additive and constitutive nature of molecules, i.e., the molecular size and shape. It has been significantly correlated to many physicochemical properties used in physicochemical-activity studies. If a biological response of a series of closely related compounds is significantly correlated to $\chi$ it is mirroring the same additive and constituitive molecular property shown in those physical properties studied thus far. Logically, it would appear that these empirical properties are intermediary measures of the very fundamental molecular structure encoded in $\chi$ and the measured biological response.

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# Quantitative Structure-Activity Relationships. 1. The Modified Free-Wilson Approach 

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#### Abstract

The relationships between the linear free energy related Hansch model and the mathematical models of Free-Wilson and Bocek-Kopecky are reviewed and discussed. Some examples are given to illustrate the theoretically derived relationships and to demonstrate scope and limitations of each mathematical model. The modified Free-Wilson approach is shown to be completely equivalent to a nonparabolic Hansch approach; it can be used to study additivity or nonadditivity of group contributions and to control and improve the fitting of Hansch equations. The Bocek-Kopecky approach is related to the parabolic form of the Hansch approach; its practical use is limited by the great number of variables involved.


In 1964 three different approaches for studying quantitative structure-activity relationships were developed: Hansch's linear multiple regression model ${ }^{1-3}$ (e.g., eq 1),
$\log 1 / C=k_{1} \pi^{2}+k_{2} \pi+k_{3} \sigma+k_{4} E_{s}+k_{5}$
Free-Wilson's additive model ${ }^{4}$ (eq 2), and Boček-
biological act. $=$ overall av $+\Sigma$ (group
contributions)
Kopecky's interaction model ${ }^{5,6}$ (eq 3 ).

$$
\begin{equation*}
\log 1 / C=b_{\mathbf{X}}+b_{\mathbf{Y}}+e_{\mathbf{X}} e_{\mathbf{Y}}+k \tag{3}
\end{equation*}
$$

Free and Wilson's model is based on the assumption that each substituent makes an additive and constant contribution to the biological activity regardless of substituent variation in the rest of the molecule. The values of the individual group contributions are calculated by regression analysis (an excellent introduction into Hansch and Free-Wilson analysis for people not familiar with mathematics is given in ref 7).

Bocek and Kopeckýs interaction model may be interpreted as a Free-Wilson-like additive model with an additional term exey accounting for possible interactions between substituents X and Y. Although BocekKopecky's model is cited in almost every review article on
quantitative structure-activity relationships, it has found no practical use due to the great number of parameters involved.

Singer and Purcell ${ }^{8}$ studied the relationships among the linear free energy based Hansch approach and the two mathematical models. They could demonstrate that all models are theoretically interrelated but Free-Wilson's model is appropriate only in the case of additivity of group contributions while Bocek-Kopecky's interaction model also holds in the case of parabolic dependence of biological activity on a particular physical property, e.g., Hansch's substituent constant $\pi$. In view of these relationships between the mathematical models and the Hansch approach they proposed the use of $\log 1 / C$ values instead of linear values ${ }^{4}$ as biological response parameters in FreeWilson analysis. In the following years there has been some discussion whether $\log 1 / C$ or $C$ should be used, 9,10 but today in most instances $\log 1 / C$ is being used. It should be noted that Bruice et al. ${ }^{11}$ were the first ones who used $\log 1 / C$ values and an additive model to calculate the activity of thyroxine analogues (eq 4).
$\log \%$ thyroxine-like activity $=k \Sigma f+c$
Cammarata and Yau ${ }^{12}$ and Fujita and Ban ${ }^{13}$ used a modified Free-Wilson approach (eq 5)
$\log 1 / C=\sum_{i} a_{i}+\mu$
where $a_{i}=$ group contribution of substituent $X_{i}$, based on $a_{\mathrm{H}}=0.00$ (which is a definition), and $\mu=\log 1 / C$ calculated of the unsubstituted compound. ${ }^{13}$

The Free-Wilson model in its classical form is based on symmetry equations; therefore, the constant term is the overall average of the biological activities.

Fujita and Ban ${ }^{13}$ described a modified Free-Wilson model without symmetry equations. In this modified model the activity contribution of each substituent is relative to H and the constant term $\mu$, obtained by the least-squares method, is a theoretically predicted activity value of the unsubstituted compound (all $R=H$ ) itself. It must be noted that also in Hansch analysis the constant term is the theoretically predicted activity value of the unsubstituted compound provided all structural parameters $\phi j$ are based on $\phi \mathrm{H}=0.00$ (e.g., $\pi$ or $\sigma$, but not log $P$ or $E_{\mathrm{s}}$ ). On the other hand, the constant term in the original Free-Wilson model is interpreted as the activity value of a hypothetical "naked" compound (all R absent); if the group contributions of all hydrogen substituents under consideration are added to this term, the $\mu$ value of the Fujita-Ban model results. In a similar manner the group contributions of the classical Free-Wilson model can be transformed to Fujita-Ban $a_{i}$ values by subtracting the group contributions of the corresponding hydrogen substituents. Therefore the Fujita-Ban modification is a simple linear transformation of the classical Free-Wilson model (a detailed description of this modified model is given in ref 13).

Cammarata and Yau ${ }^{12}$ used a similar, but statistically different, modified Free-Wilson model. Like the model of Fujita and Ban all activity contributions refer to H but the constant term is not obtained by the least-squares method; the observed activity value of the unsubstituted compound is arbitrarily taken as the constant term. Since all observed $\log 1 / C$ values include an unknown experimental error $\epsilon i$, the constant term includes the experimental error $\epsilon \mathrm{H}$ of the observed $\log 1 / C$ value of the unsubstituted compound; this is inconsistent with the definition of the least-squares method.

In accordance with these facts the Fujita-Ban modification gives better results than the Cammarata modification (only in the case of $\epsilon \mathrm{H} \rightarrow 0$ both models give similar results; for further discussion see below, example 1).

The Free-Wilson model and the nonparabolic form of Hansch's linear multiple regression model (e.g., eq 6) have
$\log 1 / C=k_{1} \pi+k_{2} \sigma+k_{3} E_{\mathrm{s}}+k_{4}$
been shown to be practically interrelated ${ }^{12-15}$ and theoretically equivalent; ${ }^{16}$ based on the assumption that the group contributions $a_{i}$ of substituents $X_{i}$ can be interpreted by a weighted sum of physical properties $\phi j$ of the substituents, eq 7 can be derived (compare eq 3 and 4 i of ref
$a_{i}=\sum_{j} b_{j} \phi_{j}$
16), in which $b_{j}$ are coefficients of different physical properties $\phi_{j}$ (e.g., substituent parameters like $\pi, \sigma$, or $E_{\mathrm{s}}$ ).

Substitution of $a_{i}$ by $\sum b_{j \phi j}$ (eq 7) in the Free-Wilson type eq 5 gives eq 8 .
$\log 1 / C=\sum_{i} \sum_{j} b_{j} \phi_{j}+\mu$
Equation 8 may be written in the form of eq 9 or in the

$$
\begin{equation*}
\log 1 / C=b_{1} \sum_{i} \phi_{1}+b_{2} \sum_{i} \phi_{2}+\ldots+b_{n} \sum_{i} \phi_{n}+\mu \tag{9}
\end{equation*}
$$

more common form of eq 10 , which are both Hansch type
$\log 1 / C=b_{1} \phi_{1}+b_{2} \phi_{2}+\ldots+b_{n} \phi_{n}+\mu$

## equations.

Cammarata ${ }^{16}$ stated that a number of statistically based structure-activity studies would support the equivalence of the additive and the linear multiple regression model, but this has never been done. On the contrary, some confusion has been added in the meantime; in comparing the Hansch and the Free-Wilson models practically, Cammarata ${ }^{16}$ used a modified Free-Wilson approach ${ }^{12}$ which did not give the best possible correlation between observed and calculated $\log 1 / C$ values (see example 1). Furthermore, Cammarata's assumption that the additive model also holds in case of parabolic dependence of $\log 1 / C$ on a particular physical property ${ }^{16,17}$ and in case of interactions between substituents ${ }^{17}$ is not generally true; these assumptions are valid only if a higher order term or an interaction term gives a somewhat better correlation but the absolute contribution of this term to $\log 1 / C$ values is small.

