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Electrolyte Diffusion into Water

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The diffusion coefficient $D(C)$ for a very dilute aqueous electrolyte is represented, in a first approximation, by $D(C) = D(0)(1 - k\sqrt{C})$, where C is the electrolyte concentration and k is a positive parameter. For the experimental setup in which, initially, a sharp boundary is formed between an electrolyte solution and pure water, the Gosting−Fujita method is not applicable for solving free diffusion with this concentration-dependent D, because D is not regular at $C = 0$. In this paper, another perturbation method which uses k as a perturbation parameter is applied to solve this diffusion problem correct up to second order in k, and the apparent diffusion coefficient D_j defined in the Rayleigh fringe method for measuring transport processes in solutions is calculated as a function of the position in the diffusion boundary. It is shown that the result does not support Albright and Miller's finding from numerical solutions that the position at which D_j agrees with $\bar{D}$ is independent of k. Here, $\bar{D}$ is the D value at the mean concentration.

Introduction

Diffusion of an electrolyte in very dilute aqueous solutions is governed by the diffusion coefficient $D(C)$ which, in a first approximation, is represented by

$$D(C) = D(0)(1 - k\sqrt{C}) \tag{1.1}$$

where C is the concentration of the electrolyte, and $D(0)$ and k are system-dependent (positive) parameters. The dependence of D on $\sqrt{C}$ is usually considered to make the diffusion boundary considerably skew from the ideal situation in which D is constant, and the skewness has been the subject of theoretical and experimental interest in the studies of liquid diffusion.

Albright and Miller[1] approached both analytically and numerically the one-dimensional diffusion equation with $D(C)$ given by eq 1.1 under the experimental setup that, initially, a dilute electrolyte solution is separated by a sharp boundary from water in an infinitely long cell. For the analytical treatment they applied the perturbation method of Gosting and Fujita,[2] which uses the fact that the diffusion boundary tends to be ideal with the diffusion coefficient $D(\bar{C})$ as a dimensionless variable $\Delta C/2\bar{C}$ goes to zero. Here, $\bar{C} = (C_B + C_T)/2$ and $\Delta C = C_B - C_T$, with C_B and C_T denoting the initial concentrations of the bottom and top layers of solution. Thus, this method views deviations of the diffusion boundary from the ideal one by a concentration dependence of $D(C)$ as perturbations due to nonzero $\Delta C/2\bar{C}$. After expanding $D(C)$ in powers of $C - \bar{C}$ about $C = \bar{C}$, it expresses C as a power series of $\Delta C/2\bar{C}$ and successively determines from the diffusion equation the coefficients of the series as functions of time t and position x (actually, of a combined single variable $x/\sqrt{t}$). In the Albright−Miller problem, C_T is zero, so that $\Delta C/2\bar{C} = 1$, which is the worst case for the convergence of the Gosting−Fujita method. However, more important is the fact that the $D(C)$ of eq 1.1 is not a regular function of C at $C = 0$, since dD/dC diverges there. Hence, the expansion of D in powers of $C - \bar{C}$ fails to be valid at $C = 0$. This means that formal application of the Gosting−Fujita perturbation approach as it stands may not hold when D depends on $\sqrt{C}$ and $C_T = 0$.

In reality, eq 1.1 should be regarded as a limiting form of D as C approaches zero. It is more practical to add at least one term linear in C to its right-hand side. Albright and Miller[3] worked out this case again using the Gosting−Fujita perturbation method for the analytic treatment. However, for the above reason the solution they present seems to have no theoretical value when $C_T = 0$.

In the present paper, the Albright−Miller problem is attacked by another perturbation method which utilizes the idea that the diffusion boundary tends to be ideal with the diffusion coefficient $D(0)$ as k diminishes to zero, so that deviations of the diffusion boundary from the ideal one by a concentration dependence of $D(C)$ may be treated as perturbations due to nonzero k. Thus, it expands C in powers of k (actually, $k\sqrt{C_B}$) and successively determines the expansion coefficients from the diffusion equation. It should be noted that though both are perturbation methods the present method differs from the Gosting−Fujita one in the basic idea.

