Enhancement of Internal Signal Stochastic Resonance by Noise Modulation in the CSTR System

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A two-variable model, which was proposed to describe a first-order, exothermic, irreversible reaction A → B carried out in a continuous stirred tank reactor (CSTR), is investigated when the control parameter is perturbed by Gaussian shot noise, which contains a constant delay τ, near the supercritical Hopf bifurcation point. Noise-induced coherent oscillation (NICO) is observed, the strength of which goes through a maximum with the increment of the noise intensity, showing the occurrence of internal signal stochastic resonance (ISSR). Furthermore, the NICO strength also shows resonance behavior with the variation of τ, indicating an accessible way to enhance ISSR by noise modulation. These behaviors are also observed when the noise delay is exponentially distributed.

The phenomenon of stochastic resonance (SR) has been extensively studied in the last decade.[1] The main result of SR, which is in some ways considered counterintuitive, shows the constructive role of noise, since the response of the system to external periodic signal may be enhanced with the addition of an optimized amount of noise. In this sense, the main fingerprint of SR is the appearance of a maximum in the output signal-to-noise ratio (SNR) at a nonzero noise level.

In spite of the fact that the original SR model was concerned with a bistable system subjected to external periodic signal and additive Gaussian white noise,[2] it is known now that there are different situations where SR appears. For example, the nonlinear system can be monostable,[3] excitable,[4] spatial extended,[5] or nondynamic;[6] the signal can be aperiodic or chaotic;[7] the noise can be colored, multiplicative,[8] or even the intrinsic randomness of a deterministic chaotic system which case leads to the so-called noise-free SR.[9] Very recently, it was reported that the external signal can be replaced by “internal signal”, as, for instance, the deterministic oscillation of the system.[10] When the control parameter is randomly modulated near the bifurcation point between a stable node and a stable limit cycle, noise-induced coherent oscillation (NICO) is observed, the strength of which goes through a maximum with the increment of the noise intensity, showing the occurrence of SR, which may be named *internal signal stochastic resonance* (ISSR).

Now another interesting and important tendency in the study of SR has been to enhance the strength of it. For example, some scientists have studied the behavior called array-enhanced stochastic resonance (AESR);[11] i.e., the maximum SNR in the SR curve can be enhanced if a nonlinear dynamic element is coupled with other elements in a one-dimensional array. Another clue to enhance the strength of SR is to change the quality of noise. According to the study of Nozaki et al., for $1/f^{\beta}$ colored noise, the case $\beta = 1$ is the most favorable for the enhancement of SR in the FHN neuron model.[12] Neiman et al.[13] have investigated the influence of the correlation time of the colored noise and they found, as stated by them, memory effects of SR. In the threshold-free model,[6] the authors also found that the bandwidth of the noise can considerably influence the output SNR. Chow et al. reported that the aperiodic stochastic resonance (ASR) can be enhanced by modulation of the noise intensity by the input signal or the unit’s output rate signal.[14]

Although SR has found great applications in different fields of science, there are so far only a few reports concerned with the study of SR in chemical reaction systems,[15,16] and to our knowledge, none with ISSR. However, chemical reactions often occur far from equilibrium and show complex nonlinear spatiotemporal behavior, such as multistability, oscillation, deterministic chaos, chemical waves, and so on. Therefore, one expects that SR, as well as ISSR or even noise-free SR, should be rather common here. In the present work, we will study the ISSR behavior of a two-variable model, which has been proposed to describe a first-order, exothermic, irreversible reaction A → B carried out in a continuous stirred tank reactor (CSTR), when the control parameter is random modulated near a supercritical Hopf bifurcation point.

Here we would rather depict the character of the special type of shot noise $\xi(t)$ used in the present work. The noise spikes are Gaussian distributed, but each spike has a constant delay τ, i.e.

$$\xi(t) = \sum_{j=0}^{\infty} \xi_j \Gamma(t - j\tau)$$

where j is an integer, and $\Gamma(x) = 1$ for $0 \le x < \tau$ and 0 otherwise; ξ_i is Gaussian-distributed random value with zero mean and unit variance:

$$\langle \xi_i \rangle = 0, \quad \langle \xi_i \xi_j \rangle = \delta_{ij}$$

In the numerical scheme, we choose $\tau = N\Delta t$, here N is a positive integer and Δt is the simulation time step. If $N = 1$, the noise changes each time step and has no time correlation so that one recovers a Gaussian white noise. In fact, $1/\tau$

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measures the frequency bandwidth of the noise. Since the experimental realization of noise must be limited by a bandwidth, it should be rather important to investigate the influence of τ on the observed noisy dynamic behavior. In the present paper, we find that the strength of the NICO not only shows a resonant behavior with the noise intensity, i.e., ISSR occurs, but also on the variation of τ, i.e., the NICO strength is remarkably enhanced for an optimized τ.

