# P- $\rho$-T Data for Carbon Dioxide from (310 to 450 ) K up to 160 MPa 

Ivan D. Mantilla, Diego E. Cristancho, Saquib Ejaz, and Kenneth R. Hall*<br>Artie McFerrin Department of Chemical Engineering, Texas A\&M University, College Station, Texas 77843-3122<br>Mert Atilhan<br>Chemical Engineering Department, Qatar University, Doha, Qatar

Gustavo A. Iglesias-Silva
Departamento de Ingeniería Química, Instituto Tecnológico de Celaya, México


#### Abstract

This paper presents $P-\rho-T$ data for pure carbon dioxide measured with a high-pressure single-sinker magnetic suspension densimeter (MSD). The data cover four isotherms (310, 350, 400, 450) K. The MSD technique yields data with less than 0.03 \% relative uncertainty over the pressure range of (10 to 200) MPa. A comparison of the experimental data to the equation of state developed by Span and Wagner indicates that the equation and the data are consistent within the low range of pressure. The reference equation has a relative uncertainty of $\pm 0.03 \%$ to $\pm 0.05 \%$ below 30 MPa . At higher pressures, the density predictions of this model agree with the experimental data with a maximum relative deviation of $0.1 \%$.


## Introduction

Carbon dioxide is the simplest and best known molecule with a strong quadrupole moment. It is an important reference and testing fluid for calibration purposes and for extending physical models beyond nonpolar frontiers. Span and Wagner ${ }^{1}$ developed the most accurate reference equation of state (EOS) for carbon dioxide. The relative uncertainty of its density predictions ranges from $\pm 0.03 \%$ to $\pm 0.05 \%$ between (240 and 523) K at pressures up to 30 MPa . At higher pressures, up to 200 MPa , the EOS uncertainty increases to $\pm 0.1 \%$.

This work provides new density data for pure carbon dioxide measured with a single-sinker magnetic suspension densimeter (MSD) up to 160 MPa . Of particular importance are the highpressure data (above 10 MPa ) for which the total relative uncertainty is less than $0.03 \%$. The relative uncertainty increases to $0.05 \%$ at lower pressures up to 7 MPa and greater than 0.1 $\%$ at the lowest pressures reported here.

## Experimental Section

The reported data include isothermal density data for carbon dioxide at $(310,350,400,450) \mathrm{K}$ up to 160 MPa . Matheson Tri-Gas provided the carbon dioxide with a mole fraction purity of $99.999 \%$. The mass and volume of the titanium sinker, determined using the procedure described by McLinden and Splett, ${ }^{2}$ are 30.39159 g and $6.741043 \mathrm{~cm}^{3}$, respectively. Patil et al. ${ }^{3}$ provide a complete and detailed description of this specific MSD. A PRT (Minco Products model S1059PA5X6 Platinum Resistance Thermometer) calibrated at fixed temperature points defined by ITS-90 and by a calibrated PRT traceable to NIST provides temperatures with a readability of 2.5 mK and repeatability of 0.5 mK , for a total uncertainty of $\pm 3 \mathrm{mK}$. Pressure measurements use two Digiquartz transducers (40 and

* To whom correspondence should be addressed. E-mail: krhall@tamu.edu. Phone: (979) 845 3357. Fax: (979) 8456446.

200 MPa ) from Paroscientific Inc. with uncertainties of $\pm 0.01$ \% full scale.

McLinden et al. ${ }^{4}$ suggest a procedure to evaluate the force transmission error (FTE) in MSD systems. Using that work as a guide, a comprehensive FTE analysis of the TAMU instrument recently appeared. ${ }^{5}$ The coupling factor $(\phi-1)$ for the MSD is $(195,189,204$, and 194$) \times 10^{-6}$ for $(310,350,400,450) \mathrm{K}$, respectively. The reproducibility of these values is less than $\pm$ $2 \times 10^{-6}$ for each isotherm. The data reported here have these coupling factors applied.

## Results and Analysis

Table 1 contains the four sets of isothermal data, each of which contains several density measurement cycles (raw data) at each pair of conditions $(T, P)$. The raw data have been adjusted to nominal temperatures and pressures with the mean density point taken as the most likely value to report. The data compare well to values predicted by the Span and Wagner ${ }^{1}$ reference equation of state as implemented in RefProp 8.0. ${ }^{6}$ The last column in the table shows the percent deviation between data and the equation.

