

# Temperature Dependence of Vapor Pressures over Saturated Aqueous Solutions at Invariant Points of the NaCl + KNO<sub>3</sub> + H<sub>2</sub>O, NaCl + Na<sub>2</sub>CO<sub>3</sub> + H<sub>2</sub>O, and NaCl + Na<sub>2</sub>SO<sub>4</sub> + H<sub>2</sub>O Systems

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**ABSTRACT:** Vapor pressures of water over saturated solutions with regard to two salts, in the NaCl + KNO<sub>3</sub> + H<sub>2</sub>O, NaCl + Na<sub>2</sub>CO<sub>3</sub> + H<sub>2</sub>O, and NaCl + Na<sub>2</sub>SO<sub>4</sub> + H<sub>2</sub>O systems are reported in the (278 to 323) K temperature range. The determined vapor pressures were used to obtain the water activities, the molar enthalpies of vaporization and the osmotic coefficients. Compositions of invariant points from the literature were correlated as a function of temperature with the aid of quadratic equations. These equations served to determine the molar enthalpies of solution.

## 1. INTRODUCTION

The thermodynamic analysis of aqueous solutions of inorganic salts is of vital importance in geological and in many industrial processes such as salt precipitation, evaporation of water from brines, concentration, dilution, and mixing. There is no surprise that considerable attention was directed in the literature to the binary, ternary, and multicomponent inorganic salt–water systems. Thermal, volumetric, activities, solubilities, and many other physical properties have been experimentally determined, mostly in binary systems and evidently much less in the systems with three or more components. Solubilities (phase diagrams) are better documented in the literature than other physicochemical properties.<sup>1–8</sup> (see for example the remarkable investigation of Freeth<sup>9</sup>). In the absence of reliable data, the desired properties are estimated from various thermodynamic models (only a short list is mentioned here).<sup>10–23</sup> Values of parameters used in these models are taken from isopiestic or other measurements performed in binary and ternary systems to give activity and osmotic coefficients.<sup>12,24–38</sup> From the ternary systems investigated here, the most complete thermodynamic analysis was performed by Rard et al.<sup>39</sup> for the NaCl + Na<sub>2</sub>SO<sub>4</sub> + H<sub>2</sub>O system. The above-mentioned investigations are concerned with different ranges of temperature and salts concentration, but evidently their attention was rarely directed to the eutonic solutions where solutions are saturated with both salts (invariant points of the system).

This investigation is a continuation of our previous work<sup>40</sup> which dealt with aqueous ternary systems of NaCl with KCl, KBr, and NaNO<sub>3</sub> and KCl with NaBr, KBr, KI, KNO<sub>3</sub>, and K<sub>2</sub>SO<sub>4</sub> at invariant points. These systems as well as those considered here have a considerable technological and geochemical interest taking into account that in many natural brines the predominant ions are sodium, potassium, chloride, nitrate, and sulfate.

The vapor pressures over saturated solutions in the NaCl + KNO<sub>3</sub> + H<sub>2</sub>O, NaCl + Na<sub>2</sub>CO<sub>3</sub> + H<sub>2</sub>O, and NaCl + Na<sub>2</sub>SO<sub>4</sub> + H<sub>2</sub>O systems in the  $T = 278$  K to  $T = 322$  K temperature range are considered here. To the best of the author's knowledge, the vapor pressures over the eutonic solutions in these systems are

unknown in the literature. Determined vapor pressures are expressed also in terms of the water activities  $a_1$  and by combining them with solubilities (taken from the literature) in terms of the osmotic coefficients  $\phi$ . From the temperature dependence of vapor pressures  $p(T)$  and solubilities  $m_i(T)$  the molar enthalpies of vaporization and the molar enthalpies of solution at invariant points are evaluated. The results from this and from a previous investigation<sup>40</sup> are compared for the following systems NaCl + NaNO<sub>3</sub> + H<sub>2</sub>O and NaCl + KNO<sub>3</sub> + H<sub>2</sub>O and NaCl + Na<sub>2</sub>SO<sub>4</sub> + H<sub>2</sub>O and KCl + K<sub>2</sub>SO<sub>4</sub> + H<sub>2</sub>O.

## 2. EXPERIMENTAL SECTION

Purum p.a. reagents (>99 mass percent) sodium chloride, potassium nitrate sodium carbonate, and sodium sulfate were from Sigma. All reagents were used in the experiments without further purification.

The vapor pressures over saturated solutions were determined using the Rotronic Hygroskop DT1 instrument which was equipped with a measuring station WA-14TH. The saturation of solutions was assured by the prolonged mixing of the aqueous phase with excess amounts of both solid salts. The compositions of the solid phases were not determined. The thermal stability of the measuring system is estimated to be  $\pm 0.05$  K and the sensitivity of the used electronic hygrometer is about  $\pm 0.003$  kPa at  $T = 298.15$  K. The employed procedure is similar to that described in ref 41.