The calculation here is made up to second order in $k\sqrt{C_B}$. The solution obtained is used to calculate D_j as a function of the position in the diffusion boundary, and the result is compared with that obtained by Albright and Miller[1] from numerical integrations. Here, D_j is an apparent diffusion coefficient appearing in the Rayleigh fringe method[4] for measuring diffusion in solutions (see below for its definition).

Basic Equations

The diffusion equation to be solved is

$$\frac{\partial C}{\partial t} = \frac{\partial}{\partial x}\left[D(0)(1 - k\sqrt{C})\frac{\partial C}{\partial x}\right] \tag{2.1}$$

and the associated initial and boundary conditions are

$$C = C_B\ (t = 0;\ 0 < x < \infty), \qquad C = 0\ (t = 0;\ -\infty < x < 0) \tag{2.2}$$

$$C = C_B\ (t > 0;\ x = \infty), \qquad C = 0\ (t > 0;\ x = -\infty) \tag{2.3}$$

If a dimensionless concentration c is introduced by

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$$c = C/C_B \tag{2.4}$$

and a dimensionless variable z by[5]

$$z = x/(2\sqrt{D(0)t}) \tag{2.5}$$

eq 2.1 changes to an ordinary differential equation

$$\frac{d}{dz}\left[(1 - \epsilon\sqrt{c})\frac{dc}{dz}\right] + 2z\frac{dc}{dz} = 0 \tag{2.6}$$

and the initial and boundary conditions are reduced to

$$c = 1\ (z = \infty), \qquad c = 0\ (z = -\infty) \tag{2.7}$$

where ϵ is defined by

$$\epsilon = k\sqrt{C_B} \tag{2.8}$$

and can be taken as smaller than unity to meet the physical requirement that D be positive. We note that Albright and Miller[1] used $\bar{D}$ for $D(0)$ in defining the variable z, where $\bar{D}$ is the D value at the mean concentration $c = {}^1/_2$, i.e.,

$$\bar{D} = D(0)(1 - 0.7071\epsilon) \tag{2.9}$$

The ordinary differential equation derived from eq 2.1 with z thus defined is different in form from eq 2.6. Hence, our solutions presented below do not hold as they stand if the z in them are simply replaced by Albright and Miller's $z = x/(2\sqrt{\bar{D}t})$.

We begin with the substitution

$$n = \sqrt{c} \tag{2.10}$$

Then eq 2.6 changes to

$$\frac{d}{dz}\left[n(1 - \epsilon n)\frac{dn}{dz}\right] + 2zn\frac{dn}{dz} = 0 \tag{2.11}$$

and eq 2.7 to

$$n = 1\ (z = \infty), \qquad n = 0\ (z = -\infty) \tag{2.12}$$

Equation 2.10 removes nonregularity from eq 2.6, but introduces the problem that the coefficient of the highest derivative in eq 2.11, i.e., $n(1 - \epsilon n)$, vanishes at $n = 0$ and $n = 1/\epsilon$. Fortunately, however, $c = 0$ occurs at $z = -\infty$ and $n = 1/\epsilon$ is located outside the interested range of n ($0 < n < 1$) because $\epsilon < 1$. Therefore, we may treat eq 2.11 by the ordinary perturbation method, in which an independent variable is expanded in powers of a small parameter contained in the equation considered.

We express $n(z)$ as

$$n(z) = n_0(z) + n_1(z) + n_2(z)\epsilon + n_2(z)\epsilon^2 + ... \tag{2.13}$$

Then, eq 2.10 gives

$$c(x,t) = u_0(z) + 2u_1(z)\epsilon + u_2(z)\epsilon^2 + ... \tag{2.14}$$

with

$$u_0(z) = n_0(z)^2 \tag{2.15}$$

$$u_1(z) = n_0(z)\, n_1(z) \tag{2.16}$$

$$u_2(z) = 2n_0(z)\, n_2(z) + n_1(z)^2 \tag{2.17}$$

which indicates that $c(x,t)$ can be determined up to second order in ϵ by calculating $u_0(z)$, $u_1(z)$, and $u_2(z)$.