Figure 1 is a $\log$-linear plot ${ }^{7}$ (linear scale delimited by the $\pm 0.1$ band and logarithmic scale beyond that) that compares these new data to other data used in developing the reference equation of state. The data by Michels et al. ${ }^{8}$ range from (313 to 423 ) K in temperature and from (8 to 180) MPa in pressure. Likewise, the data by Juza et al. ${ }^{9}$ range from ( 323.15 to 448.15 ) K and from ( 70 to 200) MPa. The data by Klimeck et al. ${ }^{10}$ range from (313 to 430) K and from (4 to 30) MPa. And the data by Gokmenoglu et al. ${ }^{11}$ range from ( 314 to 425 ) K in temperature and from (13 to 67) MPa in pressure. The development of the reference EOS excluded the latter data. The zero deviation line represents the prediction of the equation of state.

Table 1. $P, \rho, T$ Data Measured for Carbon Dioxide

| $P$ | $\rho(\exp )$ | $\rho$ (EOS) |  |
| :---: | :---: | :---: | :---: |
| MPa | $\mathrm{kg} \cdot \mathrm{m}^{-3}$ | $\mathrm{kg} \cdot \mathrm{m}^{-3}$ | $100 \cdot\left(\rho_{\text {exp }}-\rho_{\text {Eos }}\right) / \rho_{\text {exp }}$ |
| $T=310 \mathrm{~K}$ |  |  |  |
| 1.998 | 37.614 | 37.603 | 0.029 |
| 5.002 | 116.171 | 116.197 | -0.022 |
| 10.014 | 686.160 | 686.431 | -0.039 |
| 19.987 | 856.152 | 856.161 | -0.001 |
| 29.966 | 921.817 | 921.860 | -0.005 |
| 49.929 | 999.875 | 1000.131 | -0.026 |
| 75.042 | 1062.566 | 1063.061 | -0.047 |
| 100.055 | 1108.290 | 1108.777 | -0.044 |
| 125.012 | 1144.813 | 1145.222 | -0.036 |
| 139.289 | 1162.991 | 1163.314 | -0.028 |
| 149.970 | 1175.569 | 1175.839 | -0.023 |
| 159.844 | 1186.558 | 1186.760 | -0.017 |
| $T=350 \mathrm{~K}$ |  |  |  |
| 2.011 | 32.349 | 32.353 | -0.014 |
| 5.001 | 89.638 | 89.641 | -0.003 |
| 9.986 | 228.244 | 228.298 | -0.024 |
| 19.981 | 613.586 | 613.738 | -0.025 |
| 30.013 | 758.897 | 759.108 | -0.028 |
| 50.017 | 884.809 | 884.832 | -0.003 |
| 74.904 | 969.636 | 969.722 | -0.009 |
| 99.977 | 1027.564 | 1027.687 | -0.012 |
| 124.895 | 1071.804 | 1071.896 | -0.009 |
| $T=400 \mathrm{~K}$ |  |  |  |
| 0.999 | 13.455 | 13.464 | -0.062 |
| 1.998 | 27.430 | 27.436 | -0.022 |
| 4.998 | 72.779 | 72.772 | 0.010 |
| 10.021 | 161.960 | 161.938 | 0.014 |
| 14.986 | 267.104 | 267.101 | 0.001 |
| 20.028 | 381.082 | 381.115 | -0.009 |
| 25.054 | 482.518 | 482.526 | -0.002 |
| 29.994 | 561.435 | 561.412 | 0.004 |
| 35.005 | 623.368 | 623.290 | 0.012 |
| 39.985 | 672.037 | 671.953 | 0.012 |
| 49.967 | 745.445 | 745.244 | 0.027 |
| 59.920 | 799.163 | 798.946 | 0.027 |
| 69.997 | 842.031 | 841.747 | 0.034 |
| 79.880 | 876.766 | 876.413 | 0.040 |
| 89.950 | 906.957 | 906.558 | 0.044 |
| 99.838 | 932.857 | 932.415 | 0.047 |
| 109.931 | 956.349 | 955.880 | 0.049 |
| 120.079 | 977.599 | 977.117 | 0.049 |
| 126.010 | 989.102 | 988.620 | 0.049 |
| 139.527 | 1013.251 | 1012.762 | 0.048 |
| $T=450 \mathrm{~K}$ |  |  |  |
| 4.998 | 62.269 | 62.269 | 0.000 |
| 9.998 | 131.635 | 131.605 | 0.023 |
| 19.983 | 284.802 | 284.880 | -0.027 |
| 29.992 | 430.142 | 430.143 | 0.000 |
| 49.933 | 626.132 | 625.718 | 0.066 |
| 75.024 | 762.158 | 761.715 | 0.058 |
| 99.833 | 847.163 | 846.528 | 0.075 |
| 122.193 | 903.768 | 902.952 | 0.090 |