It is the purpose of this paper to give a precise practical comparison of the mathematical models and the linear multiple regression model. The following examples shall demonstrate the interrelationships of these models, the Hansch approach in its linear (e.g., eq 6) and parabolic form (e.g., eq 1), the modified Free-Wilson approach ${ }^{13}$ (eq 5), and the Bocek-Kopecky approach (eq 3). It will be shown how the modified Free-Wilson model may be used to study additivity or nonadditivity, e.g., parabolic dependence of $\log 1 / C$ on $\log P$ or $\pi$. In addition the modified Free-Wilson approach may be used to control and to improve the fitting of Hansch equations because in case of additivity of group contributions this approach always gives a "maximal correlation" between calculated and observed $\log 1 / C$ values, a fact which is predictable from eq 5 ( $\sum b_{j} \phi_{j}$ may be interpreted as a great number of known and unknown physical parameters in optimal weighted combination for each position of substitution).

Example 1 was chosen from the literature to demonstrate the close numerical equivalence of the additive and the linear multiple regression model in such cases where both methods give good fitting results. Example 1 also will demonstrate that in the case of additivity of group contributions the modified Free-Wilson approach (eq 5) gives an equal or even better correlation of observed and calculated $\log 1 / C$ values than the Hansch approach; the modified Free-Wilson approach gives the "upper limit" of correlation which may be obtained with an additive, nonparabolic approach.

Hansch and Lien, ${ }^{18}$ Cammarata, ${ }^{16}$ and Unger and Hansch ${ }^{19}$ analyzed the adrenergic blocking potencies of several $N, N$-dimethyl-2-bromophenethylamines ${ }^{20}$ by Hansch and Free-Wilson analysis (the compounds and their $\log 1 / C$ values are presented together with some structural parameters in Table I).

Cammarata ${ }^{16}$ used a modified Free-Wilson approach to calculate $\log 1 / C$ values for this group of compounds with de novo group contributions. However, he could not obtain the best possible correlation between observed and calculated $\log 1 / C$ values ( $r \geq 0.96$ ) because he used the observed $\log 1 / C$ value of the unsubstituted compound ( $\mu$ $=7.46$ ) as the basis for his analysis instead of the calculated (theoretical) one as proposed by Fujita and Ban ${ }^{13}$ (it was Cammarata's misfortune that the greatest deviation between observed and calculated $\log 1 / C$ values is obtained for the unsubstituted compound, see Table II). A reexamination of the compounds of Table I with the

Table I. Antagonism of $N, N$-Dimethyl-2-bromophenethylamines vs. Adrenaline in the Rat

| $\begin{gathered} \mathrm{X} \\ (\text { meta }) \end{gathered}$ | $\begin{gathered} \mathrm{Y} \\ \text { (para) } \end{gathered}$ |  |  |  | $\log _{\operatorname{obsd}^{d}} 1 / C$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\pi^{a}$ | $\sigma^{+b}$ | $E_{\text {s }}{ }^{\text {meta }} \mathrm{c}$ |  |
| H | F | 0.15 | -0.07 | 1.24 | 8.16 |
| H | Cl | 0.70 | 0.11 | 1.24 | 8.68 |
| H | Br | 1.02 | 0.15 | 1.24 | 8.89 |
| H | I | 1.26 | 0.14 | 1.24 | 9.25 |
| H | $\mathrm{CH}_{3}$ | 0.52 | -0.31 | 1.24 | 9.30 |
| F | H | 0.13 | 0.35 | 0.78 | 7.52 |
| Cl | H | 0.76 | 0.40 | 0.27 | 8.16 |
| Br | H | 0.94 | 0.41 | 0.08 | 8.30 |
| I | H | 1.15 | 0.36 | -0.16 | 8.40 |
| $\mathrm{CH}_{3}$ | H | 0.51 | -0.07 | 0.00 | 8.46 |
| Cl | F | 0.91 | 0.33 | 0.27 | 8.19 |
| Br | F | 1.09 | 0.34 | 0.08 | 8.57 |
| $\mathrm{CH}_{3}$ | F | 0.66 | -0.14 | 0.00 | 8.82 |
| Cl | Cl | 1.46 | 0.51 | 0.27 | 8.89 |
| Br | Cl | 1.64 | 0.52 | 0.08 | 8.92 |
| $\mathrm{CH}_{3}$ | Cl | 1.21 | 0.04 | 0.00 | 8.96 |
| Cl | Br | 1.78 | 0.55 | 0.27 | 9.00 |
| Br | Br | 1.96 | 0.56 | 0.08 | 9.35 |
| $\mathrm{CH}_{3}$ | Br | 1.53 | 0.08 | 0.00 | 9.22 |
| $\mathrm{CH}_{3}$ | $\mathrm{CH}_{3}$ | 1.03 | -0.38 | 0.00 | 9.30 |
| Br | $\mathrm{CH}_{3}$ | 1.46 | 0.10 | 0.08 | 9.52 |
| H | H | 0.00 | 0.00 | 1.24 | 7.46 |

${ }^{a}$ See ref 21. ${ }^{b}$ See ref 22. ${ }^{c}$ See ref 23. ${ }^{d}$ See ref 20 .
correct form of the modified Free-Wilson approach ( $a_{\mathrm{H}}$ $=0.00 ; \mu=\log 1 / C$ calcd of the unsubstituted compound, see ref 13) gave a better correlation ( $r=0.969 ; s=0.194$ ) than the analysis by Cammarata $(r=0.911 ; s=0.214) .{ }^{16}$ [We could not reproduce these values; from the $\log 1 / C$ values given by Cammarata (ref 16, Table I) we calculated $r=0.902 ; s=0.324$.] The matrix used for Free-Wilson analysis and the calculated $\log 1 / C$ values are given in Table II; the corresponding group contributions are listed in Table III.

From the numerous Hansch equations derived by Hansch and Lien, ${ }^{18}$ Cammarata, ${ }^{16}$ and Unger and Hansch, ${ }^{19}$ five different equations (eq 11-15) were selected.
$\log 1 / C=1.15( \pm 0.19) \pi-1.47( \pm 0.38) \sigma^{+}+$
7.82

$$
\begin{equation*}
n=22 ; r=0.944 ; s=0.197 \tag{11}
\end{equation*}
$$

$\log 1 / C=0.791( \pm 0.150) \pi_{\mathrm{m}}+1.479( \pm 0.139)$

$$
\begin{align*}
& \pi_{\mathrm{p}}-1.004( \pm 0.302) \sigma_{\mathrm{m}}-1.993( \pm 0.402) \\
& \sigma_{\mathrm{p}}+7.914  \tag{12}\\
& \quad n=22 ; r=0.946 ; s=0.203
\end{align*}
$$

$\log 1 / C=0.747( \pm 0.123) \pi_{m}-0.911( \pm 0.249)$

$$
\begin{equation*}
\sigma_{\mathrm{m}}+1.666( \pm 0.124) r_{\mathrm{v}, \mathrm{p}}+5.769 \tag{13}
\end{equation*}
$$

$$
n=22 ; r=0.961 ; s=0.168
$$

$\log 1 / C=0.82( \pm 0.27) \pi-1.02( \pm 0.45) \sigma^{+}+0.62$

$$
\begin{equation*}
( \pm 0.43) r_{u, p}+7.06 \tag{14}
\end{equation*}
$$

$$
n=22 ; r=0.964 ; s=0.164
$$

$\log 1 / C=0.83( \pm 0.27) \pi_{\mathrm{m}}+1.33( \pm 0.20) \pi_{\mathrm{p}}-$ $0.92( \pm 0.50) \sigma_{\mathrm{m}}{ }^{+}-1.89( \pm 0.57) \sigma_{\mathrm{p}}{ }^{+}+$ 7.80

$$
\begin{equation*}
n=22 ; r=0.966 ; s=0.164 \tag{15}
\end{equation*}
$$

An additional Hansch equation was derived to describe

Table II. Free-Wilson Matrix Used for Calculation of de Novo Group Contributions. Calculated Log $1 / C$ Values ${ }^{a}$

${ }^{a}$ For values of group contributions see Table III. ${ }^{b}$ Single point determinations.
$\log 1 / C$ values by $\pi, \sigma^{+}$, and $E_{\mathrm{s}^{\text {meta }}}$ (see Table I).
$\log 1 / C=1.26( \pm 0.19) \pi-1.46( \pm 0.34) \sigma^{+}+0.21$

$$
\begin{equation*}
( \pm 0.17) E_{\mathrm{s}}^{\text {meta }}+7.62 \tag{16}
\end{equation*}
$$

$$
n=22 ; r=0.959 ; s=0.173
$$

In order to calculate group contributions $a_{i}$ for each substituent, based on $a \mathrm{H}=0.00$, the following equations were derived from eq 11-16.