Introducing eq 2.13 into eq 2.11, rearranging the result in powers of ϵ, and equating the coefficients of successive terms to zero, we obtain the following set of equations for $u_0(z)$, $u_1(z)$, and $u_2(z)$:

$$d^2u_0/dz^2 + 2z\, du_0/dz = 0 \tag{2.18}$$

$$d^2u_1/dz^2 + 2z\, du_1/dz = ({}^1/_3)\, d^2u_0{}^{3/2}/dz^2 \tag{2.19}$$

$$d^2u_2/dz^2 + 2z\, du_2/dz = 2\, d^2(u_0{}^{1/2}u_1)/dz^2 \tag{2.20}$$

The associated boundary conditions are

$$u_0(\infty) = 1, \qquad u_1(\infty) = u_2(\infty) = 0 \tag{2.21}$$

$$u_0(-\infty) = u_1(-\infty) = u_2(-\infty) = 0 \tag{2.22}$$

Solutions

Solving the above set of equations is just standard mathematical exercises, so only the important consequences are presented below.

First, as is well documented in textbooks,[5] $u_0(z)$ is given by

$$u_0(z) = E(z)/2 \tag{3.1}$$

with

$$E = 1 + \Phi(z) \tag{3.2}$$

where $\Phi(z)$ is the error function defined by

$$\Phi(z) = \frac{2}{\sqrt{\pi}}\int_0^z \exp(-w^2)\, dw \tag{3.3}$$

Equation 3.1 gives

$$du_0/dz = (2/\sqrt{\pi})\exp(-z^2) \tag{3.4}$$

which is graphed in Figure 1.

Next, $u_1(z)$ is represented by

$$u_1(z) = -\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{z}\exp(-w^2)\, f(w)\, dw + \frac{1}{\sqrt{2}}[E(z)]^{3/2} - 0.309E(z) \tag{3.5}$$

where

$$f(z) = \int_{-\infty}^{z} w\sqrt{E(w)}dw \tag{3.6}$$

It follows from eq 3.5 that

$$du_1/dz = \exp(-z^2)[-(1/\sqrt{2\pi})f(z) + ({}^1/_2\sqrt{2\pi})\sqrt{E(z)} - 0.349] \tag{3.7}$$

which is graphed in Figure 2. The curve consists of a sharp positive peak accompanying a pair of nearly symmetric negative troughs on both sides of it. It should be noted that the maximum is not much shifted from $z = 0$ and its height is only about one-fifth of the maximum of du_0/dz.

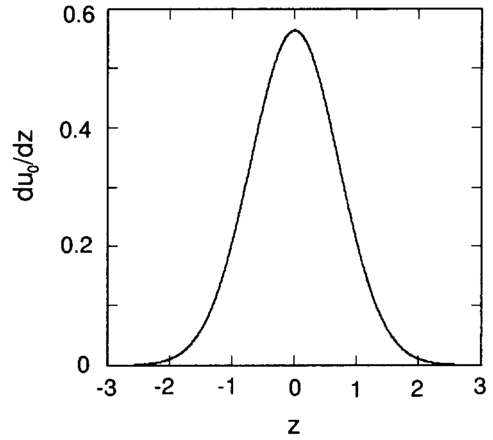

Figure 1. Distribution of du_0/dz over the z axis. This is a Gaussian distribution function.

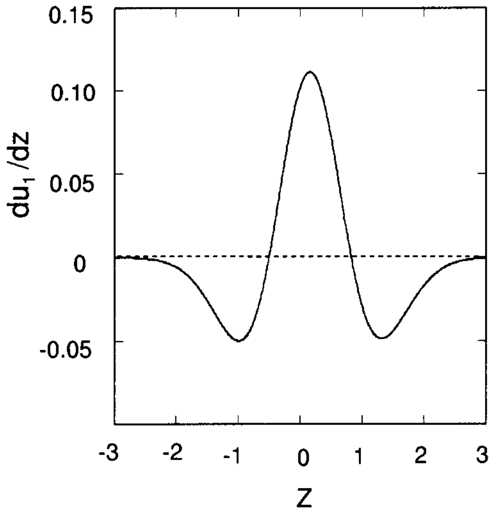

Figure 2. Distribution of du_1/dz over the z axis. The area enclosed by the solid curve above and below the dashed line is zero because of the boundary conditions for $u_1(z)$.