## 3. RESULTS AND DISCUSSION

If the aqueous solution is saturated with two nonvolatile solutes (i.e., at constant chemical potentials  $\mu_i$ ) then the change of vapor pressure  $p(T)$  with temperature  $T$  can be expressed by

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the Clausius–Clapeyron equation in the form

$$\left(\frac{dp}{dT}\right)_{\mu_i} = \frac{x_1 \Delta \overline{H}_1 + x_2 \Delta \overline{H}_2 + x_3 \Delta \overline{H}_3}{T[x_1 \Delta \overline{V}_1 + x_2 \Delta \overline{V}_2 + x_3 \Delta \overline{V}_3]}$$

$$\Delta \overline{H}_1 = \overline{H}_1^G - \overline{H}_1^L; \quad \Delta \overline{V}_1 = \overline{V}_1^G - \overline{V}_1^L$$

$$\Delta \overline{H}_2 = \overline{H}_2^S - \overline{H}_2^L; \quad \Delta \overline{V}_2 = \overline{V}_2^S - \overline{V}_2^L$$

$$\Delta \overline{H}_3 = \overline{H}_3^S - \overline{H}_3^L; \quad \Delta \overline{V}_3 = \overline{V}_3^S - \overline{V}_3^L$$

$$x_1 + x_2 + x_3 = 1; \quad i = 1, 2, 3 \quad (1)$$

which generalizes the Model and Reid<sup>42</sup> expression for binary systems.  $\Delta \overline{V}_i$  and  $\Delta \overline{H}_i$  represent changes in the molar volume and the molar enthalpy of vaporization. G, L, and S denote the gaseous, liquid and solid phases respectively and  $x_i$  are mole fractions of components in the solution,  $i = 1, 2, 3$ , where 1-water, 2-salt 1, and 3-salt 2. Thus, the pressure–temperature change is related to the change in the volume and the enthalpy associated with evaporating  $x_1$  moles of water while simultaneously crystallizing  $x_2$  moles of salt 1 and  $x_3$  moles of salt 2. However, taking into account that the mole fractions of salts, even at saturation, are small as compared with that of water and that the molar volume of water in the gaseous phase (ideal gas) is considerably larger than the molar volumes of all components in the solid and liquid phases, we have

$$\sum_{i=1}^3 x_i \Delta \overline{H}_i \approx x_1 \Delta \overline{H}_1 = x_1 \Delta \overline{H}$$

$$\sum_{i=1}^3 x_i \Delta \overline{V}_i \approx x_1 \Delta \overline{V}_1 \approx x_1 \overline{V}_1^G = x_1 RT/p \quad (2)$$

It follows from eqs 1 and 2 that the Clausius–Clapeyron can be approximated by

$$\left(\frac{d \ln p}{dT}\right)_{m_2, m_3} = \frac{\Delta \overline{H}}{RT^2} \quad (3)$$

where the apparent molar enthalpy change is associated with the evaporation of water and simultaneous crystallization of salts. The mole fractions  $x_i$  are interrelated with molalities by  $m_i = 1000x_i/(1 - x_i)/M_1$ ,  $i = 2$  and  $3$  and  $M_1$  is the molar mass of water. Thus, the vapor pressure  $p(T)$  is an explicit function of temperature and composition at the invariant point,  $p = f(T, m_2, m_3)$ . If over the considered temperature range the apparent molar enthalpy change depends linearly on  $T$ , then the integral form of eq 3 is

$$\ln[p(T; m_2, m_3)/\text{kPa}] = A + B(T/K)^{-1} + C \ln(T/K) \quad (4)$$

where

$$\frac{\Delta \overline{H}}{R} = [C(T/K) - B] \quad (5)$$

The parameters  $A$ ,  $B$ , and  $C$  for the investigated systems were evaluated by an unweighted multivariate least-squares method using the vapor pressure values from Table 1.

The composition of the invariant points as a function of temperature, taken from the literature, was approximated by the quadratic equation

$$\{m_i/\text{mol} \cdot \text{kg}^{-1}\}_{m_{j \neq i}} = a_i + b_i(T/K) + c_i(T/K)^2 \quad (6)$$

**Table 1.** Vapor Pressures  $p$  of Saturated Aqueous Solutions at Temperatures  $T$

T/K	p/kPa	T/K	p/kPa	T/K	p/kPa
NaCl + KNO <sub>3</sub> + H <sub>2</sub> O					
280.45	0.720	294.25	1.704	308.65	3.869
281.85	0.790	296.15	1.905	309.55	4.041
284.55	0.940	298.85	2.250	312.25	4.650
286.35	1.058	299.75	2.362	314.95	5.318
289.55	1.283	301.65	2.620	316.55	5.756
290.75	1.385	303.25	2.886	321.45	7.212
293.45	1.639	305.15	3.187		
NaCl + Na <sub>2</sub> CO <sub>3</sub> + H <sub>2</sub> O					
280.25	0.706	294.45	1.779	312.25	5.086
280.45	0.716	298.65	2.334	312.55	5.161
284.95	0.988	298.75	2.322	316.85	6.384
285.45	1.008	302.95	2.996	317.05	6.533
289.65	1.320	303.25	3.023	321.45	8.199
289.85	1.340	307.55	3.929	321.85	8.273
294.25	1.794	308.05	4.023		
NaCl + Na <sub>2</sub> SO <sub>4</sub> + H <sub>2</sub> O					
280.15	0.708	298.45	2.477	312.25	5.578
284.55	0.981	298.65	2.504	312.55	5.669
284.95	1.003	302.95	3.273	316.65	7.188
289.35	1.379	303.15	3.303	317.15	7.322
289.35	1.366	307.55	4.289		
293.95	1.867	307.85	4.344		
293.95	1.853	312.05	5.574		

where the coefficients  $a_i$ ,  $b_i$ , and  $c_i$  were also determined by an unweighted multivariate least-squares method.

