

# Monte Carlo Simulations of the Liquid–Vapor Interface of Lennard–Jones Diatomics for the Direct Determination of the Interfacial Tension Using the Test-Area Method<sup>†</sup>

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We have performed Monte Carlo simulations to examine the vapor–liquid interface of systems of homonuclear diatomic Lennard–Jones (LJ) molecules. The test-area (TA) method developed by Gloor et al. (*J. Chem. Phys.*, **2005**, *123*, 134703) is used to determine the vapor–liquid interfacial tension; the coexistence properties, such as densities and vapor pressures, are also reported. The TA method involves a thermodynamic (free energy perturbation) route to the tension which from a computational point of view offers some advantages over the more common mechanical (virial) route of Kirkwood and Buff (*J. Chem. Phys.* **1949**, *17*, 338). Simulation data are reported for LJ diatomics with different bond lengths  $L^* = L/\sigma = 0.3292, 0.505, 0.6, 0.63, 0.67, 0.793$ , and 1.0 (with  $\sigma$  representing the diameter of a LJ segment) over the whole range of liquid–vapor coexistence temperatures. The simulated vapor–liquid interfacial tensions for systems with values of the parameters chosen to represent real fluids (e.g., nitrogen, fluorine, chloride, and ethane) are in very good agreement with the experimental data.

## 1. Introduction

The evaluation of interfacial properties is key in many industrial processes and biological phenomena. From a more fundamental point of view, the understanding of interfacial behavior between a liquid and a vapor phase is of central importance in the description of physical phenomena, for example, capillarity, surfactant behavior, and adhesion between surfaces and liquid droplets. The interfacial tension between a liquid and a vapor can be measured experimentally in a straightforward manner, although certain limitations are encountered, particularly for low values of the tension. A detailed discussion of the various theoretical and computational approaches has been presented by Rowlinson and Widom<sup>1</sup> in their excellent monograph. The most successful theories include the square gradient theory (SGT), originating from the early work of van der Waals<sup>2</sup> and Rayleigh<sup>3</sup> as rediscovered by Cahn and Hilliard,<sup>4</sup> and the more recent approaches based on density functional theory (DFT), as popularized for fluid systems by Evans<sup>5</sup> and others. Fluid interfaces have been simulated at the microscopic level since the early days of the development of the Monte Carlo (MC) and molecular dynamics (MD) techniques which followed the introduction of electronic computers. A number of the key contributions in the area can be attributed to Rowlinson and co-workers including seminal studies of vapor–liquid interfaces for both planar surfaces<sup>6,7</sup> and drops.<sup>8</sup> It is thus a particularly honor for us to present our work in a Special Issue of the *Journal of Chemical and Engineering Data* dedicated to Sir John, who has made such important fundamental advances in broad areas of experimental, theoretical, and simulation research of fluids and fluid mixtures.

There are various options for the determination of the surface tension of fluid interfaces by molecular simulation; the reader is directed to ref 9 for a recent review of the common techniques. We only provide a brief discussion here, highlighting some of the important issues. The most widely employed method to calculate the surface tension is based on a mechanical route, which was first introduced by Kirkwood and Buff.<sup>10</sup> According to the mechanical approach, the surface tension,  $\gamma$ , can be obtained from the appropriate tensorial components of the pressure as

$$\gamma = \int_{-\infty}^{\infty} [P_N - P_T(z)] dz \quad (1)$$

where  $P_N$  is the component of the pressure tensor normal to the surface, while  $P_T$  is the tangential component. Here, the  $z$  direction is taken to be orthogonal to the plane of the interface. An alternative methodology for the calculation of surface tension is the finite size scaling approach introduced by Binder,<sup>11</sup> usually implemented in the grand canonical ensemble; the advantage of this approach is that one can estimate values of the interfacial tension close to the critical point. The other class of technique is based on a thermodynamic route where the surface contribution to the free energy is evaluated directly from a perturbative scheme. The test-area (TA) method,<sup>9</sup> which has gained popularity in recent years, belongs to this class of thermodynamic approaches.

At first sight the TA method for the direct determination of interfacial tension bears much resemblance to the approach of Bennett<sup>12</sup> for the calculation of the difference in free energy from the simultaneous simulation of two (or more) distinct systems; if the systems have different interfacial areas, one can estimate the corresponding surface tension (and surface free energy).<sup>13,14</sup> In the case of the TA approach, however, the

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derivative of the free energy with respect to the interfacial area is estimated directly (using a finite difference method) from the simulation of a single (inhomogeneous) system by computing the change in free energy due to small test (ghost) perturbations in the area; the change in free energy is determined from the general perturbation expressions developed by Zwanzig.<sup>15</sup> One should note that there is a close similarity between the TA formalism and related approaches employing the so-called expanded ensemble<sup>16,17</sup> and wandering interface<sup>18</sup> methods. Detailed comparisons of the various methods were made by Errington and Kofke.<sup>16</sup>

The change in the Helmholtz free energy  $F$  of a system in the canonical ensemble (where the number of particles  $N$ , volume  $V$ , and temperature  $T$  are held constant) due to a perturbation from a state 0 to a state 1 can be defined in terms of the corresponding partition functions  $Q_0$  and  $Q_1$  as

$$\Delta F_{0 \rightarrow 1} = F_1 - F_0 = -k_B T \ln \left( \frac{Q_1}{Q_0} \right) = -k_B T \ln \left( \frac{Z_1}{Z_0} \right) \quad (2)$$

where  $k_B$  is the Boltzmann constant, and the partition function is

$$Q = \frac{1}{N! \Lambda^{3N}} \int \exp \left( -\frac{U}{k_B T} \right) \mathbf{d}\mathbf{r}^N = \frac{Z}{N! \Lambda^{3N}} \quad (3)$$