From eq 11: $a_{i}=1.15 \pi-1.47 \sigma^{+}$
From eq 12: $a_{\text {meta }}=0.791 \pi_{\mathrm{m}}-1.004 \sigma_{\mathrm{m}}$

From eq 13: $\quad a_{\text {meta }}=0.747 \pi_{m}-0.911 \sigma_{m}$

From eq 14: $a_{\text {meta }}=0.82 \pi_{\mathrm{m}}-1.02 \sigma_{\mathrm{m}}{ }^{+}$
$a_{\text {para }}=0.82 \pi_{\mathrm{p}}-1.02 \sigma_{\mathrm{p}}{ }^{+}+0.62$ $\left(r_{\mathrm{v}, \mathrm{p}}-r_{\mathrm{v}, \mathrm{H}}\right)$
From eq 15: $a_{\text {meta }}=0.83 \pi_{\mathrm{m}}-0.92 \sigma_{\mathrm{m}}{ }^{+}$

From eq 16: $a_{\text {meta }}=1.26 \pi_{\mathrm{m}}-1.46 \sigma_{\mathrm{m}}{ }^{+}+0.21$
$\left(E_{\mathrm{s}}{ }^{\text {meta }}-E_{\mathrm{sH}}\right)$
$a_{\text {para }}=1.26 \pi_{\mathrm{p}}-1.46 \sigma_{\mathrm{p}}{ }^{+}$
A comparison of Hansch-derived group contributions (eq 17-27) and de novo group contributions calculated by the modified Free-Wilson approach is given in Table III (the original structural parameters used by each author were taken for the calculations; note that Cammarata ${ }^{16}$ and Unger and Hansch ${ }^{19}$ used different $\pi$ values).
The correlation of de novo group contributions calculated by Free-Wilson analysis and the Hansch-derived group contributions is very good; eq 11-16 are "optimally adapted". The residual variance in $\log 1 / C$ values not

Table III. Group Contributions (Based on $a_{H}=0.00$ ) for Substituents X and Y of the $N, N$-Dimethyl-2-bromophenethylamines, Calculated by Free-Wilson and Hansch Analysis

|  | Free-Wilson <br> parameters | Hansch-derived group contributions, calcd from eq |  |  |  |  |  |
| :--- | :---: | ---: | ---: | ---: | ---: | ---: | ---: |
| Substituent | 17 | 18,19 | 20,21 | 22,23 | 24,25 | 26,27 |  |
| $m-\mathrm{H}$ | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| $m-\mathrm{F}$ | $-0.30^{a}$ | -0.37 | -0.21 | -0.19 | -0.25 | -0.21 | -0.44 |
| $m-\mathrm{Cl}$ | 0.21 | 0.29 | 0.18 | 0.19 | 0.22 | 0.26 | 0.17 |
| $m-\mathrm{Br}$ | 0.43 | 0.48 | 0.42 | 0.41 | 0.35 | 0.40 | 0.34 |
| $m-\mathrm{I}$ | $0.58^{a}$ | 0.79 | 0.65 | 0.62 | 0.58 | 0.62 | 0.63 |
| $m-\mathrm{CH}_{3}$ | 0.45 | 0.69 | 0.48 | 0.45 | 0.49 | 0.49 | 0.48 |
| $p-\mathrm{H}$ | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| $p-\mathrm{F}$ | 0.34 | 0.28 | 0.10 | 0.45 | 0.36 | 0.33 | 0.29 |
| $p-\mathrm{Cl}$ | 0.77 | 0.64 | 0.58 | 0.92 | 0.80 | 0.72 | 0.72 |
| $p-\mathrm{Br}$ | 1.02 | 0.95 | 1.05 | 1.08 | 1.09 | 1.07 | 1.07 |
| $p-\mathrm{I}$ | $1.43^{a}$ | 1.24 | 1.31 | 1.30 | 1.37 | 1.41 | 1.38 |
| $p-\mathrm{CH}$ | 1.26 | 1.05 | 1.11 | 1.28 | 1.22 | 1.28 | 1.11 |
| Unsubst | 7.82 | 7.82 | 7.91 | 7.77 | 7.80 | 7.80 | 7.88 |
| compd |  |  |  |  |  |  |  |

${ }^{a}$ Single point determinations.
Table IV. Fungistatic Activity of Phenyl Ethers of Glycerol and Propylene Glycols ${ }^{a}$


| Compd no. | R | X | Y | Log $P$ | $\sigma$ | $E_{\mathrm{s}}{ }^{\text {ortho }}$ | $\begin{gathered} \log 1 / C \\ \text { obsd } \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 2-Me | OH | OH | 1.38 | -0.14 | 1.24 | 2.26 |
| 2 | 2 -Cl | OH | OH | 1.29 | 0.21 | 1.51 | 2.31 |
| 3 | $4-\mathrm{Cl}$ | OH | OH | 1.40 | 0.23 | 2.48 | 2.31 |
| 4 | 2,6-Cl | OH | OH | 1.88 | 0.42 | 0.54 | 2.37 |
| 5 | 2,4- $\mathrm{Cl}_{2}$ | OH | OH | 1.99 | 0.44 | 1.51 | 2.61 |
| 6 | $2-\mathrm{Me}, 4-\mathrm{Cl}$ | OH | OH | 2.08 | 0.09 | 1.24 | 2.33 |
| 7 | $3-\mathrm{Me}, 4-\mathrm{Cl}$ | OH | OH | 1.91 | 0.16 | 2.48 | 2.90 |
| 8 | $2-\mathrm{Me}, 6-\mathrm{Cl}$ | OH | OH | 1.97 | 0.07 | 0.27 | 2.33 |
| 9 | 2,6-Me ${ }_{2}, 4-\mathrm{Cl}$ | OH | OH | 2.76 | -0.05 | 0.00 | 2.76 |
| 10 | $3,5-\mathrm{Me}_{2}, 4-\mathrm{Cl}$ | OH | OH | 2.42 | 0.09 | 2.48 | 3.24 |
| 11 | 2,6- $\mathrm{Cl}_{2}, 4-\mathrm{Me}$ | OH | OH | 2.40 | 0.25 | 0.54 | 3.10 |
| 12 | 2 -Me | OH | H | 2.14 | -0.14 | 1.24 | 2.46 |
| 13 | $2-\mathrm{Cl}$ | OH | H | 2.05 | 0.21 | 1.51 | 2.84 |
| 14 | $4-\mathrm{Cl}$ | OH | H | 2.16 | 0.23 | 2.48 | 2.81 |
| 15 | $2,6-\mathrm{Cl}_{2}$ | OH | H | 2.64 | 0.42 | 0.54 | 3.04 |
| 16 | $2,4-\mathrm{Cl}_{2}$ | OH | H | 2.75 | 0.44 | 1.51 | 3.35 |
| 17 | $2-\mathrm{Me}, 4-\mathrm{Cl}$ | OH | H | 2.84 | 0.09 | 1.24 | 3.30 |
| 18 | $3-\mathrm{Me}, 4-\mathrm{Cl}$ | OH | H | 2.67 | 0.16 | 2.48 | 3.30 |
| 19 | $2-\mathrm{Me}, 6-\mathrm{Cl}$ | OH | H | 2.73 | 0.07 | 0.27 | 2.70 |
| 20 | 2,6-Me $2,4-\mathrm{Cl}$ | OH | H | 3.52 | -0.05 | 0.00 | 3.51 |
| 21 | $3,5-\mathrm{Me}_{2}, 4-\mathrm{Cl}$ | OH | H | 3.18 | 0.09 | 2.48 | 3.68 |
| 22 | $2,6-\mathrm{Cl}_{2}, 4-\mathrm{Me}$ | OH | H | 3.16 | 0.25 | 0.54 | 3.47 |
| 23 | 2 Me | H | OH | 2.34 | -0.14 | 1.24 | 2.79 |
| 24 | $4-\mathrm{Cl}$ | H | OH | 2.36 | 0.23 | 2.48 | 3.07 |
| 25 | $2-\mathrm{Me}, 6-\mathrm{Cl}$ | H | OH | 2.93 | 0.07 | 0.27 | 2.78 |
| 26 | $2,6-\mathrm{Me}_{2}, 4-\mathrm{Cl}$ | H | OH | 3.72 | -0.05 | 0.00 | 3.51 |
| 27 | $3,5-\mathrm{Me}_{2}, 4-\mathrm{Cl}$ | H | OH | 3.38 | 0.09 | 2.48 | 3.93 |
| 28 | $2,6-\mathrm{Cl}_{2}, 4-\mathrm{Me}$ | H | OH | 3.36 | 0.25 | 0.54 | 3.67 |