Thermodynamically, the solubility-against temperature relation is of the form<sup>43</sup>

$$\frac{\partial \ln \{m_i\}_{m_{j \neq i}}}{\partial(1/T)} = - \frac{\Delta H_i}{R \left(1 + \frac{\partial \ln \gamma_i}{\partial \ln m_i}\right)_{m_{j \neq i}}} \quad (7)$$

where  $\Delta H_i$  is the molar enthalpy of solution of component  $i$  and  $\gamma_i$  is its activity coefficient (the hydration of the salt in the solid phase and the type of electrolyte are ignored in eq 7). If the change of activity coefficients with concentration near the invariant points is also neglected then using eqs 6 and 7, the apparent molar enthalpy of solution can be expressed in terms of  $a_i$ ,  $b_i$ , and  $c_i$  coefficients as

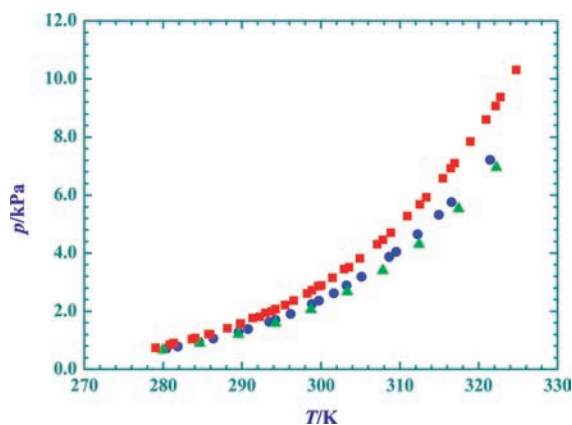
$$\Delta \overline{H}_i = RT^2 \left\{ \frac{b_i + 2c_i(T/K)}{a_i + b_i(T/K) + c_i(T/K)^2} \right\} \quad (8)$$

and its value has only an indicative character.

**NaCl + KNO<sub>3</sub> + H<sub>2</sub>O System.** Vapor pressures of water over solutions with the invariant points composition of all investigated systems are presented in Table 1 and the coefficients  $A$ ,  $B$ , and  $C$  with corresponding mean standard deviations in Table 2. In Figure 1 are compared vapor pressures of NaCl + KNO<sub>3</sub> + H<sub>2</sub>O with those of NaCl + NaNO<sub>3</sub> + H<sub>2</sub>O and KCl + KNO<sub>3</sub> + H<sub>2</sub>O reported in our previous investigation.<sup>40</sup> As can be observed, that irrespective of temperature,  $p(\text{KCl} + \text{KNO}_3 + \text{H}_2\text{O}) > p(\text{NaCl}$

**Table 2.** Coefficients of eq 4: Vapor Pressures of Saturated Aqueous Solutions

system	A	B	C	$\sigma(p)/\text{kPa}$
NaCl + KNO <sub>3</sub> + H <sub>2</sub> O	80.699	-7908.0	-9.374	0.020
NaCl + Na <sub>2</sub> CO <sub>3</sub> + H <sub>2</sub> O	59.159	-7156.5	-6.028	0.028
NaCl + Na <sub>2</sub> SO <sub>4</sub> + H <sub>2</sub> O	100.53	-9199.1	-12.073	0.019

**Figure 1.** Vapor pressures  $p$  of saturated aqueous solutions as a function of temperature  $T$ . Sodium chloride + sodium nitrate, green triangles; potassium chloride + potassium nitrate, red squares, ref 40; sodium chloride + potassium nitrate, blue circles, this work.

+ KNO<sub>3</sub> + H<sub>2</sub>O) >  $p(\text{NaCl} + \text{NaNO}_3 + \text{H}_2\text{O})$ . Thus, the vapor pressure of water is always higher in the systems with potassium salts than with sodium salts which means that water passes into the gaseous phase more easily in the case of potassium than sodium salts.

The determined vapor pressures of water  $p$  can be also expressed in terms of the water activity  $a_1(T, m_2, m_3) = p(T, m_2, m_3)/p^*(T)$  and by a value of the osmotic coefficient  $\phi = \phi(T, m_2, m_3)$

$$\phi(T, m_2, m_3) = -\frac{1000}{M_1(\nu_2 m_2 + \nu_3 m_3)} \ln a_1(T, m_2, m_3) \quad (9)$$

where  $p^*(T)$  are vapor pressures of pure water at a given  $T$  and they are available from the Saul and Wagner equation.<sup>44</sup>  $M_1$  is the molar mass of water and  $\nu_i$ ,  $i = 2$  and  $3$ , denotes the stoichiometric coefficients,  $\nu_i = 2$  for 1:1 electrolytes and  $\nu_i = 3$  for 1:2 electrolytes. It follows from eq 9 that the accuracy of the determined osmotic coefficients depends on how accurately the composition of solutions at invariant points is known.

The phase diagram of the NaCl + KNO<sub>3</sub> + H<sub>2</sub>O system is relatively simple<sup>1-3</sup> but the scattering of experimental solubilities of NaCl prevents from fitting them using eq 6. However, the overall change of  $m_{\text{NaCl}}$  with  $T$  is rather small so the average value:  $m_{\text{NaCl}} = (6.50 \pm 0.21) \text{ mol} \cdot \text{kg}^{-1}$  was used in the calculations. On the contrary, potassium nitrate solubility increases significantly with an increase of temperature and its solubility can be represented up to 373 K by

$$\begin{aligned} \{m_{\text{KNO}_3}/\text{mol} \cdot \text{kg}^{-1}\}_{m_{\text{NaCl}}} &= 170.495 - 1.2283(T/\text{K}) \\ &+ 0.002246(T/\text{K})^2 \\ \sigma(\{m_{\text{KNO}_3}/\text{mol} \cdot \text{kg}^{-1}\}_{m_{\text{NaCl}}}) &= 0.36 \end{aligned} \quad (10)$$