Table 2. Second and Third Virial Coefficients for $\mathrm{CO}_{2}$

| $\frac{T}{\mathrm{~K}}$ | $\frac{B}{\mathrm{~cm}^{3} \cdot \mathrm{~mol}^{-1}}$ | $\frac{C}{\left(\mathrm{~cm}^{3} \cdot \mathrm{~mol}^{-1}\right)^{2}}$ |
| :---: | :---: | :---: |
| 310 | -111.85 | 4133 |
| 350 | -83.70 | 3464 |
| 400 | -59.70 | 2838 |
| 450 | -43.10 | 2515 |

The predictions are consistent with current experimental densities within the relative uncertainties claimed for the reference EOS over the whole range of pressures studied, namely, $\pm 0.03 \%$ to $\pm 0.05 \%$ below 30 MPa and $\pm 0.1 \%$ beyond that. However, in the high-pressure region the current uncertainties are lower than those of the data used to develop


Figure 1. Percent deviation of experimental data compared to Span and Wagner equation of state. $\mathbf{\Delta}$, This work; $\bigcirc$, Klimeck; ${ }^{10} \diamond$, Michels; ${ }^{8} \square$, Juza; ${ }^{9} \times$, Gokmenoglu. ${ }^{11}$


Figure 2. Second virial coefficients for $\mathrm{CO}_{2}$. $\mathbf{\Delta}$, This work; O, Holste; ${ }^{12}$ $\square$, Duschek; ${ }^{13} \Delta$, Patel; $;{ }^{14}+$, Butcher, ${ }^{15}-$, Dadson; ${ }^{16} \times$, Huff; ${ }^{17} \diamond$, Michels; ${ }^{18}$ *, Waxman; ${ }^{19}$---, EOS prediction. ${ }^{1}$


Figure 3. Absolute deviations of second virial coefficients from values predicted by the Span et al. equation of state $\Delta B=B_{\exp }-B_{\text {Eos. }} \mathbf{\Delta}$, This work; O, Holste; ${ }^{12} \square$, Duschek; ${ }^{13} \Delta$, Patel; ${ }^{14}+$, Butcher; ${ }^{15}$-, Dadson; ${ }^{16}$ $\times$, Huff, ${ }^{17} \diamond$, Michels; ${ }^{18} *$, Waxman. ${ }^{19}$
the EOS. It is possible that a refit using the data reported here would improve the predictive capabilities of the EOS at high pressures.

The linear behavior of the compressibility factor of carbon dioxide at low pressures was used to determine the second and third virial coefficients with an uncertainty of $2.5 \mathrm{~cm}^{3} \cdot \mathrm{~mol}$ and


Figure 4. Third virial coefficients for $\mathrm{CO}_{2}$. 4 , This work; $\bigcirc$, Holste; ${ }^{12} \square$, Duschek; ${ }^{13} \diamond$, Michels; ${ }^{18}+$, Butcher; ${ }^{15}---$, EOS prediction. ${ }^{1}$


Figure 5. Absolute deviations of third virial coefficients from values predicted by the Span et al. equation of state $\Delta C=C_{\exp }-C_{\text {Eos. }} \mathbf{\Delta}$, This work; O, Holste; ${ }^{12} \square$, Duschek; ${ }^{13} \diamond$, Michels; ${ }^{18}+$, Butcher. ${ }^{15}$
$250\left(\mathrm{~cm}^{3} \cdot \mathrm{~mol}\right)^{2}$, respectively. These values appear in Table 2. Figures 2 and 3 present the second virials and their deviations using Span and Wagner ${ }^{1}$ as the reference. Figures 4 and 5 present the same information for the third virials. The current values are consistent with previous ones generated using lowpressure experiments (lower uncertainty), and their trends with temperature follow the equation of state predictions.

## Conclusions

This paper presents accurate $P \rho T$ data for carbon dioxide using a high-pressure single-sinker MSD within an experimental uncertainty of $3 \times 10^{-4} \mathrm{~kg} \cdot \mathrm{~m}^{3}$ in density for pressures greater than 7 MPa and up to $5 \times 10^{-4} \mathrm{~kg} \cdot \mathrm{~m}^{3}$ for pressures between 5 and 7 MPa . The Span and Wagner ${ }^{1}$ EOS agrees well with the data up to 160 MPa .

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