${ }^{a}$ All data taken from Hansch and Lien; ${ }^{24}$ the $E_{\mathrm{s}}$ values were taken from ref 23.
explained by Free-Wilson analysis must be due to nonadditivity and/or biological variance and/or experimental error.

Craig and Hansch ${ }^{15}$ stated that the additive model and the linear multiple regression model give different quantitative results. This statement is valid only if the group contributions are not transformed to $a_{H}=0.00$ and/or in case of nonadditivity of group contributions. Provided that the biological activity data are accurate and reliable and that both models give good correlations between observed and calculated $\log 1 / C$ values, which are basic conditions for comparison of both models, the group contributions calculated by both models also show good correlation. In Free-Wilson analysis there may be some deviations due to single point determinations because experimental error or biological variance is completely
reflected in a group contribution derived from only one $\log 1 / C$ value.
Example 2 demonstrates how the modified Free-Wilson approach may be used to control the fitting of a Hansch equation and to improve the Hansch equation by comparison of de novo group contributions and Hanschderived group contributions.
From the extensive work of Hansch and Lien ${ }^{24}$ on structure-activity relationships in antifungal agents one set of compounds, substituted phenyl ethers of glycerol and propylene glycols, was reexamined by the modified Free-Wilson approach. Structures of the compounds are given in Table IV together with some structural parameters. The matrix used for Free-Wilson analysis and the calculated $\log 1 / C$ values are given in Table $V$ (as a simplification the assumption was made that substituents

Table V. Free-Wilson Matrix Used for Calculation of de Novo Group Contributions. Calculated Log $1 / C$ Values ${ }^{a}$

| R |  |  |  |  | $\mathrm{X}=\mathrm{OH}$ | $\mathrm{Y}=\mathrm{OH}$ | $\begin{gathered} \log 1 / C \\ \text { obsd } \end{gathered}$ | $\log 1 / C$ caled | $\Delta$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| O-Me | $m-\mathrm{Me}$ | $p-\mathrm{Me}$ | $o-\mathrm{Cl}$ | $p-\mathrm{Cl}$ |  |  |  |  |  |
| 1 |  |  |  |  | 1 | 1 | 2.26 | 2.08 | -0.18 |
|  |  |  | 1 |  | 1 | 1 | 2.31 | 2.15 | -0.16 |
|  |  |  |  | 1 | 1. | 1 | 2.31 | 2.40 | 0.09 |
|  |  |  | 2 |  | 1 | 1 | 2.37 | 2.42 | 0.05 |
|  |  |  | 1 | 1 | 1 | 1 | 2.61 | 2.67 | 0.06 |
| 1 |  |  |  | 1 | 1 | 1 | 2.33 | 2.60 | 0.27 |
|  | 1 |  |  | 1 | 1 | 1 | 2.90 | 2.81 | -0.09 |
| 1 |  |  | 1 |  | 1 | 1 | 2.33 | 2.35 | 0.02 |
| 2 |  |  |  | 1 | 1 | 1 | 2.76 | 2.79 | 0.03 |
|  | 2 |  |  | 1 | 1 | 1 | 3.24 | 3.23 | -0.01 |
|  |  | 1 | 2 |  | 1 | 1 | 3.10 | 3.02 | -0.08 |
| 1 |  |  |  |  | 1 |  | 2.46 | 2.62 | 0.16 |
|  |  |  | 1 |  | 1 |  | 2.84 | 2.69 | -0.15 |
|  |  |  |  | 1 | 1 |  | 2.81 | 2.94 | 0.13 |
|  |  |  | 2 |  | 1 |  | 3.04 | 2.96 | -0.08 |
|  |  |  | 1 | 1 | 1 |  | 3.35 | 3.21 | -0.14 |
| 1 |  |  |  | 1 | 1 |  | 3.30 | 3.14 | -0.16 |
|  | 1 |  |  | 1 | 1 |  | 3.30 | 3.35 | 0.05 |
| 1 |  |  | 1 |  | 1 |  | 2.70 | 2.89 | 0.19 |
| 2 |  |  |  | 1 | 1 |  | 3.51 | 3.33 | -0.18 |
|  | 2 |  |  | 1 | 1 |  | 3.68 | 3.77 | 0.09 |
|  |  | 1 | 2 |  | 1 |  | 3.47 | 3.56 | 0.09 |
| 1 |  |  |  |  |  | 1 | 2.79 | 2.72 | -0.07 |
|  |  |  |  | 1 |  | 1 | 3.07 | 3.05 | -0.02 |
| 1 |  |  | 1 |  |  | 1 | 2.78 | 3.00 | 0.22 |
| 2 |  |  |  | 1 |  | 1 | 3.51 | 3.44 | -0.07 |
|  | 2 |  |  | 1 |  | 1 | 3.93 | 3.87 | -0.06 |
|  |  | 1 | 2 |  |  | 1 | 3.67 | 3.67 | 0.00 |

${ }^{a}$ For group contributions, see Table VI; $\mu=3.07$.
give equal contributions in the ortho and ortho position and likewise in the meta and meta' position; a coefficient of 2 was given to ortho,ortho'- and meta,meta'-disubstituted compounds).

Hansch and Lien ${ }^{24}$ derived eq 28 (compounds 19 and $\log 1 / C=0.691( \pm 0.14) \log P+0.428( \pm 0.51)$ $\sigma+1.213$

25 were excluded) for the compounds of Table IV. For all compounds of Table IV we obtained eq 29.

$$
\begin{gather*}
\log 1 / C=0.665( \pm 0.149) \log P+0.500( \pm 0.568) \\
\sigma+1.235  \tag{29}\\
\quad n=28 ; r=0.879 ; s=0.241
\end{gather*}
$$

Free-Wilson analysis (for matrix see Table V) gave a significant better correlation of observed and calculated $\log 1 / C$ values ( $n=28 ; r=0.967 ; s=0.145$ ). The high correlation coefficient demonstrates that additivity of group contributions is fulfilled for this set of compounds.