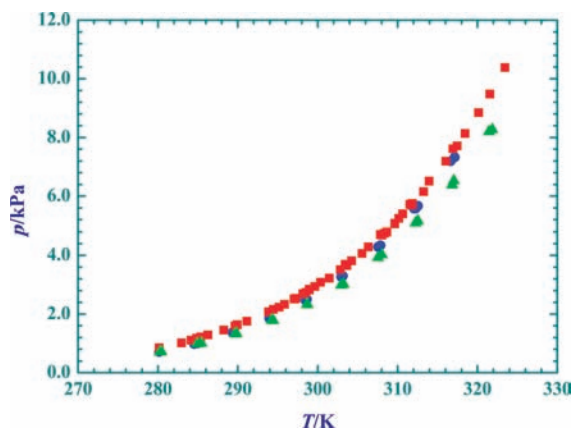
**Table 3.** Solubilities  $m_2$  and  $m_3$ , Vapor Pressures  $p$ , Water Activities  $a_1$ , and Osmotic Coefficients  $\phi$  at Temperatures  $T$ 

$T/\text{K}$	$m_2/\text{mol} \cdot \text{kg}^{-1}$	$m_3/\text{mol} \cdot \text{kg}^{-1}$	$p/\text{kPa}$	$a_1$	$\phi$
NaCl + KNO <sub>3</sub> + H <sub>2</sub> O					
278.15	6.50 <sup>a</sup>	2.57	0.613	0.703	0.94
283.15	6.50	2.73	0.857	0.699	0.94
288.15	6.50	3.00	1.181	0.693	0.93
293.15	6.50	3.39	1.605	0.687	0.90
298.15	6.50	3.89	2.154	0.680	0.87
303.15	6.50	4.50	2.857	0.673	0.83
308.15	6.50	5.22	3.745	0.665	0.79
313.15	6.50	6.05	4.855	0.657	0.75
318.15	6.50	7.00	6.231	0.648	0.71
323.15	6.50	8.06	7.917	0.639	0.67
NaCl + Na <sub>2</sub> CO <sub>3</sub> + H <sub>2</sub> O					
278.15	5.58	0.59	0.616	0.701	1.53
283.15	5.27	0.91	0.889	0.704	1.47
288.15	4.88	1.33	1.262	0.707	1.40
293.15	4.40	1.85	1.764	0.710	1.32
298.15	3.69	2.59	2.432	0.712	1.24
303.15	3.88	2.46	3.305	0.714	1.24
308.15	4.07	2.33	4.435	0.716	1.22
313.15	4.26	2.21	5.879	0.718	1.21
318.15	4.43	2.10	7.702	0.719	1.21
323.15	4.60	1.99	9.978	0.720	1.20
NaCl + Na <sub>2</sub> SO <sub>4</sub> + H <sub>2</sub> O					
278.15	5.93	0.18	0.612	0.701	1.59
283.15	5.81	0.33	0.885	0.720	1.45
288.15	5.63	0.56	1.258	0.738	1.30
293.15	5.50	0.73	1.762	0.754	1.19
298.15	5.56	0.69	2.432	0.767	1.12
303.15	5.63	0.66	3.309	0.780	1.04
308.15	5.69	0.62	4.444	0.790	0.99
313.15	5.75	0.60	5.894	0.799	0.94
318.15	5.81	0.57	7.725	0.806	0.90
323.15	5.88	0.55	10.009	0.811	0.87

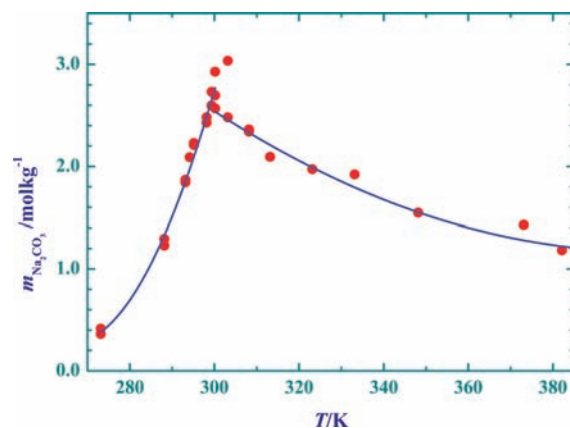
<sup>a</sup> See text.

Using eqs 4, 6, and 10, the compositions of the invariant points, vapor pressures, activities of water, and osmotic coefficients in 5 K intervals are calculated and they are presented in Table 3. As can be observed, with increasing  $T$ , the content of sodium chloride is constant, that of potassium nitrate strongly increases and water activities and osmotic coefficients decrease. The apparent molar enthalpy of solution of potassium nitrate as a function of temperature is positive ( $\Delta\bar{H} = 21.1 \text{ kJ} \cdot \text{mol}^{-1}$  at 298.15 K) and has a maximum at about 313.15 K. The apparent molar enthalpy of vaporization in the NaCl + KNO<sub>3</sub> + H<sub>2</sub>O system at 298.15 K is  $\Delta\bar{H} = 42.5 \text{ kJ} \cdot \text{mol}^{-1}$ .

