(V_{2}^{(2)}-V_{1}^{(2)}\right)\left(\frac{\partial U^{(2)}}{\partial V^{(2)}}\right)_{T} \tag{8}
\end{equation*}
$$

In the present analysis, we set the internal energy of one of these states $U_{1}{ }^{(2)}$ to an arbitrary value of zero at the reference temperature. Then, internal energy increments are calculated at each temperature and density on the basis of differences in $U$ from this reference state. Because the present two-phase measurements are at relatively high densities, it is appropriate to select the vapor phase at saturation for the low density state 1 in eq 8. At each temperature considered, $U_{1}{ }^{(2)}$ and $V_{1}^{(2)}$ are calculated with the equation of state of Bücker and Wagner ${ }^{10}$ at the saturated vapor state with the constraint that $U_{1}{ }^{(2)}=0$ at the reference temperature.

A temperature of 245 K was selected as the reference temperature in the present analysis, where vapor pressures can be directly measured with low uncertainty and the properties of the vapor phase approach those of the ideal gas. The accurate vapor pressure data of Glos et al. ${ }^{17}$ in the vicinity of 245 K form the basis for analysis of the derived vapor pressure data
from the present calorimetric measurements. Bücker and Wagner ${ }^{10}$ discuss an equation of the form

$$
\begin{equation*}
\ln \left(\frac{P_{\sigma}}{P_{\mathrm{c}}}\right)=\frac{1}{1-\tau}\left(C_{1} \tau+C_{2} \tau^{1.5}+C_{3} \tau^{2.5}+C_{4} \tau^{4.5}\right) \tag{9}
\end{equation*}
$$

where $\tau=\left(1-T / T_{\mathrm{c}}\right), T_{\mathrm{c}}=407.81 \mathrm{~K}, P_{\mathrm{c}}=3629 \mathrm{kPa}$, and the $C_{i}$ are empirical coefficients. Equation 9 accurately represents the best available vapor pressure data for isobutane, including the data of Glos et al., ${ }^{17}$ over a wide range of temperatures from near the triple point to the critical point. Equation 9 is selected as the functional form to represent vapor pressure during the present analysis at temperatures from the triple point to 262 K . The derivative of vapor pressure with respect to temperature, as represented by eq 9 , is also required for evaluation of eq 6

$$
\begin{array}{r}
\frac{\mathrm{d} P_{\sigma}}{\mathrm{d} T}=-\frac{P_{\sigma}}{T}\left[\ln \left(\frac{P_{\sigma}}{P_{\mathrm{c}}}\right)+C_{1}+1.5 C_{2} \tau^{0.5}+2.5 C_{3} \tau^{1.5}+\right. \\
\left.4.5 C_{4} \tau^{3.5}\right] \tag{10}
\end{array}
$$

Equation 7 was evaluated from two-phase calorimetric data along an isochore at temperatures from (114 to 250) K with the calorimeter filled with $n=0.70101 \mathrm{~mol}$ isobutane. The change in internal energy along this isochore was determined from a function of temperature fitted to this calorimetric data over the temperature range from (114 to 316) K. The energy needed to increase the temperature of the sample by 1 K is given by

$$
\begin{equation*}
\frac{Q}{n \Delta T}=a_{0}+a_{1} T+a_{2} T^{2}+a_{3} T^{3} \tag{11}
\end{equation*}
$$

where $Q / \mathrm{J}$ is the energy pulse, $n / \mathrm{mol}$ is the amount of substance, and $T / \mathrm{K}$ is the sample temperature. The coefficients are given by $a_{0}=8.1730643 \cdot 10^{1}, a_{1}=1.68991666 \cdot 10^{-1}, a_{2}=$ $-2.29275433 \cdot 10^{-4}$, and $a_{3}=1.07685751 \cdot 10^{-6}$. The change of internal energy along this isochore is given by the expression

$$
\begin{equation*}
\Delta U=\int_{T_{1}}^{T_{2}} \frac{Q}{n \Delta T} \mathrm{~d} T \tag{12}
\end{equation*}
$$

Although the exact cell volume varies with temperature and pressure, we may approximate the density as a function of temperature only. The density of this pseudoisochore at temperatures from (116 to 316) K was fit to the equation

$$
\begin{equation*}
\rho=b_{0}+b_{1} T+b_{2} T^{2} \tag{13}
\end{equation*}
$$

where $\rho$ is in mol $\cdot \mathrm{L}^{-1}$ and the coefficients are given by $b_{0}=$ $5.29491605 \cdot 10^{2}, \quad b_{1}=-1.50381468 \cdot 10^{-2}$, and $b_{2}=$ $-2.01968831 \cdot 10^{-5}$.
The internal energy of the saturated vapor was calculated from the equation of state of Bücker and Wagner ${ }^{10}$ for isobutane. Previous analysis ${ }^{20}$ has shown that the results are insensitive to the choice of a gas-phase equation of state as long as it reproduces the correct behavior of the second virial coefficient. A value of $\left(\partial U^{(2)} / \partial V^{(2)}\right)_{T}$ was calculated at the reference temperature of 245 K with the ancillary equation for vapor