Finally, $u_2(z)$ is conveniently represented as the sum of three terms:

$$u_2(z) = -0.564p(z) + 0.167q(z) - 0.437r(z) \tag{3.8}$$

with

$$p(z) = \int_{-\infty}^{z} \exp(-w^2)\{Ga(w) - [f(w)]^2 + f(w)\,E(w) - 2\int_{-\infty}^{w} yGa(y)\,dy + 0.986\}\,dw \tag{3.9}$$

$$q(z) = [E(z)]^2 - 2.00E(z) - (8/\sqrt{\pi})\int_{-\infty}^{z} \exp(-w^2)\,g(w)\,dw \tag{3.10}$$

$$r(z) = (6\sqrt{2})\{-(1/\sqrt{2\pi})\int_{-\infty}^{z} (-w^2)f(w)\,dw + (\sqrt{2}/12)[E(z)]^{3/2} - 0.309E(z)\} \tag{3.11}$$

where

$$g(z) = \int_{-\infty} wE(w)\,dw \tag{3.12}$$

and

$$Ga(z) = -0.2[1 + \tanh(z + 0.2)] + 0.057[\Phi(z - 0.75) + \Phi(|z - 0.75|)] \tag{3.13}$$

The function $Ga(z)$ is an approximation to $G(z)$ defined by

$$G(z) = \frac{1}{\sqrt{\pi E(z)}}\int_{-\infty}^{z} \exp(-w^2)\,f(w)\,dw \tag{3.14}$$

and has been introduced to simplify the numerical evaluation of $p(z)$. Figure 3 compares $Ga(z)$ with the exact $G(z)$. Though the agreement of the two functions leaves something to be desired, both give $p(z)$ almost indistinguishable values over the entire range of z.

The derivatives dp/dz, dq/dz, and dr/dz are given by

$$dp/dz = \exp(-z^2)\{Ga(z) - [f(z)]^2 + f(z)\,E(z) - 2\int_{-\infty}^{z} wGa(w)\,dw + 0.986\} \tag{3.15}$$

$$dq/dz = \exp(-z^2)[(4/\sqrt{\pi})E(z) - (8/\sqrt{\pi})g(z) - 2.257] \tag{3.16}$$

$$dr/dz = (6/\sqrt{\pi})\exp(-z^2)[-f(z) + (^1/_2)\sqrt{E(z)} - 0.874] \tag{3.17}$$

Figure 4 shows the du_2/dz vs z plot calculated by inserting these three equations into

$$du_2/dz = -0.564(dp/dz) + 0.167(dq/dz) - 0.437(dr/dz) \tag{3.18}$$

It has a general feature similar to the du_1/dz curve, except that the two troughs have different depth. The height and location of the maximum are comparable to those of du_1/dz. This implies that the contribution of the ϵ^2 term to the concentration gradient dc/dz (and c as well) becomes significant with increasing ϵ.

Results and Discussion

Concentration and Concentration Gradient Distributions. With the substitution of the above analytical results into eq 2.14 and

$$dc/dz = du_0/dz + 2\epsilon(du_1/dz) + \epsilon^2(du_2/dz) + ... \tag{4.1}$$

the distributions of c and dc/dz over the z axis for a given ϵ can be calculated. Figure 5 shows the dc/dz distributions for $\epsilon = 0$, 0.2, 0.4, and Figure 6 the corresponding c distributions. These graphs are meaningful only for ϵ at which contributions of terms higher than ϵ^2 may be neglected. Thus, those for $\epsilon = 0.4$ may be accepted with reservation until at least the order of magnitude of u_3 is estimated.

The curves for nonzero ϵ reveal expected deviations from the ideal situation ($\epsilon = 0$). Especially, those in Figure 5 show clearly enhanced sharpening with increasing ϵ. We observe the c distribution for $\epsilon = 0.5$ reported by Albright and Miller[1] to exhibit more marked downward deviations than ours on the side below $c = {}^1/_2$.