Here,  $Z = \int \exp(-U/k_B T) \mathbf{d}\mathbf{r}^N$  is the configurational integral,  $U(\mathbf{r}^N)$  is the configurational energy which is a function of the configurational space  $\mathbf{r}^N$  (and corresponds to the total potential energy resulting from the interactions between all of the particles in the system in the absence of external fields), and  $\Lambda$  is the de Broglie wavelength which incorporates all of the kinetic contributions. If the configurational energy  $U_1$  of state 1 is treated as a perturbation to the initial configurational energy  $U_0$  ( $U_1 = U_0 + \Delta U$ , where  $\Delta U$  is the change in the potential energy due to the perturbation), then the ratio of configurational integrals in eq 2 can be expressed as

$$\frac{Z_1}{Z_0} = \frac{\int \exp \left( -\frac{U_1}{k_B T} \right) \mathbf{d}\mathbf{r}^N}{\int \exp \left( -\frac{U_0}{k_B T} \right) \mathbf{d}\mathbf{r}^N} = \frac{\int \exp \left( -\frac{U_0}{k_B T} \right) \exp \left( -\frac{\Delta U}{k_B T} \right) \mathbf{d}\mathbf{r}^N}{\int \exp \left( -\frac{U_0}{k_B T} \right) \mathbf{d}\mathbf{r}^N} \quad (4)$$

The probability of a particular configuration of  $N$  particles being in a state  $\mathbf{r}^N$  is proportional to

$$P(\mathbf{r}^N) = \frac{1}{N! \Lambda^{3N} Q} \exp \left( -\frac{U}{k_B T} \right) = \frac{1}{Z} \exp \left( -\frac{U}{k_B T} \right) \quad (5)$$

and the corresponding ensemble average of any property  $M$  is thus given by

$$\langle M \rangle = \int M P(\mathbf{r}^N) \mathbf{d}\mathbf{r}^N = \frac{1}{Z} \int M \exp \left( -\frac{U}{k_B T} \right) \mathbf{d}\mathbf{r}^N \quad (6)$$

so that eq 4 can be expressed as the Boltzmann factor of the change  $\Delta U$  in configurational energy averaged over the unperturbed system 0:

$$\frac{Z_1}{Z_0} = \left\langle \exp \left( -\frac{\Delta U}{k_B T} \right) \right\rangle_0 \quad (7)$$

Inserting the result of eq 7 in eq 2 for the perturbation in the Helmholtz free energy  $\Delta F$  we obtain the following simple relation:

$$\Delta F_{0 \rightarrow 1} = -k_B T \ln \left\langle \exp \left( -\frac{\Delta U}{k_B T} \right) \right\rangle_0 \quad (8)$$

The equation is the general result of Zwanzig<sup>15</sup> that forms the basis of high-temperature perturbation theories; it is important to emphasize at this point that eq 8 is exact in the limit of infinitesimal perturbations.

We now turn to the thermodynamic definition of surface tension which in the case of a planar interface in the canonical ensemble can be expressed as<sup>1</sup>

$$\gamma = \left( \frac{\partial F}{\partial A} \right)_{N,V,T} = \lim_{\Delta A \rightarrow 0} \left( \frac{\Delta F}{\Delta A} \right)_{N,V,T} \quad (9)$$

The TA method consists in performing very small test changes (perturbations) in the interfacial area of the system (by increasing/decreasing the box dimensions in the plane of the interface with a corresponding decrease/increase in the box length normal to the interface) keeping the number of particles  $N$ , volume  $V$ , and temperature  $T$  constant. The change in Helmholtz free energy due to the test perturbation in the area is obtained from eq 8 as the Boltzmann factor of the test change in the configurational energy averaged over the configurations of the unperturbed reference system. The surface tension can then simply be estimated from eq 9 from an extrapolation to infinitesimal area perturbations:

$$\gamma = \lim_{\Delta A \rightarrow 0} \left( \frac{\Delta F_{0 \rightarrow 1}}{\Delta A} \right)_{N,V,T} = \lim_{\Delta A \rightarrow 0} \frac{-k_B T \ln \left\langle \exp \left( -\frac{\Delta U}{k_B T} \right) \right\rangle_0}{\Delta A} \quad (10)$$

A similar form of the Zwanzig relation is commonly used to estimate the chemical potential with test perturbations in the number of particles (c.f., the test-particle method of Widom<sup>19</sup>) or to determine the pressure<sup>20–22</sup> and pressure tensor<sup>23,24</sup> with test-volume perturbations.

Originally, the TA approach was implemented in combination with Metropolis MC simulations to determine the surface tension for the vapor–liquid interface of Lennard–Jones (LJ) and square-well systems.<sup>9</sup> The estimates obtained with the TA method were found to be in good agreement with those obtained from a mechanical route and MD simulation. The TA method and its variants have been used extensively to determine the interfacial tensions of systems with realistic potentials, for example,  $n$ -alkanes,<sup>25</sup> water,<sup>26</sup> binary mixtures of  $n$ -alkanes,<sup>27</sup>

high-pressure aqueous solutions of methane<sup>28</sup> and acid gases,<sup>29,30</sup> ionic liquids,<sup>31</sup> and model fluids including chains of LJ segments,<sup>17,32,33</sup> the Widom–Rowlinson mixture,<sup>34</sup> Mie  $n - 6$  particles,<sup>35</sup> and drops of LJ fluids.<sup>36</sup> The method is now also being extended to treat the fluid–fluid and fluid–solid interfaces exhibited by systems with hard-core interactions such as hard-sphere chains,<sup>37,38</sup> primitive models of liquid crystals,<sup>39–42</sup> model colloid–polymer mixtures,<sup>43,44</sup> and mixtures of prolate and oblate particles.<sup>45,46</sup>