Table 3. Vapor Pressures, $P_{\sigma}$, Derived from the Present Saturated Liquid Heat Capacity, $C_{\sigma}$, Data for Isobutane at Temperatures from (113.73 to 261) $K^{a}$

| $T$ | $P$ | $P_{\text {calcd } \sigma}$ | $\Delta P_{\sigma}$ | $100 \Delta P_{\sigma} / P_{\sigma}$ | $T$ | $P$ | $P_{\text {calcd }, \sigma}$ | $\Delta P_{\sigma}$ | $100 \Delta P_{\sigma} / P_{\sigma}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| K | Pa | Pa | Pa | \% | K | Pa | Pa | Pa | \% |
| 113.730 | 0.02279 | 0.02192 | 0.00087 | 3.969 | 189.000 | 1626.41049 | 1623.32844 | 3.08205 | 0.190 |
| 115.000 | 0.03158 | 0.03043 | 0.00116 | 3.800 | 191.000 | 1915.53605 | 1912.25654 | 3.27951 | 0.171 |
| 117.000 | 0.05198 | 0.05020 | 0.00178 | 3.547 | 193.000 | 2247.24439 | 2243.77333 | 3.47106 | 0.155 |
| 119.000 | 0.08400 | 0.08131 | 0.00269 | 3.310 | 195.000 | 2626.43606 | 2622.78122 | 3.65484 | 0.139 |
| 121.000 | 0.13340 | 0.12941 | 0.00400 | 3.088 | 197.000 | 3058.40015 | 3054.57098 | 3.82916 | 0.125 |
| 123.000 | 0.20838 | 0.20255 | 0.00583 | 2.879 | 199.000 | 3548.82741 | 3544.83487 | 3.99254 | 0.113 |
| 125.000 | 0.32041 | 0.31204 | 0.00838 | 2.684 | 201.000 | 4103.82266 | 4099.67888 | 4.14378 | 0.101 |
| 127.000 | 0.48536 | 0.47351 | 0.01184 | 2.501 | 203.000 | 4729.91612 | 4725.63412 | 4.28201 | 0.091 |
| 129.000 | 0.72481 | 0.70831 | 0.01650 | 2.329 | 205.000 | 5434.07386 | 5429.66713 | 4.40673 | 0.081 |
| 131.000 | 1.06781 | 1.04514 | 0.02266 | 2.169 | 207.000 | 6223.70714 | 6219.18925 | 4.51788 | 0.073 |
| 133.000 | 1.55288 | 1.52217 | 0.03071 | 2.018 | 209.000 | 7106.68070 | 7102.06485 | 4.61585 | 0.065 |
| 135.000 | 2.23060 | 2.18952 | 0.04108 | 1.876 | 211.000 | 8091.32001 | 8086.61853 | 4.70148 | 0.058 |
| 137.000 | 3.16655 | 3.11227 | 0.05428 | 1.744 | 213.000 | 9186.41729 | 9181.64115 | 4.77614 | 0.052 |
| 139.000 | 4.44488 | 4.37402 | 0.07086 | 1.620 | 215.000 | 10401.23649 | 10396.39485 | 4.84164 | 0.047 |
| 141.000 | 6.17249 | 6.08104 | 0.09144 | 1.504 | 217.000 | 11745.51713 | 11740.61685 | 4.90027 | 0.042 |
| 143.000 | 8.48380 | 8.36709 | 0.11672 | 1.395 | 219.000 | 13229.47696 | 13224.52222 | 4.95474 | 0.037 |
| 145.000 | 11.54633 | 11.39893 | 0.14740 | 1.293 | 221.000 | 14863.81362 | 14858.80551 | 5.00812 | 0.034 |
| 147.000 | 15.56691 | 15.38264 | 0.18426 | 1.198 | 223.000 | 16659.70510 | 16654.64135 | 5.06374 | 0.030 |
| 149.000 | 20.79878 | 20.57071 | 0.22807 | 1.109 | 225.000 | 18628.80922 | 18623.68406 | 5.12516 | 0.028 |
| 151.000 | 27.54952 | 27.26989 | 0.27963 | 1.025 | 227.000 | 20783.26210 | 20778.06614 | 5.19597 | 0.025 |
| 153.000 | 36.18970 | 35.85000 | 0.33970 | 0.948 | 229.000 | 23135.67561 | 23130.39592 | 5.27969 | 0.023 |
| 155.000 | 47.16255 | 46.75353 | 0.40902 | 0.875 | 231.000 | 25699.13386 | 25693.75426 | 5.37960 | 0.021 |
| 157.000 | 60.99438 | 60.50612 | 0.48826 | 0.807 | 233.000 | 28487.18896 | 28481.69039 | 5.49857 | 0.019 |
| 159.000 | 78.30589 | 77.72787 | 0.57802 | 0.744 | 235.000 | 31513.85573 | 31508.21692 | 5.63881 | 0.018 |
| 161.000 | 99.82434 | 99.14557 | 0.67877 | 0.685 | 237.000 | 34793.60588 | 34787.80420 | 5.80168 | 0.017 |
| 163.000 | 126.39642 | 125.60558 | 0.79084 | 0.630 | 239.000 | 38341.36125 | 38335.37381 | 5.98743 | 0.016 |
| 165.000 | 159.00203 | 158.08761 | 0.91443 | 0.578 | 241.000 | 42172.48660 | 42166.29166 | 6.19494 | 0.015 |
| 167.000 | 198.76863 | 197.71912 | 1.04952 | 0.531 | 243.000 | 46302.78172 | 46296.36030 | 6.42142 | 0.014 |
| 169.000 | 246.98628 | 245.79037 | 1.19591 | 0.487 | 245.000 | 50748.47300 | 50741.81088 | 6.66212 | 0.013 |
| 171.000 | 305.12326 | 303.77007 | 1.35319 | 0.445 | 247.000 | 55526.20462 | 55519.29458 | 6.91004 | 0.012 |
| 173.000 | 374.84214 | 373.32143 | 1.52071 | 0.407 | 249.000 | 60653.02933 | 60645.87377 | 7.15557 | 0.012 |
| 175.000 | 458.01628 | 456.31867 | 1.69760 | 0.372 | 251.000 | 66146.39886 | 66139.01269 | 7.38617 | 0.011 |
| 177.000 | 556.74657 | 554.86379 | 1.88278 | 0.339 | 253.000 | 72024.15413 | 72016.56807 | 7.58606 | 0.011 |
| 179.000 | 673.37842 | 671.30350 | 2.07492 | 0.309 | 255.000 | 78304.51524 | 78296.77940 | 7.73584 | 0.010 |
| 181.000 | 810.51883 | 808.24631 | 2.27252 | 0.281 | 257.000 | 85006.07136 | 84998.25919 | 7.81217 | 0.009 |
| 183.000 | 971.05340 | 968.57949 | 2.47391 | 0.255 | 259.000 | 92147.77047 | 92139.98306 | 7.78742 | 0.008 |
| 185.000 | 1158.16323 | 1155.48597 | 2.67726 | 0.232 | 261.000 | 99748.90918 | 99741.27984 | 7.62933 | 0.008 |
| 187.000 | 1375.34164 | 1372.46101 | 2.88063 | 0.210 |  |  |  |  |  |