Apparent Diffusion Coefficient

When the Rayleigh fringe method[4] is used to study free diffusion in solutions it is usual to evaluate an apparent diffusion coefficient D_j defined below. This is based on the expectation that D_j as a function of j, i.e., the position in the diffusion boundary, allows us to accurately estimate the diffusion coefficient at the mean concentration of a given solution. Albright and Miller[1] confirmed this with their numerical solutions to eq

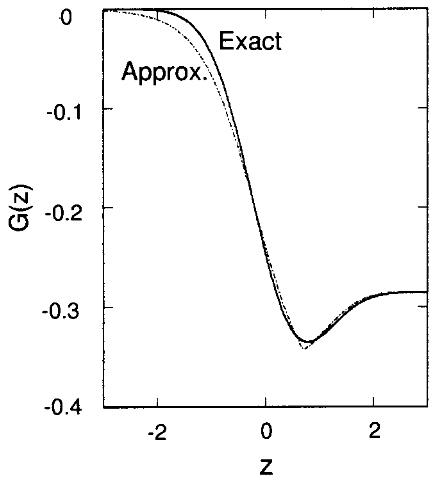

Figure 3. Comparison of *Ga*(*z*) (dotted line) with *G*(*z*) (solid line).

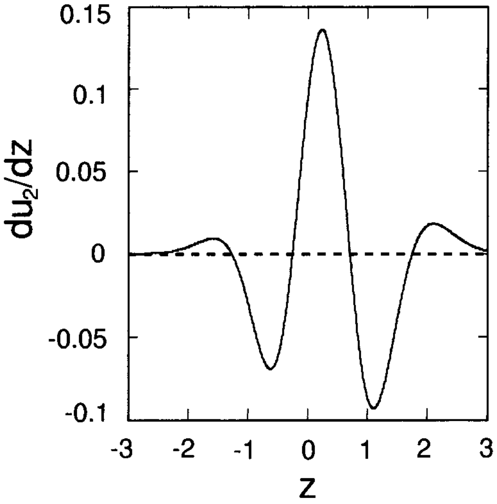

Figure 4. Distribution of du_2/dz over the *z* axis. The area enclosed by the solid curve above and below the dashed line is zero because of the boundary conditions for $u_2(z)$.

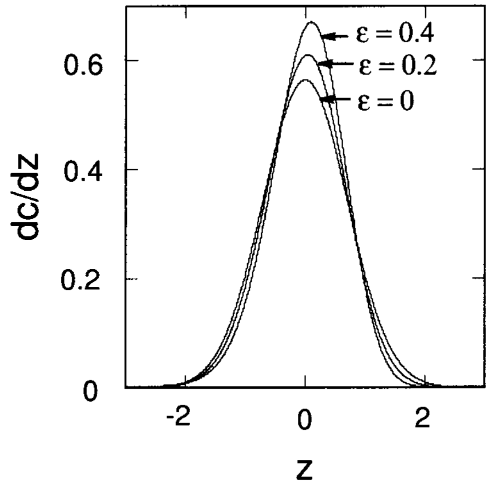

Figure 5. Distributions of concentration gradient d*c*/d*z* for different ϵ.

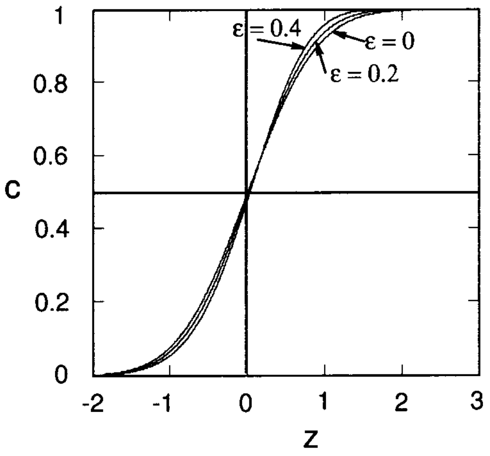

Figure 6. Concentration distributions corresponding to Figure 5.