According to the irreversible A $\rightarrow$ B reaction carried out in CSTR, the total mass flow (with rate j) carries heat and component A and B continuously into and out of the reaction vessel; heat is removed through a cooling coil which is maintained at a temperature T_c; component A reacts to form B and heat is released. The mass balance and the energy balance for component A can be described by two coupled first-order differential equations as follows:[13]

$$\dot{x} = 1 - x - xDE(y) \tag{1.1}$$

$$\dot{y} = -\beta y + xBDE(y) \tag{1.2}$$

where $E(y) = \exp[y/(1 + \eta y)]$, and x and y are dimensionless concentration and temperature, respectively; for certain choices of the physical quantities, the parameters B, D, β, and η are related to the control parameters j and T_c via the following equations:

$$\eta = \frac{T^*}{8827}, D = \frac{8.2365 \times 10^{10} e^{-1/\eta}}{j}, B = \frac{271.46}{\eta T^*},$$
$$\beta = 1 + \frac{4.08}{j}, T^* = \frac{885.8j + 11.02T_c}{2.7j + 11.02}$$

For more details of the CSTR equations and the physical quantities, one can turn to ref 17 and references there in. Figure 1a shows the bifurcation diagram for eq 1 with the variation of j, keeping $T_c = 292$ K. The parameter path contains a saddle-node bifurcation value of periodic orbits (j_{snp}) and a supercritical and subcritical Hopf bifurcation value: j_h and $j_{h'}$. Within the bistable region between j_{snp} and $j_{h'}$, the stable node is surounded by a smaller unstable limit cycle (the dot line) and a larger stable limit cycle (the dash-dot line). Notice that this diagram is the same as that in ref 17 and we redraw it here just for convenience.

To study the ISSR behavior, we choose the control parameter j to be located in the right vicinity of the supercritical Hopf bifurcation point j_h and perturb it with duration-modulated noise as described above, as:

$$j = j_0[1 + 2D\xi(t)] \tag{2}$$

with $j_0 = 0.998$. First, we choose $\tau = \Delta t$. For $D = 0$, the deterministic oscillation is absent. If D exceeds a certain level however, an obvious peak appears in the output power spectrum, indicating the occurrence of NICO. With the increment of noise level, the NICO strength is first considerably enhanced and finally annihilated into the noise background. In addition, for suitable noise level, higher harmonics up to $7\omega_0$ (here ω_0 is the fundamental frequency of the NICO) appear in the power spectrum and their strength also goes through nonmonotonical variation. Obviously, the dependence of the NICO strength on noise intensity shows the characteristic of SR. In Figure 1b, the power spectra for different noise levels are present, noting that each curve is obtained by averaging over 50 independent runs. The ISSR behaviors are depicted in Figure 1c, where the NICO strength is measured by $SNR/\Delta\omega$; here $\Delta\omega$ is the width of the NICO peak at the half of its height. The higher harmonics also show ISSR behavior, as, for instance, the ISSR curve for $2\omega_0$ harmonic and $3\omega_0$ harmonic are shown in Figure 1c too. It is found that all the harmonic oscillations reach the maximum strength at a same noise level $D = 0.06$. Second, we fix $D = 0.015$ (now only the ω_0 component is present in the power spectra for $\tau = \Delta t$) and change τ. When τ increases, we find that the NICO strength is first remarkably enhanced, and also higher harmonics appear in the power spectra, while thereafter it decreases. It seems that here τ plays a similar role as the noise intensity. The power spectra and NICO strength of the ω_0 component for different values of τ are shown in Figure 2, a and b, respectively.

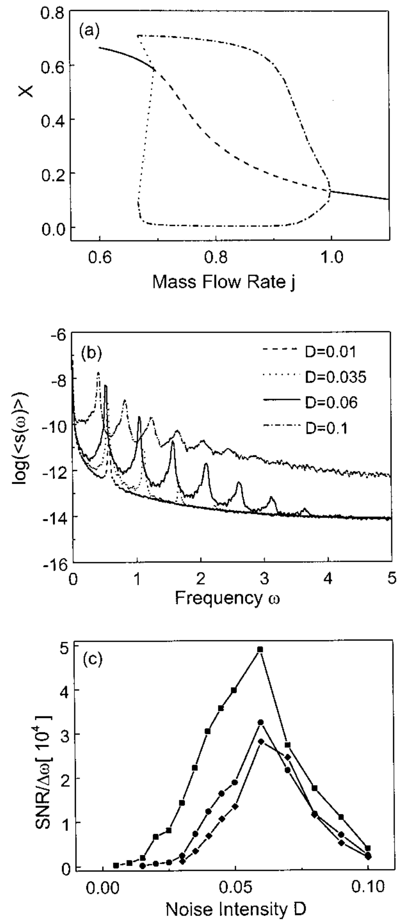

Figure 1. (a) Bifurcation diagram of the CSTR model. The stable (completely unstable) stationary states are denoted with a solid (dashed) line; the stable (unstable) limit cycles are denoted with a dot-dashed (dotted) line. (b) Average power spectra at different noise level. For $D = 0.06$, the NICO strength reaches the maximum and even $7\omega_0$ component is observed. (c) The dependence of the NICO strength of ω_0 (squares), $2\omega_0$ (circles), and $3\omega_0$ (diamonds) components on noise intensity. Notice the strength of the $2\omega_0$ ($3\omega_0$) components are multiplied by 5 (20), respectively.