By substituting $\pi$ for $\log P$ eq 30 can be derived from
$a_{i}=0.665 \pi+0.500 \sigma$
eq 29 to calculate group contributions.
A comparison of these Hansch-derived group contributions with de novo group contributions (Table VI) shows great deviations, especially for $0-\mathrm{Me}$ and $\mathrm{o}-\mathrm{Cl}$ (values calculated from eq 30 are too high) and for $p-\mathrm{Me}$ (value calculated from eq 30 is too small). Since these deviations may be due to an unfavorable steric effect of the ortho substituents, a further Hansch analysis with $\log P$ and $E_{\mathrm{s}}{ }^{\text {ortho }}$ (Table IV) was run (eq 31).

$$
\begin{align*}
\log 1 / C=0.741 & ( \pm 0.109) \log P+0.214( \pm 0.078) \\
E_{\mathrm{s}}^{\text {ortho }}+ & +0.846  \tag{31}\\
n & =28 ; r=0.942 ; s=0.170
\end{align*}
$$

Table VI. Group Contributions (Based on $a_{\mathrm{H}}=0.00$ ) for Substituents R, X, and Y, Calculated by Free-Wilson and Hansch Analysis

|  |  |  | Group contributions, <br> calcd from |  |  |  |
| :--- | ---: | :---: | ---: | ---: | ---: | :---: |
|  |  |  |  | Eq 32, | Free- <br> Wilson |  |
| Substituent | $\pi^{a}$ | $\sigma^{b}$ | Eq 30 | 33 | analysis |  |
| $o-\mathrm{Me}$ | 0.68 | -0.14 | 0.38 | 0.24 | 0.20 |  |
| $m-\mathrm{Me}$ | 0.51 | -0.07 | 0.30 | 0.38 | 0.41 |  |
| $p-\mathrm{Me}$ | 0.52 | -0.17 | 0.26 | 0.39 | 0.59 |  |
| $o-\mathrm{Cl}$ | 0.59 | 0.21 | 0.50 | 0.23 | 0.27 |  |
| $p-\mathrm{Cl}$ | 0.70 | 0.23 | 0.58 | 0.52 | 0.52 |  |
| $\mathrm{X}=\mathrm{OH}$ | -0.96 |  | -0.64 | -0.71 | -0.65 |  |
| $\mathrm{Y}=\mathrm{OH}$ | -0.76 |  | -0.51 | -0.56 | -0.54 |  |

${ }^{a}$ Calculated from $\log P$ values. ${ }^{24} \quad{ }^{b}$ From $\sigma$ values of Table IV.

Equations 32 and 33 were derived from eq 31 to calculate group contributions.
$a_{\text {meta,para }}=0.741 \pi$
$a_{\text {ortho }}=0.741 \pi+0.214\left(E_{\mathrm{s}}\right.$ ortho $\left.-E_{\mathrm{s}, \mathrm{H}}\right)$
A comparison of group contributions calculated from eq 32 and 33 with the de novo group contributions (Table VI) shows good correlation (the deviation found for $p$-Me may be due to the small number of compounds bearing $p-\mathrm{Me}$ ). This good correlation is in agreement with the fact that the $\sigma$ term in eq 29 is statistically not significant ( $t_{\sigma}=$ $1.811 ; p>0.05$ ) while the $E_{s}{ }^{\text {ortho }}$ term in eq 31 is statistically significant ( $t$ Es ortho $=5.658 ; p<0.001$ ).
A combination of $\log P, \sigma$, and $E_{s}{ }^{\circ}$ ortho gives no further improvement of correlation (eq 34). Again the $\sigma$ term is

$$
\begin{gather*}
\log 1 / C=0.751( \pm 0.104) \log P+0.366( \pm 0.383) \\
\sigma+0.205( \pm 0.074) E_{\mathrm{s}} \text { ortho }+0.782  \tag{34}\\
n=28 ; r=0.950 ; s=0.161
\end{gather*}
$$

Table VII. Hansch and Free-Wilson Analysis of Butyrylcholinesterase Inhibitory Activity of 1-Decyl-3-carbamoylpiperidines

| $\mathrm{R}_{1}$ | $\mathrm{CH}_{3}\left(\mathrm{CH}_{2}\right)_{9} \mathrm{~N}^{-}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathrm{R}_{2}$ | $\mathrm{p} I_{50}$ obsd | Eq 35 | Eq 36 | FreeWilson analysis ${ }^{b}$ |
| H | H | 4.21 | 4.19 | 4.21 | 4.25 |
| H | $\mathrm{CH}_{3}$ | 4.46 | 4.47 | 4.46 | 4.46 |
| H | $\mathrm{C}_{2} \mathrm{H}_{5}$ | 4.86 | 4.76 | 4.86 | 4.78 |
| $\mathrm{CH}_{3}$ | $\mathrm{CH}_{3}$ | 4.66 | 4.76 | 4.66 | 4.66 |
| $\mathrm{CH}_{3}$ | $\mathrm{C}_{2} \mathrm{H}_{5}{ }^{\text {a }}$ | 5.01 | 5.04 | 5.00 | 4.99 |
| $\mathrm{C}_{2} \mathrm{H}_{5}$ | $\mathrm{C}_{2} \mathrm{H}_{5}$ | 5.28 | 5.33 | 5.29 | 5.32 |
| $\mathrm{C}_{3} \mathrm{H}_{7}$ | $\mathrm{C}_{3} \mathrm{H}_{7}$ | 5.98 | 5.90 | 5.98 | 5.98 |

${ }^{a}$ Compound not included in Free-Wilson analysis and calculation of eq 35 and 36 . ${ }^{b}$ See ref 25.
not significant ( $t_{\sigma}=1.973 ; p>0.05$ ) while the $E_{\mathrm{s}}$ ortho term is significant ( $t_{E_{s} \text { ortho }}=5.677 ; p<0.001$ ).

Example 3 is taken from Clayton and Purcell's comparison of Hansch and Free-Wilson analysis. 25 They studied structure-activity relationships for butyrylcholinesterase inhibitory activity of 1-decyl-3carbamoylpiperidines by both methods and derived several Hansch equations (e.g., eq 35 and 36 ) and also de novo

$$
\begin{align*}
& \mathrm{p} I_{50}=0.570 \pi+4.187  \tag{35}\\
& \quad n=6 ; r=0.992 ; s=0.087 ; F=268 \\
& \mathrm{p} I_{50}=-0.119 \pi^{2}+1.205 \pi+0.687 \sigma^{*}+ \\
& \quad 3.533  \tag{36}\\
& \quad n=6 ; r=0.999 ; s=0.045 ; F=8984
\end{align*}
$$

group contributions which fit the experimental data very well (Table VII).

Cammarata ${ }^{16}$ concluded that this example would give evidence for the validity of Free-Wilson additivity in cases of parabolic dependence of $\log 1 / C$ values on $\log P$ or $\pi$ (eq 36). However, from Table VII one can see that $\mathrm{p} \mathrm{I}_{50}$ may be described well by a nonparabolic equation (eq 35); if there is any parabolic dependence, its numerical influence is so small that additivity of group contributions is not perturbed. Despite the enormously high $F$ value eq 36 may be a chance correlation; ${ }^{26}$ therefore, it is not possible to demonstrate validity or nonvalidity of FreeWilson additivity in cases of parabolic dependence of log $1 / C$ values on $\pi$ with this example.

Example 4 was chosen from a paper of Hansch et al. ${ }^{27}$ on parabolic dependence of drug activity on lipophilic character to demonstrate complete breakdown of the Free-Wilson model and to demonstrate the applicability of a Bocek-Kopecky-like model in cases of real nonadditivity of group contributions.
Hansch et al. ${ }^{27}$ derived for a group of $N, N$-diacylureas (for structures see Table VIII) eq 37.
$\log 1 / C=-0.177( \pm 0.087)(\log P)^{2}+0.599$

$$
\begin{align*}
& ( \pm 0.222) \log P+1.893  \tag{37}\\
& \quad n=13 ; r=0.918 ; s=0.079
\end{align*}
$$

For Free-Wilson analysis $N, N$-diacetylurea was taken as basis compound; therefore, all group contributions refer to $a \mathrm{COCH}_{3}=0.00$ and $\mu=\log 1 / C$ calcd of $N, N$-diacetylurea (in all cases where no "unsubstituted" compound with $\mathrm{R}=\mathrm{H}$ is included, any compound may be chosen as basis compound in the modified Free-Wilson approach ${ }^{28}$ ).
For Bocek-Kopecky analysis a total number of ten variables would have been needed; therefore the simplifying assumption was made that the interactions between substituents $R_{1}$ and $R_{2}$ depend only on the number of carbon atoms of these substituents (the matrix used for the Free-Wilson analysis and the Bocek-Kopecky-like analysis is given in Table VIII).