2.1. In what follows, we examine what happens to D_j when our perturbation solution is used.

In circumstances where concentration is proportional to refractive index, the following set of relations[1] is the basis for computing D_j:

$$c(z_j) = j/J \tag{5.1}$$

$$c(z_j) - {}^1/_2 = {}^1/_2\Phi(z_j^*) \tag{5.2}$$

$${}^1/_2 - c(z_{J-j}) = {}^1/_2\Phi(z_j^*) \tag{5.3}$$

Here, J is the total number of Rayleigh fringes, j and $J - j$ are the fringe numbers for Creeth's symmetric fringes[4] which define a pair of concentrations located above and below at an equal distance from $c = {}^1/_2$, and z_j and z_{J-j} are the z coordinates at these fringes (in practice, since $D(0)$ is unknown in advance, they are replaced by $x_j/\sqrt{t}$ and $x_{J-j}/\sqrt{t}$). Furthermore, z_j^* is a variable as a function of which D_j is to be evaluated.

When the concentration distribution is antisymmetric about the origin of z (or the concentration gradient distribution is symmetric about $z = 0$), as observed in free diffusion of binary or multicomponent systems with constant diffusion coefficients (main-term and cross-term ones), we have $z_{J-j} = -z_j$ so that eq 5.3 is not needed. This is the case that Albright and Sherill[6] have summarized methods for analyzing Rayleigh fringe data on liquid diffusion. However, since z_{J-j} may not be equal to $-z_j$ for concentration-dependent diffusion, eq 5.3 must be retained. In this case, z_j and z_{J-j} determined as functions of z_j^* from eq 5.2 and 5.3 are used to calculate D_j, which is defined by

$$D_j = D(0)\left(\frac{z_j - z_{J-j}}{2z_j^*}\right)^2 \tag{5.4}$$

With the analytical expression for $c(z)$ obtained above, this calculation can be carried through up to second order in ϵ.

Because $c(z)$ reduces to $u_0(z)$ at $\epsilon = 0$ (see eq 2.14), it follows from eq 3.1 and 5.2 that z_j tends to z_j^* as $\epsilon \to 0$. Thus, we start assuming the series

$$z_j = z_j^* + \epsilon v_1(z_j^*) + \epsilon^2 v_2(z_j^*) + ... \tag{5.5}$$

Then, we find from eq 5.2 and 5.3 that z_{J-j} is represented by

$$z_{J-j} = -z_j^* + \epsilon v_1(-z_j^*) + \epsilon^2 v_2(-z_j^*) + ... \tag{5.6}$$

so that eq 5.4 gives

$$D_j/D(0) = [1 + \epsilon V_1(z_j^*) + \epsilon^2 V_2(z_j^*) + ...]^2 \tag{5.7}$$

with

$$V_1(z) = [v_1(z) - v_1(-z)]/(2z) \qquad (5.8)$$

$$V_2(z) = [v_2(z) - v_2(-z)]/(2z) \qquad (5.9)$$

To find the expressions for $v_1(z)$ and $v_2(z)$ we substitute eq 5.5 for z_j in $c(z_j) = u_0(z_j) + 2\epsilon u_1(z_j) + \epsilon^2 u_2(z_j) + ...$ (see eq 2.14), expand each term on the right-hand side in powers of ϵ, and use the results to arrange the left-hand side of eq 5.2 as a power series of ϵ. Comparison of both sides then yields

$$v_1(z_j^*)(\mathrm{d}u_0/\mathrm{d}z)_{z=z_j^*} + 2u_1(z_j^*) = 0 \qquad (5.10)$$

$$v_2(z_j^*)(\mathrm{d}u_0/\mathrm{d}z)_{z=z_j^*} + (^1/_2)v_1(z_j^*)^2(\mathrm{d}^2u_0/\mathrm{d}z^2)_{z=z_j^*} + 2v_1(z_j^*)(\mathrm{d}u_1/\mathrm{d}z)_{z=z_j^*} + u_2(z_j^*) = 0 \qquad (5.11)$$

These allow $v_1(z_j^*)$ and $v_2(z_j^*)$, and hence $D_j/D(0)$ as a function of z_j^*, to be determined when the expressions for u_0, u_1, and u_2 are inserted.