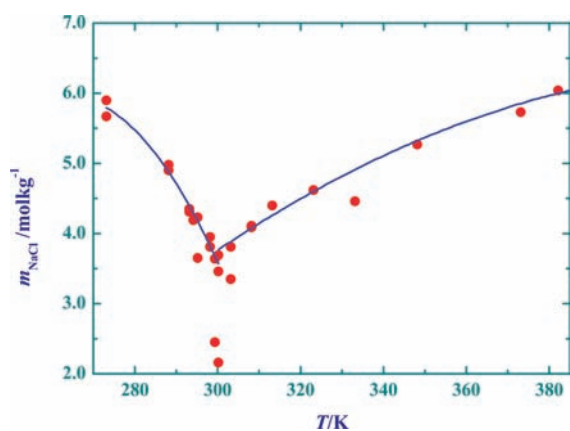
**NaCl + Na<sub>2</sub>CO<sub>3</sub> + H<sub>2</sub>O System.** The vapor pressures of water over invariant points in the NaCl + Na<sub>2</sub>CO<sub>3</sub> + H<sub>2</sub>O system are plotted in Figure 2 together with those of the NaCl + Na<sub>2</sub>SO<sub>4</sub> + H<sub>2</sub>O and KCl + K<sub>2</sub>SO<sub>4</sub> + H<sub>2</sub>O (taken from our previous work<sup>40</sup>). In systems as shown previously, potassium salts have higher vapor pressures than sodium salts  $p(\text{KCl} + \text{K}_2\text{SO}_4 + \text{H}_2\text{O}) > p(\text{NaCl} + \text{Na}_2\text{SO}_4 + \text{H}_2\text{O}) > p(\text{NaCl} + \text{Na}_2\text{CO}_3 + \text{H}_2\text{O})$ .



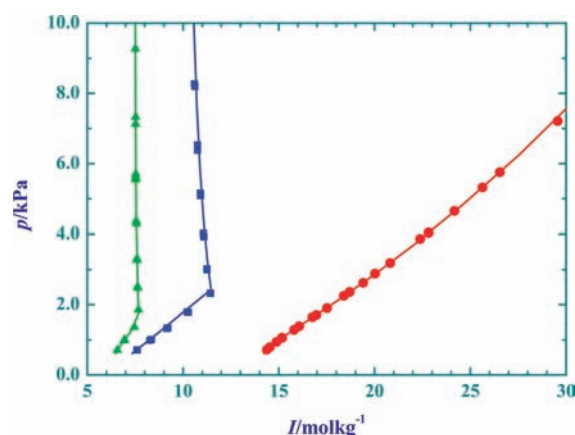
**Figure 2.** Vapor pressures  $p$  of saturated aqueous solutions as a function of temperature  $T$ . Potassium chloride + potassium sulfate, red squares, ref 40; sodium chloride + sodium carbonate, green triangles; sodium chloride + sodium sulfate, blue circles, this work.



**Figure 4.** Sodium carbonate content in solutions with the invariant point composition in the sodium chloride + sodium carbonate + water system as a function of temperature  $T$ . Red circles, refs 1–3 and 9; continuous lines are calculated in this work.



**Figure 3.** Sodium chloride content in solutions with the invariant point composition in the sodium chloride + sodium carbonate + water system as a function of temperature  $T$ . Red circles, refs 1–3 and 9; continuous lines are calculated in this work.



**Figure 5.** Vapor pressure of water over saturated solutions  $p$  as a function of the total ionic strength  $I$ . Red circles, NaCl + KNO<sub>3</sub> + H<sub>2</sub>O; blue squares, NaCl + Na<sub>2</sub>CO<sub>3</sub> + H<sub>2</sub>O; green triangles, NaCl + Na<sub>2</sub>SO<sub>4</sub> + H<sub>2</sub>O.

The phase diagram of this system is more complicated because in the NaCl + Na<sub>2</sub>CO<sub>3</sub> + H<sub>2</sub>O system, Na<sub>2</sub>CO<sub>3</sub>·10H<sub>2</sub>O is stable to about 298 K and at higher temperatures above 303 K Na<sub>2</sub>CO<sub>3</sub>·7H<sub>2</sub>O is stable. In between, over a small temperature range, from about (298 to 303) K Na<sub>2</sub>CO<sub>3</sub>·7H<sub>2</sub>O exists.<sup>1–3,9</sup> In the Na<sub>2</sub>CO<sub>3</sub> + H<sub>2</sub>O system, the transformation in the solid phase from Na<sub>2</sub>CO<sub>3</sub>·10H<sub>2</sub>O to Na<sub>2</sub>CO<sub>3</sub>·7H<sub>2</sub>O occurs at 305.15 K and from Na<sub>2</sub>CO<sub>3</sub>·7H<sub>2</sub>O to Na<sub>2</sub>CO<sub>3</sub>·H<sub>2</sub>O at 308.55 K.<sup>8</sup> Since near 300 K the exact location of the saturated solutions is uncertain (Figures 3 and 4), it is assumed that only two branches of the solubility curve exist, which means that the existence of sodium carbonate heptahydrate is completely neglected.

In the region of sodium carbonate decahydrate existence, the contents of sodium chloride and sodium carbonate in saturated solutions can be expressed by

$$\begin{aligned} \{m_{\text{NaCl}}/\text{mol}\cdot\text{kg}^{-1}\}_{m_{\text{Na}_2\text{CO}_3}} &= -117.584 + 0.9376(T/\text{K}) \\ &\quad - 0.001779(T/\text{K})^2 \\ \sigma(\{m_{\text{NaCl}}/\text{mol}\cdot\text{kg}^{-1}\}_{m_{\text{Na}_2\text{CO}_3}}) &= 0.09 \end{aligned} \quad (11)$$