Due to the parabolic character of eq 37 Free-Wilson analysis gave only a correlation which is statistically not significant ( $n=13 ; r=0.822 ; s=0.135 ; F=2.93 ; p>0.05$ ). A Bocek-Kopecky-like analysis with the same matrix plus an interaction term $N_{1} \cdot N_{2}$ gave a better correlation which is statistically significant ( $n=13 ; r=0.982 ; s=0.049 ; F$ $=26.55 ; p<0.001$ ). The values found for de novo group contributions and the coefficient of the interaction term are given in Table IX.

That the interaction term $N_{1} \cdot N_{2}$ is physically meaningful can be shown by the following considerations. Since log $P$ for this group of compounds can be calculated from the total number of carbon atoms of substituents $R_{1}$ and $R_{2}$ by eq 38 (see ref 27), it is possible to correlate $\log 1 / C$ to $N^{2}$ and $N$ (eq 39).
$\log P=0.50 N-2.60$

$$
\begin{gather*}
\log 1 / C=-0.044( \pm 0.022) N^{2}+0.760( \pm 0.334)  \tag{38}\\
N-0.862  \tag{39}\\
\quad n=13 ; r=0.918 ; s=0.079
\end{gather*}
$$

Table VIII. Hypnotic Activity of $N, N^{\prime}$-Diacylureas $\mathrm{R}_{1} \mathrm{NHCONHR}_{2}$; Structures and Matrix Used for Free-Wilson and Boček-Kopecky Analysis ${ }^{\text {a }}$

| $\mathrm{R}_{1}, \mathrm{R}_{2}$ | Free-Wilson matrix |  |  |  |  | Interaction term $N_{1} \cdot N_{2}{ }^{a}$ | $N_{1}{ }^{\text {a }}$ | $N_{2}{ }^{\text {a }}$ | $\log 1 / C$ obsd ${ }^{\text {b }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathrm{C}_{3}$ | $\mathrm{C}_{4}$ | $\mathrm{C}_{5}$ | $\mathrm{C}_{6}$ | $\mathrm{C}_{3}$ |  |  |  |  |
| Acetyl, propionyl | 1 |  |  |  |  | 6 | 2 | 3 | 1.84 |
| Propionyl, propionyl | 2 |  |  |  |  | 9 | 3 | 3 | 2.06 |
| Acetyl, butyryl |  | 1 |  |  |  | 8 | 2 | 4 | 2.16 |
| Propionyl, butyryl | 1 | 1 |  |  |  | 12 | 3 | 4 | 2.23 |
| Acetyl, valeryl |  |  | 1 |  |  | 10 | 2 | 5 | 2.27 |
| Butyryl, butyryl |  | 2 |  |  |  | 16 | 4 | 4 | 2.40 |
| Propionyl, valeryl | 1 |  | 1 |  |  | 15 | 3 | 5 | 2.35 |
| Acetyl, hexanoyl |  |  |  | 1 |  | 12 | 2 | 6 | 2.46 |
| Butyryl, valeryl |  | 1 | 1 |  |  | 20 | 4 | 5 | 2.38 |
| Propionyl, hexanoyl | 1 |  |  | 1 |  | 18 | 3 | 6 | 2.25 |
| Acetyl, heptanoyl |  |  |  |  | 1 | 14 | 2 | 7 | 2.55 |
| Valeryl, valeryl |  |  | 2 |  |  | 25 | 5 | 5 | 2.32 |
| Butyryl, hexanoyl |  | 1 |  | 1 |  | 24 | 4 | 6 | 2.28 |

[^0]Table IX. Group Contributions and Interaction Terms Calculated by Free-Wilson, Boček-Kopecký, and Hansch Analysis (Based on $a_{\mathrm{COCH}_{3}}=0.00$ )

| Substituent | Group contributions $a_{i}$ and interaction term $N_{1} \cdot N_{2}$ calcd by |  |  |
| :---: | :---: | :---: | :---: |
|  | FreeWilson analysis | BocekKopecký analysis | Hansch analysis (eq 42) |
| Acetyl | 0.00 | 0.00 | 0.00 |
| Propionyl | 0.00 | 0.43 | 0.54 |
| Butyryl | 0.15 | 0.91 | 0.99 |
| Valeryl | 0.19 | 1.25 | 1.36 |
| Hexanoyl | 0.24 | 1.49 | 1.63 |
| Heptanoyl | $0.51{ }^{\text {a }}$ | $1.81{ }^{\text {a }}$ | 1.82 |
| Interaction term |  | $-0.085 N_{1} \cdot N_{2}$ | $-0.088 N_{1} \cdot N_{2}{ }^{\text {b }}$ |

${ }^{a}$ Single point determinations. ${ }^{b}$ See eq 40.
Substitution of $N$ by $\left(N_{1}+N_{2}\right)$ like Singer and Purcell ${ }^{8}$ have done in their comparison of the Hansch and Bocek-Kopecky approach gives eq 40 .

$$
\begin{align*}
& \log 1 / C=-0.044\left(N_{1}{ }^{2}+N_{2}{ }^{2}\right)-0.760\left(N_{1}+\right. \\
& \left.\quad N_{2}\right)-0.862 \tag{40}
\end{align*}
$$

Equation 40 is a Boček-Kopecky-like equation (eq 3) with additive parameters $b \mathrm{X}=-0.044 N_{1}{ }^{2}+0.760 N_{1}$ and $b_{\mathrm{Y}}=-0.044 \mathrm{~N}_{2}{ }^{2}+0.760 \mathrm{~N}_{2}$ (corresponding to Free-Wilson group contributions) and an interaction term exey $=$ $-0.088 \mathrm{~N}_{1} \cdot \mathrm{~N}_{2}$.
Equation 41 can be derived from eq 40 to calculate group
$b_{i}=-0.044 N_{i}^{2}+0.760 N_{i}$
contributions for each substituent R. Group contributions $a_{i}$ based on $a \mathrm{COCH}_{3}=0.00$ can be calculated from eq 42.
$a_{i}=-0.044 N_{i}{ }^{2}+0.760 N_{i}-1.344$
The values of Table IX show the numerical equivalence of Bocek-Kopecky analysis and parabolic Hansch analysis and the complete breakdown of Free-Wilson additivity as predicted by Singer and Purcell. 8

Example 5 is a reinvestigation of the correlations given by Bruice et al. ${ }^{11}$ and Hansch and Fujita ${ }^{1}$ for the thyroxine-like activity of compounds of structure I. Bruice

et al. ${ }^{11}$ derived eq 4
$\log \%$ thyroxine-like activity $=k \Sigma f+c$
where
$\Sigma f=f_{\mathrm{X}_{1}}+f_{\mathrm{X}_{2}}+f_{\mathrm{Y}_{1}}+f_{\mathrm{Y}_{2}}+f_{\mathrm{OR}}{ }^{\prime}+f_{\mathrm{R}}$
(except for H it was implicated that $f \mathrm{x}_{1}=f \mathrm{X}_{2}$ and $f \mathrm{Y}_{1}=$ $f \mathrm{Y}_{2}$ ).
From their $f$ values we calculated for compounds 1-14 the values of $k$ and $c$ (structures and values of $\log A$ and $\Sigma f$ are given in Table X).
$\log A=2.804( \pm 0.853) \Sigma f+13.116$

$$
\begin{equation*}
n=14 ; r=0.900 ; s=0.523 \tag{44}
\end{equation*}
$$

Equation 45 was obtained by excluding compound 6 , which is not well predicted by eq 44.

$$
\begin{align*}
\log A=3.178 & ( \pm 0.314) \Sigma f+14.873  \tag{45}\\
n & =13 ; r=0.989 ; s=0.183
\end{align*}
$$