In the original work with the TA approach,<sup>9</sup> the central difference (CD) approximation was taken from two simultaneous test perturbations from the reference state to provide more accurate values for the surface tension; the first deformation involved an increase in the surface area  $A_1 = A_0 + \Delta A$  corresponding to a free-energy change of  $\Delta F_{0 \rightarrow 1} = F_1(A_1) - F_0(A_0)$ , and the second a decrease in the surface area  $A_{-1} = A_0 - \Delta A$  corresponding to a free-energy change of  $\Delta F_{0 \rightarrow -1} = F_{-1}(A_{-1}) - F_0(A_0)$ .

In the current work we study the vapor–liquid interfacial tension obtained with the TA method from MC simulations of the vapor–liquid equilibria (VLE) of two center homonuclear LJ molecules (2CLJ) for a range of bond lengths. Accurate estimates for the surface tension are obtained from an extrapolation to infinitesimal area changes  $\Delta A = 0$  from a linear fit of the average of the results obtained from a series of positive and negative perturbations in the surface area. Our results are compared with those obtained by other techniques where possible. A comparison with experimental results of real diatomic fluids is also made using values of intersegment parameters from the literature.

## 2. Simulation of the Interfacial Tension

The 2CLJ diatomics studied here comprise two spherically truncated sites characterized by a 12–6 LJ site–site potential with a cutoff radius of  $6\sigma$ :

$$u_{\text{ST}}(r) = \begin{cases} 4\epsilon \left[ \left( \frac{\sigma}{r_{ij}} \right)^{12} - \left( \frac{\sigma}{r_{ij}} \right)^6 \right] & r_{ij} \leq 6\sigma \\ 0 & r_{ij} > 6\sigma \end{cases} \quad (11)$$

where  $\sigma$  is the diameter of each site,  $\epsilon$  is the well-depth energy parameter, and  $r_{ij}$  is the distance between site  $i$  of one molecule and site  $j$  on another. No long-range corrections are considered as these are negligible for this range of interaction.<sup>9,47,48</sup> The variables are described in reduced units for convenience, being defined in terms of the LJ size and energy parameters: the bond length  $L^* = L/\sigma$ , the temperature  $T^* = k_B T/\epsilon$ , the density  $\rho^* = \rho\sigma^3$ , the pressure  $P^* = P\sigma^3/\epsilon$ , the energy  $E^* = E/\epsilon$ , the surface tension  $\gamma^* = \gamma\sigma^2/\epsilon$ , the box dimensions  $l^* = l/\sigma$ , the interface area  $A^* = A/\sigma^2$ , the distance from the interface  $z^* = z/\sigma$ , and the interface width  $D^* = D/\sigma$ .

The vapor–liquid coexistence of systems of 2CLJ with bond lengths  $L^*$  of 0.3292, 0.505, 0.60, 0.63, 0.67, 0.793, and 1.0 is considered. Metropolis Monte Carlo simulations (MC–NVT)<sup>49</sup> are carried out in elongated boxes of dimensions  $V = l_x \times l_y \times l_z = 17.5\sigma \times 17.5\sigma \times 52.5\sigma$ . Periodic boundary conditions are used in each direction. The number of particles  $N$  is chosen so that  $\rho$  is close to the critical density in each system. We perform the MC analogue of a temperature quench spinodal separation,<sup>50</sup> and the system is equilibrated for at least  $5 \cdot 10^5$  cycles until the liquid and vapor phases are separated by two interfaces. A cycle consists in  $N$  trial particle moves, where a move can be either translational or orientational with the same probability. The

magnitude of the particle moves is adjusted to give a  $\sim 30\%$  to  $35\%$  acceptance rate. Runs are performed for each system at temperatures ranging from  $T_r = T/T_c = 0.5$  to  $0.95$  (where  $T_c$  is the critical temperature), with increments of  $\Delta T^* = 0.1$ . For each new temperature,  $2.5 \cdot 10^5$  equilibration cycles are performed followed by  $5 \cdot 10^5$  production cycles.

A density profile along the long axis of the simulation box is obtained for each temperature. For this purpose, the simulation cell is divided into a histogram over 1050 slabs along the  $z$  direction. The vapor and liquid coexistence densities,  $\rho_v^*$  and  $\rho_l^*$ , are determined directly from the density profiles, taking averages in regions sufficiently removed from the interface. Estimates of the errors of the coexistence densities are computed as the standard deviations from the mean densities. The reduced interfacial thickness  $D^*$  and the position of the Gibbs dividing surface  $z_0$  are obtained from a fit of the density profiles to the following algebraic expression:

$$\rho^*(z^*) = \frac{1}{2}(\rho_l^* + \rho_v^*) - \frac{1}{2}(\rho_l^* - \rho_v^*) \tanh\left(\frac{z^* - z_0^*}{D^*}\right) \quad (12)$$