For comparison with Albright and Miller's report we have converted the calculated $D_j/D(0)$ to $D_j(z_j^*)/\bar{D}$($\bar{D}$ is defined by eq 2.9), and show the results for three ϵ values in Figure 7. It is seen that as z_j^* increases, $D_j/\bar{D}$ for a given ϵ starts from a value slightly smaller than unity, rises, intersects the line for $D_j/\bar{D} = 1$ (i.e., the line for $\epsilon = 0$), reaches a maximum, and falls. This behavior is consistent with what Albright and Miller derived from numerical integrations, except that they found no maxima and the intersection with the line for $\epsilon = 0$ to occur at the same $z_j^* = 0.63$, regardless of ϵ. In the comparison, however, we must note that their z_j^* is not the same as ours, being larger by a constant factor of $(1 - 0.7071\epsilon)^{1/2}$ than the latter. Our "cross points" read off Figure 7 are $z_j^* = 0.722$, 0.680, 0.625 for $\epsilon = 0.1$, 0.2, 0.3, respectively. When corrected for this difference, the Albright–Miller z_j^* values are about 0.75, 0.73, 0.71, which are definitely larger than 0.63. Thus, our analytical solution does not lend complete support to Albright and Miller's findings, but it confirms that D_j gives a very close approximation to $\bar{D}$ (see Appendix).

As Figure 6 shows, the concentrations at $|z_j^*|$ larger than 1.7 are almost indistinguishable from zero or unity. Since D_j at z_j^* > 1.7 depends on them, its computation should suffer considerable errors unless they are determined very accurately. This fact may account for the presence of a maximum in D_j from our perturbation solution.

Appendix

Albright and Miller[1] refer to their numerical investigation of the diffusion equation with a linearly concentration-dependent D:

$$D(c) = D(0)(1 - \beta c) \qquad (A.1)$$

where β is a positive parameter corresponding to ϵ above and c is a reduced concentration defined by eq 2.4. Though reported only for $\beta = 0.5$ their $D_j/\bar{D}$ starts from a value slightly above unity, decreases monotonically, and crosses the line for $\epsilon = 0$ as z_j^* increases. Interestingly, this behavior is just opposite to what we observe for the $\sqrt{c}$-dependent D, and suggests that D_j not only gives a good approximation to $\bar{D}$ but also is sensitive to the form of concentration dependence of D. In what follows, we examine what our perturbation method predicts for D_j when D is governed by eq A.1.

The problem is to solve the diffusion equation

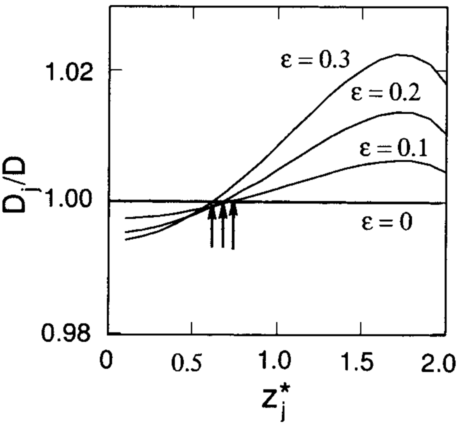

Figure 7. Variations of $D_j/\bar{D}$ with z_j^* for small ϵ. Arrows indicate cross points.

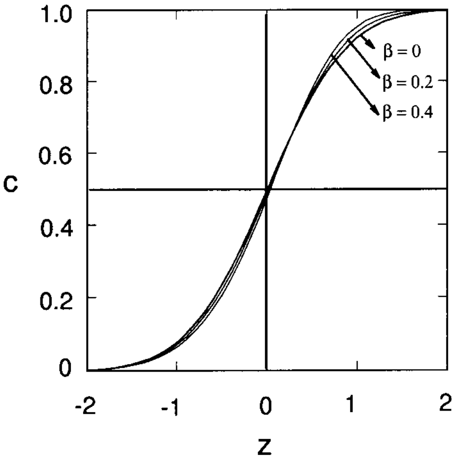

Figure 8. Concentration distributions for different β.