Hansch and Fujita ${ }^{1}$ derived for compounds 1-9 eq 46

$$
\begin{gather*}
\log A=-1.134 \pi_{x}^{2}+7.435 \pi_{x}-16.323 \sigma_{x}- \\
0.287  \tag{46}\\
n=9 ; r=0.884 ; s=0.660
\end{gather*}
$$

( $\pi$ and $\sigma$ values are given in Table X). The high coeffi-

Table X. Thyroxine-Like Activity on Rodents

|  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Compd no. | $\mathrm{X}_{1}{ }^{\text {a }}$ | $\mathrm{X}_{2}{ }^{\text {a }}$ | $\mathrm{Y}_{1}$ | $\mathrm{Y}_{2}$ | $\pi_{x}{ }^{\text {b }}$ | $\pi^{c}$ | $\sigma_{x}{ }^{b}$ | $E_{\text {s, } \mathrm{X}_{1}}{ }^{\text {d }}$ | $D^{e}$ | $\Sigma f$ | $\log A$ obsd $^{f}$ |
| 1 | H | I | I | I | 1.15 | 3.45 | 0.28 | 1.24 | 0 | -3.74 | 2.74 |
| 2 | H | Br | I | I | 0.85 | 3.15 | 0.23 | 1.24 | 0 | -3.96 | 2.30 |
| 3 | I | I | I | I | 2.30 | 4.60 | 0.56 | -0.16 | 0 | -4.04 | 2.00 |
| 4 | F | I | I | I | 1.36 | 3.66 | 0.34 | 0.78 | 0 | -4.28 | 1.45 |
| 5 | Br | Br | I | I | 1.70 | 4.00 | 0.46 | 0.08 | 0 | -4.48 | 1.00 |
| 6 | H | F | I | I | 0.21 | 2.51 | 0.06 | 1.24 | 0 | -3.98 | 0.38 |
| 7 | F | F | I | I | 0.42 | 2.72 | 0.12 | 0.78 | 0 | -4.52 | 0.26 |
| 8 | Cl | Cl | I | I | 1.30 | 3.60 | 0.46 | 0.27 | 0 | -4.68 | -0.05 |
| 9 | H | H | I | I | 0.00 | 2.30 | 0.00 | 1.24 | 0 | -4.80 | -0.43 |
| 10 | H | I | Br | Br | 1.15 | 2.85 | 0.28 | 1.24 | 2 | -4.02 | 2.11 |
| 11 | H | Br | Br | Br | 0.85 | 2.55 | 0.23 | 1.24 | 2 | -4.24 | 1.64 |
| 12 | I | I | Br | Br | 2.30 | 4.00 | 0.56 | -0.16 | 2 | -4.32 | 1.10 |
| 13 | Br | Br | Br | Br | 1.70 | 3.40 | 0.46 | 0.08 | 2 | -4.76 | -0.29 |
| 14 | I | I | Cl | Cl | 2.30 | 3.60 | 0.56 | -0.16 |  | -4.96 | -1.00 |

[^1]cients found for the $\pi$ and $\sigma$ terms are little reliable because $\pi$ and $\sigma$ are correlated for this group of compounds (eq 47). Therefore eq 46 may be a chance correlation. ${ }^{26}$
\[

$$
\begin{array}{rl}
\pi_{x}=3.713 & ( \pm 0.793) \sigma_{x}-0.003  \tag{47}\\
n & n 9 ; r=0.973 ; s=0.184
\end{array}
$$
\]

In order to find a better correlation of structure and activity for this group of compounds some Free-Wilson analyses were run to prove additivity of group contributions (the matrices used are not given in detail because the Free-Wilson analyses were only an aid to derive the final Hansch equation).

A first Free-Wilson analysis of compounds 1-9 (it was implicated that $a \mathrm{X}_{1}=a \mathrm{X}_{2}$ ) gave eq 48. The low correlation

$$
\begin{align*}
& \log A=\Sigma a_{i}+0.67  \tag{48}\\
& n=9 ; r=0.739 ; s=1.062 \\
& {\left[F_{\mathrm{X}}\right]=-0.22 ;\left[\mathrm{Cl}_{\mathrm{X}}\right]=-0.36 ;\left[\mathrm{Br}_{\mathrm{X}}\right]=0.46} \\
& \quad\left[\mathrm{I}_{\mathrm{X}}\right]=0.95\left(\text { all values based on } a_{\mathrm{H}}=0.00\right)
\end{align*}
$$

coefficient of eq 48 indicates that there is no additivity of group contributions; therefore, a Bocek-Kopecky-like analysis was run with $\pi^{2}$ as interaction term (eq 49), which

$$
\begin{align*}
& \log A=-1.35 \pi_{\mathbf{x}}^{2}+\Sigma a_{i}+0.05  \tag{49}\\
& n=9 ; r=0.931 ; s=0.479 \\
& {\left[F_{\mathrm{X}}\right]=0.13 ;\left[\mathrm{Cl}_{\mathrm{X}}\right]=1.09 ;\left[\mathrm{Br}_{\mathrm{X}}\right]=2.59 ;\left[\mathrm{I}_{\mathrm{X}}\right]=} \\
& \left.4.41 \text { (all values based on } a_{\mathrm{H}}=0.00\right)
\end{align*}
$$

gave a correlation coefficient much better than that of eq 48. The interaction term in eq 49 is similar to the parabolic term of eq 46, which seems to support the biological significance of eq 46.

A second Free-Wilson analysis was run with compounds 1-14 (it was implicated that $a \mathrm{X}_{1}=a \mathrm{X}_{2}$ and $a \mathrm{Y}_{1}=a \mathrm{Y}_{2}$ ).

$$
\begin{aligned}
& \log A=\Sigma a_{i}+1.28 \\
& n=14 ; r=0.726 ; s=1.080 \\
& {\left[\mathrm{~F}_{\mathrm{X}}\right]=-0.54 ;\left[\mathrm{Cl}_{\mathrm{X}}\right]=-0.66 ;\left[\mathrm{Br}_{\mathrm{X}}\right]=-0.10 ;} \\
& \quad\left[\mathrm{I}_{\mathrm{X}}\right]=0.49\left(\text { values based on } a_{\mathrm{H}}=0.00\right) \\
& {\left[\mathrm{Br}_{\mathrm{Y}}\right]=-0.21 ;\left[\mathrm{Cl}_{\mathrm{Y}}\right]=-1.63} \\
& \left.\quad \text { (values based on } a_{\mathrm{I}}=0.00\right)
\end{aligned}
$$

The corresponding Bocek-Kopeckýy analysis with $\pi^{2}$ (note that $\pi=\pi_{x}+\pi y$ ) as interaction term gave eq 51 .

$$
\begin{align*}
& \log A=0.002 \pi^{2}+\Sigma a_{i}+1.28  \tag{51}\\
& n=14 ; r=0.730 ; s=1.160 \\
& {\left[\mathrm{~F}_{\mathrm{X}}\right]=-0.59 ;\left[\mathrm{Cl}_{\mathrm{X}}\right]=-0.67 ;\left[\mathrm{Br}_{\mathrm{X}}\right]=-0.11 ;} \\
& \quad\left[\mathrm{I}_{\mathrm{X}}\right]=0.48\left(\text { values based on } a_{\mathrm{H}}=0.00\right) \\
& {\left[\mathrm{Br}_{\mathrm{Y}}\right]=-0.21 ;\left[\mathrm{C} l_{\mathrm{Y}}\right]=-1.63} \\
& \left.\quad \text { (values based on } a_{\mathrm{I}}=0.00\right)
\end{align*}
$$

Equation 51 demonstrates that there is no significant contribution of $\pi^{2}$, but there is also no additivity of group contributions as can be seen from the low correlation coefficients of eq 50 and 51 .