In Table 1 we compile the VLE data obtained for each system after equilibration, including the saturation temperature  $T^*$ , vapor pressure  $P_v^*$ , coexistence densities for the liquid and the vapor phases,  $\rho_l^*$  and  $\rho_v^*$ , and interfacial thickness  $D^*$ . The coexistence envelopes for the various systems are depicted in Figure 1 and compared with the data reported by Dubey et al.<sup>51</sup> and Kriebel et al.<sup>52</sup> obtained from MD simulations, showing good agreement. The MD data of Kriebel et al. extend to higher temperatures (possibly due to system size effects and the expected overestimate of the critical point) and are confined to a narrower range of temperatures. The density-profile data from our MC simulations for the 2CLJ system with a bond length of  $L^* = 0.6$  for temperatures ranging from  $T^* = 1.4$  to  $2.2$  are compared with the hyperbolic tangent description of eq 12 in Figure 2; a small degree of structure is seen in the dense phase close to the interface for the lowest temperature examined. The temperature dependence of the interfacial width  $D^*$  is shown in Figure 3 for different values of  $L^*$ ; the maximum interfacial width exhibits an inverse correlation with the bond length. At low temperatures, the same sharp interface, with  $D^* \approx 0.5$ , is observed for the systems with the various bond lengths. The vapor pressure is calculated using the virial route<sup>49</sup> by undertaking separate MC–NVT simulations of homogeneous systems at the corresponding vapor densities. To correlate the coexistence densities, the phase equilibrium data are fitted to the following expression as in ref 53:

$$\rho_{\pm} = \rho_c + C_2 \left| 1 - \frac{T^*}{T_c^*} \right| \pm \frac{1}{2} B_0 \left| 1 - \frac{T^*}{T_c^*} \right|^{\beta} \quad (13)$$

Here,  $\beta$  is set to its universal value ( $\beta = 0.325$ , as determined from renormalization group theory),  $\rho_+$  and  $\rho_-$  represent the vapor and liquid phase densities, respectively, and  $C_2$  and  $B_0$  are adjustable parameters, the optimal values of which are reported in Table 2 together with the corresponding values of the critical temperature and density. The critical pressure  $P_c$  is calculated from eq 14 with the corresponding value of the critical temperature:

$$\ln P_v^* = a + b/(T^* + c) \quad (14)$$

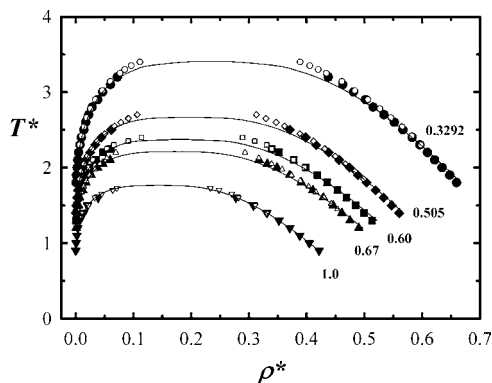
**Table 1. Reduced Temperature ( $T^*$ ), Vapor Pressure ( $P_v^*$ ), Vapor and Liquid Coexistence Densities ( $\rho_l^*$  and  $\rho_v^*$ ), Interfacial Width ( $D^*$ ), and the Surface Tension ( $\gamma^*$ ) for the VLE of 2CLJ Fluids for Various Bond Lengths  $L^*$  Obtained from MC-NVT Simulations and the TA Method<sup>a</sup>**