$$\partial c/\partial t = \partial/\partial x[D(0)(1 - \beta c)\partial c/\partial x] \qquad (A.2)$$

under the same initial and boundary conditions as eqs 2.2 and 2.3 by expanding c in powers of β as

$$c(x,t) = c_0(z) + \beta c_1(z) + \beta^2 c_2(z) + ... \qquad (A.3)$$

where z is defined by eq 2.5. We have determined the solution up to second order in β. The results are

$$c_0(z) = E(z)/2 \qquad (A.4)$$

$$c_1(z) = (^1/_2)E(z)^2 - E(z) - (4/\sqrt{\pi})\int_{-\infty}^{z}\exp(-z^2)\, g(z)\, \mathrm{d}z \qquad (A.5)$$

$$c_2(z) = (1/\sqrt{\pi})\int_{-\infty}^{z}\exp(-z^2)\, I(z)\, \mathrm{d}z - (^1/_2\sqrt{\pi})\int_{-\infty}^{z}\exp(-z^2)\, J(z)\, \mathrm{d}z + 0.0359E(z) \qquad (A.6)$$

where

$$I(z) = F(z) - 2\int_{-\infty}^{z} zF(z)\, \mathrm{d}z,$$
$$J(z) = E(z)\, g(z) - 2\int_{-\infty}^{z} zE(z)\, g(z)\, \mathrm{d}z \qquad (A.7)$$

with

$$F(z) = 1.5/[1 + \exp(-3z + 0.6)] \qquad (A.8)$$

and $E(z)$ and $g(z)$ defined in the text.

Figure 8 shows the concentration distributions for three values of β. The skewness of the curve even for $\beta = 0.4$ is not as marked as expected from Albright and Miller's curve for $\beta = 0.5$. Especially at c below $c = {}^1/_2$ the deviations from the ideal situation are unexpectedly small.

For the D under consideration eqs 5.10 and 5.12 must be replaced by

$$v_1(z_j^*)(\mathrm{d}u_0/\mathrm{d}z)_{z=z_j^*} + c_1(z_j^*) = 0 \tag{A.9}$$

$$v_2(z_j^*)(\mathrm{d}c_0/\mathrm{d}z)_{z=z_j^*} + ({}^1/_2)v_1(z_j^*)^2(\mathrm{d}^2c_0/\mathrm{d}z^2)_{z=z_j^*} + v_1(z_j^*)(\mathrm{d}c_1/\mathrm{d}z)_{z=z_j^*} + c_2(z_j^*) = 0 \tag{A.10}$$

In Figure 9, the calculated $D_j/\bar{D}$ for different β are plotted against z_j^*, where $\bar{D}$ is the D value at the mean concentration $c = {}^1/_2$, i.e.,

$$\bar{D} = 1 - ({}^1/_2)\beta \tag{A.11}$$

For either nonzero β indicated, $D_j/\bar{D}$ shows a variation with z_j^* very similar to that reported by Albright and Miller, but it crosses the line for $\beta = 0$ at a z_j^* value which differs for different β. The crossover points read off Figure 9 are $z_j^* = 1.07$, 0.893 for $\epsilon = 0.2$, 0.4, respectively. When corrected for the difference between our and Albright–Miller's z_j^*, 1.07 and 0.893 change to 1.18 and 1.12, respectively, which are as twice as larger than their z_j^* value that Albright and Miller found for $\beta = 0.5$.

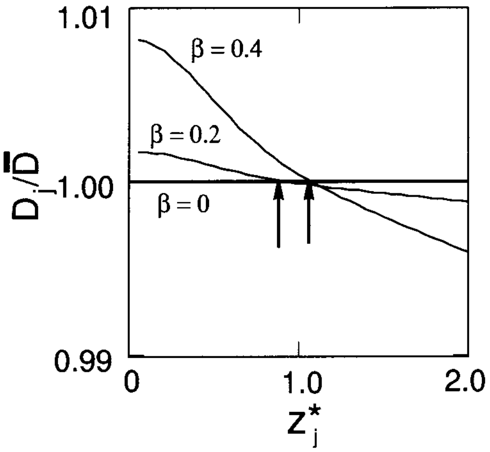

Figure 9. Variations of $D_j/\bar{D}$ with z_j^* for small β. Arrows indicate cross points.

References and Notes

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