${ }^{a}$ These derived values are compared with the values $P_{\text {calcd }, \sigma}$ calculated with the vapor pressure expression of Bücker and Wagner. ${ }^{10}$
pressure of Bücker and Wagner ${ }^{10}$ for isobutane. Application of the method described above to calculate vapor pressures from the measured internal energy increments resulted in optimum coefficients for the vapor pressure expression of eq 9 . These coefficients are given by $C_{1}=-6.90872652, C_{2}=1.52675920$, $C_{3}=-1.53031020$, and $C_{4}=-2.36936121$. It must be emphasized that these coefficients are optimized for the temperature range from the triple point to 262 K , and eq 9 should not be used with these coefficients at temperatures above 262 K . At temperatures above 262 K , the ancillary expression of Bücker and Wagner ${ }^{10}$ is more accurate. Table 3 provides the vapor pressures calculated from internal energy increments, along with values calculated with the ancillary equation of Bücker and Wagner ${ }^{10}$ for isobutane. Figure 7 shows deviations between the present vapor pressures derived from $C_{\sigma}$, along with data from the literature, relative to the ancillary equation of Bücker and Wagner ${ }^{10}$ at temperatures from the triple point to 262 K . We note that the present values are in excellent agreement with the direct measurements of Glos et al. ${ }^{17}$ at temperatures up to 262 K . Because the ancillary equation of Bücker and Wagner ${ }^{10}$ is based on their own vapor pressure measurements, ${ }^{17}$ a high level of agreement with our derived vapor pressures provides strong evidence that any inconsistencies between them are inconsequential. This finding is most important for the calculations of $C_{\sigma}$ by using eq 5 because two important derivatives in this equation were calculated with the ancillary equation of Bücker and Wagner. ${ }^{10}$ Figure 8 focuses