$T^*$	$P_v^*$	$\rho_l^*$	$\rho_v^*$	$D^*$	$\gamma^*$
$L^* = 0.3292$					
$N = 3790$					
1.8	0.0012561(1)	0.659(2)	0.0007(1)	0.658	2.651(38)
1.9	0.0026009(4)	0.648(3)	0.0014(1)	0.706	2.478(63)
2.0	0.0049666(7)	0.634(2)	0.0026(2)	0.775	2.282(54)
2.1	0.007478(1)	0.622(2)	0.0037(1)	0.865	2.099(51)
2.2	0.010464(2)	0.609(2)	0.0050(1)	0.944	1.896(27)
2.3	0.016291(5)	0.596(2)	0.0076(2)	1.017	1.724(38)
2.4	0.021535(8)	0.583(2)	0.0098(2)	1.156	1.538(42)
2.5	0.03124(1)	0.567(2)	0.0141(6)	1.317	1.365(37)
2.6	0.03931(2)	0.552(2)	0.0174(4)	1.444	1.195(29)
2.7	0.05125(2)	0.534(1)	0.0225(6)	1.568	1.029(44)
2.8	0.06515(3)	0.515(1)	0.0285(3)	1.731	0.857(44)
2.9	0.08542(4)	0.502(1)	0.0381(3)	1.931	0.708(29)
3.0	0.10366(4)	0.481(1)	0.0466(3)	2.292	0.559(26)
3.1	0.12915(9)	0.463(1)	0.0602(9)	2.486	0.452(54)
$L^* = 0.505$					
$N = 3312$					
1.4	0.00084792(6)	0.560(2)	0.0006(1)	0.629	1.890(47)
1.5	0.0017036(3)	0.547(3)	0.0012(1)	0.714	1.726(58)
1.6	0.0036549(6)	0.533(2)	0.0024(2)	0.804	1.553(51)
1.7	0.006058(1)	0.520(2)	0.0037(2)	0.920	1.368(42)
1.8	0.009475(3)	0.504(1)	0.0056(3)	1.048	1.217(32)
1.9	0.013767(4)	0.489(1)	0.0079(5)	1.243	1.032(45)
2.0	0.021744(7)	0.475(2)	0.0123(3)	1.386	0.886(41)
2.1	0.03178(1)	0.456(1)	0.0179(5)	1.536	0.726(39)
2.2	0.04413(2)	0.439(1)	0.0249(5)	1.824	0.587(24)
2.3	0.05960(3)	0.417(1)	0.0341(3)	2.131	0.431(36)
2.4	0.07633(7)	0.398(1)	0.0442(3)	2.481	0.321(28)
2.5	1.0017(1)	0.371(1)	0.062(1)	3.089	0.195(27)
$L^* = 0.60$					
$N = 3168$					
1.3	0.00101975(7)	0.513(2)	0.0008(2)	0.677	1.524(67)
1.4	0.0016192(2)	0.499(2)	0.0012(1)	0.818	1.381(32)
1.5	0.0035293(6)	0.484(2)	0.0024(2)	0.941	1.199(45)
1.6	0.007080(1)	0.469(2)	0.0047(3)	1.036	1.055(40)
1.7	0.012330(3)	0.453(2)	0.0080(3)	1.183	0.890(35)
1.8	0.018039(5)	0.436(1)	0.0114(3)	1.380	0.722(48)
1.9	0.02824(1)	0.419(1)	0.0178(3)	1.580	0.597(33)
2.0	0.03894(1)	0.399(1)	0.0244(3)	1.853	0.458(28)
2.1	0.05447(6)	0.378(1)	0.0349(8)	2.256	0.316(33)
2.2	0.07190(5)	0.353(1)	0.0474(3)	2.962	0.197(22)
2.25	0.0860(1)	0.340(1)	0.0604(4)	3.313	0.157(31)
$L^* = 0.63$					
$N = 2894$					
1.2	0.00047601(6)	0.511(2)	0.00040(7)	0.642	1.559(44)
1.3	0.0015805(3)	0.496(2)	0.0012(1)	0.693	1.417(52)
1.4	0.0026909(7)	0.482(2)	0.0020(2)	0.834	1.269(49)
1.5	0.004396(1)	0.467(2)	0.0031(4)	1.005	1.092(27)
1.6	0.008534(1)	0.452(1)	0.0058(2)	1.124	0.935(32)
1.7	0.014408(6)	0.434(2)	0.0095(2)	1.283	0.795(29)
1.8	0.021896(8)	0.417(1)	0.0142(3)	1.533	0.613(12)
1.9	0.03276(2)	0.399(1)	0.0212(5)	1.800	0.499(29)
2.0	0.04607(2)	0.376(1)	0.0302(8)	2.123	0.359(28)
2.1	0.06352(3)	0.351(1)	0.0433(3)	2.616	0.230(22)
2.2	0.0855(1)	0.318(1)	0.063(1)	3.350	0.126(28)
$L^* = 0.67$					
$N = 2846$					
1.2	0.0010741(2)	0.490(3)	0.0009(1)	0.636	1.419(59)
1.3	0.0016953(2)	0.475(2)	0.0013(2)	0.805	1.276(46)
1.4	0.0040594(9)	0.461(2)	0.0030(4)	0.869	1.091(36)
1.5	0.006988(1)	0.446(2)	0.0050(3)	1.041	0.923(30)
1.6	0.009745(2)	0.429(1)	0.0066(3)	1.289	0.803(33)
1.7	0.017947(9)	0.411(1)	0.0121(3)	1.446	0.646(29)
1.8	0.02638(1)	0.391(1)	0.0176(3)	1.744	0.500(40)
1.9	0.03951(2)	0.371(1)	0.0268(7)	2.002	0.362(31)
2.0	0.05433(4)	0.349(1)	0.0377(7)	2.662	0.258(25)
2.05	0.06499(5)	0.332(1)	0.0468(3)	2.859	0.179(34)
2.1	0.07739(9)	0.317(1)	0.0594(9)	3.214	0.142(15)

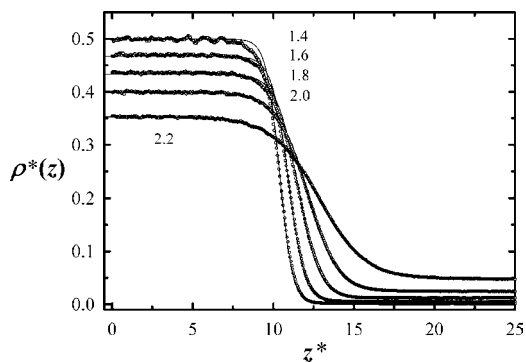
Table 1. Continued

$T^*$	$P_V^*$	$\rho_l^*$	$\rho_v^*$	$D^*$	$\gamma^*$
$L^* = 0.793$					
$N = 2605$					
1.0	0.00030394(4)	0.467(2)	0.00031(8)	0.545	1.346(25)
1.1	0.00064492(3)	0.451(2)	0.0006(1)	0.716	1.192(24)
1.2	0.0021685(5)	0.436(2)	0.0019(3)	0.789	1.040(23)
1.3	0.0036612(7)	0.420(2)	0.0030(2)	0.919	0.886(25)
1.4	0.007089(2)	0.403(1)	0.0055(3)	1.163	0.732(22)
1.5	0.012964(6)	0.385(1)	0.0098(3)	1.353	0.589(16)
1.6	0.020367(7)	0.366(1)	0.0152(8)	1.686	0.452(26)
1.7	0.03013(2)	0.345(1)	0.0224(4)	2.058	0.325(21)
1.8	0.04582(4)	0.318(1)	0.0358(4)	2.631	0.199(19)
$L^* = 1.0$					
$N = 2000$					
0.9	0.00016502(1)	0.421(3)	0.00018(5)	0.605	1.144(41)
1.0	0.0007482(1)	0.405(3)	0.0008(1)	0.737	0.960(24)
1.1	0.0021139(4)	0.389(2)	0.0020(3)	0.880	0.803(32)
1.2	0.004138(1)	0.371(2)	0.0037(3)	1.111	0.661(26)
1.3	0.007953(3)	0.352(1)	0.0068(3)	1.373	0.519(27)
1.4	0.01627(1)	0.331(1)	0.0140(5)	1.623	0.378(21)
1.5	0.02423(1)	0.308(1)	0.0205(8)	2.269	0.256(19)
1.6	0.04129(4)	0.277(1)	0.0389(8)	2.811	0.141(16)

<sup>a</sup> The total number of particles in the parallelepiped cell is  $N$ . The errors are estimated from the standard deviation of the mean determined from 10 subaverages of  $5 \cdot 10^3$  cycles each. For example, 1.23(4) represents  $1.23 \pm 0.04$ .