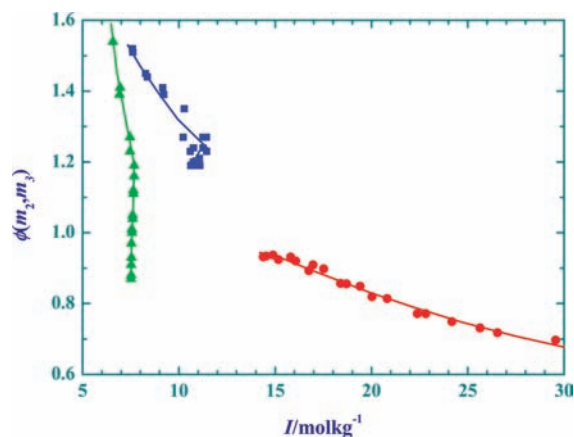
$$\begin{aligned} \{m_{\text{Na}_2\text{CO}_3}/\text{mol}\cdot\text{kg}^{-1}\}_{m_{\text{NaCl}}} &= 150.04 - 1.1270(T/\text{K}) \\ &\quad + 0.002120(T/\text{K})^2 \\ \sigma(\{m_{\text{Na}_2\text{CO}_3}/\text{mol}\cdot\text{kg}^{-1}\}_{m_{\text{NaCl}}}) &= 0.07 \end{aligned} \quad (12)$$

and when the solid phase exists with sodium carbonate monohydrate we have

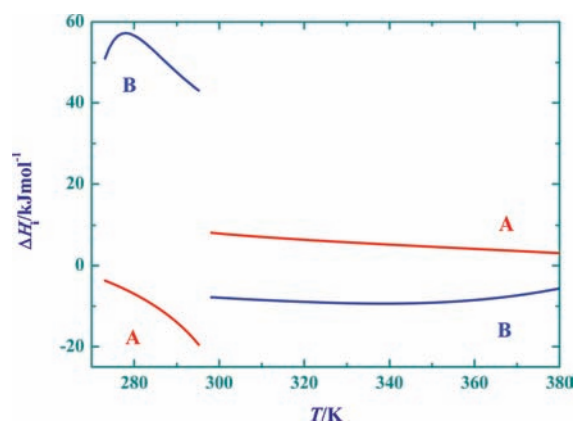
$$\begin{aligned} \{m_{\text{NaCl}}/\text{mol}\cdot\text{kg}^{-1}\}_{m_{\text{Na}_2\text{CO}_3}} &= -21.930 + 0.1315(T/\text{K}) \\ &\quad - 0.00153(T/\text{K})^2 \\ \sigma(\{m_{\text{NaCl}}/\text{mol}\cdot\text{kg}^{-1}\}_{m_{\text{Na}_2\text{CO}_3}}) &= 0.07 \end{aligned} \quad (13)$$

$$\begin{aligned} \{m_{\text{Na}_2\text{CO}_3}/\text{mol}\cdot\text{kg}^{-1}\}_{m_{\text{NaCl}}} &= 22.488 - 0.1061(T/\text{K}) \\ &\quad + 0.00132(T/\text{K})^2 \\ \sigma(\{m_{\text{Na}_2\text{CO}_3}/\text{mol}\cdot\text{kg}^{-1}\}_{m_{\text{NaCl}}}) &= 0.07 \end{aligned} \quad (14)$$

In all systems, as is observed in Figures 1 and 2, the vapor pressure of water over invariant points  $p(T, m_2, m_3)$  is a



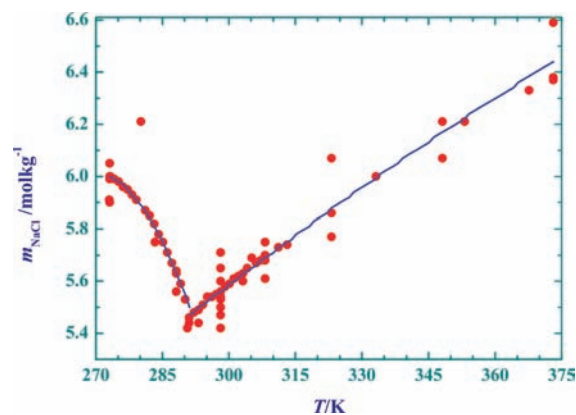
**Figure 6.** Osmotic coefficient of saturated solutions  $\phi(m_2, m_3)$  as a function of the total ionic strength  $I$ . Red circles, NaCl + KNO<sub>3</sub> + H<sub>2</sub>O; blue squares, NaCl + Na<sub>2</sub>CO<sub>3</sub> + H<sub>2</sub>O; green triangles, NaCl + Na<sub>2</sub>SO<sub>4</sub> + H<sub>2</sub>O.



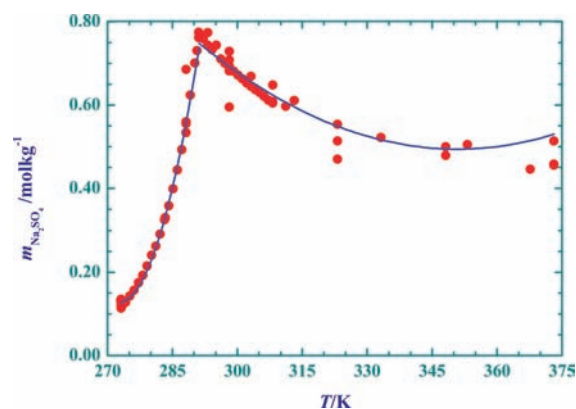
**Figure 7.** Apparent molar enthalpies of solution of components in the NaCl + Na<sub>2</sub>CO<sub>3</sub> + H<sub>2</sub>O system as a function of temperature  $T$  (eq 8). A, sodium chloride; B, sodium carbonate.

monotonic increasing function with concave upward curvature ( $\partial p(T, m_2, m_3)/\partial T > 0$  and  $\partial^2 p(T, m_2, m_3)/\partial T^2 > 0$ ). The existence of different hydrates of salts in the solid phase on the pressure  $p(T, m_2, m_3)$  curve is unseen. However, the situation is completely different if  $p(T, m_2, m_3)$  is plotted as a function of the total ionic strength  $I$  where  $I = m_2 + m_3$  for 1:1 electrolytes and  $I = m_2 + 3m_3$  for 1:2 electrolytes (Figure 5). As is evident, in systems with two hydrates in the solid phase, the curve of  $p(T, m_2, m_3)$  has two branches with different curvatures. The same is true when the osmotic coefficients  $\phi(T, m_2, m_3)$  are considered as a function of the ionic strength  $I$  (Figure 6).