Next a Free-Wilson analysis was run to prove the implication that substituent contributions are equal in positions $\mathrm{X}_{1}$ and $\mathrm{X}_{2}$ (it was implicated that $a \mathrm{X}_{1} \neq a \mathrm{X}_{2}$ and $a Y_{1}=a_{2}$; compound 8 was excluded because no differ-
entiation is possible between Cl in position $\mathrm{X}_{1}$ and $\mathrm{X}_{2}$ ).

$$
\begin{align*}
& \log A=\Sigma a_{i}-0.43  \tag{52}\\
& n=13 ; r=0.981 ; s=0.387 \\
& {\left[\mathrm{~F}_{\mathrm{X}_{1}}\right]=-0.83 ;\left[\mathrm{Br}_{\mathrm{X}_{1}}\right]=-1.62 ;\left[\mathrm{I}_{\mathbf{X}_{1}}\right]=-0.70} \\
& {\left[\mathrm{~F}_{\mathrm{X}_{2}}\right]=1.17 ;\left[\mathrm{Br}_{\mathrm{X}_{2}}\right]=2.79 ;\left[\mathrm{I}_{\mathbf{X}_{2}}\right]=3.07} \\
& \quad \text { (all values based on } a_{\mathrm{H}}=0.00 \text { ) } \\
& {\left[\mathrm{Br}_{\mathrm{Y}}\right]=-0.39 ;\left[\mathrm{Cl} \mathrm{Y}_{\mathrm{Y}}\right]=-1.47} \\
& \left.\quad \text { (values based on } a_{\mathrm{I}}=0.00\right)
\end{align*}
$$

Of course a Free-Wilson analysis with only 13 compounds and 8 group contributions ( $=9$ unknowns, including $\mu$ ) is strictly forbidden and it is not surprising to get a correlation coefficient of 0.981 in such an analysis. But a closer look shows that there are great differences for the group contributions in position $X_{1}$ and position $X_{2}$ : all $X_{1}$ values are negative and all $X_{2}$ values are positive. Since the $X_{2}$ values are most probably $\pi$-correlated, also the $\mathrm{X}_{1}$ values should be $\pi$-correlated but there may be a negative steric contribution for $\mathrm{X}_{1}$.

A Hansch analysis of compounds 1-9 with $\pi_{x}$ and $E_{\mathrm{s}, \mathrm{X}_{1}}$ as structural parameters gave eq 53. It must be noted that

$$
\begin{align*}
& \log A=2.43( \pm 0.88) \pi_{\mathrm{x}}+2.66( \pm 1.17) E_{\mathrm{s}, \mathrm{x}_{1}}- \\
& \quad 3.42  \tag{53}\\
& n=9 ; r=0.940 ; s=0.439 ; F=22.81 ; p<0.005
\end{align*}
$$

Jorgensen et al. ${ }^{30}$ discussed a hypothetical thyroid hormone receptor with a binding receptor, a functional receptor, and a proximal steric block for substituent $\mathrm{X}_{1}$. Corresponding to this model $E_{\mathbf{s}}$ values were used only for position $\mathbf{X}_{1}$; in cases of different substituents $\mathbf{X}_{1}$ and $\mathbf{X}_{2}$ the smaller substituent was taken as $X_{1}$, since there seems to be no steric hindrance for substituents in position $\mathrm{X}_{2}$.

Hansch analysis of compounds 1-13 gave eq 54; an indicator variable ${ }^{31} D$ was included in order to account for all polar, electronic, and steric changes that may occur if $\mathrm{Y}=\mathrm{I}$ is replaced by $\mathrm{Y}=\mathrm{Br}$. Equations 53 and 54

$$
\begin{align*}
& \log A=2.54( \pm 0.71) \pi_{x}+2.91( \pm 0.83) E_{\mathrm{s}, \mathrm{X}_{1}}- \\
& \quad 0.35( \pm 0.30) D-3.72  \tag{54}\\
& n=13 ; r=0.942 ; s=0.408 ; F=23.45 ; p<0.001
\end{align*}
$$

demonstrate that doing something forbidden may come to a good end; eq 48-52 should not be discussed further; they were only presented to show how Free-Wilson analysis and Bocek-Kopecky analysis can be used to prove additivity of group contributions and to derive better fitting Hansch equations in cases where no simple correlation of $\log 1 / C$ with $\pi$ and/or $\sigma$ is given. A more detailed discussion of structure-activity relationships of thyroxine analogues will be given in the following paper. ${ }^{32}$

## Discussion

The preceding examples show clearly the close numerical equivalence of the linear free energy related Hansch approach and the mathematical models of Free and Wilson and Bocek and Kopecky. We hope that these examples give enough evidence to end the discussion whether Free-Wilson additivity also holds in cases of parabolic dependence of biological activity on a particular physical parameter or not; the statements made by Singer and Purcell ${ }^{8}$ that Free-Wilson's model corresponds to the nonparabolic Hansch approach and Bocek-Kopeckýs model corresponds to the parabolic Hansch approach are established by our examples as a fact. Cammarata's opinion that the Free-Wilson approach also holds in cases
of parabolic dependence of biological activity ${ }^{16,17}$ is valid only in these special instances where the numerical contribution of a parabolic term or an interaction term is small.

If the modified ${ }^{13}$ Free-Wilson approach (all $a_{i}$ based on $a_{\mathrm{H}}=0.00$ and $\mu=\log 1 / C$ calcd of the unsubstituted compound) is used, the de novo group contributions are, within the experimental error and/or the biological variance, identical with Hansch-derived group contributions (provided both models give satisfactory correlations between observed and calculated $\log 1 / C$ values). In cases of significant parabolic dependence of biological activity on a particular physical parameter an interaction term has to be included in the Free-Wilson analysis. The de novo group contributions and the coefficient of the interaction term calculated by such a Bocek-Kopecky-like approach are identical with the corresponding Hansch-derived values, if the Hansch equation is transformed in a manner as Singer and Purcell ${ }^{\circ}$ have shown.

Based on this equivalence the Free-Wilson approach can be used to check additivity of group contributions for a given set of compounds, if the number of variables necessary is not too great compared to the number of compounds involved. A good correlation between observed and calculated $\log 1 / C$ values gives evidence that there may exist a nonparabolic relationship between appropriate structural parameters and biological activity. On the other hand, an unsatisfactory correlation gives evidence for a parabolic dependence, an interaction between several substituents or the nonvalidity of a simplifying assumption (e.g., assumed equivalence of ortho and ortho' substituents; compare example 5).

If the modified Free-Wilson approach is used only to derive a significant Hansch correlation, no care has to be taken if the Free-Wilson analysis per se gives a statistical significant result (of course such a "forbidden" FreeWilson analysis must not be used to make predictions for new compounds or to calculate Hansch equations for the de novo group contributions). A close inspection of such Free-Wilson derived group contributions may give some ideas of which structural parameters should be used in doing a Hansch analysis, as was shown in examples 2 and 5. The final Hansch equation has to be examined very carefully on its statistical significance and its physical and biochemical meaning with the criteria given by Unger and Hansch ${ }^{19}$ in order not to abuse this method to generate some more "statistical unicorns".

The goal of every Free-Wilson or Bocek-Kopecky analysis should be the derivation of a significant Hansch equation which gives us a better understanding of how
drugs act at the molecular level.

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[^0]:    ${ }^{a} N_{1}, N_{2}=$ number of carbon atoms of substituents $\mathrm{R}_{1}, \mathrm{R}_{2}$. ${ }^{b}$ See ref 29.

[^1]:    ${ }^{a}$ For different substituents $X_{1}$ and $X_{2}$ the smaller substituent was taken as $X_{1}$ and the greater substituent was taken as $X_{2}$.
    ${ }^{b}$ For better comparison original $\pi$ and $\sigma$ values used by Hansch and Fujita ${ }^{1}$ were taken; other values gave similar results.
    ${ }^{c} \pi=\pi_{x}+\pi_{y} . \quad{ }^{d}$ See ref 23. $\quad$ Indicator variable $(D=0$ for $\mathrm{Y}=\mathrm{I} ; D=1$ for $\mathrm{Y}=\mathrm{Br}$ ). f See ref 11 .