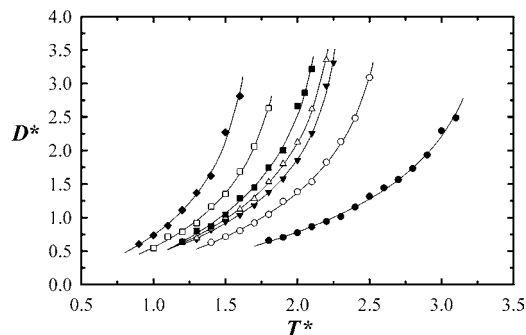
**Figure 1.** Temperature–density representation of the VLE obtained from MC-NVT simulations of 2CLJ fluids for different bond lengths.  $L^* = 0.3292$ : ●, this work, ○, ref 52;  $L^* = 0.505$ : ◆, this work, ◇, ref 52;  $L^* = 0.60$ : ■, this work, □, ref 51;  $L^* = 0.67$ : ▲, this work, △, ref 52;  $L^* = 1.0$ : ▼, this work, ▽, 51. The continuous curves correspond to the correlation of eq 13 to our simulation data.



**Figure 2.** Density profiles of the VLE obtained from MC-NVT simulations of 2CLJ fluids for systems with a bond length of  $L^* = 0.60$  for five different temperatures (the reduced values of which are labeled on the figure): ○, simulation results; —, eq 12.

Here  $a$ ,  $b$ , and  $c$  are obtained by fitting this expression to the vapor pressure data; the optimal values for these parameters are also given in Table 2.

Once equilibrated, the inhomogeneous systems consist of phase-separated vapor and liquid regions in direct contact,



**Figure 3.** Dependence of the reduced interface width ( $D^*$ ) on the reduced temperature ( $T^*$ ) for the VLE obtained from MC-NVT simulations of 2CLJ fluids for different bond lengths: ◆,  $L^* = 1.0$ ; □,  $L^* = 0.793$ ; ■,  $L^* = 0.67$ ; △,  $L^* = 0.63$ ; ▼,  $L^* = 0.60$ ; ○,  $L^* = 0.505$ ; ●,  $L^* = 0.3292$ ; —, a temperature dependent correlation.

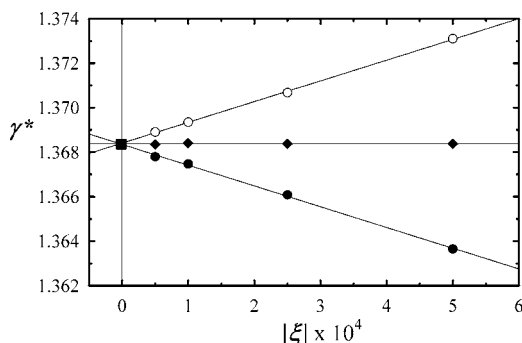
contained within a parallelepiped simulation box of dimensions  $V = l_x l_y l_z$ . The elongation of the rectangular box in the  $z$  direction, where  $3l_x = 3l_y = l_z$ , ensures that two vapor–liquid interfaces develop parallel to the  $x$ – $y$  plane. The initial equilibrated reference state (0) corresponds to an average interfacial area of  $A_0 = 2l_x l_y$ . The dimensionless perturbation parameter  $\xi$  is set at the start of the simulation, and the corresponding small fractional change in the dimensionless area is obtained as  $\Delta A^* = A_0^* \xi$ . The perturbation is performed in such a way that the scaled dimensions along the  $x$  and  $y$  axes are increased (or decreased) in the same proportion, that is,  $l_{x,1}^* = l_{x,0}^*(1 + \xi)^{1/2} = l_{y,1}^* = l_{y,0}^*(1 + \xi)^{1/2}$ . To keep the overall volume constant, the dimension in the  $z$  direction is decreased (or increased) accordingly to  $l_{z,1}^* = l_{z,0}^*(1 + \xi)^{-1}$ . Thus the perturbed state corresponds to a new surface area of  $A_1^* = A_0^*(1 + \xi)$ .

Positive and negative TA perturbations are undertaken simultaneously once every 10 cycles to evaluate the Boltzmann factors of the difference in configurational energy which are required to determine the change in free energy and thus the surface tension. Once independent values of surface tension for the positive and negative change in surface area are obtained from eq 10 for several values of the perturbation parameter, an average value is calculated for each one. The final estimate of

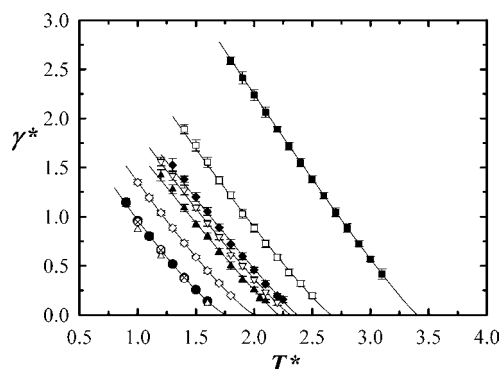
**Table 2. Reduced Critical Temperature  $T_c^*$ , Pressure  $P_c^*$ , and Density  $\rho_c^*$  Obtained from MC-NVT Simulations of 2CLJ Fluids for Various Bond Lengths  $L^*$  Compared with the MD Values Obtained by Dubey et al.<sup>51</sup> and Kriebel et al.<sup>52 a</sup>**