Since solubilities are known from eqs 11–14, it is possible to determine the apparent molar enthalpy of solution of sodium chloride and sodium carbonate using eq 8. The results of calculations are plotted in Figure 7. When sodium carbonate decahydrate exists in the solid phase  $\overline{\Delta H}_{\text{NaCl}} < 0$  and  $\partial \overline{\Delta H}_{\text{NaCl}}/\partial T < 0$ ; the apparent molar enthalpy of solution of sodium carbonate is large and positive,  $\overline{\Delta H}_{\text{Na}_2\text{CO}_3} > 0$ , and has a maximum near 278 K. In the sodium carbonate monohydrate region,  $\overline{\Delta H}_{\text{NaCl}} > 0$  and  $\partial \overline{\Delta H}_{\text{NaCl}}/\partial T < 0$  and  $\overline{\Delta H}_{\text{Na}_2\text{CO}_3} < 0$  when its change with  $T$  is small (at 298.15 K  $\overline{\Delta H}_{\text{NaCl}} = 8.1 \text{ kJ} \cdot \text{mol}^{-1}$  and  $\overline{\Delta H}_{\text{Na}_2\text{CO}_3} = -7.8 \text{ kJ} \cdot \text{mol}^{-1}$ ).  $\phi(T, m_2, m_3)$  decreases with



**Figure 8.** Sodium chloride content in solutions with the invariant point composition in the sodium chloride + sodium sulfate + water system as a function of temperature  $T$ . Red circles, refs 1–3; continuous lines are calculated in this work.



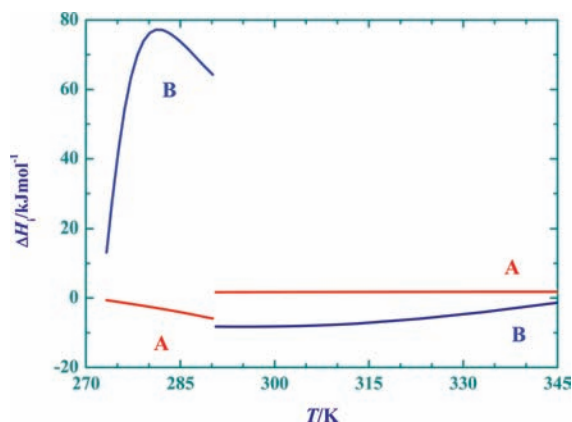
**Figure 9.** Sodium carbonate content in solutions with the invariant point composition in the sodium chloride + sodium sulfate + water system as a function of temperature  $T$ . Red circles, refs 1–3; continuous lines are calculated in this work.

temperature,  $\partial \phi(T, m_2, m_3)/\partial T < 0$  (Table 3) and the apparent molar enthalpy of vaporization in the NaCl + Na<sub>2</sub>CO<sub>3</sub> + H<sub>2</sub>O system at 298.15 K is  $\overline{\Delta H} = 44.5 \text{ kJ} \cdot \text{mol}^{-1}$ .

**NaCl + Na<sub>2</sub>SO<sub>4</sub> + H<sub>2</sub>O System.** As expected, the vapor pressures of water over invariant points in the NaCl + Na<sub>2</sub>SO<sub>4</sub> + H<sub>2</sub>O system are lower than the corresponding KCl + K<sub>2</sub>SO<sub>4</sub> + H<sub>2</sub>O system<sup>40</sup> (Figure 2).

Results of many determinations of solubility in the NaCl + Na<sub>2</sub>SO<sub>4</sub> + H<sub>2</sub>O system are compiled in refs 1–3. There is general agreement that Na<sub>2</sub>SO<sub>4</sub> · 10H<sub>2</sub>O exists up to about 291 K whereas anhydrous sodium sulfate exists above this temperature (Figures 8 and 9). However, the accuracy of these determinations is probably not specially high. Even at 298.15 K, the scattering of solubilities of NaCl is unacceptable large (Figure 8). Nevertheless, the transformation of the decahydrate to the anhydrous salt is clearly manifested if the vapor pressure of water over saturated solutions  $p(T, m_2, m_3)$  is related to the total ionic strength of solutions  $I$  (Figure 5). The same is observed in Figure 6 in the case of the osmotic coefficients  $\phi(T, m_2, m_3)$ .

In the region of sodium sulfate decahydrate existence, the contents of sodium chloride and sodium sulfate in the saturated



**Figure 10.** Apparent molar enthalpies of solution of components in the NaCl + Na<sub>2</sub>SO<sub>4</sub> + H<sub>2</sub>O system as a function of temperature  $T$  (eq 8). A, sodium chloride; B, sodium sulfate.

solutions are given by

$$\begin{aligned} \{m_{\text{NaCl}}/\text{mol}\cdot\text{kg}^{-1}\}_{m_{\text{Na}_2\text{SO}_4}} &= -82.789 + 0.6558(T/\text{K}) \\ &\quad - 0.001211(T/\text{K})^2 \\ \sigma(\{m_{\text{NaCl}}/\text{mol}\cdot\text{kg}^{-1}\}_{m_{\text{Na}_2\text{SO}_4}}) &= 0.01 \end{aligned} \quad (15)$$

$$\begin{aligned} \{m_{\text{Na}_2\text{SO}_4}/\text{mol}\cdot\text{kg}^{-1}\}_{m_{\text{NaCl}}} &= 130.130 - 0.9542(T/\text{K}) \\ &\quad + 0.001751(T/\text{K})^2 \\ \sigma(\{m_{\text{Na}_2\text{SO}_4}/\text{mol}\cdot\text{kg}^{-1}\}_{m_{\text{NaCl}}}) &= 0.01 \end{aligned} \quad (16)$$

In this region, the molality of NaCl in the saturated solution decreases when the temperature increases and that of Na<sub>2</sub>SO<sub>4</sub> increases, but sodium chloride continues to be the predominant component in the mixture (Figures 8 and 9).