Figure 7. Deviations between vapor pressures $\left(P_{\sigma}\right)$ and the vapor pressure expression of Bücker and Wagner ${ }^{10}$ for isobutane ( $P_{\sigma, \text { calcd }}$ ) at temperatures from $(113.707$ to 260$) \mathrm{K}$ :,$- P_{\sigma}$ values derived from the present saturated liquid heat capacity, $C_{\sigma}$, data. Published data for $P_{\sigma}$ are also shown: $\Delta$, Glos et al.; ${ }^{17}+$, Tickner and Lossing; ${ }^{19} \diamond$, Aston et al.; ${ }^{12}$---, uncertainty bounds for the data of Glos et al. ${ }^{17}$
on the vapor pressure deviations at the lowest temperatures, from the triple point to 160 K . The present values are in slightly better agreement with the data of Tickner and Lossing ${ }^{19}$ (orifice-mass selective detector) than the direct pressure measurements of Glos


Figure 8. Deviations between vapor pressures $\left(P_{\sigma}\right)$ and the vapor pressure expression of Bücker and Wagner ${ }^{10}$ for isobutane ( $P_{\sigma, \text { calcd }}$ ) at temperatures from $(113.707$ to 160$) \mathrm{K}:-, P_{\sigma}$ values derived from the present saturated liquid heat capacity, $C_{\sigma}$, data. Published data for $P_{\sigma}$ are also shown: $\triangle$, Glos et al. $;^{17}+$, Tickner and Lossing; ${ }^{19}---$, uncertainty bounds for the data of Glos et al. ${ }^{17}$
et al. ${ }^{17}$ at temperatures below 136 K . It appears that this indirect technique was less sensitive to impurities at the lowest pressures. However, agreement is excellent ( $<0.2 \mathrm{~Pa}$ ) and within claimed uncertainties for all measurements at temperatures below 140 K. It is also apparent in Figure 8 that there were no data available at temperatures below 120 K prior to the present work. On the basis of the present vapor pressure results, the triple-point pressure of isobutane is 0.023 Pa at the triple-point temperature of 113.73 K , reported above. This agrees well with the value of 0.02 Pa reported by Glos et al. ${ }^{17}$
Assessment of Uncertainties. Uncertainty in $C_{V}$ is limited by the uncertainty of the temperature increase measurement and the change-of-volume work adjustment. In the following discussion, the definition for the expanded uncertainty, which is two times the standard uncertainty, corresponds to a coverage factor $k=2$ and thus a two standard deviation estimate. Recent triplepoint measurements of $\operatorname{argon}^{8}$ (Supporting Information) indicate that the calibration of the platinum resistance thermometer presently has an uncertainty of 0.003 K near the ITS-90 fixed point for the triple point of argon ( 83.8058 K ). Different sources of uncertainty, including calibration of the platinum resistance thermometer, radiation to or from the thermometer head, and shifts in the resistance at temperature scale fixed points contribute to an expanded uncertainty of 0.01 at 100 K to 0.03 at 345 K for the absolute temperature measurement during $C_{V}$ measurements. The uncertainty of the temperature increase also depends on reproducibility of temperature measurements. Between heating intervals, temperatures are measured while the cell temperature drifts linearly with time for a period of 20 min . The temperatures assigned to the beginning ( $T_{1}$ ) and to the end $\left(T_{2}\right)$ of a heating interval are determined by extrapolation of a linear temperature drift (approximately $\left(\left(-1 \cdot 10^{-3}\right)\right.$ to $\left.\left.\left(0.5 \cdot 10^{-3}\right)\right) \mathrm{K} \cdot \mathrm{min}^{-1}\right)$ to the midpoint time of the interval. This procedure leads to an uncertainty of ( 0.01 to 0.04 ) K for the extrapolated temperatures $T_{1}$ and $T_{2}$, depending on the standard deviation of the linear fit. In all cases, values from ( 0.002 to $0.006) \mathrm{K}$ were obtained for the uncertainty of the temperature rise, $\Delta T=T_{2}-T_{1}$. For a typical experimental value of $\Delta T=$ 4 K , this corresponds to a relative uncertainty of between 0.05 $\%$ and $0.15 \%$.
The uncertainty of the change-of-volume work adjustment has a larger influence on the single-phase values because two-
phase experiments are performed over a smaller pressure interval. The ratio of change-of-volume work to total applied heat ranges from 0.11 for the highest density isochore to 0.005 at the lowest densities. Estimated uncertainties of 2.3 \% to 3.0 \% in the change-of-volume work adjustment are due to both the deviation of the calculated pressure derivatives and the uncertainty of the volume change. This leads to a relative uncertainty of $0.2 \%$ in $C_{V}$ for the lowest density isochore up to $0.3 \%$ for the highest density.