$L^*$		$T_c^*$	$P_c^*$	$\rho_c^*$	$C_2$	$B_0$	$a$	$b$	$c$
0.3292	MC (this work)	3.4028	0.2110	0.2339	0.2122	0.8510	3.2700	-15.2145	-0.2502
	MD <sup>52</sup>	3.5436		0.24524					
0.505	MC (this work)	2.6664	0.1454	0.1965	0.1839	0.7217	3.7714	-15.2591	0.0108
	MD <sup>52</sup>	2.8001		0.20566					
0.60	MC (this work)	2.3739	0.1163	0.1801	0.1793	0.6742	3.4391	-12.8749	-0.0706
	MD <sup>51</sup>	2.27 ± 0.01		0.197 ± 0.01					
0.63	MC (this work)	2.3156	0.1161	0.1768	0.1665	0.6507	3.8175	-14.0955	0.0451
0.67	MC (this work)	2.2121	0.1079	0.1711	0.1702	0.6397	5.6755	-21.5023	0.5089
	MD <sup>52</sup>	2.3355		0.17526					
0.793	MC (this work)	2.0066	0.08838	0.1588	0.1506	0.5857	3.5285	-12.2067	0.0434
1.0	MC (this work)	1.7638	0.07212	0.1438	0.1368	0.5312	2.7684	-8.8874	-0.1173
	MD <sup>51</sup>	1.78 ± 0.01		0.149 ± 0.01					

<sup>a</sup> The fitted parameters  $B_0$  and  $C_2$  (of eq 13) and  $a$ ,  $b$ , and  $c$  (of eq 14) are also included.



**Figure 4.** Values of the reduced surface tension of the VLE obtained from MC-NVT and the TA approach in simulations of 2CLJ fluids with a bond length of  $L^* = 0.505$  at a reduced temperature of  $T^* = 1.7$ . Data are shown for different values of the perturbation factor:  $\circ$ , expansive perturbations,  $\xi > 0$ ;  $\bullet$ , compressive perturbations,  $\xi < 0$ ; and  $\blacklozenge$ , average of the positive and negative perturbations.  $\blacksquare$  corresponds to the tension  $\gamma^*$  obtained by extrapolation to infinitesimal perturbations.



**Figure 5.** Dependence of the reduced surface tension ( $\gamma^*$ ) on the reduced temperature ( $T^*$ ) for the VLE obtained from MC-NVT simulations and the TA method of 2CLJ fluids for different bond lengths:  $\blacksquare$ ,  $L^* = 0.3292$ ;  $\square$ ,  $L^* = 0.505$ ;  $\blacklozenge$ ,  $L^* = 0.60$ ;  $\nabla$ ,  $L^* = 0.63$ ;  $\blacktriangle$ ,  $L^* = 0.67$ ;  $\diamond$ ,  $L^* = 0.793$ ;  $\bullet$ ,  $L^* = 1.0$  (this work);  $\triangle$ , MD;<sup>48</sup>  $\circ$ , MC expanded ensemble;<sup>17</sup>  $\times$ , MC TA;<sup>17</sup> —, correlation of eq 15.

the vapor–liquid interfacial tension is determined from an extrapolation to  $\xi \rightarrow 0$  with the linear fit of the averages obtained for the different values of  $\xi^{23}$  (see Figure 4). The corresponding values of the surface tension are reported in Table 1 and are also depicted in Figure 5 for the 2CLJ systems with the various bond lengths. The errors are obtained by calculating the standard deviation of the mean determined from 10 blocks averages of entire data. The surface tension data obtained from the TA approach are also correlated with the expression of Guggenheim:<sup>54</sup>

$$\gamma^* = \gamma_0(1 - T^*/T_c^*)^\mu \quad (15)$$

**Table 3. Parameters Corresponding to the Guggenheim Relation (eq 15) for the Vapor–Liquid Interfacial Tension of 2CLJ Fluids for Various Bond Lengths  $L^*$**

$L^*$	$\gamma_0$	$\mu$
0.329	5.946	1.098
0.505	4.310	1.133
0.60	3.508	1.086
0.63	3.538	1.133
0.67	3.176	1.076
0.793	3.110	1.207
1.0	2.755	1.252

**Table 4. 2CLJ Bond Length ( $L^*$ ), Segment Energy ( $\epsilon$ ), and Size ( $\sigma$ ) Parameters for Selected Real Substances**

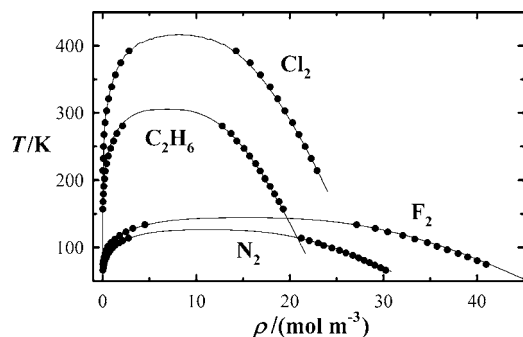
fluid	$L^*$	$\epsilon/k_B$ (K)	$\sigma$ (Å)
N <sub>2</sub>	0.329	36.67	3.308
F <sub>2</sub>	0.505	53.47	2.832
C <sub>2</sub> H <sub>6</sub>	0.505	112.2	3.64
Cl <sub>2</sub>	0.63	178.3	3.332

where  $\gamma_0$  and  $\mu$  are seen to provide a good overall description of the tension for these systems; the corresponding values of  $\gamma_0$  and  $\mu$  are given in Table 3. One should note that the universal value of  $\mu = 1.23$  was not found to provide an adequate representation of the data for the particles with the shorter bond lengths over the entire temperature range. The results corresponding to the tangent LJ diatomic system with  $L^* = 1.0$  are compared with existing data,<sup>48,17</sup> and good agreement is apparent.