One should note that the type of noise used above is not so realistic, for in addition to random components, it also contains a periodic component, i.e., the distance τ between the two successive spikes ξ_i and ξ_{i+1} is constant. However, realistic shot noise has typically distributed delays and in many cases the distribution may be close to an exponential. Thus, it is interesting and important to further extend our study for the case of shot noise with this type of delay distribution. In our present work, the delay time τ is randomly chosen to be $n\tau_0$, where $n = 1, 2, ..., 10$, and the frequency counts of $n\tau_0$ decreases exponentially with the increment of n as, for instance, shown in Figure 3a, where $\tau_0 = 10\Delta t$. Substituting this type of noise into eq 1, we

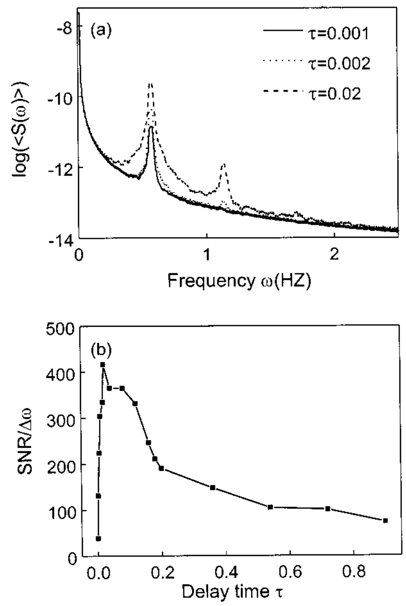

Figure 2. (a) Average power spectra for different choices of τ. The numerical time step is $\Delta t = 0.001$. (b) Dependence of the NICO strength on τ. The maximum is reached at $\tau \simeq 20\Delta t$.

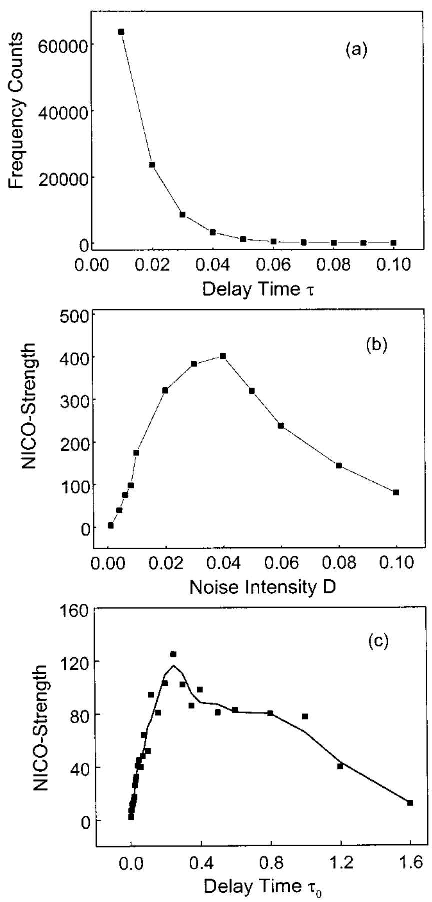

Figure 3. (a) Illustration of the exponential distribution of noise delay τ which is randomly chosen to be $n\tau_0$, where $n = 1, 2, ..., 10$ and $\tau_0 = 10\Delta t = 0.01$; (b) ISSR behavior for $\tau_0 = 10\Delta t$. (c) Dependence of NICO strength on τ_0, where the maximum enhancement is reached at $\tau_0 \simeq 250\Delta t$.

find that NICO behavior also exists. The dependence of the NICO strength on the noise intensity is shown in Figure 3b, which demonstrates the occurrence of ISSR. When we keep the noise intensity constant and change τ_0, a similar effect as that depicted in Figure 2b is also observed, i.e., the NICO strength can be remarkably enhanced for some intermidate values of τ_0. An example is given in Figure 3c, where $D = 0.002$ and one sees that the NICO strength is enhanced for about 100 times for $\tau_0 \simeq 250\ \Delta t$.

To conclude, we have reported the ISSR behavior near a supercritical Hopf bifurcation point in a chemical reaction system carrying out in a CSTR. We have used a type of shot noise for which the delay time between the successive spikes is constant or exponentially distributed. What interests us most is that for both cases the strength of the ISSR can be remarkably enhanced by suitable alternation of the delay time, indicating an accessible way to enhance ISSR or maybe even SR. The robustness of this behavior to the distribution of the delay time indicates it may be applicable to many real systems.

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