The energy applied to the calorimeter is the integral of the product of voltage and current from the initial to the final heating time. Voltage and current are measured 20 times during a heating interval, which lasts 20 min . The measurements of the electrical quantities have a relative uncertainty of $0.01 \%$. However, we must account for the effect of radiation heat losses or gains, which occur when a time-dependent lag of the controller leads to a small temperature difference of about 20 mK between the cell and the radiation shield at the beginning and end of a heating period. Because heat transfer by radiation is proportional to $\left(T_{1}{ }^{4}-T_{2}{ }^{4}\right) \approx 4 \cdot T^{3} \cdot \Delta T$, radiation losses should increase substantially with the cell temperature. Therefore, the relative uncertainty in the applied heat is estimated to be 0.02 $\%$ for lower temperatures and up to $0.10 \%$ for the highest temperatures. This leads to a relative uncertainty in $C_{V}$ between $0.04 \%$ and $0.2 \%$.

The energy applied to the empty calorimeter has been measured in repeated experiments and fitted to a function of temperature; ${ }^{3}$ its relative uncertainty is less than $0.02 \%$. Its influence on the uncertainty of the heat capacity is reduced because the ratio of the heat applied to the empty calorimeter to the total heat varies only from 0.35 to 0.70 for the singlephase runs and from 0.61 to 0.62 for the two-phase runs. The mass of each sample was determined with a relative uncertainty of $0.01 \%$ by differential weighing before and after trapping the sample in the collection cylinder. The density calculated from this mass and the cell volume has a relative uncertainty of $0.03 \%$. For pressures, the uncertainty of 7 kPa for the pressure transducer is added to the cross term for the pressure derivative in the change-of-volume work adjustment. However, the uncertainty of neither $P$ nor $\rho$ contributes appreciably to the combined uncertainty for molar heat capacity. When the various sources of experimental uncertainty are combined using a root-sum-of-squares formula, the expanded uncertainty (i.e., a coverage factor $k=2$ and thus a two-standard deviation estimate) is estimated to be $0.7 \%$ for $C_{V}, 0.5 \%$ for $C_{V}{ }^{(2)}$, and $0.7 \%$ for $C_{\sigma}$.

## Conclusions

Calorimetric measurements were conducted on liquid isobutane in equilibrium with its vapor and on compressed liquid isobutane along isochores. The heat capacity, $C_{V}$, was measured as a function of temperature and density at 171 single-phase and 94 two-phase states for isobutane. Molar heat capacity results are reported for two-phase $\left(C_{V}^{(2)}\right)$, saturated liquid $\left(C_{\sigma}\right)$, and single-phase $\left(C_{V}\right)$ isochores. Temperatures ranged from the triple point of isobutane, near 114 K , to the upper temperature limit of the calorimeter of 345 K at pressures up to 35 MPa . Vapor pressure data are reported that are based on measurements of $C_{V}^{(2)}$ along a two-phase isochore. Measurements were also made to determine the triple-point temperature of (113.707 $\pm$ $0.030) \mathrm{K}$ and enthalpy of fusion of $(4494 \pm 20) \mathrm{J} \cdot \mathrm{mol}^{-1}$ for isobutane near its triple point. The principal sources of uncertainty are the temperature rise measurement and the change-of-volume work adjustment. The expanded uncertainty
(i.e., a coverage factor $k=2$ and thus a two-standard deviation estimate) for values of $C_{V}^{(2)}$ is estimated to be $0.5 \%$, for $C_{\sigma}$ it is $0.7 \%$, and for $C_{V}$ it is $0.7 \%$. These measurements agree well with published heat capacity measurements and with the Helmholtz free energy model of Bücker and Wagner, ${ }^{10}$ who had developed their model independently of this study. The present study suggests that on the basis of the presently available experimental thermodynamic data, a revision of the model of Bücker and Wagner ${ }^{10}$ is not needed at this time.

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## Supporting Information Available:

Measurement of the triple-point temperature of pure (0.999999 mol fraction) argon and its offset of 3 mK above that of the fixed point on the ITS-90 temperature scale. This material is available free of charge via the Internet at http://pubs.acs.org.

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