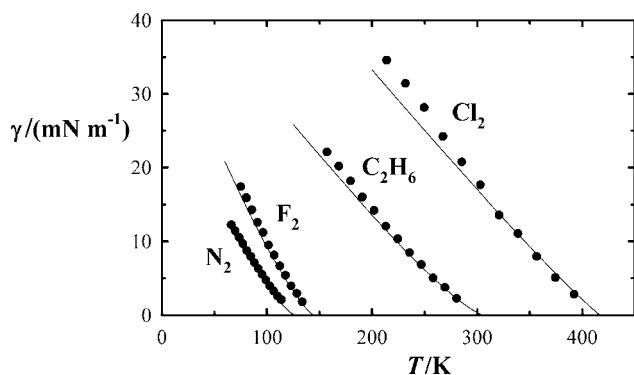
### 3. Real Substances

The 2CLJ model has been shown to provide a good representation of the intermolecular potential of small apolar molecules and can be used as a reference for perturbation approaches that incorporate polar<sup>55</sup> or quadrupolar interactions.<sup>56</sup> Here we use the parametrization of the 2CLJ model for real systems and compare the simulation results with the experimental values for the vapor–liquid coexistence densities and surface tension. A number of studies have been carried out to describe the VLE of real substances with 2CLJ systems.<sup>57–64</sup>

We have chosen to examine nitrogen, fluorine, chlorine, and ethane, the site–site LJ parameters for which are given in Table 4, taken from the work of McGuigan et al.<sup>61</sup> and Spyriouni et al.<sup>62</sup> In Figures 6 and 7 we show a comparison of our simulated values for the vapor–liquid coexistence densities and interfacial tension with the corresponding experimental data.<sup>65–67</sup> No attempt has been made to reparameterize the potential parameters or to perform a comparative evaluation of the potential functions. From Figure 6 we can conclude that the potentials used provide a reliable description of VLE densities. From Figure 7 one can also see that there is a good representation of



**Figure 6.** Comparison of the experimental saturated vapor and liquid densities of real fluids with the values obtained from MC-NVT simulation of 2CLJ models: ●, model parameters of Table 3; —, correlated experimental data from NIST webbook,<sup>65</sup> and experimental data for Cl<sub>2</sub>.<sup>66</sup>



**Figure 7.** Comparison of the experimental vapor-liquid interfacial tension of real fluids with the values obtained from MC-NVT simulation and the TA method of 2CLJ models: ●, model parameters of Table 3; —, correlated experimental data from the NIST webbook<sup>65</sup> for N<sub>2</sub>, F<sub>2</sub>, and C<sub>2</sub>H<sub>6</sub>, and Prosim Component Plus<sup>67</sup> for Cl<sub>2</sub>.

the vapor-liquid interfacial tension, with a slightly poorer description for chlorine (though in this case we should note that the comparison is with estimated values rather than with actual experimental data). Multipolar interactions arising from the uneven charge distributions in the molecules are not explicitly taken into account in these models but rather averaged into the potential parameters. In addition a united atom coarse-grained representation is made in this case. If interfacial tension data were to be used in the parameter estimation, a more robust description of the experimental data could presumably be achieved. Notwithstanding, these results confirm that simple molecular models such as the 2CLJ are appropriate not only for bulk fluid phase equilibria but also for the interfacial properties. Studies of the fluid phase equilibria and interfacial properties of these types of models for mixtures of *n*-alkanes and *n*-perfluoroalkanes including xenon as a component (the latter taking on the role of an ennobled alkane<sup>68–70</sup>) are currently being undertaken.

#### 4. Conclusions

The TA approach is seen to be an efficient and versatile method to determine the surface tension of molecular systems such as the two center LJ fluids. Vapor-liquid coexistence properties and the interfacial tension of systems of 2CLJ particles have been investigated for a range of physically reasonable bond lengths, covering temperatures over the entire fluid region. A comparison of these data with published results confirmed the adequacy of the approach. The TA method has an advantage over the mechanical approach in that it does not rely on the calculation of the components of the pressure tensor

through the use of the virial relation. Its application using MC is therefore straightforward, as only configurational energies are required. We have also shown that simple 2CLJ models can provide a good description of vapor-liquid surface tension of real diatomic fluids such as nitrogen, fluorine, chlorine, and ethane, using potential parameters refined to bulk VLE data alone. As well as providing a good analysis of the adequacy of the intermolecular potential model in describing the thermodynamic properties of real fluids, our simulation data for the 2CLJ systems will be useful in assessing the quality of theoretical developments. In future work we plan to extend the DFT free energy functionals<sup>71–75</sup> developed with the statistical associating fluid theory for potentials of variable range (SAFT-VR)<sup>76,77</sup> to molecules formed from LJ segments. The SAFT-VR formalism originally formulated for bulk fluids of molecules formed from square-well segments is generic and can easily be cast to treat the LJ,<sup>78</sup> Yukawa,<sup>79</sup> and Mie<sup>80</sup> forms of the site-site potential. The 2CLJ data reported in the current work will provide an invaluable test bed for an accurate extension of the SAFT-DFT approach.

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