Above 291 K, anhydrous sodium sulfate exists in the solid phase and the solubilities of components can be expressed by

$$\begin{aligned} \{m_{\text{NaCl}}/\text{mol}\cdot\text{kg}^{-1}\}_{m_{\text{Na}_2\text{SO}_4}} &= 0.1708 + 0.02216(T/\text{K}) \\ &\quad - 0.000017(T/\text{K})^2 \\ \sigma(\{m_{\text{NaCl}}/\text{mol}\cdot\text{kg}^{-1}\}_{m_{\text{Na}_2\text{SO}_4}}) &= 0.06 \end{aligned} \quad (17)$$

$$\begin{aligned} \{m_{\text{Na}_2\text{SO}_4}/\text{mol}\cdot\text{kg}^{-1}\}_{m_{\text{NaCl}}} &= 9.2848 - 0.05012(T/\text{K}) \\ &\quad + 0.00007143(T/\text{K})^2 \\ \sigma(\{m_{\text{Na}_2\text{SO}_4}/\text{mol}\cdot\text{kg}^{-1}\}_{m_{\text{NaCl}}}) &= 0.02 \end{aligned} \quad (18)$$

In this region, the reverse situation is observed, the content of NaCl increases, and that of Na<sub>2</sub>SO<sub>4</sub> decreases (Figures 8 and 9).

The behavior of the apparent molar enthalpy of solution of sodium chloride and sodium sulfate in the NaCl + Na<sub>2</sub>SO<sub>4</sub> + H<sub>2</sub>O system is similar to that in the NaCl + Na<sub>2</sub>CO<sub>3</sub> + H<sub>2</sub>O system (Figures 7 and 10). When sodium carbonate decahydrate exists in the solid phase  $\overline{\Delta H}_{\text{NaCl}} < 0$  and  $\partial\overline{\Delta H}_{\text{NaCl}}/\partial T < 0$ ; in the case of sodium sulfate the apparent molar enthalpy of solution is large and positive,  $\overline{\Delta H}_{\text{Na}_2\text{SO}_4} > 0$ , and has a maximum near 281 K. Changes with  $T$  in the sodium sulfate region are rather small.  $\overline{\Delta H}_{\text{NaCl}} > 0$  but  $\partial\overline{\Delta H}_{\text{NaCl}}/\partial T > 0$  and  $\overline{\Delta H}_{\text{Na}_2\text{CO}_3} < 0$  and also  $\overline{\Delta H}_{\text{Na}_2\text{CO}_3} > 0$ . At 298.15 K we have:  $\overline{\Delta H}_{\text{NaCl}} = 1.7 \text{ kJ}\cdot\text{mol}^{-1}$  and  $\overline{\Delta H}_{\text{Na}_2\text{CO}_3} = -8.2 \text{ kJ}\cdot\text{mol}^{-1}$ . Osmotic coefficients  $\phi(T, m_2, m_3)$ , as in other systems, decrease with temperature,  $\partial\phi(T, m_2, m_3)/$

$\partial T < 0$  (Table 3) and the apparent molar enthalpy of vaporization in the NaCl + Na<sub>2</sub>SO<sub>4</sub> + H<sub>2</sub>O system at 298.15 K is  $\overline{\Delta H} = 46.6 \text{ kJ}\cdot\text{mol}^{-1}$ .

Determined here, the relative humidities ( $100 \cdot a_1(T, m_2, m_3)$ ) are rather close, the highest, 77 % at 298.15 K, is in the aqueous saturated solution with NaCl + Na<sub>2</sub>SO<sub>4</sub>. In other systems we have 71 % saturated with NaCl + Na<sub>2</sub>CO<sub>3</sub> and 68 % with NaCl + KNO<sub>3</sub>. In the systems with sodium sulfate and sodium carbonate, the water activities increase with  $T$ ,  $\partial a_1(T, m_2, m_3)/\partial T > 0$ , but with potassium nitrate the opposite situation is observed  $\partial a_1(T, m_2, m_3)/\partial T < 0$  (Table 3).

## CONCLUSIONS

Vapor pressures of water over saturated solutions with regards to two salts in the NaCl + KNO<sub>3</sub> + H<sub>2</sub>O, NaCl + Na<sub>2</sub>CO<sub>3</sub> + H<sub>2</sub>O, and NaCl + Na<sub>2</sub>SO<sub>4</sub> + H<sub>2</sub>O systems were determined in the  $T = 278 \text{ K}$  to  $T = 322 \text{ K}$  temperature range. As was observed previously,<sup>40</sup> the vapor pressure of water is always higher in the systems with potassium salts than with sodium salts.

In all systems, changes in vapor pressure with temperature are monotonic functions of  $T$ . The same took place with other derived from  $p(T, m_2, m_3)$  quantities, the water activities, the osmotic coefficients and the apparent molar enthalpies of vaporization (Tables 1 and 3, Figures 1 and 2). However, if they are considered as a function of the total ionic strength  $I$ , their curves acquire a quite different character which expresses indirectly the change in composition of the solid phase with  $T$  (Figures 5 and 6). The apparent molar enthalpies of solution originate from temperature dependence of the solubilities in invariant points,  $m_1(T)$ , and therefore represents directly the changes in the solid phase (Figures 7 and 10).

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